Research Article

Fractional Order Accumulation NGM (1, 1, k) Model with Optimized Background Value and Its Application

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Received 18 June 2021; Revised 30 June 2021; Accepted 10 August 2021; Published 3 September 2021

Academic Editor: Lifeng Wu

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Aiming at the problem of unstable prediction accuracy of the classic NGM (1, 1, k) model, the modeling principle and parameter estimation method of this model are deeply analyzed in this study. Taking the minimum mean absolute percentage error as the objective function, the model is improved from the two perspectives of the construction method of the background value and the fractional order accumulation generation. The fractional order accumulation NGM (1, 1, k) model based on the optimal background value (short for the FBNGM (1, 1, k) model) is proposed in the study. The particle swarm optimization algorithm is used to estimate the parameters of the proposed model. Taking two actual cases with economic significance as examples, empirical analysis of the proposed model is conducted. The simulation and prediction results show the practicality and efficiency of the FBNGM (1, 1, k) model proposed in this study, which further broadens the application scope of the grey prediction model.

1. Introduction

The grey forecasting model has the advantages of no special requirements on the distribution of modeling data and easily understood modeling mechanism. So it is used in many fields such as environment [1, 2], agriculture [3], industry [4], energy [5, 6], economy [7, 8], and so on. The GM (1, 1) model plays an important role in the grey forecasting model. In order to improve the prediction accuracy of the GM (1, 1) model, some scholars have made a series of improvements from the perspectives of the background value [8], cumulative order [9], and discretization [10]. But the prediction accuracy of the improved model for nonhomogeneous exponential series is still not ideal, so the NGM (1, 1, k) model came into being [11].

In order to expand the application scope of the NGM (1, 1, k) model, the researchers further optimized the grey differential equation of the NGM (1, 1, k) model in practical applications and proposed the NHGM (1, 1, k) model [12] and the grey prediction model with time power term [13]. In order to eliminate the inherent bias caused by the mismatch problem,

Wang and Gong [14] proposed the intensional NGM (1, 1, k) model based on the classic NGM (1, 1, k) model. In order to compensate the lack of accuracy of GM (1, 1) in simulating nonhomogeneous exponential growth series, Cui et al. [11] constructed the NGM (1, 1, k) model to approximate the characteristics of nonhomogeneous exponential growth series by using the grey differential equation of GM (1, 1) as a deductive reasoning tool. Tong et al. [15] proposed the BNGM (1, 1, k) model with optimized grey differential equation by analyzing the cause of parameter estimation error in the classic NGM (1, 1, k) model. However, the single optimized NGM (1, 1, k) model has limited effect in improving the accuracy of model prediction. Based on fractional order accumulation, Wu et al. [16] proposed the discrete grey model with the fractional order. Zeng et al. [17] proposed the idea of multiple optimizations and verified the prediction accuracy of the dual optimized GM (1, 1) model based on function transformation, and background value reconstruction is much higher than the single optimized GM (1, 1) model through actual application examples. This shows that multiple optimizations could

complement each other through different optimization measures. It makes up for the limitations of single optimization and further improves the accuracy of the grey prediction model.

The main contributions of this study are as follows:

- (1) Based on the previous scholars' research, referring to the fractional order accumulation generation technology and the idea of optimizing the background value, the fractional order accumulation NGM (1, 1, k) model based on the optimal background value (short for FBNGM (1, 1, k) model) is constructed in this study.
- (2) Based on the actual application cases of China's electricity consumption and agricultural output value, the proposed FBNGM (1, 1, k) model is compared with the NGM (1, 1, k) model and two single optimization model in simulation and prediction accuracy. The results show that the FBNGM (1, 1, k) model has better performance in improving model accuracy than the other models, which verifies the practicality and efficiency of the proposed model.

The remainder of this study is organized as follows: the second part introduces the basic model and accuracy measure indicator; the third part introduces the FBNGM (1, 1, k) model and its parameter solution in detail; the fourth part is the empirical study, based on two actual application cases of predicting China's electricity consumption and agricultural output value; the fifth part is the conclusion.

2. Preliminary Knowledge

This section presents the fractional order accumulation and inverse accumulation generation technology of modeling sequence, the related concepts of the NGM (1, 1, k) model, and the accuracy measure indicator of the model.

2.1. Fractional Order Accumulation and Inverse Accumulation Generation Technology [17]

- (1) Assuming $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$ is a nonnegative original sequence, so $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \dots, x^{(r)}(n)\}$, $r \in R^+$ is called the r^{th} order accumulation generation sequence of $X^{(0)}$, where

$$x^{(r)}(k) = \sum_{i=1}^k \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i), \quad k = 1, 2, \dots, n. \quad (1)$$

- (2) Assuming $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$ is a nonnegative original sequence, so $X^{(-r)} = \{x^{(-r)}(1), x^{(-r)}(2), \dots, x^{(-r)}(n)\}$, $r \in R^+$ is called the r^{th} order inverse accumulation generation sequence of $X^{(0)}$, where

$$x^{(-r)}(k) = \sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} x^{(0)}(k-i),$$

$$k = 1, 2, \dots, n. \quad (2)$$

2.2. NGM (1, 1, k) Model [15]. $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}$ is the first-order accumulation generation sequence of $X^{(0)}$ and $Z^{(1)} = \{z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)\}$, $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$, $k = 2, 3, \dots, n$, is the mean generation sequence of $X^{(1)}$, so

$$x^{(0)}(k) + az^{(1)}(k) = b \frac{2k-1}{2} + c \quad (3)$$

is called the grey differential equation form of the NGM (1, 1, k) model, where $z^{(1)}(k)$ is called the background value, a is called the development coefficient, $b(2k-1)/2 + c$ is named the grey action or driving term, and the least squares estimates of the parameter a, b, c is

$$\hat{P} = (\hat{a}, \hat{b}, \hat{c})^T = (B^T B)^{-1} B^T Y, \quad (4)$$

where

$$B = \begin{pmatrix} -z^{(1)}(2) & \frac{3}{2} & 1 \\ -z^{(1)}(3) & \frac{5}{2} & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & \frac{(2n-1)}{2} & 1 \end{pmatrix}, \quad (5)$$

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}.$$

Finally,

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt + c \quad (6)$$

is called whitened differential equation form of the NGM (1, 1, k) model.

Under the initial condition $\hat{x}^{(1)}(1) = x^{(1)}(1)$, the time response function of the NGM (1, 1, k) model is

$$\hat{x}^{(1)}(k) = \left[x^{(1)}(1) - \frac{\hat{b}}{\hat{a}} - \frac{\hat{c}}{\hat{a}} + \frac{\hat{b}}{\hat{a}^2} \right] e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a}} k + \frac{\hat{c}}{\hat{a}} - \frac{\hat{b}}{\hat{a}^2}, \quad k = 2, 3, \dots, n. \quad (7)$$

The simulated and predicted values of the NGM (1, 1, k) model for the original sequence $X^{(0)}$ are

$$\begin{cases} \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k), & k = 2, 3, \dots, \\ \hat{x}^{(0)}(1) = x^{(0)}(1). \end{cases} \quad (8)$$

2.3. Accuracy Measure Indicator. In grey system theory, the absolute percentage error (APE) of each point, the mean absolute percentage error (MAPE) of all points, and the root mean square error (RMSE) of all points are usually used to measure the simulation and prediction accuracy of the grey prediction model [6].

The absolute percentage error of each point is

$$\text{APE}(k) = \frac{|\hat{x}^{(0)}(k) - x^{(0)}(k)|}{x^{(0)}(k)} \times 100\%, \quad k = 1, 2, \dots \quad (9)$$

The mean absolute percentage error of all points is

$$\text{MAPE} = \frac{1}{n-1} \sum_{k=2}^n \frac{|\hat{x}^{(0)}(k) - x^{(0)}(k)|}{x^{(0)}(k)} \times 100\%. \quad (10)$$

The root mean square error of all points is

$$\text{RMSE} = \sqrt{\frac{1}{n-1} \sum_{k=2}^n [\hat{x}^{(0)}(k) - x^{(0)}(k)]^2}. \quad (11)$$

3. Fractional Order Accumulation NGM (1, 1, k) Model with Optimal Background Value

3.1. FBNGM (1, 1, k) Model. Assuming $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \dots, x^{(r)}(n)\}$, $r \in R^+$ is the r^{th} order accumulation generation sequence of the nonnegative original sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$, and $Z^{(r)} = \{z^{(r)}(2), z^{(r)}(3), \dots, z^{(r)}(n)\}$ is the optimized background value sequence of $X^{(r)}$, among them, $z^{(r)}(k) = wx^{(1)}(k) + (1-w)x^{(1)}(k-1)$, $0 < r < 1$, $0 < w < 1$, $k = 2, 3, \dots, n$, so

$$x^{(r-1)}(k) + az^{(r)}(k) = b \frac{2k-1}{2} + c \quad (12)$$

is called the grey differential equation form of the NGM (1, 1, k) model with the fractional order and optimized

background value, denoted as the FBNGM (1, 1, k) model. Where a is called development coefficient, $z^{(r)}(k)$ is named the optimized background value, w is named the optimized background value weight, and the least squares estimate of the three parameters a, b, c is

$$\hat{P} = (\hat{a}, \hat{b}, \hat{c})^T = (B^T B)^{-1} B^T Y, \quad (13)$$

where

$$B = \begin{pmatrix} -z^{(r)}(2) & \frac{3}{2} & 1 \\ -z^{(r)}(3) & \frac{5}{2} & 1 \\ \vdots & \vdots & \vdots \\ -z^{(r)}(n) & \frac{(2n-1)}{2} & 1 \end{pmatrix}, \quad (14)$$

$$Y = \begin{pmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{pmatrix}.$$

Thus,

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = bt + c \quad (15)$$

is called the whitened differential equation form of the FBNGM (1, 1, k) model.

Under initial condition $\hat{x}^{(r)}(1) = x^{(r)}(1)$, the time response function of the FBNGM (1, 1, k) model is

$$\begin{aligned} \hat{x}^{(r)}(k) &= \left[x^{(r)}(1) - \frac{\hat{b}}{\hat{a}} - \frac{\hat{c}}{\hat{a}} + \frac{\hat{b}}{\hat{a}^2} \right] e^{-\hat{a}(k-1)} \\ &\quad + \frac{\hat{b}}{\hat{a}} k + \frac{\hat{c}}{\hat{a}} - \frac{\hat{b}}{\hat{a}^2}, \quad k = 2, 3, \dots, n. \end{aligned} \quad (16)$$

The simulated and predicted values of the FBNGM (1, 1, k) model for the original sequence $X^{(0)}$ are

$$\begin{cases} \hat{x}^{(0)}(k) = \left[\hat{x}^{(r)} \right]^{(-r)}(k) = \sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \hat{x}^{(r)}(k-i), & k = 2, 3, \dots, \\ \hat{x}^{(0)}(1) = x^{(0)}(1). \end{cases} \quad (17)$$

3.2. Parameter Solution of the FBNGM (1, 1, k) Model. The proposed FBNGM (1, 1, k) model described above is established based on unknown parameters r and w . In order to find the best fitness for the FBNGM (1, 1, k) model, MAPE

is taken as the objective function, the relationship between model parameters is taken as constraint condition, and the nonlinear programming model is conducted as follows:

$$\begin{aligned} \min f(r, w) &= \frac{1}{n-1} \sum_{k=2}^n \left| \frac{\hat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\%, \\ \text{s.t. } \left\{ \begin{aligned} &0 < r < 1, 0 < w < 1, \\ &\hat{P} = [\hat{a}, \hat{b}, \hat{c}]^T = (B^T B)^{-1} B^T Y, \\ &B = \begin{pmatrix} -z^{(r)}(2) & \frac{3}{2} & 1 \\ -z^{(r)}(3) & \frac{5}{2} & 1 \\ \vdots & \vdots & \vdots \\ -z^{(r)}(n) & \frac{(2n-1)}{2} & 1 \end{pmatrix}, Y = \begin{pmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{pmatrix}, \\ &z^{(r)}(k) = wx^{(r)}(k) + (1-w)x^{(r)}(k-1), \quad k = 2, 3, \dots, n, \\ &\hat{x}^{(r)}(k) = \left[x^{(r)}(1) - \frac{\hat{b}}{\hat{a}} - \frac{\hat{c}}{\hat{a}} + \frac{\hat{b}}{\hat{a}^2} \right] e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a}}k + \frac{\hat{c}}{\hat{a}} - \frac{\hat{b}}{\hat{a}^2}, \quad k = 2, 3, \dots, n, \\ &\hat{x}^{(0)}(k) = \left[\hat{x}^{(r)} \right]^{(-r)}(k) = \sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \hat{x}^{(r)}(k-i), \quad k = 2, 3, \dots, \\ &\hat{x}^{(0)}(1) = x^{(0)}(1). \end{aligned} \right. \end{aligned} \quad (18)$$

For the abovementioned nonlinear programming model, the particle swarm optimization algorithm [18] (PSO) is used to solve the best parameters of the proposed FBNGM (1, 1, k) model in this study.

3.3. Modeling Procedure of the FBNGM (1, 1, k) Model. The modeling procedure of the FBNGM (1, 1, k) model is presented clearly in Figure 1.

4. Empirical Applications

In this section, two cases will be used to verify the feasibility and effectiveness of the proposed FBNGM (1, 1, k) model.

4.1. Case 1. China is the most electricity consuming country in the world, accounting for 27.5% of global electricity consumption. The shortage of power supply will have a

serious negative impact on the stable development of China's economy. Therefore, the ability to reasonably predict China's electricity consumption plays an important role in formulating the Chinese government's energy policy. The national electricity consumption data from 2010 to 2016 are used for modeling in this study (<http://www.stats.gov.cn/tjsj/ndsj/2020/indexch.htm>), while the data from 2017 to 2020 are used to measure the accuracy of the model. In order to illustrate the superiority of the FBNGM (1, 1, k) model in simulation and prediction, the NGM (1, 1, k) model (abbreviated as NGM), BNGM (1, 1, k) model (abbreviated as BNGM), FNGM (1, 1, k) model (abbreviated as FNGM), and FBNGM (1, 1, k) model (abbreviated as FBNGM) together are used to simulate and predict the electricity consumption in China. The selection or optimization results of hyperparameters as well as estimation results of system parameters for all models are given in Table 1. The seeking process of hyperparameters based on PSO is shown in Figure 2. The

FIGURE 1: Modeling procedure of the FBNGM (1, 1, k) model.

TABLE 1: The parameters for all models in Case 1.

Model	Parameter				
	r	w	a	b	c
NGM	1.0000	0.5000	0.0041	3.0516	4.2635
BNGM	1.0000	0.4620	0.0043	3.0669	4.2623
FNGM	0.2091	0.5000	0.9109	6.3085	4.8645
FBNGM	0.8986	0.5017	-0.2269	-8.9817	4.1691

FIGURE 2: Track of the seeking process of PSO algorithms for the three models in Case 1.

simulation and prediction results of all models are given in Table 2 and Figures 3 and 4.

In the case, the estimation results and search process of the unknown parameters of the models are given in Table 1 and Figure 2. From Table 2 and Figures 3 and 4, it is easy to

see that the simulation accuracy of the FBNGM (1, 1, k) model is higher than that of the other three prediction models, and the prediction accuracy of the FBNGM (1, 1, k) model is higher than that of the NGM (1, 1, k) model and the BNGM (1, 1, k) model. According to the accuracy measure

TABLE 2: Simulation and prediction results of China's electricity consumption (billion kilowatt hours).

Year	Actual value	NGM		BNGM		FNGM		FBNGM	
		Forecasting value	APE (%)	Forecasting value	APE (%)	Forecasting value	APE (%)	Forecasting value	APE (%)
2010	4192	4192	0.00	4192	0.00	4192	0.00	4192	0.00
2011	4693	4695	0.04	4693	0.00	4680	0.28	4680	0.28
2012	4959	4980	0.43	4979	0.39	5000	0.82	4995	0.73
2013	5322	5265	1.08	5263	1.11	5269	1.00	5265	1.08
2014	5523	5548	0.45	5546	0.42	5530	0.12	5527	0.06
2015	5802	5830	0.49	5828	0.45	5804	0.04	5802	0.00
2016	6130	6111	0.30	6109	0.34	6112	0.29	6107	0.38
Modeling period (2010–2016)									
MAPE (%)		0.4653		0.4599		0.5393		0.4220	
RMSE (10^{-2})		0.3032		0.3033		0.3337		0.2981	
Testing period (2017–2020)									
2017	6482	6391	1.40	6388	1.45	6473	0.14	6456	0.40
2018	7151	6670	6.73	6666	6.78	6909	3.39	6867	3.97
2019	7285	6947	4.64	6942	4.71	7447	2.22	7360	1.02
2020	7511	7224	3.82	7218	3.91	8122	8.14	7957	5.94
MAPE (%)		4.1485		4.1768		3.1238		2.8328	
RMSE (10^{-2})		3.3029		3.3219		2.6123		2.6727

FIGURE 3: Simulation and prediction results for the four grey models in Case 1.

indicator, the proposed FBNGM $(1, 1, k)$ model has good performance in simulation and prediction, which shows that the FBNGM $(1, 1, k)$ model is suitable and effective for predicting China's electricity consumption.

4.2. Case 2. Agriculture is the basic industry to support the construction and development of national economy, and accurately predicting the development trend of agricultural production is the premise and foundation for formulating national economic plan and social development strategy. China is a large population country; its agriculture is related

to national livelihood and social stability and has a fundamental position and role in the national economy. In the study, country agriculture gross output value data in 2007–2014 (<http://data.stats.gov.cn/easyquery.htm>) is used to establish the model, and the data in 2015–2018 is used to test the prediction accuracy. In order to illustrate the superiority of the FBNGM $(1, 1, k)$ model in simulation and prediction, the NGM $(1, 1, k)$ model (abbreviated as NGM), BNGM $(1, 1, k)$ model (abbreviated as BNGM), FNGM $(1, 1, k)$ model (abbreviated as FNGM), and FBNGM $(1, 1, k)$ model (abbreviated as FBNGM) together are used to simulate and predict the agriculture gross output value in China.

FIGURE 4: Comparison of simulation and prediction accuracy for the four grey models in Case 1.

TABLE 3: The parameters for all models in Case 2.

Model	Parameter				
	r	w	a	b	c
NGM	1.0000	0.5000	0.0059	4.5083	2.4532
BNGM	1.0000	0.6232	0.0032	4.3929	2.4631
FNGM	0.1230	0.5000	-0.2754	-1.9803	1.2934
FBNGM	0.1041	0.5017	-0.2749	-1.8595	7.7389

FIGURE 5: Track of the seeking process of PSO algorithms for the three models in Case 2.

The selection or optimization results of hyperparameters as well as estimation results of system parameters for all models are given in Table 3. The seeking process of hyperparameters

based on PSO is shown in Figure 5. The simulation and prediction results of all models are given in Table 4 and Figures 6 and 7.

TABLE 4: Simulation and prediction results of agriculture gross output value in China (100 million yuan).

Year	Actual value	NGM		BNGM		FNGM		FBNGM	
		Forecasting value	APE (%)	Forecasting value	APE (%)	Forecasting value	APE (%)	Forecasting value	APE (%)
2007	27674	27674	0.00	27674	0.00	27674	0.00	27674	0.00
2008	32464	31044	4.38	31083	5.32	31166	4.00	31242	3.76
2009	33584	35357	5.28	35369	3.15	35627	6.09	35658	6.18
2010	38431	39646	3.16	39642	1.96	40163	4.51	40155	4.49
2011	44781	43909	1.95	43902	1.91	44586	0.44	44559	0.49
2012	49085	48147	1.91	48148	1.22	48807	0.56	48777	0.62
2013	53028	52361	1.26	52380	1.75	52756	0.51	52722	0.58
2014	55626	56550	1.66	56599	4.25	56351	1.30	56286	1.19
Modeling period (2007–2014)									
MAPE (%)		2.7990		2.7953		2.4868		2.4885	
RMSE (10^{-2})		11.6963		11.6997		11.6974		11.8765	
Testing period (2015–2018)									
2015	57774	60714	5.09	60804	5.24	59493	2.97	59324	2.68
2016	60139	64854	7.84	64996	8.08	62053	3.18	61634	2.49
2017	62099	68970	11.06	69175	11.39	63860	2.84	62929	1.34
2018	64734	73062	12.87	73340	13.30	64691	0.07	62808	2.97
MAPE (%)		9.2150		9.5027		2.2649		2.7201	
RMSE (10^{-2})		60.7154		62.6322		15.5892		17.7565

FIGURE 6: Simulation and prediction results for the four grey models in Case 2.

In the case, the estimation results and search process of the unknown parameters of the models are given in Table 3 and Figure 5. From Table 4 and Figures 6 and 7, it is easy to see that the simulation accuracy of the FBNGM (1, 1, k) model is higher than that of the other three prediction models, and the prediction accuracy of the FBNGM (1, 1, k)

model is higher than that of the NGM (1, 1, k) model and the BNGM (1, 1, k) model. According to the accuracy measure indicator, the proposed FBNGM (1, 1, k) model has good performance in simulation and prediction, which shows that the FBNGM (1, 1, k) model is suitable and effective for predicting the agriculture gross output value in China.

FIGURE 7: Comparison of simulation and prediction accuracy for the four grey models in Case 2.

5. Conclusion

In order to further improve the prediction accuracy of the NGM (1, 1, k) model, this study optimizes the model from the weight of the background value and fractional order accumulation generation. Then, the parameter solution and modeling process of the proposed FBNGM (1, 1, k) model is described in detail. In order to verify the feasibility and effectiveness of the FBNGM (1, 1, k) model proposed in this study, the novel model and other three grey forecasting models together are applied to the two actual cases with economic significance. It is obvious from the empirical results of the application examples that the simulation and prediction accuracy of the FBNGM (1, 1, k) model are higher than NGM (1, 1, k) and BNGM (1, 1, k) models. It confirms that the proposed FBNGM (1, 1, k) model has certain effectiveness in improving prediction accuracy. Therefore, the thought of optimizing the NGM (1, 1, k) model proposed in the study based on the weight of the background value and fractional order accumulation generation has a certain application value in the prediction of electricity consumption and agriculture gross output value.

Data Availability

The data used to support the findings of this study are available at <http://data.stats.gov.cn/easyquery.htm>.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Jun Zhang wrote the original draft and developed the methodology. Yanping Qin developed software and involved in visualization. Xinyu Zhang involved in modification. Bing Wang wrote the review. Dongxue Su validated the study. Huaqiong Duo updated the draft.

Acknowledgments

The relevant works conducted were supported by National Natural Science Foundation of China (32160332), Inner Mongolia Agricultural University High-Level Talents Scientific Research Project (NDYB2019-35), Key Project of the Study of Statistical Science from Statistics Bureau of Inner Mongolia Autonomous Region (TJXHKT202001), and the first batch of Industry-University Cooperative Education Project of the Ministry of Education in 2019 (201901148037).

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