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Research Article

On Fixed Point Findings for Diverse Contractions in *b* Dislocated-Multiplicative Metric Spaces

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In our present research study, we present the idea of b dislocated-multiplicative metric space (abbrev. bd-multiplicative metric space) that is generalization of b-multiplicative metric space and dislocated-multiplicative metric space. Furthermore, we prove some of the fixed point theorems in bd-multiplicative metric spaces. Also, we get common fixed point findings for fuzzy mappings in these spaces. Our findings are improved and more generalized form of several findings (see, e.g., [5, 6]).

1. Introduction

In 2008, the idea of multiplicative calculus was defined by Bashirov et al. [1] and then the conception of multiplicative metric spaces (multiplicative distance) was introduced by Çevikel and Özavşar [2]. Czerwik [3] presented concepts of b-metric space that is the popularization of metric space. Dosenovic et al. in section Future Work in [4] presented the idea of b-multiplicative metric spaces. After that, Ali et al. in [5] studied fixed point theorems for single-valued and multivalued mappings on b-multiplicative metric spaces.

Furthermore, several authors obtained some fixed point findings for mappings satisfying different contractive conditions (see, e.g., [6–8]). The idea of fuzzy mappings was initially studied by Weiss [9] and Butnariu [10]. Then, the concept of fuzzy mappings was studied by Heilpern [11]. Many of the fixed point theorems for fuzzy contraction mappings in the metric linear space were proved (e.g., [12–15]), which are the fuzzy extension for the Banach contraction principle. The concept of b-multiplicative metric spaces, as one of the useful generalizations of multiplicative metric spaces, was first used by Dosenovic et al. in [4],and Ali et al. in [5] study fixed point theorems for single-valued and multivalued mappings on b-multiplicative metric spaces.

Our findings in b-multiplicative metric space, d-multiplicative metric space, and multiplicative metric space can be obtained as corollaries of our findings.

In this part, we list some of the concepts which we will use in our major findings.

The definition of b-multiplicative metric space is given as follows.

Definition 1 (see [4, 5]). Suppose that X is a nonempty set and $s \geq 1$ is a given real number. A function $d\colon X \times X \longrightarrow [1, \infty)$ is considered as a b-multiplicative metric if it satisfies the following conditions: $\forall \eta,\ \xi,\ z \in X$,

(i) $d(\eta, \xi) \geq 1$
(ii) $d(\eta, \xi) = 1$ iff $\eta = \xi$
(iii) $d(\eta, \xi) = d(\xi, \eta)$
(iv) $d(\eta, \xi) \leq d(\eta, z)^s \cdot d(z, \xi)^s$

Example 1 (see [5]). Let $X = [0, \infty)$. Define a function $d\colon X \times X \longrightarrow [1, \infty)$, $d(\eta, \xi) = a^{(\eta-\xi)^2}$, where $a > 1$ is any fixed real number. Then, (X, d) is a b-multiplicative metric with $s = 2$.

In [11, 16], an element in any fuzzy set has a degree of belonging, a membership function may be used in order to introduce the value of degree of belonging for any element to a set, and the value of degree of belonging takes real values on the whole closed interval $[0, 1]$. The membership function is

$$\mu_A\colon X \longrightarrow [0, 1]. \tag{1}$$

Suppose (X, d) is a metric linear space. In X, a fuzzy set is a function $A\colon X \longrightarrow [0, 1]$. Thus, it is an element of I^X, where $I = [0, 1]$. If A is a fuzzy set and $\eta \in X$, then the function value $A(\eta)$ is considered as the grade of membership of η in A.

I^X denotes to the collection of all fuzzy sets in X. The α-level set of A is defined by

$$A_\alpha = \{\eta\colon A(\eta) \geq \alpha\} \ with\ \alpha \in (0, 1]\ and\ A_0 = \overline{\{\eta\colon A(\eta) > 0\}}, \tag{2}$$

whenever $\overline{\{\}}$ is the closure of set (nonfuzzy) $\{\}$.

Definition 2 (see [17]). A fuzzy set A in X is an approximate quantity if its α-level set is a nonempty compact subset (nonfuzzy) of X for each $\alpha \in [0, 1]$.

The set of all an approximate quantities denoted by $W^*(X)$ is a subcollection of $\mathfrak{F}(X)$.

Ozavsar and Cevikel [2] prove that every multiplicative contraction in a complete multiplicative metric space has a unique fixed point.

Definition 3 (see [2]). Assume that (X, d) is a multiplicative metric space. A mapping $g\colon X \longrightarrow X$ is called multiplicative contraction if

$$\exists \lambda \in [0, 1)\colon d(g\eta_1, g\eta_2) \leq d(\eta_1, \eta_2)^\lambda\ \forall \eta_1, \eta_2 \in X. \tag{3}$$

Theorem 1 (see [2]). *Assume that (X, d) is a multiplicative metric space. A mapping $g\colon X \longrightarrow X$ is called multiplicative contraction. Then, g has a unique fixed point.*

Theorem 2 (see [4]). *Suppose that (X, d) is a complete multiplicative metric space and a continuous function $g\colon X \longrightarrow X$, $\lambda \in [0, 1)$ such that*

$$d(g\eta, g\xi) \leq \{\max\{d(\eta, g\eta), d(\xi, g\xi)\}\}^\lambda. \tag{4}$$

Then, g has a unique fixed point.

In 2015, Kang et al. [18] introduced the concept of compatible mappings as follows.

Definition 4. Let (X, d) be a multiplicative metric space. The mappings $f, F\colon X \longrightarrow X$; then, (f, F) is called compatible if and only if $ft = Ft$ for some t in X implying $fFt = Fft$.

Many authors studied many fixed point theorems for compatible mappings in multiplicative metric space and employed it to prove a common fixed point theorem (see [4, 18]).

In this paper, we introduce the new notion of bd-multiplicative metric space. We prove fixed point theorems for single mappings and a common fixed point for fuzzy mappings in bd-multiplicative metric space. As illustrative application, we state some of our theorems on Cartesian product in these spaces.

2. Fixed Point Theorems in bd-Multiplicative Metric Spaces

In this part, we present the conception of bd-multiplicative metric space. Also, we introduce some of the fixed point theories and show our main findings with the help of some examples in this space.

Definition 5. Suppose that $X \neq \phi$ and $s \geq 1$ is a given real number. A function $d\colon X \times X \longrightarrow \mathbb{R}^+$ is called bd-multiplicative metric space if it satisfies the following conditions: $\forall\, \eta, \xi, z \in X$,

(i) $d(\eta, \xi) \geq 1$
(ii) $d(\eta, \xi) = 1$ implies $\eta = \xi$
(iii) $d(\eta, \xi) = d(\xi, \eta)$
(iv) $d(\eta, \xi) \leq [d(\eta, z) \cdot d(z, \xi)]^s$

Example 2. Let $X = \mathbb{R}^+ \cup \{0\}$. Define $d\colon X \times X \longrightarrow [1, \infty)$ as

$$d(\eta, \xi) = a^{(\eta+\xi)^2},\ \forall \eta, \xi \in X,\ \ a \geq 1. \tag{5}$$

Then, (X, d) is bd-multiplicative metric space with $s = 2$.

Example 3. Let $X = [1, \infty)$. Define $d\colon X \times X \longrightarrow [1, \infty)$ as

$$d(\eta, \xi) = a^{((\eta-1)+(\xi-1))^2},\ \forall \eta, \xi \in X,\ \ a \geq 1. \tag{6}$$

Then, (X, d) is bd-multiplicative metric space with $s = 2$.

Definition 6. Let (X, d) be an bd-multiplicative metric space. We say that $\{\eta_n\}$ converges to η if and only if

$$d(\eta_n, \eta) \longrightarrow_{bd} 1,\ as\, n \longrightarrow \infty. \tag{7}$$

Definition 7. Let (X, d) be an bd-multiplicative metric space. We say that $\{\eta_n\}$ is bd-multiplicative Cauchy if and only if

$$d(\eta_n, \eta_m) \longrightarrow_{bd} 1,\ as\ n, m \longrightarrow \infty. \tag{8}$$

Definition 8. An bd-multiplicative metric space (X, d) is complete if every multiplicative Cauchy sequence in X is convergent.

Now, we state the following lemma without proof.

Lemma 1. *Suppose that* (X,d) *is bd-multiplicative metric space. Then, any subsequence of convergent sequence in* X *is convergent.*

The following theorem is the generalization of Theorem 3.2 in [2].

Theorem 3. *Suppose that* (X,d) *is a complete bd-multiplicative metric space and a continuous function* $g\colon D \longrightarrow X$; $D \subseteq X$ *satisfies*

$$d(g\eta, g\xi) \leq d(\eta,\xi)^k, \tag{9}$$

where η, $\xi \in D$ *and* $k \in [0, 1/s)$. *Then,* g *has a unique fixed point.*

Proof. Let η_0 be an arbitrary point in X; then by hypothesis, there exists η_1 such that $\eta_1 = g\eta_0$. In a similar way, one can obtain a sequence $\{\eta_n\} \subseteq X$ such that

$$\begin{aligned}
\eta_n &= g\eta_{n-1} = g^n \eta_0,\\
d(\eta_n, \eta_{n+m}) &\leq d(\eta_n, \eta_{n+1})^{s^n} \cdot d(\eta_{n+1}, \eta_{n+2})^{s^{n+1}} \ldots d\\
&\quad \cdot (\eta_{n+m-1}, \eta_{n+m})^{s^{n+m-1}}\\
&= d(g^n\eta_0, g^n\eta_1)^{s^n} \cdot d\left(g^{n+1}\eta_0, g^{n+1}\eta_1\right)^{s^{n+1}} \ldots d\\
&\quad \cdot \left(g^{n+m-1}\eta_0, g^{n+m-1}\eta_1\right)^{s^{n+m-1}}\\
&\leq d(\eta_0,\eta_1)^{(ks)^n} \cdot d(\eta_0,\eta_1)^{(ks)^{n+1}} \ldots d\\
&\quad \cdot (\eta_0,\eta_1)^{(ks)^{n+m-1}}\\
&\leq d(\eta_0,\eta_1)^{(ks)^n/1-ks}.
\end{aligned} \tag{10}$$

Then,

$$d(\eta_n, \eta_{n+m}) \leq d(\eta_0,\eta_1)^{(ks)^n/1-ks}. \tag{11}$$

As $n \longrightarrow \infty$ in (11) and $k < 1/s \Rightarrow ks < 1$, then $\{\eta_n\}$ is a multiplicative Cauchy sequence.

Since (X,d) is complete, then $\{\eta_n\}$ is convergent such that $\lim_{n\longrightarrow\infty}\eta_n = \eta^*$. However,

$$\begin{aligned}
\eta^* &= \lim_{n\longrightarrow\infty}\eta_{n+1}\\
&= \lim_{n\longrightarrow\infty} g\eta_n\\
&= g\lim_{n\longrightarrow\infty}\eta_n\\
&= g\eta^*.
\end{aligned} \tag{12}$$

Therefore, η^* is a fixed point of g. Suppose that $g\eta^* = \eta^*$, $g\overline{\eta} = \overline{\eta}$, and $\overline{\eta} \neq \overline{\eta}$.

$$d(\eta^*, \overline{\eta}) = d(g\eta^*, g\overline{\eta}) \leq d(\eta^*, \overline{\eta})^k \leq d(\eta^*, \overline{\eta}). \tag{13}$$

This is a contradiction with assumption; then, $\eta^* = \overline{\eta}$. Then, g has a unique fixed point. □

Example 4. Suppose that $X = [1, \infty)$, (X,d) is *bd*-multiplicative metric space and $d(\eta,\xi) = a^{((\eta-1)+(\xi-1))^2}$, where $a = 2$ with $s = 2$. Define $g\colon X \longrightarrow X$ such that $g\eta = (\eta+1)/2$:

$$\begin{aligned}
d(g\eta, g\xi) &= 2^{((\eta-1)/2+(\xi-1)/2)^2}\\
&= 2^{1/4((\eta-1)+(\xi-1))^2}\\
&= d(\eta,\xi)^{1/4}.
\end{aligned} \tag{14}$$

Then, (1) holds such that $k = 1/4$. Therefore, g has a unique fixed point $1 \in X$.

Corollary 1. *Suppose that* (X,d) *is a complete multiplicative metric space and a continuous function* $g\colon X \longrightarrow X$ *satisfies*

$$d(g\eta, g\xi) \leq d(\eta,\xi)^k, \tag{15}$$

where $\eta, \xi \in D$ *and* $k \in [0, 1)$. *Then,* g *has a unique fixed point.*

Corollary 2. *Suppose that* (X,d) *is a complete b-multiplicative metric space and a continuous function* $g\colon X \longrightarrow X$ *satisfies*

$$d(g\eta, g\xi) \leq d(\eta,\xi)^k, \tag{16}$$

where $\eta, \xi \in X$ *and* $k \in [0, 1/s)$. *Then,* g *has a unique fixed point.*

The following theorem is the generalization of Theorem 2.32 in [4].

Theorem 4. *Suppose that* (X,d) *is a complete bd-multiplicative metric space and a continuous function* $g\colon D \longrightarrow X$, $k \in [0, 1/s)$, *such that*

$$d(g\eta, g\xi) \leq \{\max\{d(\eta,\xi), d(\eta, g\eta), d(\xi, g\xi)\}\}^k. \tag{17}$$

Then, g has a unique fixed point.

Proof. Let η_0 be an arbitrary point in X; then by hypothesis, there exists η_1 such that $\eta_1 = g\eta_0$.

In a similar way, one can obtain $\eta_2 \in X$ such that $\eta_2 = g\eta_1$.

$$\begin{aligned}
d(\eta_1,\eta_2) &= d(g\eta_0, g\eta_1)\\
&\leq \max\{d(\eta_0,\eta_1), d(\eta_0, g\eta_0), d(\eta_1, g\eta_1)\}^k\\
&= \max\{d(\eta_0,\eta_1), d(\eta_0,\eta_1), d(\eta_1,\eta_2)\}^k\\
&= \max\{d(\eta_0,\eta_1), d(\eta_1,\eta_2)\}^k\\
&= d(\eta_0,\eta_1)^k.
\end{aligned} \tag{18}$$

Otherwise, we have a contradiction, that is, $d(\eta_1,\eta_2) \leq d(\eta_1,\eta_2)^k$.

$$\begin{aligned}
d(\eta_2,\eta_3) &= d(g\eta_1, g\eta_2)\\
&\leq \max\{d(\eta_1,\eta_2),\ d(\eta_1, g\eta_1),\ d(\eta_2, g\eta_2)\}^k\\
&= \max\{d(\eta_1,\eta_2),\ d(\eta_1,\eta_2),\ d(\eta_2,\eta_3)\}^k\\
&= \max\{d(\eta_1,\eta_2), d(\eta_2,\eta_3)\}^k\\
&= d(\eta_0,\eta_1)^{k^2}.
\end{aligned} \tag{19}$$

Continuing in this way, we produce a sequence $\{\eta_n\}$ in X such that $\{\eta_n\} = g(\eta_{n-1})$ and

$$\begin{aligned} d(\eta_n, \eta_{n+1}) &= d(g\eta_{n-1}, g(\eta_n)) \\ &\leq \max\{d(\eta_{n-1}, \eta_n), d(\eta_{n-1}, g\eta_{n-1}), d(\eta_n, g\eta_n)\}^k \\ &= \max\{d(\eta_{n-1}, \eta_n), d(\eta_{n-1}, \eta_n), d(\eta_n, \eta_{n+1})\}^k \\ &= \max\{d(\eta_{n-1}, \eta_n), d(\eta_n, \eta_{n+1})\}^k \\ &= \left\{d(\eta_0, \eta_1)^{k^{n-1}}\right\}^k \\ &= d(\eta_0, \eta_1)^{k^n}, \end{aligned} \tag{20}$$

for each $n \in \mathbb{N}$. It follows by induction that $d(\eta_n, \eta_{n+1}) \leq d(\eta_0, \eta_1)^{k^n}$. However,

$$\begin{aligned} d(\eta_n, \eta_{n+m}) &\leq d(\eta_n, \eta_{n+1})^{s^n} \cdot d(\eta_{n+1}, \eta_{n+2})^{s^{n+1}} \\ &\quad \ldots d(\eta_{n+m-1}, \eta_{n+m})^{s^{n+m-1}} \\ &= d(g^n\eta_0, g^n\eta_1)^{s^n} \cdot d\left(g^{n+1}\eta_0, g^{n+1}\eta_1\right)^{s^{n+1}} \\ &\quad \ldots d\left(g^{n+m-1}\eta_0, g^{n+m-1}\eta_1\right)^{s^{n+m-1}} \\ &\leq d(\eta_0, \eta_1)^{(ks)^n} \cdot d(\eta_0, \eta_1)^{(ks)^{n+1}} \\ &\quad \ldots d(\eta_0, \eta_1)^{(ks)^{n+m-1}} \\ &\leq d(\eta_0, \eta_1)^{(ks)^n\left[1+(ks)+(ks)^2+\ldots\right]} \\ &\leq d(\eta_0, \eta_1)^{(ks)^n/1-ks}. \end{aligned} \tag{21}$$

As $k \in (0, 1/s)$, $m, n \longrightarrow \infty$, $ks < 1$, and (X, d) is complete, then $\{\eta_n\}$ is a multiplicative Cauchy sequence in X and there exists $\eta^* \in X$ such that $d(\eta_n, \eta^*) \longrightarrow_{bd} 1$.

From Lemma 1, $\eta^* = \lim_{n\longrightarrow\infty}\eta_{n+1} = \lim_{n\longrightarrow\infty} g\eta_n = g\lim_{n\longrightarrow\infty}\eta_n = g\eta^*$.

Then, η^* is fixed point of g, and $g\eta^* = \eta^*$. Suppose that g has another fixed point $\overline{\eta}$ such that $g\overline{\eta} = \overline{\eta}$ and $\overline{\eta} \neq \overline{\eta}$.

$$\begin{aligned} d(\eta^*, \overline{\eta}) &= d(g\eta^*, g\overline{\eta}) \\ &\leq \max\{d(\eta^*, \overline{\eta}), d(\eta^*, g\eta^*), d(\overline{\eta}, g\overline{\eta})\}^k \\ &= \max\{d(\eta^*, \overline{\eta}), d(\eta^*, \eta^*), d(\overline{\eta}, \overline{\eta})\}^k \\ &\leq \max\{d(\eta^*, \overline{\eta}), d(\eta^*, \eta_n)^s \cdot d(\eta_n, \eta^*)^s, d(\overline{\eta}, \eta_n)^s \\ &\quad \cdot d(\eta_n, \overline{\eta})^s\}^k \\ &= \max\{d(\eta^*, \overline{\eta}), 1\}^k \\ &= d(\eta^*, \overline{\eta})^k \\ &\leq d(\eta^*, \overline{\eta}). \end{aligned} \tag{22}$$

This is a contradiction with assumption; then, $\eta^* = \overline{\eta}$. Then, g has a unique fixed point. □

Corollary 3. *Suppose that (X, d) is a complete multiplicative metric space and a continuous function g: $X \longrightarrow X$ satisfies*

$$d(g\eta, g\xi) \leq \{\max\{d(\eta, \xi), d(\eta, g\eta), d(\xi, g\xi)\}\}^k, \tag{23}$$

where η, $\xi \in X$ and $k \in [0, 1)$. Then, g has a unique fixed point.

Corollary 4. *Suppose that (X, d) is a complete b-multiplicative metric space and a continuous function g: $X \longrightarrow X$ satisfies*

$$d(g\eta, g\xi) \leq \{\max\{d(\eta, \xi), d(\eta, g\eta), d(\xi, g\xi)\}\}^k, \tag{24}$$

where η, $\xi \in X$ and $k \in [0, 1/s)$. Then, g has a unique fixed point.

3. Common Fixed Point Theorems for Fuzzy Mappings in *bd*-Multiplicative Metric Spaces

Definition 9. Suppose that X is an arbitrary set and Y is bd-multiplicative-metric space. A mapping F is stated according to be a fuzzy mapping iff F is a function from the set X into $W^*(Y)$, i.e., $F(\eta) \in W^*(Y)$, for each $\eta \in X$.

Definition 10. Suppose that (X, d) is a bd-multiplicative metric space. The functions g: $Y \subseteq X \longrightarrow X$ and G: $Y \longrightarrow W^*(Y)$. A hybrid pair (g, G) is called D-compatible iff $\{gt\} \subset Gt$ for some $t \in Y$ implies $gGt \subset Ggt$.

Definition 11. Suppose that (X, d) is a bd-multiplicative metric space. Two maps G and g are said to be occasionally coincidentally idempotent if $g^2\eta = g\eta$ for some $C(g, G)$, where $C(g, G)$ refers to the set of all coincidence points of two mappings g and G, i.e.,

$$C(g, G) = \{\eta \colon g\eta = G\eta\}. \tag{25}$$

Now, we state the following lemma without proof.

Lemma 2. *Suppose that (X, d) is a bd-multiplicative metric space and $M \subseteq W^*(X)$. Then,*

$$\overline{M} = \{\eta \in X \colon d(\eta, M) = 1\}. \tag{26}$$

Corollary 5. *Suppose that (X, d) is a bd-multiplicative metric space and $M \subseteq W^*(X)$ and $d(\eta, M) = 1$ if and only if $\eta \in \overline{M} = M$.*

Lemma 3. *Suppose that (X, d) is a bd-multiplicative metric space, G: $X \longrightarrow W^*(X)$ is a fuzzy map, and $\eta_0 \in X$. Then, there exists $\eta_1 \in X$ such that $\{\eta_1\} \subseteq G(\eta_0)$.*

Theorem 5. *Suppose that (X, d) is a complete bd-multiplicative metric space and two continuous mappings g, f: $X \longrightarrow X$ satisfy*

$$d(f\eta, f\xi) \leq d(\eta, \xi)^k \text{ and } d(g\eta, g\xi) \leq d(\eta, \xi)^k, \tag{27}$$

where $\eta, \xi \in X$ *and* $k \in [0, 1/s)$ *and two fuzzy mappings* $G, F\colon X \longrightarrow W^*(X)$, *such that*

(i) $\{GX\}_\alpha \subset f(X)$, $\{FX\}_\alpha \subset g(X)$

(ii) *The pairs* (G, g) *and* (F, f) *are D-compatible and occasionally idempotent mappings*

Then, there exists $\eta^* \in X$ *such that* $\eta^* = f\eta^* = g\eta^*$ *and* $\eta^* \in \{F\eta^*\}_\alpha \cap \{G\eta^*\}_\alpha$.

Proof. Suppose η_0 is an arbitrary point in X. Then, there is $\{\xi_1\} = \{g\eta_1\} \subset \{F\eta_0\}_\alpha$, from Lemma 3.2; then, there exists $\{\xi_2\} = \{f\eta_2\} \subset \{G\eta_1\}_\alpha$, where

$$\{\xi_{2n+1}\} = \{g\eta_{2n+1}\} \subset \{F\eta_{2n}\}_\alpha, \tag{28}$$

$$\{\xi_{2n+2}\} = \{f\eta_{2n+2}\} \subset \{G\eta_{2n+1}\}_\alpha. \tag{29}$$

From (11) in Theorem 3,

$$d(\xi_n, \xi_m) \leq d(\xi_0, \xi_1)^{(ks)^n/1-ks}. \tag{30}$$

As $n \longrightarrow \infty$, $k < 1/s \Rightarrow ks < 1$ that implies $\{\xi_n\}$ is a multiplicative Cauchy sequence.

Since (X, d) is complete, then $\{\eta_n\}$ is convergent such that

$$\lim_{n\longrightarrow\infty} d(\xi_n, \xi_m) = 1. \tag{31}$$

Next, we prove that $\eta^* \in \{Fz\}_\alpha$.

$$\begin{aligned} d(fz, f\{Fz\}_\alpha) &= d\left(f^2 z, f\{Fz\}_\alpha\right) \\ &= d(f\eta^*, f\{Fz\}_\alpha) \\ &\leq d(\eta^*, \{Fz\}_\alpha)^k \\ &\leq d(\eta^*, fz)^{ks} \cdot d(fz, \{Fz\}_\alpha)^{ks} \\ &= d(\eta^*, fz)^{ks} \cdot d(\eta^*, \{Fz\}_\alpha)^{ks} \\ &= d(\eta^*, \eta^*)^{ks} \cdot d(\eta^*, \{Fz\}_\alpha)^{ks}. \\ &\Downarrow \\ d(\eta^*, \{Fz\}_\alpha)^{k-ks} &\leq d(\eta^*, \eta^*)^{ks} \\ &\leq d(\eta^*, \xi_{2n+2})^{ks^2} \cdot d(\xi_{2n+2}, \eta^*)^{ks^2} \\ &= d(\xi_{2n+2}, \eta^*)^{ks^2} \cdot d(\xi_{2n+2}, \eta^*)^{ks^2}. \end{aligned} \tag{32}$$

From Lemma 1, ξ_{2n+2} is a convergent sequence, i.e., $d(\xi_{2n+2}, \eta^*) \longrightarrow 1$ as $n \longrightarrow \infty$,i.e.,

$$\begin{aligned} d(\eta^*, \{Fz\}_\alpha)^{k-ks} &\leq 1. \\ d(\eta^*, \{Fz\}_\alpha) &\leq 1. \end{aligned} \tag{33}$$

Then, we have $d(\eta^*, \{Fz\}_\alpha) = 1$ and Corollary 5 illustrates that $\eta^* \in \overline{\{Fz\}_\alpha} = \{Fz\}_\alpha$.

Since $\eta^* = fz \in \{Fz\}_\alpha \subset g(X)$, there exists $w \in X$ suchthat $\eta^* = gw$.

Similar to the previous steps, we can prove that $\eta^* = gw \in \{Gw\}_\alpha$.

As two pairs (G, g) and (F, f) are D-compatible,

$$\begin{aligned} \{\eta^*\} = \{fz\} \subset \{Fz\}_\alpha \text{ and } \{\eta^*\} \\ = \{gw\} \subset \{Gw\}_\alpha. \end{aligned} \tag{34}$$

Moreover,

$$\{f\eta^*\} = \{ffz\} \subset \{fFz\}_\alpha \subset \{Ffz\}_\alpha = \{F\eta^*\}_\alpha. \tag{35}$$

$$\begin{aligned} \{g\eta^*\} = \{ggw\} \subset \{gGw\}_\alpha \subset \{Ggw\}_\alpha \\ = \{G\eta^*\}_\alpha. \end{aligned} \tag{36}$$

Now, we show that $\eta^* = f\eta^*$ and $\eta^* = g\eta^*$. Since $\{\xi_n\}$ and $\{\eta_n\}$ are convergent sequences, from Lemma 1,

$$\begin{aligned} \eta^* &= \lim_{n\longrightarrow\infty} \xi_{2n+2} \\ &= \lim_{n\longrightarrow\infty} f\eta_{2n+2} \\ &= f \lim_{n\longrightarrow\infty} \eta_{2n+2} \\ &= f\eta^*. \end{aligned} \tag{37}$$

$$\begin{aligned} \eta^* &= \lim_{n\longrightarrow\infty} \xi_{2n+1} \\ &= \lim_{n\longrightarrow\infty} g\eta_{2n+1} \\ &= g \lim_{n\longrightarrow\infty} \eta_{2n+1} \\ &= g\eta^*. \end{aligned} \tag{38}$$

Then, $\eta^* = f\eta^* = g\eta^*$ and $\eta^* \in \{F\eta^*\}_\alpha \cap \{G\eta^*\}_\alpha$. □

Example 5. Suppose that $X = [0, 1]$, (X, d) is a bd-multiplicative metric space defined by $d(\eta, \xi) = a^{(\eta+\xi)^2}$, and $a > 1$. Define maps $g, f\colon X \longrightarrow X$ as $g\eta = \eta^2$, $f\eta = \eta^3 \,\forall\, \eta$, and $\xi \in X$. Also, define two fuzzy mappings $G, F\colon X \longrightarrow W^*(X)$ as

$$(F\eta)(\xi) = \begin{cases} 0 & if \quad 0 \leq \xi < 1/5 \\ 1/3 & if \quad 1/5 \leq \xi \leq \eta^3 \\ 2/3 & if \quad \eta^3 < \xi < 4/5 \\ 1 & if \quad 4/5 \leq \xi \leq 1 \end{cases}, \tag{39}$$

and

$$(G\eta)(\xi) = \begin{cases} 0 & if \quad 0 \leq \xi < 1/4 \\ 1/6 & if \quad 1/4 \leq \xi \leq \eta^2 \\ 1/9 & if \quad \eta^2 < \xi < 6/5 \\ 1 & if \quad 6/5 \leq \xi \leq 1 \end{cases}. \tag{40}$$

Now, for $\alpha = 1/3$, $f\{F\eta\}_{1/3} = [(1/125), \eta^9] \subset [(1/5), \eta^3] = \{Ff\eta\}_{1/3}$ and for $\alpha = 1/6$, $g\{G\eta\}_{1/6} = [(1/16), \eta^4] \subset [(1/4), \eta^4] = \{Gg\eta\}_{1/6}$; i.e., (g, G) and (f, F) are D-compatible. Finally, $f1 = ff1 \in [(1/5), 1] = Ff1$ and $gg1 = g1 \in [1/4, 1] = Gg1$; i.e., (f, F) and (g, G) are occasionally coincidentally idempotent. Then, $1 = f1 = g1 \in [1/5, 1] \cap [1/4, 1] = \{F1\}_{1/3} \cap \{G1\}_{1/6}$ is a common fixed point.

Example 6. Suppose that $X = [2/5, 3/2]$, (X, d) is a bd-multiplicative metric space defined by $d(\eta, \xi) = 2^{((\eta - (2/5)) + (\xi - (2/5)))^2}$. Define maps $g, f\colon X \longrightarrow X$ as $g\eta = 1/2(\eta + 2/5)$, $f\eta = (\eta + 2)/6 \,\forall\, \eta$, and $\xi \in X$. Also, define two fuzzy mappings $G, F\colon X \longrightarrow W^*(X)$ as

$$(F\eta)(\xi) = \begin{cases} 0 & if \quad 0 \le \xi < 1/5 \\ 1/3 & if \quad 1/5 \le \xi < (\eta + 2)/6, \\ 2/3 & if \quad (\eta + 2)/6 \le \xi \le 1 \end{cases} \tag{41}$$

and

$$(G\eta)(\xi) = \begin{cases} 0 & \text{if } 0 \le \xi < \dfrac{1}{4} \\ \dfrac{1}{6} & \text{if } \dfrac{1}{4} \le \xi < \dfrac{1}{2}\left(\eta + \dfrac{2}{5}\right). \\ \dfrac{1}{4} & \text{if } \dfrac{1}{2}\left(\eta + \dfrac{2}{5}\right) \le \xi \le 1 \end{cases} \tag{42}$$

Now, for $\alpha = 2/3$,

$$f\{F\eta\}_{2/3} = [(\eta + 14)/36, 1/2] \subset [(\eta + 14)/36, 1] = \{Ff\eta\}_{2/3}, \tag{43}$$

and for $\alpha = 1/4$,

$$\begin{aligned} g\{G\eta\}_{1/4} &= [(1/4\eta + 1/10), 7/20] \subset [(1/4\eta + 1/10), 1] \\ &= \{Gg\eta\}_{1/4}. \end{aligned} \tag{44}$$

Hence, (g, G) and (f, F) are D-compatible. Finally,

$$\begin{aligned} f2/5 &= ff2/5 \in [4/10, 1] \\ &= Ff2/5 \end{aligned} \tag{45}$$

and

$$\begin{aligned} gg2/5 &= g2/5 \in [2/10, 1] \\ &= Gg2/5. \end{aligned} \tag{46}$$

Therefore, (f, F) and (g, G) are occasionally coincidentally idempotent. Furthermore,

$$\begin{aligned} d(f\eta, f\xi) &= 2^{(((\eta+2/6)-(2/5))+((\xi+2/6)-(2/5)))^2} \\ &= 2^{\frac{1}{36}((\eta - (2/5)) + (\xi - (2/5)))^2} \\ &= d(\eta, \xi)^{1/36}. \end{aligned} \tag{47}$$

Then, $2/5 = f2/5 = g2/5 \in [4/10, 1] \cap [2/10, 1] = \{F2/5\}_{2/3} \cap \{G2/5\}_{1/4}$ is a common fixed point.

We concluded the following corollary when we set $f = g$ in Theorem 5.

Corollary 6. *Let $Y \subset X$, and suppose continuous mapping $g\colon Y \longrightarrow X$ satisfies $d(g\eta, g\xi) \le d(\eta, \xi)^k$, where $\eta, \xi \in Y$, $k \in [0, 1/s)$ and fuzzy mapping $G\colon Y \longrightarrow W^*(X)$ such that*

(i) $\{GY\}_\alpha \subset g(Y)$

(ii) The pair (G, g) is D-compatible and occasionally idempotent mappings

Then, there exists $\eta^* \in Y$ such that $\eta^* = g\eta^* \in \{G\eta^*\}_\alpha$.

Theorem 6. *Let $Y \subset X$ and suppose two continuous mappings g, $f\colon Y \longrightarrow X$, satisfy*

$$d(f\eta, f\xi) \le d(\eta, \xi)^k \text{and}\, d(g\eta, g\xi) \le d(\eta, \xi)^k, \tag{48}$$

where η, $\xi \in Y$, $k \in [0, 1/s)$, and $\{F_n\}_\alpha\colon Y \longrightarrow W^*(X)$ such that $\forall\, \eta \in Y$,

(i) $\{F_l Y\}_\alpha \subset f(Y)$ and $\{F_k Y\}_\alpha \subset g(Y)$

(ii) The pairs (F_l, f) and (F_k, g) are D-compatible and occasionally idempotent mappings

Then, there exists $\eta^* \in Y$ such that $\eta^* = f\eta^* = g\eta^*$ and $\eta^* \in \cap_{n=0}^{\infty}\{F_n\eta^*\}_\alpha$.

Proof. The proof of this theorem is completed, when putting $F_l = G$ and $F_k = F$ in Theorem 5. □

Remark 1. If $F_l = G$ and $F_k = F$, then Theorem 6 implies Theorem 5.

Theorem 7. *Suppose that (X, d) is a complete bd-multiplicative metric space and two continuous mappings g, $f\colon X \longrightarrow X$ satisfy*

$$\begin{aligned} d(f\eta, f\xi) &\le \{\max\{d(\eta, \xi), d(\eta, f\eta), d(\xi, f\xi)\}\}^k, \\ d(g\eta, g\xi) &\le \{\max\{d(\eta, \xi), d(\eta, g\eta), d(\xi, g\xi)\}\}^k, \end{aligned} \tag{49}$$

where η, $\xi \in X$, $k \in [0, 1/s)$, and two fuzzy mappings F, $G\colon X \longrightarrow W^*(X)$, such that

(i) $\{GX\}_\alpha \subset f(X)$, $\{FX\}_\alpha \subset g(X)$

(ii) The pairs (F, f) and (G, g) are D-compatible and occasionally idempotent mappings

Then, there exists $\eta^* \in X$: $\eta^* = f\eta^* = g\eta^*$ and $\eta^* \in \{F\eta^*\}_\alpha \cap \{G\eta^*\}_\alpha$.

Proof. Let η_0 be an arbitrary point in X; then, there exists $\xi_1 = g\eta_1 \in \{F\eta_0\}_\alpha$ and from Lemma 3, there exists $\{\xi_2\} = \{f\eta_2\} \subset \{G\eta_1\}_\alpha$ such that

$$\begin{aligned} \{\xi_{2n+1}\} &= \{g\eta_{2n+1}\} \subset \{F\eta_{2n}\}_\alpha, \\ \{\xi_{2n+2}\} &= \{f\eta_{2n+2}\} \subset \{G\eta_{2n+1}\}_\alpha. \end{aligned} \tag{50}$$

Since $\xi_1 = g\eta_1 \in \{F\eta_0\}$, there exists $\xi_2 = f\eta_2 \subset \{G\eta_1\}_\alpha$, and from Theorem 4,

$$d(\xi_n, \xi_m) \le d(\xi_0, \xi_1)^{(ks)^n/1-ks}. \tag{51}$$

As $n \longrightarrow \infty$, $k < 1/s \Rightarrow ks < 1$ that implies $\{\xi_n\}$ is a multiplicative Cauchy sequence. Since (X, d) is complete, then $\{\eta_n\}$ is convergent such that

$$\lim_{n\longrightarrow\infty} d(\xi_n, \xi_m) = 1. \tag{52}$$

As $\{\xi_{2n+2}\}$ is a Cauchy sequence in $f(X)$ and $f(X)$ is joint orbitally complete, then there exists $\eta^* \in X$ such that

$$\lim_{n\longrightarrow\infty} \xi_{2n+2} = \eta^*, \quad \eta^* = fz\ \forall z \in X. \tag{53}$$

We prove that $\eta^* \in \{Fz\}_\alpha$ and

$$\begin{aligned} d(\eta^*, \{Fz\}_\alpha) &= d(fz, \{Fz\}_\alpha) \leq d(fz, \eta^*)^s \cdot d(\eta^*, \{Fz\}_\alpha)^s \\ &= d(ffz, fz)^s \cdot d(\eta^*, \{Fz\}_\alpha)^s \\ &= d\left(f^2 z, f^2 z\right)^s \cdot d(\eta^*, \{Fz\}_\alpha)^s \\ &\leq \left\{\max\left\{d(fz, fz), d\left(fz, f^2 z\right), d\left(fz, f^2 z\right)\right\}\right\}^{ks} \\ &\quad \cdot d(\eta^*, \{Fz\}_\alpha)^s \\ &= \{\max\{d(fz, fz), d(fz, fz) \cdot d(fz, fz)\}\}^{ks} \\ &\quad \cdot d(\eta^*, \{Fz\}_\alpha)^s \\ &= \{\max\{d(\eta^*, \eta^*), d(\eta^*, \eta^*), d(\eta^*, \eta^*)\}\}^{ks} \\ &\quad \cdot d(\eta^*, \{Fz\}_\alpha)^s \\ &= d(\eta^*, \eta^*)^{ks} \cdot d(\eta^*, \{Fz\}_\alpha)^s. \\ d(\eta^*, \{Fz\}_\alpha)^{1-s} &\leq d(\eta^*, \eta^*)^{ks} \\ &\leq \left(d(\eta^*, \xi_{2n+2})^s \cdot d(\xi_{2n+2}, \eta^*)^s\right)^{ks}. \end{aligned} \tag{54}$$

As $n \longrightarrow \infty$, then $d(\eta^*, \xi_{2n+2}) \longrightarrow_{bd} 1$. However, ξ_{2n+2} is a convergent sequence; i.e.,

$$\begin{aligned} d(\eta^*, \{Fz\}_\alpha)^{1-s} &\leq 1, \\ d(\eta^*, \{Fz\}_\alpha) &\leq 1. \end{aligned} \tag{55}$$

Then, we have $d(\eta^*, \{Fz\}_\alpha) = 1$ and Corollary 5 illustrates that $\eta^* \in \overline{\{Fz\}_\alpha} = \{Fz\}_\alpha$.

Since $\{\eta^*\} = \{fz\} \subset \{Fz\}_\alpha \subset g(X)$, there exists $w \in X$ such that $\eta^* = gw$.

Similar to the previous steps, we can prove that $\eta^* = gw \in \{Gw\}_\alpha$.

As two pairs (G, g) and (F, f) are D-compatible,

$$\begin{aligned} \{\eta^*\} &= \{fz\} \subset Fz \quad \text{and}\ \{\eta^*\} \\ &= \{gw\} \subset \{Gw\}_\alpha, \end{aligned} \tag{56}$$

and therefore

$$\begin{aligned} \{f\eta^*\} &= \{ffz\} \subset \{fFz\}_\alpha \subset \{Ffz\}_\alpha \\ &= \{F\eta^*\}_\alpha, \end{aligned} \tag{57}$$

$$\begin{aligned} \{g\eta^*\} &= \{ggw\} \subset \{gGw\}_\alpha \subset \{Ggw\}_\alpha \\ &= \{G\eta^*\}_\alpha. \end{aligned} \tag{58}$$

Now, we show that $\eta^* = f\eta^*$ and $\eta^* = g\eta^*$. Since $\{\xi_n\}$ and $\{\eta_n\}$ are convergent sequences, then

$$\begin{aligned} \eta^* &= \lim_{n\longrightarrow\infty} \xi_{2n+2} \\ &= \lim_{n\longrightarrow\infty} f\eta_{2n+2} \\ &= f \lim_{n\longrightarrow\infty} \eta_{2n+2} \\ &= f\eta^*, \end{aligned} \tag{59}$$

$$\begin{aligned} \eta^* &= \lim_{n\longrightarrow\infty} \xi_{2n+1} \\ &= \lim_{n\longrightarrow\infty} g\eta_{2n+1} \\ &= g \lim_{n\longrightarrow\infty} \eta_{2n+1} \\ &= g\eta^*. \end{aligned} \tag{60}$$

Then, f, g, F, and G have a common fixed point. □

Theorem 8. *Suppose that (X, d) is a complete bd-multiplicative metric space, $Y \subset X$, and two continuous mappings g, $f\colon Y \longrightarrow X$ satisfy*

$$\begin{aligned} d(f\eta, f\xi) &\leq \{\max\{d(\eta, \xi), d(\eta, f\eta), d(\xi, f\xi)\}\}^k, \\ d(g\eta, g\xi) &\leq \{\max\{d(\eta, \xi), d(\eta, g\eta), d(\xi, g\xi)\}\}^k, \end{aligned} \tag{61}$$

where η, $\xi \in Y$, $k \in [0, 1/s)$, and $\{F_n\}\colon Y \longrightarrow W^*(X)$, such that

(i) $\{F_l Y\}_\alpha \subset f(Y)$, $\{F_k Y\}_\alpha \subset g(Y)$, $k = 2n+1$, $l = 2n+2$, and $n \in \mathbb{N}$

(ii) The pairs (F_k, f) and (F_l, g) are D-compatible and occasionally idempotent mappings

Then, there exist $\eta^* \in Y$ such that $\eta^* = f\eta^* = g\eta^*$ and $\eta^* \in \cap_{n=0}^{\infty} \{F_n \eta^*\}_\alpha$.

4. Applications

In this section, we give some applications on our main results. We state some of our theorems on Cartesian product without proof.

Theorem 9. *Suppose that (X, d) is a complete bd-multiplicative metric space. The map $g\colon D^2 = D \times D \longrightarrow D^2$, $D \subseteq X$ satisfies*

$$d(g(a, c), g(b, d)) \leq d((a, c), (b, d))^k, \tag{62}$$

where (a, c), $(b, d) \in D^2$ and $k \in [0, 1/s)$. Then, g has a unique fixed point.

The next example illustrates the previous theory.

Example 7. Suppose that $X = [1, \infty)$ and $D = \{(1, \eta)\colon \eta \in X\}$. Define $d\colon D^2 \longrightarrow [1, \infty)$ as

$$d((a, c), (b, d)) = a^{((c/b) - 1 + (d/a) - 1)^2}, \tag{63}$$

$\forall\, (a, c), (b, d) \in D$, $a \geq 1$. Then, (X, d) is a complete bd-multiplicative metric space with $s = 2$.

Let $g: D \longrightarrow D$ be a function defined by $g((1,\eta)) = (1, (\eta+1)/2)$. Then, condition (3) holds. However,

$$\begin{aligned} d(g(1,\eta), g(1,\xi)) &= d(1, (1+\eta)/2), (1, (1+\xi)/2) \\ &= a^{((\eta-1)/2+\xi-1/2)^2} \\ &= a^{1/4(\eta-1+\xi-1)^2} \\ &= d((1,\eta),(1,\xi))^{1/4}, \end{aligned} \tag{64}$$

with $k = 1/4$. It is obvious that $(1,1) \in D^2$ is a unique fixed point of a map g.

Theorem 10. *Suppose that (X,d) is a complete bd-multiplicative metric space and $g: D^2 \longrightarrow D^2$, $k \in [0,1/s)$, such that*

$$\begin{aligned} d(g(a,c), g(b,d)) \leq \{&\max\{d((a,c),(b,d)), d((a,c), g(a,c)), \\ &\cdot d((b,d), g(b,d))\}\}^k. \end{aligned} \tag{65}$$

Then, g has a unique fixed point.

Theorem 12. *Suppose that (X,d) is a complete bd-multiplicative metric space and two continuous mappings $g, \quad f: D^2 \longrightarrow D^2$ satisfy*

$$\begin{aligned} d(f(a,c), \quad f(b,d)) &\leq d((a,c),(b,d))^k, \\ d(g(a,c), \quad g(b,d)) &\leq d((a,c),(b,d))^k, \end{aligned} \tag{66}$$

where $(a,c),(b,d) \in D^2$, $k \in [0,1/s)$, and two fuzzy mappings $G, \quad F: D^2 \longrightarrow W^*(X)$, such that

(i) $\{GD^2\}_\alpha \subset f(D^2)$ and $\{FD^2\}_\alpha \subset g(D^2)$

(ii) The pairs (F,f) and (G,g) are D-compatible and occasionally idempotent mappings

Then f, g, F, and G have a common fixed point.

Theorem 13. *Suppose that (X,d) is a complete bd-multiplicative metric space, $D \subset X$, and two mappings $g, \quad f: D^2 \longrightarrow D^2$ satisfy*

$$\begin{aligned} d(f(a,c), \ f(b,d)) &\leq d((a,c),(b,d))^k, \\ d(g(a,c), g(b,d)) &\leq d((a,c),(b,d))^k, \end{aligned} \tag{67}$$

where $(a,c),(b,d) \in D^2$, $k \in [0,1/s)$, and $\{F_n\}: D^2 \longrightarrow W^*(X)$ such that

(i) $\{F_l D^2\}_\alpha \subset f(D^2)$, $\{F_k D^2\}_\alpha \subset g(D^2)$, $k = 2n+1$, $l = 2n+2$, and $n \in \mathbb{N}$

(ii) The pairs (F_k, f) and (F_l, g) are D-compatible and occasionally idempotent mappings

Then, there exists $\eta^* \in D \times D$ such that $\eta^* = f\eta^* = g\eta^*$ and $\eta^* \in \cap_{n=0}^{\infty}\{F_n\eta^*\}_\alpha$.

Theorem 14. *Suppose that (X,d) is a complete bd-multiplicative metric space and two mappings $g, \quad f: D^2 \longrightarrow D^2$, $D \subset X$, satisfy*

$$\begin{aligned} d(f(a,c), f(b,d)) &\leq \{\max\{d((a,c),(b,d)), d((a,c), f(a,c)), d((b,d), f(b,d))\}\}^k, \\ d(g(a,c), g(b,d)) &\leq \{\max\{d((a,c),(b,d)), d((a,c), g(a,c)), d((b,d), g(b,d))\}\}^k, \end{aligned} \tag{68}$$

where $(a,c),(b,d) \in D^2$, $k \in [0,1/s)$, $\{F_n\}_\alpha: D^2 \longrightarrow W^*(X)$, such that

(i) $\{F_l D^2\}_\alpha \subset f(D^2)$, $\{F_k D^2\}_\alpha \subset g(D^2)$, $k = 2n+1$, $l = 2n+2$, and $n \in \mathbb{N}$

(ii) The pairs (F_k, f) and (F_l, g) are D-compatible and occasionally idempotent mappings

Then, there exists $\eta^* \in D^2$ such that $\eta^* = f\eta^* = g\eta^*$ and $\eta^* \in \cap_{n=0}^{\infty}\{F_n\eta^*\}_\alpha$.

5. Conclusion

In this paper, we introduced the concept of *bd*-multiplicative metric spaces. We studied some of the fixed point theorems in these spaces. Also, we obtain common fixed point theorems for fuzzy mappings in complete *bd*-multiplicative metric spaces. Finally, we get some of applications on our main findings. We hope that our presented idea herein will be a source of motivation for other researchers to extend and improve these findings for their applications.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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