Research Article

Cohen Positive Strongly p -Summing and p -Dominated m -Homogeneous Polynomials

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We introduce the concepts of Cohen positive strongly p -summing and positive p -dominated m -homogeneous polynomials. The version of Pietsch's domination theorem for the first class among other results and a Bu-type theorem is proved, as well as some inclusions with other known spaces. Moreover, we present a characterization of these classes in tensor terms.

1. Introduction

The identity from ℓ_1 into ℓ_2 is absolutely 2-summing, but its adjoint is not. Cohen introduced the concept of Cohen strongly p -summing linear operators in [1] to study the conjugate of absolutely p^* -summing linear operators. This paper had an enormous fame; in view of this, many analogies in the multilinear and the polynomial cases were investigated [2–5]. In this article, we extend this concept to positive m -homogeneous polynomials.

This work is organized as follows. In Section 2, we introduce the class of Cohen positive strongly p -summing m -homogeneous polynomials and prove that this class is related to different classes of p -summability $(\Pi_p^+(E, F), \mathcal{D}_p^{m+}(^mX; F), \mathcal{D}_p^+(\widehat{\otimes}_{\pi, s}^m X; F))$. By means of the adjoint operator, the associated symmetric m -linear operator and the linearization of these polynomials. Section 3 is essentially dedicated to the study of positive p -dominated m -linear operators and polynomials. This section includes our main theorem (the Bu-type theorem) which serves as a focal point in the proof of the relationship between positive p -dominated polynomials and those defined in Section 2. In Section 4, we present some applications based on Proposition 3.

Finally, Section 5 highlights the classes defined earlier in terms of tensor product and proves a duality between these classes and tensor spaces equipped with adequate norms through a linear mapping.

Firstly, we consider $X, X_1, \dots, X_m, Y$ and $E, E_1, \dots, E_m, F, G$ are Banach spaces and Banach lattices, respectively, for $m \in \mathbb{N}^*$.

Let $\mathcal{L}(X_1, \dots, X_m; Y)$ is the Banach space of all continuous m -linear operators from $X_1 \times \dots \times X_m$ to Y . If $Y = \mathbb{K}$, we write $\mathcal{L}(X_1, \dots, X_m)$. In the case when $X_1 = \dots = X_m = X$, we simply write $\mathcal{L}(^mX; Y)$. For a Banach space X , we denote X^* and B_X are the topological dual and the closed unit ball, respectively. For $1 \leq p \leq \infty$, let p^* be its conjugate, that is, $(1/p) + (1/p^*) = 1$.

Let E^+ be the positive cone of the Banach lattice E such that $\{x \in E, x \geq 0\}$. For $x \in E$, let $x^+ := \sup\{x, 0\}$ and $x^- := \sup\{-x, 0\}$ be the positive part and the negative part of x , respectively.

For any $x \in E$, we have $x = x^+ - x^-$ and $|x| = x^+ + x^-$.

Recall that an m -linear operator $T: E_1 \times \dots \times E_m \longrightarrow F$ is called positive if $T(x^1, \dots, x^m) \in F^+$, whenever $x^1 \in E_1^+, \dots, x^m \in E_m^+$. Let $\mathcal{L}^+(E_1, \dots, E_m; F)$ be the collection of all positive continuous m -linear operators from

$E_1 \times \cdots \times E_m$ to F under the norm $\|T\| = \sup_{x^i \in B_{E_i}^+} \|T(x^1, \dots, x^m)\|$.

We denote by $\ell_p^n(X)$ the space of all sequences $(x_i)_{i=1}^n$ in X with the norm,

$$\|(x_i)_{i=1}^n\|_p = \left(\sum_{i=1}^n \|x_i\|^p \right)^{1/p}, \quad (1)$$

and by $\ell_{p,\text{weak}}^n(X)$ the space of all sequences $(x_i)_{i=1}^n$ in X with the norm,

$$w_p((x_i)_{i=1}^n) = \|(x_i)_{i=1}^n\|_{\ell_{p,\text{weak}}^n(X)} = \sup_{\varphi \in B_{X^*}} \left(\sum_{i=1}^n |\langle x_i, \varphi \rangle|^p \right)^{1/p}. \quad (2)$$

Then, $\ell_{p,\text{weak}}^n(X)$ is a Banach space with respect to the norm w_p . For a Banach lattice E , we define

$$\ell_{p,|\text{weak}|}^n(E) := \{(x_i)_{i=1}^n : (|x_i|)_{i=1}^n \in \ell_{p,\text{weak}}^n(E)\}, \quad (3)$$

$$\|(x_i)_{i=1}^n\|_{\ell_{p,|\text{weak}|}^n(E)} = w_p((|x_i|)_{i=1}^n).$$

Let $B_{E^*}^+ = B_{E^*} \cap E^{*+}$. If $x_1, \dots, x_n \geq 0$, we have that

$$\|(x_i)_{i=1}^n\|_{\ell_{p,|\text{weak}|}^n(E)} = \sup_{\xi \in B_{E^*}^+} \left(\sum_{i=1}^n \langle x_i, \xi \rangle^p \right)^{1/p}. \quad (4)$$

The definition of m -homogeneous polynomial can be seen, for instance, in [3, 6, 7].

A map $P: X \rightarrow Y$ is an m -homogeneous polynomial if there exists a unique symmetric m -linear operator $\hat{P}: X \times \cdots \times X \rightarrow Y$ such that $P(x) = \hat{P}(x, \dots, x)$, for all $x \in X$. Both are related by the polarization formula:

$$\hat{P}(x^1, \dots, x^m) = \frac{1}{m!2^m} \sum_{\substack{\epsilon_j = \pm 1 \\ 1 \leq j \leq m}} \epsilon_1 \cdots \epsilon_m P\left(\sum_{j=1}^m \epsilon_j x^j\right). \quad (5)$$

P is bounded on the unit ball of X if and only if $\hat{P}$ is bounded on the unit ball of $X \times \cdots \times X$. These norms are related by the inequalities $\|P\| \leq \|\hat{P}\| \leq (m^m/m!)\|P\|$. $\mathcal{P}(^m X; Y)$ denotes the Banach space of all continuous m -homogeneous polynomials from X to Y with the norm $\|P\| = \sup_{\|x\| \leq 1} \|P(x)\| = \inf\{C: \|P(x)\| \leq C\|x\|^m, x \in X\}$.

For the scalar-valued case $Y = \mathbb{R}$ or $\mathbb{C}$, we use the simplified notation $\mathcal{P}(^m X)$.

By $X_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi X_m$, we denote the completed projective tensor product of $X_1, \dots, X_m$. If $X = X_1 = \cdots = X_m$, we write $\widehat{\otimes}_\pi^m X$. By $\widehat{\otimes}_s^m X := X \widehat{\otimes}_s \cdots \widehat{\otimes}_s X$, the m -fold symmetric tensor product of X , that is, the set of all elements $u \in \widehat{\otimes}_s^m X$ of the form

$$u = \sum_{i=1}^n x_i \otimes \cdots \otimes x_i \quad (n \in \mathbb{N}, x_i \in X, 1 \leq i \leq n). \quad (6)$$

By $\widehat{\otimes}_{\pi,s}^m X$, we denote the closure of $\widehat{\otimes}_s^m X$ in $\widehat{\otimes}_\pi^m X$. Consider the canonical polynomial $\delta_m: X \rightarrow \widehat{\otimes}_\pi^m X$ defined by $\delta_m(x) = x \otimes \cdots \otimes x$, and we have the following factorization $P = \tilde{P} \circ \delta_m$.

Recalling some definitions we need in the sequel.

Let $1 \leq p < \infty$. An operator $u: E \rightarrow X$ is said to be positive p -summing [8] if there exists a constant $C > 0$ such that, for all $n \in \mathbb{N}$, $x_1, \dots, x_n \in E$, the inequality

$$\|u(x_i)_{i=1}^n\|_p \leq C \|(x_i)_{i=1}^n\|_{\ell_{p,|\text{weak}|}^n(E)}. \quad (7)$$

For $p = \infty$,

$$\sup_{1 \leq i \leq n} \|u(x_i)\| \leq C \|(x_i)_{i=1}^n\|_{\ell_{\infty,|\text{weak}|}^n(E)}. \quad (8)$$

The space of positive p -summing operators from E into X given by $\Pi_p^+(E; X)$, $t \cdot \Pi_p^+(E; X)$ becomes a Banach space with norm $\pi_p^+(\cdot)$ defined by the infimum of the constants $C > 0$ that verify inequality (7). We have $\Pi_\infty^+(E; X) = \mathcal{L}(E; X)$.

The next definition is due to Achour and Saadi in [3]. Let $1 < p \leq +\infty$. An m -homogeneous polynomial $P: X \rightarrow Y$ is Cohen strongly p -summing if $\exists C > 0$ such that, for all $x_1, \dots, x_n \in X$ and $y_1^*, \dots, y_n^* \in Y^*$,

$$\sum_{i=1}^n |\langle P(x_i), y_i^* \rangle| \leq C \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \sup_{y \in B_Y} \|(y_i^*(y))\|_{\ell_{p^*}^n}. \quad (9)$$

The class of such polynomials is denoted by $\mathcal{P}_{\text{Coh},p}(^m X; Y)$; it is equipped with the norm $d_p(\cdot)$, i.e., the smallest constant C such that inequality (9) holds. For $p = 1$, we have $\mathcal{P}_{\text{Coh},1}(^m X; Y) = \mathcal{P}(^m X; Y)$.

2. Cohen Positive Strongly p -Summing m -Homogeneous Polynomials

In this section, we give a natural generalization of the notion of Cohen positive strongly p -summing to the category of homogeneous polynomial mappings. As in the multilinear case, there is an analogue connection between polynomial mapping and its linearization and associated symmetric m -linear operator.

Definition 1 (see [4]). Let $1 \leq p \leq +\infty$. An m -linear operator $T: X_1 \times \cdots \times X_m \rightarrow F$ is Cohen positive strongly p -summing multilinear operator if $\exists C > 0$, such that, for all $x_1^j, \dots, x_n^j \in X_j$, $1 \leq j \leq m$, and any $y_1^*, \dots, y_n^* \in F^*$,

$$\|\langle T(x_1^1, \dots, x_n^1), y_1^* \rangle\|_{\ell_1^n} \leq C \left(\sum_{i=1}^n \prod_{j=1}^m \|x_i^j\|_{X_j}^p \right)^{1/p} \|(y_i^*)_{i=1}^n\|_{\ell_{p^*,|\text{weak}|}^n(F^*)}. \quad (10)$$

Moreover, the class of all Cohen positive strongly p -summing m -linear operators from $X_1 \times \cdots \times X_m$ into F is given by $\mathcal{D}_p^{m+}(X_1, \dots, X_m; F)$. This space is a Banach space with the norm $d_p^{m+}(\cdot)$ which is the smallest constant C such that inequality (10) holds. For $m = 1$, it is the space of positive strongly p -summing operators linear operators [2].

Now, we introduce the notion of Cohen positive strongly p -summing m -homogeneous polynomials, and we study some fundamental properties.

Definition 2. Let $1 < p \leq +\infty$ and $m \in \mathbb{N}$. An m -homogeneous polynomial $P: X \longrightarrow F$ is Cohen positive strongly p -summing if $\exists C > 0$, such that, for all $x_1, \dots, x_n \in X$ and any $y_1^*, \dots, y_n^* \in F^*$,

$$\sum_{i=1}^n |\langle P(x_i), y_i^* \rangle| \leq C \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \|(y_i^*)_{i=1}^n\|_{\ell_{p^*, \text{weak}}^m(F^*)}. \quad (11)$$

The class of such polynomials is given by $\mathcal{P}_{\text{Coh}, p}^+({}^m X; F)$. It is a Banach space with the norm $d_p^+(\cdot)$ which is the smallest constant C such that inequality (11) holds, for $p = 1$, we have $\mathcal{P}_{\text{Coh}, 1}^+({}^m X; F) = \mathcal{P}({}^m X; F)$.

Remark 1. Fix $m \in \mathbb{N}$; an m -homogeneous polynomial $P: X \longrightarrow F$ is Cohen positive strongly p -summing if and only if $\exists C > 0$ satisfying

$$\sum_{i=1}^n |\langle P(x_i), y_i^* \rangle| \leq C \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} w_{p^*}((y_i^*)_{i=1}^n), \quad (12)$$

holds for every $x_1, \dots, x_n \in X$ and $y_1^*, \dots, y_n^* \in F^{**}$.

Example 1. We use the same example used in [3], with slight adjustments.

Let $1 < p \leq +\infty$, $m \in \mathbb{N}$, and $u: X \longrightarrow F$ be a positive strongly p -summing operator. For $\varphi \in X^*$, the functional

$$\begin{aligned} P: X &\longrightarrow F \\ x &\mapsto \varphi^{m-1}(x)u(x), \end{aligned} \quad (13)$$

is Cohen positive strongly p -summing.

For $x_1, x_2, \dots, x_n \in X$ and $y_1^*, y_2^*, \dots, y_n^* \in F^*$, because $u \in \mathcal{D}_p^+(X, F)$, we have by [2]

$$\sum_{i=1}^n |\langle u(x_i), y_i^* \rangle| \leq d_p^+(u) \|(x_i)_{i=1}^n\|_p \|(y_i^*)_{i=1}^n\|_{\ell_{p^*, \text{weak}}^m(F^*)}. \quad (14)$$

Then,

$$\begin{aligned} \sum_{i=1}^n |\langle P(x_i), y_i^* \rangle| &= \sum_{i=1}^n |\langle \varphi^{m-1}(x_i)u(x_i), y_i^* \rangle| \\ &= \sum_{i=1}^n |\langle u(\varphi^{m-1}(x_i)x_i), y_i^* \rangle| \\ &\leq d_p^+(u) \|(\varphi^{m-1}(x_i)x_i)_{i=1}^n\|_p \|(y_i^*)_{i=1}^n\|_{\ell_{p^*, \text{weak}}^m(F^*)} \\ &\leq d_p^+(u) \left(\sum_{i=1}^n \|(\varphi^{m-1}(x_i)x_i)_{i=1}^n\|^p \right)^{1/p} \|(y_i^*)_{i=1}^n\|_{\ell_{p^*, \text{weak}}^m(F^*)} \\ &\leq d_p^+(u) \|\varphi\|^{m-1} \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \|(y_i^*)_{i=1}^n\|_{\ell_{p^*, \text{weak}}^m(F^*)}. \end{aligned} \quad (15)$$

Hence, P is Cohen positive strongly p -summing and $d_p^+(P) \leq d_p^+(u) \|\varphi\|^{m-1}$.

Proposition 1 (ideal property). Let $P \in \mathcal{P}({}^m X; F)$, S in $L^+(F; G)$, and R in $\mathcal{L}(Y; X)$.

If P is Cohen positive strongly p -summing polynomial, then SPR is Cohen positive strongly p -summing polynomial and $d_p^+(\text{SPR}) \leq d_p^+(P) \|R\|^m \|S\|$.

Proof. Let $(x_i)_{i=1}^n \subset E$ and $(y_i^*)_{i=1}^n \subset G^{**}$

$$\begin{aligned}
\sum_{i=1}^n |\langle SPR(x_i), y_i^* \rangle| &= \sum_{i=1}^n |\langle PR(x_i), S^*(y_i^*) \rangle| \\
&\leq d_p^+(P) \left(\sum_{i=1}^n \|R(x_i)\|^{mp} \right)^{1/p} \sup_{y^{**} \in B_{F^{**}}^+} \left(\sum_{i=1}^n \langle S^*(y_i^*), y^{**} \rangle^{p^*} \right)^{1/p^*} \\
&\leq d_p^+(P) \|R\|^m \|S\| \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \sup_{y^{**} \in B_{F^{**}}^+} \left(\sum_{i=1}^n \langle y_i^*, \frac{S^{**}(y^{**})}{\|S^{**}\|} \rangle^{p^*} \right)^{1/p^*} \\
&\leq d_p^+(P) \|R\|^m \|S\| \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \sup_{\varphi \in B_{G^*}^+} \left(\sum_{i=1}^n \langle y_i^*, \varphi \rangle^{p^*} \right)^{1/p^*}.
\end{aligned} \tag{16}$$

Thus, SPR is Cohen positive strongly p -summing polynomial and $d_p^+(SPR) \leq d_p^+(P) \|R\|^m \|S\|$. $\square$

Next, we present a version of Pietsch's domination theorem, for our class of polynomials.

Theorem 1 (domination theorem). *An m -homogeneous polynomial $P: X \rightarrow F$ is Cohen positive strongly p -summing ($1 < p \leq +\infty$) if and only if there is a Radon probability measure μ on $B_{F^{**}}^+$, such that, for all $x \in X$ and $y^* \in F^*$, we have*

$$|\langle P(x), y^* \rangle| \leq C \|x\|^m \left(\int_{B_{F^{**}}^+} (|y^*(y^{**})|)^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \tag{17}$$

In addition, in this case,

$$d_p^+(P) = \inf \{ C > 0, \text{ for all } C \text{ verifying inequality (17)} \}. \tag{18}$$

For the proof of this theorem, we need to give a small review about $R-S$ -abstract (p, p^*) -summing operators.

Let the sets be X, Y , and E , the Banach space $\mathcal{H}$ is the family of mappings from X to Y and G and the compact Hausdorff topological space K . Assume that the maps

$$\begin{aligned}
R: K \times E \times G &\longrightarrow [0, \infty), \\
S: \mathcal{H} \times E \times G &\longrightarrow [0, \infty),
\end{aligned} \tag{19}$$

satisfy.

For all $f \in \mathcal{H}$, there is an $x_0 \in E$, such that

$$R(\varphi, x_0, b) = S(f, x_0, b) = 0, \tag{20}$$

for any $\varphi \in K$ and $b \in G$.

The mapping $R_{x,b}: K \rightarrow [0, \infty)$ defined by $R_{x,b}(\varphi) = R(\varphi, x, b)$ is continuous, for any $x \in E$ and $b \in G$, and $R(\varphi, x, \eta b) \leq \eta R(\varphi, x, b)$ and $\eta S(f, x, b) \leq S(f, x, \eta b)$, for any $\varphi \in K$, $x \in E$, $0 \leq \eta \leq 1$, $b \in G$, and $f \in \mathcal{H}$.

Definition 3 (see [9]). Let R and S be as above and $0 < p < \infty$. A mapping $f \in \mathcal{H}$ is said to be $R-S$ -abstract p -summing if $\exists C > 0$ such that

$$\left(\sum_{j=1}^m S(f, x_j, b_j)^p \right)^{1/p} \leq C \sup_{\varphi \in K} \left(\sum_{j=1}^m R(\varphi, x_j, b_j)^p \right)^{1/p}, \tag{21}$$

for all $x_1, \dots, x_m \in E$, $b_1, \dots, b_m \in G$, and $m \in \mathbb{N}^*$.

Theorem 2 (see [9]). Consider R and S be before, $0 < p < \infty$ and $f \in \mathcal{H}$. Then, f is $R-S$ -abstract p -summing $\Leftrightarrow, >0$ and a Borel probability measure μ on K such that

$$S(f, x, b) \leq C \left(\int_K R(\varphi, x, b)^p d\mu \right)^{1/p}, \tag{22}$$

where $\forall x \in E$ and $b \in G$. Furthermore, the infimum of such constant C equals $\pi_{RS,p}(f)$.

For our proof, we also need to recall Theorem 4.6 in [9].

Theorem 3 (see [9]). A map $f \in \mathcal{H}$ is $R_1, \dots, R_t-S$ -abstract $(p_1, \dots, p_t)$ -summing $\Leftrightarrow, >0$, and a Borel probability measures μ_j on K_j , such that

$$S(f, x^{(1)}, \dots, x^{(r)}, b^{(1)}, \dots, b^{(t)}) \leq C \prod_{j=1}^t \left(\int_{K_j} R_j(\varphi, x^{(1)}, \dots, x^{(r)}, b^{(j)})^{p_j} d\mu_j \right)^{1/p_j}, \tag{23}$$

where $\forall x^{(l)} \in E_l$, $l = 1, \dots, r$, and $b^{(j)} \in G_j$ with $j = 1, \dots, t$.

Proof. (proof of the domination theorem). Assume that P is Cohen positive strongly p -summing polynomial, by selecting the parameters,

$$\left\{ \begin{array}{l} t = 2 \text{ and } r = m - 1, \\ E = X, \\ K_1 = B_{X^*}^+ \text{ and } K_2 = B_{F^{**}}^+, \\ G = F, \\ q = 1 \text{ } q_1 = p \text{ and } q_2 = p^*, \\ \mathcal{H} = \mathcal{P}({}^m X, F), \\ S(P, x, y^*) = |\langle P(x), y^* \rangle|, \\ R_1(\varphi, x) = \|x\|^m, \\ R_2(\varphi, x, y^*) = (\varphi|y^*|). \end{array} \right. \quad (24)$$

Hence, $P: X \longrightarrow F$ is Cohen positive strongly p -summing $\Leftrightarrow$ and P is $R_1 - R_2 - S$ -abstract (p, p^*) -summing. The previous theorem tells us that P is $R_1 - R_2 - S$ -abstract (p, p^*) -summing $\Leftrightarrow$ there is $C > 0$, and there are probability measures μ_k on K_k , $k = 1, 2$, such that

$$S(P, x, y^*) \leq C \left(\int_{K_1} R_1(\varphi, x)^p d\mu_1 \right)^{1/p} \left(\int_{K_2} R_2(\varphi, x, y^*)^{p^*} d\mu_2 \right)^{1/p^*}, \quad (25)$$

i.e.,

$$\begin{aligned} |\langle P(x), y^* \rangle| &\leq C \left(\int_{B_{X^*}^+} \|x\|^{mp} d\mu_1 \right)^{1/p} \left(\int_{B_{F^{**}}^+} (\varphi|y^*|^{p^*}) d\mu_2 \right)^{1/p^*} \\ &\leq C \|x\|^m \left(\int_{B_{F^{**}}^+} (\varphi|y^*|)^{p^*} d\mu_2 \right)^{1/p^*}. \end{aligned} \quad (26)$$

Proposition 2. Consider $1 < p_1 < p_2 \leq +\infty$.

If $P \in \mathcal{P}_{\text{Coh}, p_2}^+({}^m X; F)$, then $P \in \mathcal{P}_{\text{Coh}, p_1}^+({}^m X, F)$ and $d_{p_1}^+(P) \leq d_{p_2}^+(P)$.

Proof. Assume that $P \in \mathcal{P}_{\text{Coh}, p_1}^+({}^m X, F)$; the Pietsch domination (Theorem 1) provides a Radon probability measure μ on $B_{F^{**}}^+$ such that $\forall (x, y^*) \in (X \times F^*)$ for which

$$\begin{aligned} |\langle P(x), y^* \rangle| &\leq d_{p_2}^+(P) \|x\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p_2^*} d\mu(y^{**}) \right)^{1/p_2^*} \\ &\leq d_{p_2}^+(P) \|x\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p_1^*} d\mu(y^{**}) \right)^{1/p_1^*}. \end{aligned} \quad (27)$$

So, $P \in \mathcal{P}_{\text{Coh}, p_1}^+({}^m X, F)$ and $d_{p_1}^+(P) \leq d_{p_2}^+(P)$. $\square$

Definition 4 (see [10]). Given a continuous m -homogeneous polynomial $P \in \mathcal{P}({}^m X, Y)$ between the Banach spaces X and Y , the adjoint of P is the following continuous linear operator:

$$\begin{aligned} P^*: Y^* &\longrightarrow \mathcal{P}({}^m X), \\ P^*(y^*)(x) &= y^*(P(x)). \end{aligned} \quad (28)$$

We have $\|P^*\| = \|P\|$.

Proposition 3. Let $1 < p \leq +\infty$. The m -homogeneous polynomial $P \in \mathcal{P}({}^m X; F)$ is Cohen positive strongly

p -summing if and only if P^* is positive p -summing, and we have $d_p^+(P) = \pi_{p^*}^+(P^*)$.

Proof. Suppose that $P \in \mathcal{P}_{\text{Coh},p}^+({}^m X; F)$; we have by (17) $|\langle P(x), y^* \rangle| \leq d_p^+(P) \|x\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}$, for $x \in X$ and $y^* \in F^*$.

Taking the supremum, we obtain

$$\sup_{\|x\| \leq 1} |P^*(y^*)(x)| \leq d_p^+(P) \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \quad (29)$$

Then,

$\|P^*(y^*)\| \leq d_p^+(P) \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}$. By the Pietsch domination theorem in Proposition 3.4 in [2], $P^* \in \Pi_{p^*}^+(F^*, \mathcal{P}({}^m X))$. Conversely, let $P^* \in \Pi_{p^*}^+(F^*, \mathcal{P}({}^m X))$ we have, for $x \in X$ and $y^* \in F^*$,

$$|\langle P(x), y^* \rangle| = |P^*(y^*)(x)| \leq \|P^*(y^*)\| \|x\|^m. \quad (30)$$

Using the theorem of Pietsch domination for positive p^* -summing linear operators, we find

$$|\langle P(x), y^* \rangle| \leq C \|x\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}, \quad (31)$$

$P \in \mathcal{P}_{\text{coh},p}^+({}^m X, F).$

□

Proposition 4. If $1 < p \leq +\infty$,

$$\mathcal{P}_{\text{Coh},p}({}^m X; F) \subseteq \mathcal{P}_{\text{Coh},p}^+({}^m X; F). \quad (32)$$

Proof. Take any P in $\mathcal{P}_{\text{Coh},p}({}^m X; F)$; hence, its adjoint operator P^* is in $\Pi_{p^*}(F^*, \mathcal{P}({}^m X))$ by Proposition 1 in [11], and again by Proposition 1 in [8], P^* is in $\Pi_{p^*}(F^*, \mathcal{P}({}^m X))$; then, $P \in \mathcal{P}_{\text{Coh},p}^+({}^m X; F)$. □

Proposition 5. The following properties are equivalent:

The polynomial $P \in \mathcal{P}({}^m X; F)$ is Cohen positive strongly p -summing

Its associated symmetric m -linear operator $\hat{P} \in \mathcal{L}({}^m X; F)$ is Cohen positive strongly p -summing

Proof. First, suppose that $\hat{P}$ is Cohen positive strongly p -summing; let $x_1, \dots, x_n \in X$ and $y_1^*, \dots, y_n^* \in F^*$; then,

$$\begin{aligned} \sum_{i=1}^n |\langle P(x_i), y_i^* \rangle| &= \sum_{i=1}^n |\langle \hat{P}(x_i, \dots, x_i), y_i^* \rangle| \\ &\leq d_p^+(\hat{P}) \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \| (y_i^*)_{i=1}^n \|_{\ell_{p^*, |\text{weak}|}^m(F^*)}. \end{aligned} \quad (33)$$

Therefore, P is Cohen positive strongly p -summing and $d_p^+(P) \leq d_p^+(\hat{P})$.

Conversely, suppose that $P \in \mathcal{P}_{\text{Coh},p}^+({}^m X; F)$. Let $x^j \in X$ such that $\|x^j\| \leq 1$ ($1 \leq j \leq m$) and $y^* \in F^*$; we have, by (5),

$$\begin{aligned} |\langle \hat{P}(x^1, \dots, x^m), y^* \rangle| &= \left| \left\langle \frac{1}{m!2^m} \sum_{\substack{\epsilon_j = \pm 1 \\ 1 \leq j \leq m}} \epsilon_1 \cdots \epsilon_m P \left(\sum_{j=1}^m \epsilon_j x^j \right), y^* \right\rangle \right| \\ &\leq \frac{1}{m!2^m} \sum_{\substack{\epsilon_j = \pm 1 \\ 1 \leq j \leq m}} \left| \left\langle P \left(\sum_{j=1}^m \epsilon_j x^j \right), y^* \right\rangle \right|, \text{ by the domination theorem} \\ &\leq \frac{1}{m!2^m} \sum_{\substack{\epsilon_j = \pm 1 \\ 1 \leq j \leq m}} d_p^+(P) \left\| \sum_{j=1}^m \epsilon_j x^j \right\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*} \\ &\leq \frac{1}{m!2^m} d_p^+(P) \sum_{\substack{\epsilon_j = \pm 1 \\ 1 \leq j \leq m}} \left(\sum_{i=1}^m \|x^i\| \right)^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*} \\ &\leq \frac{1}{m!2^m} d_p^+(P) 2^m m^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*} \\ &\leq \frac{m^m}{m!} d_p^+(P) \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \end{aligned} \quad (34)$$

Furthermore, for every $x^j \in X$ ($1 \leq j \leq m$), $x^j \neq 0$, we have

$$\left| \langle \hat{P} \left(\frac{x^1}{\|x^1\|}, \dots, \frac{x^m}{\|x^m\|} \right), y^* \rangle \right| \leq \frac{m^m}{m!} d_p^+(P) \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \quad (35)$$

Thus,

$$|\langle \hat{P}(x^1, \dots, x^m), y^* \rangle| \leq \frac{m^m}{m!} d_p^+(P) \prod_{j=1}^m \|x^j\| \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \quad (36)$$

Using [4], $\hat{P}$ is Cohen positive strongly p -summing; furthermore, $d_p^+(\hat{P}) \leq (m^m/m!)d_p^+(P)$. $\square$

Corollary 1. *Let $1 < p < \infty$. If the polynomial $P \in \mathcal{P}(^m X; F)$ is Cohen positive strongly p -summing, then its associated symmetric m -linear operator $\hat{P} \in \mathcal{P}(^m X; F)$ is multiple p -convex. If the polynomial $P \in \mathcal{P}(^m X; c_0)$ is Cohen positive strongly p -summing, then its associated symmetric m -linear operator $\hat{P}$ is in $\mathcal{L}(^m X; c_0)$.*

Proof. The proof of this corollary is an adaptation of the proof of Proposition 3.3 in [4]. $\square$

Proposition 6. *Let $1 < p \leq \infty$. Let $P: X \rightarrow F$ be a m -homogeneous polynomial and $\tilde{P}$ its linearization. The following properties are equivalent*

- (i) *The polynomial P belongs to $\mathcal{P}_{coh,p}^+(^m X; F)$*
- (ii) *The operator $\tilde{P}$ belongs to $\mathcal{D}_p^+(\widehat{\otimes}_{\pi,s}^m X; F)$*
- (iii) *$\exists Z$ is a Banach space, $u \in \mathcal{D}_p^+(Z, F)$, and $Q \in \mathcal{P}(^m X; Z)$ such that $P = uQ$*

Proof. (i) $\Rightarrow$ (ii) Assume that P is Cohen positive strongly p -summing; let $v \in \widehat{\otimes}_{\pi,s}^m X$ such that $v \neq 0$ and $y^* \in F^*$, and assume that $v = \sum_{i=1}^n x_i \otimes \dots \otimes x_i$; then,

$$\begin{aligned} |\langle \tilde{P}(v), y^* \rangle| &\leq \sum_{i=1}^n |\langle P(x_i, \dots, x_i), y^* \rangle| \\ &\leq \sum_{i=1}^n d_p^+(P) \|x_i\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*} \\ &= d_p^+(P) \sum_{i=1}^n \|x_i\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \end{aligned} \quad (37)$$

Selecting the infimum over any represents of v , we find

$$|\langle \tilde{P}(v), y^* \rangle| \leq d_p^+(P) \|v\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}, \quad (38)$$

and by Theorem 4.13 in [2], $\tilde{P}$ is positive strongly p -summing.

(ii) $\Rightarrow$ (iii) From the factorization, $P = \tilde{P} \circ \delta_m$ gives the result directly.

(iii) $\Rightarrow$ (i) Assume that there exists a Banach space Z , $u \in \mathcal{D}_p^+(Z, F)$ and $Q \in \mathcal{P}(^m X; Z)$, such that $P = uQ$. Then,

$$\|Q(x)\| \leq \|Q\| \|x\|^m. \quad (39)$$

Using Theorem 4.13 in [2],

$$|\langle u(x), y^* \rangle| \leq d_p^+(u) \|x\| \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}. \quad (40)$$

Hence,

$$\begin{aligned} |\langle P(x), y^* \rangle| &= |\langle u(Q(x)), y^* \rangle| \\ &\leq d_p^+(u) \|Q(x)\| \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*} \\ &\leq d_p^+(u) \|Q\| \|x\|^m \left(\int_{B_{F^{**}}^+} (|y^*|(y^{**}))^{p^*} d\mu(y^{**}) \right)^{1/p^*}, \end{aligned} \quad (41)$$

and according to the domination theorem, $P \in \mathcal{P}_{coh,p}^+(^m X; F)$. $\square$

3. Positive p -Dominated m -Homogeneous Polynomials

In this section, we give a natural generalization of the notion of positive p -dominated to the category of homogeneous polynomial and the m -linear operators.

3.1. Positive p -Dominated m -Linear Operators

Definition 5. An m -linear operator $T: E_1 \times \dots \times E_m \rightarrow Y$ is positive p -dominated if there is a positive constant $C > 0$ such that

$$\left(\sum_{i=1}^n \|T(x_i^1, \dots, x_i^m)\|^{p/m} \right)^{m/p} \leq C \| (x_i^1)_{i=1}^n \|_{\ell_{p, |\text{weak}|}^n(E_1)} \cdots \| (x_i^m)_{i=1}^n \|_{\ell_{p, |\text{weak}|}^n(E_m)}, \quad (42)$$

$\forall (x_i^j)_{i=1}^n \subset E_j$, $1 \leq j \leq m$ and $m \in \mathbb{N}$. We denote $\mathcal{L}_d^{p+}(E_1, \dots, E_m; Y)$ the space of all positive p -dominated m -linear operators and the infimum of all constants C verifying (42) is denoted $\pi_p^{d+}(T)$. For all $p \geq m$, π_p^{d+} is a norm on $\mathcal{L}_d^{p+}(E_1, \dots, E_m; Y)$, whereas for $p < m$, it is only a quasi-norm.

For $m = 1$, the space of all positive p -dominated operators is nothing other than the space of positive p -summing operators.

We should stress out that the above definition makes sense for Banach spaces Y , but the more interesting case occurs when we consider a Banach lattice F , since this assumption allows to establish an inclusion relation between the classes of positive dominated multilinear operators and Cohen positive strongly summing multilinear operators and later on for polynomials.

The next theorem is an analogous Pietsch's domination-type theorem for m -linear positive p -dominated operators. The proof of this result is an adaptation of that of Theorem 4.7 in [9], so we do not present it here.

Theorem 4 (domination theorem for positive p -dominated multilinear operators). *An m -linear operator $T: E_1 \times \dots \times E_m \rightarrow Y$ is positive p -dominated if and only if there is Radon probability measures $\mu_j \in B_{E_j}^+$, ($1 \leq j \leq m$) and $K > 0$, and*

$$\|T(x^1, \dots, x^m)\| \leq K \prod_{j=1}^m \left(\int_{B_{E_j}^+} \langle x^{j*}, x^j \rangle^p d\mu_j(x^{j*}) \right)^{1/p}, \quad (43)$$

for all $x^j \in E_j^+$. Moreover, the smallest constants K is $\pi_p^{d+}(T)$.

The following theorem is a version of the Bu-theorem [12]; in [13], Achour and Mezrag proved an inclusion theorem between p -dominated operators and Cohen strongly q -summing m -linear operators; in our work, we prove a positive analogue of Theorem 3.2 in [13]. This theorem plays an important role in the proof of the relationship between Cohen positive strongly summing and positive dominated polynomials.

Theorem 5. *Let $1 < p, q < \infty$. Let $\ell_2^{n_1}, \dots, \ell_2^{n_m}$ and F be Banach lattices. Then,*

$$\mathcal{L}_d^{p+}(\ell_2^{n_1}, \dots, \ell_2^{n_m}; F) \subseteq \mathcal{D}_q^{m+}(\ell_2^{n_1}, \dots, \ell_2^{n_m}; F), \quad (44)$$

for all $T \in \mathcal{L}_d^{p+}(\ell_2^{n_1}, \dots, \ell_2^{n_m}; F)$ $d_q^{m+}(T) \leq \pi_p^{d+}(T)$.

For the proof of this theorem, we need to recall the Khintchine [14] and the Kahane [13] multiple inequalities.

3.1.1. Multiple Khintchine's Inequality. If $0 < p < \infty$, then

$$\begin{aligned} & A_p^m \left(\sum_{i_1, \dots, i_m=1}^{n_1, \dots, n_m} |a_{i_1, \dots, i_m}|^2 \right)^{1/2} \\ & \leq \left(\int_0^1 \cdots \int_0^1 |r_{i_1}(t_1) \cdots r_{i_m}(t_m) a_{i_1, \dots, i_m}|^p dt_1 \cdots dt_m \right)^{1/p} \\ & \leq B_p^m \left(\sum_{i_1, \dots, i_m=1}^{n_1, \dots, n_m} |a_{i_1, \dots, i_m}|^2 \right)^{1/2}, \end{aligned} \quad (45)$$

for each choice of the scalars $(a_{i_1, \dots, i_m})_{i_1, \dots, i_m=1}^{n_1, \dots, n_m}$, A_p and B_p are the Khintchine constants, and by $(r_n)_{n \in \mathbb{N}}$ we denote the sequence of Rademacher function such that $r_n: [0, 1] \rightarrow \mathbb{R}$ as $r_n(t) = \sin(2^n \pi t)$.

3.1.2. Multiple Kahane's Inequality. Let $1 \leq p < \infty$. Let $\text{Rad}_p(\mu, X)$ be the vector space of all almost unconditionally summable sequences (x_n) in the Banach space X . It is a Banach space under the norm:

$$\|(x_n)\|_p = \left(\int_0^1 \left\| \sum_{n=1}^{\infty} r_n(t) x_n \right\|^p d\mu(t) \right)^{1/p}. \quad (46)$$

Let $1 \leq p, q < \infty$. Let X be a Banach space and $(x_{i_1, \dots, i_n})$ be in X . We have

$$\begin{aligned} & \left\| \sum_{i_1, \dots, i_n} x_{i_1, \dots, i_n} \otimes_{j=1}^n r_{i_j} \right\|_{L_p(\otimes_{1 \leq i \leq n} \mu_i, X)} \\ & \leq K_{p,q}^n \left\| \sum_{i_1, \dots, i_n} x_{i_1, \dots, i_n} \otimes_{j=1}^n r_{i_j} \right\|_{L_q(\otimes_{1 \leq i \leq n} \mu_i, X)}, \end{aligned} \quad (47)$$

where $K_{p,q}$ is the simple constant of Kahane's inequality ($\mu_i = \mu$).

Proof. Let $T \in \mathcal{L}_d^{p+}(\ell_2^{n_1}, \dots, \ell_2^{n_m}; F)$; then, by the domination theorem for positive p -dominated m -linear operators, there are probability measures μ_j on $B_{\ell_2^{n_j}}^+$, such that, for any $(x^1, \dots, x^m) \in \ell_2^{n_1} \times \dots \times \ell_2^{n_m}$, we have $\ell_2^{n_j}$

$$\|T(x^1, \dots, x^m)\| \leq \pi_p^{d+}(T) \prod_{j=1}^m \left(\int_{B_{\ell_2^{n_j}}^+} \langle x^*, x^j \rangle^p d\mu_j(x^*) \right)^{1/p}. \quad (48)$$

For every $(x_i^j)_{i=1}^n \subset \ell_2^{n_j}$, for $j = 1, \dots, m$ and $(y_i^*)_{i=1}^n \subset F^*$, we have

$$(x_i^1, \dots, x_i^m) = \left(\sum_{k_1=1}^{n_1} x_{i,k_1}^1 e_{k_1}, \dots, \sum_{k_m=1}^{n_m} x_{i,k_m}^m e_{k_m} \right). \quad (49)$$

Then,

$$\sum_{i=1}^n |\langle T(x_i^1, \dots, x_i^m), y_i^* \rangle| = \sum_{i=1}^n \left| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} x_{i,k_1}^1 \cdots x_{i,k_m}^m \langle T(e_{k_1}, \dots, e_{k_m}), y_i^* \rangle \right|. \quad (50)$$

Using Hölder's inequality, we obtain

$$\begin{aligned} & \sum_{i=1}^n |\langle T(x_i^1, \dots, x_i^m), y_i^* \rangle| \\ & \leq \sum_{i=1}^n \left[\left(\sum_{k_1=1}^{n_1} |x_{i,k_1}^1|^2 \right)^{1/2} \cdots \left(\sum_{k_m=1}^{n_m} |x_{i,k_m}^m|^2 \right)^{1/2} \right. \\ & \quad \left. \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} \left(|\langle T(e_{k_1}, \dots, e_{k_m}), y_i^* \rangle|^2 \right)^{1/2} \right] \\ & \leq \sum_{i=1}^n \left(\prod_{j=1}^m \|x_i^j\|_{\ell_2^{n_j}} \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} \left(|\langle T(e_{k_1}, \dots, e_{k_m}), y_i^* \rangle|^2 \right)^{1/2} \right). \end{aligned} \quad (51)$$

By the multiple Khintchine inequality, on the last inequality, we obtain

$$\begin{aligned} & \sum_{i=1}^n |\langle T(x_i^1, \dots, x_i^m), y_i^* \rangle| \\ & \leq \sum_{i=1}^n \left(\prod_{j=1}^m \|x_i^j\|_{\ell_2^{n_j}} A_{q^*}^m \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1} \cdots r_{k_m} \langle T(e_{k_1}, \dots, e_{k_m}), y_i^* \rangle \right\|_{L_{q^*}([0,1])} \right) \\ & \leq A_{q^*}^m \left(\sum_{i=1}^n \prod_{j=1}^m \|x_i^j\|^q \right)^{1/q} \left(\sum_{i=1}^n \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1} \cdots r_{k_m} \langle T(e_{k_1}, \dots, e_{k_m}), y_i^* \rangle \right\|_{L_{q^*}([0,1])}^{q^*} \right)^{1/q^*}. \end{aligned} \quad (52)$$

By Fubini's theorem

$$\begin{aligned} & \sum_{i=1}^n |\langle T(x_i^1, \dots, x_i^m), y_i^* \rangle| \\ & \leq A_{q^*}^m \left(\sum_{i=1}^n \prod_{j=1}^m \|x_i^j\|^q \right)^{1/q} \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1} \cdots r_{k_m} \langle T(e_{k_1}, \dots, e_{k_m}), y_i^* \rangle \right\|_{L_{q^*}([0,1], \ell_{q^*}^m)} \\ & \leq A_{q^*}^m \left(\sum_{i=1}^n \prod_{j=1}^m \|x_i^j\|^q \right)^{1/q} \|y_i^*\|_{\ell_{q^*, \text{weak}}^{n_i}(F^*)} \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1} \cdots r_{k_m} T(e_{k_1}, \dots, e_{k_m}) \right\|_{L_{q^*}([0,1])}. \end{aligned} \quad (53)$$

For the quantity,

$$\left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1} \cdots r_{k_m} T(e_{k_1}, \dots, e_{k_m}) \right\|_{L_{q^*}([0,1], F)}, \quad (54)$$

using the multiple Kahane's inequality

$$\begin{aligned} & \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1}(t_1) \cdots r_{k_m}(t_m) T(e_{k_1}, \dots, e_{k_m}) \right\|_{L_{q^*}([0,1], F)} \\ & \leq K_{p, q^*}^m \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1}(t_1) \cdots r_{k_m}(t_m) T(e_{k_1}, \dots, e_{k_m}) \right\|_{L_p([0,1], F)} \\ & \leq K_{p, q^*}^m \left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} T(r_{k_1}(t_1)e_{k_1}, \dots, r_{k_m}(t_m)e_{k_m}) \right\|_{L_p([0,1], F)} \\ & \leq \pi_p^{d^+}(T) \left(\int_0^1 \prod_{j=1}^m \left(\int_{B_{\ell_2^{n_j}}^+} \left| \sum_{k_j=1}^{n_j} x^*(e_{k_j}) r_{k_j}(t_j) \right|^p d\mu_j(x^*) dt_j \right)^{1/p} \right) \\ & \leq \pi_p^{d^+}(T) \left(\prod_{j=1}^m \left(\int_0^1 \int_{B_{\ell_2^{n_j}}^+} \left| \sum_{k_j=1}^{n_j} x^*(e_{k_j}) r_{k_j}(t_j) \right|^p d\mu_j(x^*) dt_j \right)^{1/p} \right). \end{aligned} \quad (55)$$

Applying Khintchine's inequality,

$$\begin{aligned} & \leq B_p^m \pi_p^{d^+}(T) \prod_{j=1}^m \left(\int_{B_{\ell_2^{n_j}}^+} \left(\sum_{k_j=1}^{n_j} |x^*(e_{k_j})|^2 \right)^{p/2} d\mu_j(x^*) \right)^{1/p} \\ & \leq B_p^m \pi_p^{d^+}(T) \prod_{j=1}^m \left(\int_{B_{\ell_2^{n_j}}^+} \|x^*\|_{\ell_2^{n_j}}^p d\mu_j(x^*) \right)^{1/p} \\ & \leq B_p^m \pi_p^{d^+}(T). \end{aligned} \quad (56)$$

Consequently,

Exploiting (53) and (57) gives

$$\left\| \sum_{k_1, \dots, k_m=1}^{n_1, \dots, n_m} r_{k_1} \cdots r_{k_m} T(e_{k_1}, \dots, e_{k_m}) \right\|_{L_{q^*}([0,1], F)} \leq K_{p, q^*}^m B_p^m \pi_p^{d^+}(T). \quad (57)$$

$$\sum_{i=1}^n |\langle T(x_i^1, \dots, x_i^m), y_i^* \rangle| \leq (A_{q^*} K_{p, q^*} B_p)^m \pi_p^{d^+}(T) \left(\sum_{i=1}^n \prod_{j=1}^m \|x_i^j\|^q \right)^{1/q} \| (y_i^*)_{i=1}^n \|_{\ell_{q^*, |\text{weak}|}^m(F^*)}. \quad (58)$$

Thus, $T \in \mathcal{D}_q^{m+}(\ell_2^{n_1}, \dots, \ell_2^{n_m}; F)$ and $d_q^{m+}(T) \leq (A_{q^*} K_{p,q^*} B_p)^m \pi_p^{d+}(T)$.

And, this completes the proof. $\square$

3.2. Positive p -Dominated m -Homogeneous Polynomials

Definition 6. Let $1 \leq p < \infty$; an m -homogeneous polynomial $P: E \rightarrow Y$ is positive p -dominated if $\exists C > 0$ such that, for any $n \in \mathbb{N}^*$ and any $(x_i)_{i=1}^n \subset E^+$,

$$\left(\sum_{i=1}^n \|P(x_i)\|^{p/m} \right)^{m/p} \leq C \sup_{x^* \in B_{E^*}^+} \left(\sum_{i=1}^n \langle x^*, x_i \rangle^p \right)^{m/p}. \quad (59)$$

We denote $\mathcal{P}_{d,p}^+({}^m E; Y)$ the space of positive p -dominated polynomials from E into Y , and by $\delta_p^+(\cdot)$, the norm defined by the infimum of any constants verifying the above inequality, for $p \geq m$, but for $p < m$, it is only a quasi-norm.

Theorem 6. (domination theorem for positive p -dominated m -homogeneous polynomials). Let $1 \leq p < \infty$; an m -homogeneous polynomial $P: E \rightarrow Y$ is positive p -dominated if there are $K > 0$ and a probability measure μ on $B_{E^*}^+$ such that

$$\|P(x)\| \leq K \left(\int_{B_{E^*}^+} \langle x^*, x \rangle^p d\mu(x^*) \right)^{m/p}, \quad (60)$$

$\forall x \in E^+$. Moreover, the smallest K is $\delta_p^+(P)$.

In Proposition 7, we study the relationship between positive p -dominated m -linear operators and positive p -dominated m -homogeneous polynomials

Proposition 7. Let $P: E \rightarrow Y$ be an m -homogeneous polynomial and $\hat{P}$ be its corresponding symmetric m -linear operator such that $P(x) = \hat{P}(x, \dots, x)$.

(1) If $\hat{P}$ is positive p -dominated operator, then P is p -dominated and

$$\delta_p^+(P) \leq \pi_p^{d+}(\hat{P}). \quad (61)$$

(2) If P is positive p -dominated, then $\hat{P}$ is positive p -dominated and

$$\begin{aligned} (a) \text{ For } p \geq m, \pi_p^{d+}(\hat{P}) &\leq (m^m/m!) \delta_p^+(P) \\ (b) \text{ For } p < m, \pi_p^{d+}(\hat{P}) &\leq (m^m/m!) 2^{((m^2/p)-m)} \delta_p^+(P) \end{aligned}$$

Proof. (1) Suppose that $\hat{P}$ is positive p -dominated; let $(x_i)_{i=1}^n \subset E^+$; then,

$$\begin{aligned} \left(\sum_{i=1}^n \|P(x_i)\|^{p/m} \right)^{m/p} &= \left(\sum_{i=1}^n \|\hat{P}(x_i, \dots, x_i)\|^{p/m} \right)^{m/p} \\ &\leq \pi_p^{d+}(\hat{P}) \|(x_i)_{i=1}^n\|_{\ell_{p, \text{weak}}^n(E)}^m. \end{aligned} \quad (62)$$

Therefore, P is positive p -dominated and $\delta_p^+(P) \leq \pi_p^{d+}(\hat{P})$.

(2) For $1 \leq j \leq m$ and $n \in \mathbb{N}$, let $x^j \in E_j$ satisfy $\|x^j\| \leq 1$:

$$\left(\|\hat{P}(x^1, \dots, x^m)\|^{p/m} \right)^{m/p} = \left(\left\| \frac{1}{m! 2^m} \sum_{\epsilon_j = \pm 1} \epsilon_1 \dots \epsilon_m P \left(\sum_{j=1}^m \epsilon_j x^j \right) \right\|^{p/m} \right)^{m/p} \leq \frac{1}{m! 2^m} \left(\sum_{\epsilon_j = \pm 1} \left\| P \left(\sum_{j=1}^m \epsilon_j x^j \right) \right\|^{p/m} \right)^{m/p}. \quad (63)$$

By Theorem 6,

$$\begin{aligned} &\leq \frac{1}{m! 2^m} \delta_p^+(P) \sum_{\epsilon_j = \pm 1} \left(\int_{B_{E^*}^+} \langle x^*, \sum_{j=1}^m \epsilon_j x^j \rangle^p d\mu(x^*) \right)^{m/p} \\ &\leq \frac{1}{m! 2^m} \delta_p^+(P) \sum_{\epsilon_j = \pm 1} \left(\left\| \sum_{j=1}^m \epsilon_j x^j \right\|^p \int_{B_{E^*}^+} \langle x^*, \frac{\sum_{j=1}^m \epsilon_j x^j}{\left\| \sum_{j=1}^m \epsilon_j x^j \right\|} \rangle^p d\mu(x^*) \right)^{m/p} \\ &\leq \frac{1}{m! 2^m} \delta_p^+(P) \sum_{\epsilon_j = \pm 1} \left(\sum_{j=1}^m \|x^j\| \right)^m \left(\int_{B_{E^*}^+} \langle x^*, y \rangle^p d\mu(x^*) \right)^{m/p} \\ &\leq \frac{1}{m! 2^m} \delta_p^+(P) 2^m m^m \left(\int_{B_{E^*}^+} \langle x^*, y \rangle^p d\mu(x^*) \right)^{m/p} \\ &\leq \frac{m^m}{m!} \delta_p^+(P) \left(\int_{B_{E^*}^+} \langle x^*, y \rangle^p d\mu(x^*) \right)^{m/p}. \end{aligned} \quad (64)$$

For $p < m$, $1 \leq j \leq m$, and $n \in \mathbb{N}$, let $x_1^j, \dots, x_n^j \in E_j$ satisfy $\|(x_i^j)_{i=1}^n\|_{\ell_{p,\text{weak}}^n}^n = 1$:

$$\begin{aligned}
 \left(\sum_{i=1}^n \|\widehat{P}(x_i^1, \dots, x_i^m)\|^{p/m} \right)^{m/p} &\leq \frac{1}{m! 2^m} \left(\sum_{i=1}^n \sum_{\epsilon_j = \pm 1} \|P(\epsilon_1 x_i^1, \dots, \epsilon_m x_i^m)\|^{p/m} \right)^{m/p} \\
 &\leq \frac{1}{m! 2^m} \delta_p^+(P) \left(\sum_{\epsilon_j = \pm 1} \left\| (\epsilon_1 x_i^1, \dots, \epsilon_m x_i^m)_{i=1}^n \right\|_{\ell_{p,\text{weak}}^n}^p \right)^{m/p} \\
 &\leq \frac{1}{m! 2^m} \delta_p^+(P) m^m \left(\sum_{\epsilon_j = \pm 1} 1 \right)^{m/p} \\
 &\leq \frac{m^m}{m!} 2^{(m^2/p) - m} \delta_p^+(P).
 \end{aligned} \tag{65}$$

□

In Corollary 2, we give an inclusion between the two classes of polynomials defined earlier.

Corollary 2. Let $1 < p, q < \infty$, and ℓ_2^n Banach lattice space; then,

$$\begin{aligned}
 \mathcal{P}_{d,p}^+(^m \ell_2^n; F) &\subseteq \mathcal{P}_{\text{Coh},q}^+(^m \ell_2^n; F), \\
 d_q^+(P) &\leq \delta_p^+(P).
 \end{aligned} \tag{66}$$

Proof. Let $P \in \mathcal{P}_{d,p}^+(^m \ell_2^n; F)$; from Proposition 7, the associate symmetric of P denoted $\widehat{P}$ belongs to $\mathcal{L}_d^{p+}(^m \ell_2^n; F)$, and by Theorem 5, $\widehat{P}$ is in $\mathcal{D}_q^{m+}(^m \ell_2^n; F)$, so Proposition 5 finishes the proof. □

4. Applications

Let K be a compact Hausdorff space and $(K, \mathcal{A}, \mu)$ a finite measure space.

As usual $C(K)$ denotes the space of continuous functions on K , and $\mathcal{M}(K)$ denotes the Banach space of all regular Borel measures on K , and we have $\mathcal{M}(K) \cong (C(K))^*$.

We shall write $L^p(\mu, Y)$ for the space of measurable functions on K with

$$\|f\|_p = \left(\int_K \|f(t)\|^p d\mu \right)^{1/p}, \quad \text{for } 1 \leq p < +\infty, \tag{67}$$

and for $p = +\infty$,

$$\|f\|_\infty = \inf \{M \geq 0 : \|f(t)\| \leq M \quad \mu \text{ almost every where}\}. \tag{68}$$

Let $1 \leq p < \infty$. An operator $u: E \rightarrow X$ is said to be p -concave if there exists a constant $C > 0$ such that, for all $n \in \mathbb{N}$, $x_1, \dots, x_n \in E$, the inequality

$$\|u(x_i)_{i=1}^n\|_p \leq C \left\| \left(\sum_{i=1}^n |x_i|^p \right)^{1/p} \right\| \tag{69}$$

holds.

Let $\mathcal{C}_p(E; X)$ be the space of p -concave operators from E into X . $\mathcal{C}_p(E, X)$ becomes a Banach space with norm defined by the infimum of the constant $C > 0$ that satisfy inequality (69).

The following results are a conclusion of the work done by Blasco in [8] and Proposition 3.

Proposition 8. Let $1 < p < \infty$:

- (1) $P \in \mathcal{P}_{\text{coh},p}^+(^m X, L^\infty(\mu))$ if and only if $P^* \in \mathcal{L}(L^1(\mu), tPn(^m X))$
- (2) $P \in \mathcal{P}_{\text{coh},p}^+(^m X, \mathcal{M}(K))$ if and only if $P^* \in \mathcal{C}_p(C(K), tPn(^m X))$
- (3) $P \in \mathcal{P}_{\text{coh},p}^+(^m X, L^p(\mu))$ if and only if $P^* \in L^{p^*}(\mu, \mathcal{P}(^m X))$
- (4) $P \in \mathcal{P}_{\text{coh},p}^+(^m X, \ell^{p^*})$ if and only if $P^* \in \ell^{p^*}(P(^m X))$

5. Tensorial Approach

This section is intended to present a tensorial perspective of Cohen positive strongly p -summing and positive p -dominated polynomials and m -linear operators. For doing this, we apply the standard technique of associate a linear functional on a tensor product to each multilinear operator or polynomial. In the case of Cohen positive strongly p -summing operators, we pay more attention on the details for polynomials than m -linear operators. Later, we pay more attention on positive p -dominated m -linear operators than polynomials. Let m be a nonnegative integer, X be a Banach space, and F be a Banach lattice, each m -homogeneous polynomial $P: X \rightarrow F$ defines a functional given by

$$\begin{aligned}\phi_P: (\otimes_s^m X) \otimes F^* &\longrightarrow \mathbb{R}, \\ x \otimes \cdots \otimes x \otimes y^* &\longrightarrow y^*(P(x)).\end{aligned}\quad (70)$$

The next step in our development is to define adequate norm α on the tensor space $(\otimes_s^m X) \otimes F^*$ in order that P be Cohen positive strongly p -summing (positive p -dominated) exactly when ϕ_P is bounded on $(\otimes_s^m X \otimes F^*, \alpha)$.

5.1. Cohen Positive Strongly p -Summing Polynomials from a Tensor Viewpoint. Before introducing the respecting tensor norm, let us state an equivalence of being a Cohen positive strongly p -summing polynomial. This is an immediate consequence of the duality between ℓ_∞^n and ℓ_1^n . The m -homogeneous polynomial $P: X \longrightarrow F$ is Cohen positive strongly p -summing if and only if $\exists C > 0$ such that

$$\left| \sum_{i=1}^n \lambda_i y_i^*(P(x_i)) \right| \leq \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} \|(\gamma_i^*)_{i=1}^n\|_{\ell_{p^*, |\text{weak}|}^n(F^*)}, \quad (71)$$

for all finite sequences $(\lambda_i)_{i=1}^n \subset \mathbb{R}$, $(x_i)_{i=1}^n \subset X$, and $(\gamma_i^*)_{i=1}^n \subset F^*$. In this case, $d_p^+(P)$ agrees with the sharpest constant C so that the inequality holds.

Definition 7. For each z in $(\otimes_s^m X) \otimes F$, we define

$$\Lambda_p^+(z) = \inf \left\{ \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} \|(\gamma_i^*)_{i=1}^n\|_{\ell_{p^*, |\text{weak}|}^n(F)} \right\}, \quad (72)$$

where the infimum is taken over any representations $z = \sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i$. For each z in $(\otimes_s^m X) \otimes F$, we define

$$\Lambda_p^+(z) = \inf \left\{ \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} w_{p^*}((\gamma_i^*)_{i=1}^n) \right\}, \quad (73)$$

where the infimum is taken over all representations $z = \sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i$ with $y_i \geq 0$, for all $1 \leq i \leq n$.

The next proposition is a consequence of the fact that every $y \in F$ can be represented as $y = y^+ - y^-$. Moreover, it helps to reduce the calculations related to λ_p^+ and Λ_p^+ .

Proposition 9. For every z in $(\otimes_s^m X) \otimes F$, we have

$$\lambda_p^+(z) \leq \Lambda_p^+(z) \leq 2\lambda_p^+(z). \quad (74)$$

Proof. Let z in $(\otimes_s^m X) \otimes F$. It is clear that $\lambda_p^+(z) \leq \Lambda_p^+(z)$ by (4). For the opposite inequality, we take $z = \sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i$. Then,

$$z = \sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i^+ + \sum_{i=1}^n (-\lambda_i) x_i \otimes \cdots \otimes x_i \otimes y_i^-. \quad (75)$$

Thus, we obtain a new representation for z such that

$$\begin{aligned}\Lambda_p^+(z) &\leq 2^{1/p} \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} \sup_{w^* \in B_{F^*}^{p^*}} \left(\sum_{i=1}^n w^*(y_i^+)^{p^*} + w^*(y_i^-)^{p^*} \right)^{(1/p^*)} \\ &\leq 2^{1/p} \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} \sup_{w^* \in B_{F^*}^{p^*}} \left(\sum_{i=1}^n 2w^*(|y_i|)^{p^*} \right)^{(1/p^*)} \\ &\leq 2^{(1/p)+(1/p^*)} \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} \sup_{w^* \in B_{F^*}^{p^*}} \left(\sum_{i=1}^n w^*(|y_i|)^{p^*} \right)^{(1/p^*)} \\ &\leq 2 \|(\lambda_i)_{i=1}^n\|_{\ell_\infty^n} \|(\|x_i\|)_{i=1}^n\|_{\ell_p^n} \|(\gamma_i^*)_{i=1}^n\|_{\ell_{p^*, |\text{weak}|}^n}.\end{aligned}\quad (76)$$

Therefore, $\Lambda_p^+(z) \leq 2\lambda_p^+(z)$, for all z . $\square$

Proposition 10. *The application $z \longrightarrow \lambda_p^+(z)$ is a norm on $(\widehat{\otimes}_{\pi,s}^m X) \otimes F$.*

Proof. Standard tensorial techniques can be applied to prove that λ_p^+ is nonnegative, homogeneous and verify the

triangular inequality. So, we only prove that $\lambda_p^+(z) = 0$ implies $z = 0$. Let ξ be a bounded linear functional on $\widehat{\otimes}_{\pi,s}^m X$ and $y^* \geq 0$ bounded on F^* . Then,

$$\begin{aligned} |\xi \otimes y^*(z)| &= \left| \sum_{i=1}^n \lambda_i \xi(x_i \otimes \cdots \otimes x_i) y^*(y_i) \right| \\ &\leq \|(\lambda_i)_{i=1}^n\|_\infty \sum_{i=1}^n |\xi(x_i \otimes \cdots \otimes x_i) y^*(y_i)| \\ &\leq \|(\lambda_i)_{i=1}^n\|_\infty \left(\sum_{i=1}^n |\xi(x_i \otimes \cdots \otimes x_i)|^p \right)^{1/p} \left(\sum_{i=1}^n |y^*(y_i)|^{p^*} \right)^{1/p^*} \\ &\leq \|\xi\| \|(\lambda_i)_{i=1}^n\|_\infty \left(\sum_{i=1}^n \|x_i\|^{mp} \right)^{1/p} \left(\sum_{i=1}^n |y^*(y_i)|^{p^*} \right)^{1/p^*} \\ &\leq \|\xi\| \|y^*\| \|(\lambda_i)_{i=1}^n\|_\infty \|(\|x_i\|^m)_{i=1}^n\|_{\ell_p^n} w_{p^*}((y_i)_{i=1}^n). \end{aligned} \quad (77)$$

Hence, $\lambda_p^+(z) = 0$ implies $\xi \otimes y^*(z) = 0$, for all ξ and $y^* \geq 0$. For arbitrary y^* , we have that $y^* = (y^*)^+ - (y^*)^-$. Then, $\lambda_p^+(z) = 0$ implies

$$\xi \otimes y^*(z) = \xi \otimes (y^*)^+(z) - \xi \otimes (y^*)^-(z) = 0. \quad (78)$$

Therefore, z must be zero since the set of linear functionals of the form $\xi \otimes y^*$ is a separating set for $(\widehat{\otimes}_{\pi,s}^m X) \otimes F^*$. $\square$

Next, we give the characterization of Cohen positive strongly p -summing polynomials in tensor terms.

Proposition 11 (duality). *Let $P: X \longrightarrow F$ be an m -homogeneous polynomial. The following are equivalent:*

- (i) P is Cohen positive strongly p -summing
- (ii) ϕ_P is bounded on $((\widehat{\otimes}_s^m X) \otimes F^*, \lambda_p^+)$.

Under this circumstances, $d_p^+(P) = \|\phi_P\|$.

Proof. Let $z = \sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i^*$ be any element of $(\widehat{\otimes}_s^m X) \otimes F^*$; then,

$$|\phi_P(z)| = \left| \sum_{i=1}^n \lambda_i y_i^*(P(x_i)) \right| \leq d_p^+(P) \|(\lambda_i)_{i=1}^n\|_\infty \|(\|x_i\|^m)_{i=1}^n\|_{\ell_p^n} \| (y_i^*)_{i=1}^n \|_{\ell_{p,|\text{weak}|}^n(F^*)}. \quad (79)$$

After taking the infimum over all the representations of z , we have that

$$|\phi_P(z)| \leq d_p^+(P) \lambda_p^+(z), \quad (80)$$

Holds, for all z .

Conversely, take sequences $(\lambda_i)_{i=1}^n$ in $\mathbb{R}$, $(x_i)_{i=1}^n$ in X , and $(y_i^*)_{i=1}^n$ in F^* . Hence,

$$\begin{aligned} \left| \sum_{i=1}^n \lambda_i y_i^*(P(x_i)) \right| &= |\phi_P(z)| \leq \|\phi_P\| \lambda_p^+ \left(\sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i^* \right) \\ &\leq \|\phi_P\| \|(\lambda_i)_{i=1}^n\|_\infty \|(\|x_i\|^m)_{i=1}^n\|_{\ell_p^n} \| (y_i^*)_{i=1}^n \|_{\ell_{p,|\text{weak}|}^n(F^*)}. \end{aligned} \quad (81)$$

$\square$

As we saw, the norms λ_p^+ and Λ_p^+ are equivalent, and one of them is more useful than the other depending on the

circumstances we are interested on. The norm λ_p^+ provides an isometry between polynomials and linear functionals, while Λ_p^+ reduces calculations in Proposition 12.

Proposition 12 (uniform property). *Let $R: X \longrightarrow Y$ be a bounded linear operator and $S: F \longrightarrow E$ be a positive operator. Then, the operator defined by*

$$R \otimes S: ((\otimes_s^m X) \otimes F, \Lambda_p^+) \longrightarrow ((\otimes_s^m Y) \otimes E, \Lambda_p^+), \quad (82)$$

$$x \otimes \cdots \otimes x \otimes y \mapsto R(x) \otimes \cdots \otimes R(x) \otimes S(y),$$

is bounded and $\|R \otimes S\| \leq \|R\|^m \|S\|$.

Proof. Let $z = \sum_{i=1}^n \lambda_i x_i \otimes \cdots \otimes x_i \otimes y_i$ with $y_i \geq 0$. Then,

$$\begin{aligned} \Lambda_p^+(R \otimes S(z)) &\leq \|(\lambda_i)_i\|_\infty^n \left\| \left(\|R x_i \otimes R x_i\|_{i=1}^n \right)_{i=1}^n \right\|_{\ell_p^n} w_{p^*}(S(y_i)) \\ &\leq \|R\|^m \|S\| \|(\lambda_i)_i\|_\infty^n \left\| \left(\|x_i\|^m \right)_{i=1}^n \right\|_{\ell_p^n} w_{p^*}((y_i)_{i=1}^n). \end{aligned} \quad (83)$$

After taking the infimum over all the representations of z , we obtain $\Lambda_p^+(R \otimes S(z)) \leq \|R\|^m \|S\| \Lambda_p^+(z)$. $\square$

As it is expected, previous proposition is also true if Λ_p^+ is replaced by λ_p^+ , but this change involves a factor of 2 in the inequality.

5.2. Cohen Positive Strongly p -Summing m -Linear Operators from a Tensor Viewpoint. Bearing in mind the previous section, we are able to develop analogous ideas to

characterize Cohen positive strongly p -summing m -linear operators. As before, throughout this section, $X_1, \dots, X_m$ are Banach spaces and F is a Banach lattice. Next, recall that each m -linear operator $T: X_1 \times \cdots \times X_m \longrightarrow F$ defines a linear functional by

$$\begin{aligned} \phi_T: X_1 \otimes \cdots \otimes X_m \otimes F^* &\longrightarrow \mathbb{R}, \\ x^1 \otimes \cdots \otimes x^m \otimes y^* &\mapsto y^*(T(x^1, \dots, x^m)). \end{aligned} \quad (84)$$

In this case, for each z in $X_1 \otimes \cdots \otimes X_m \otimes F$, we define

$$\lambda_p^{m+}(z) = \inf \left\{ \|(\lambda_i)_i\|_\infty^n \left\| \left(\|x_i^1\| \cdots \|x_i^m\| \right)_{i=1}^n \right\|_{\ell_p^n} \| (y_i)_{i=1}^n \|_{\ell_{p^*}^{[weak]}(F)} \right\}, \quad (85)$$

where the infimum is taken over all representations, $z = \sum_{i=1}^n \lambda_i x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i$.

Using usual tensor techniques and slight modifications of the proof of Proposition 10, let us prove that the assignment $z \mapsto \lambda_p^{m+}(z)$ is a norm on $X_1 \otimes \cdots \otimes X_m \otimes F$. The respective duality result is exhibited in the next proposition and requires an analogous expression to (71).

Proposition 13 (duality). *Let $T: X_1 \times \cdots \times X_m \longrightarrow F$ be an m -linear operator. Then, the following are equivalents:*

- (i) T is Cohen positive strongly p -summing
- (ii) ϕ_T is bounded on $(X_1 \otimes \cdots \otimes X_m \otimes F^*, \lambda_p^{m+})$

Under this circumstances, $d_p^{m+}(T) = \|\phi_T\|$.

Proof. For any $z = \sum_{i=1}^n \lambda_i x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i^*$ in $X_1 \otimes \cdots \otimes X_m \otimes F^*$, we have

$$\begin{aligned} |\phi_T(z)| &= \left| \sum_{i=1}^n \lambda_i y_i^*(T(x_i^1, \dots, x_i^m)) \right| \\ &\leq d_p^{m+}(T) \|(\lambda_i)_i\|_\infty^n \left\| \left(\|x_i^1\| \cdots \|x_i^m\| \right)_{i=1}^n \right\|_{\ell_p^n} \| (y_i^*)_{i=1}^n \|_{\ell_{p^*}^{[weak]}(F^*)}. \end{aligned} \quad (86)$$

Hence, for any z , we have

$$|\phi_T(z)| \leq d_p^{m+}(T) \lambda_p^{m+}(z). \quad (87)$$

On the other direction, let $(\lambda_i)_{i=1}^n$ in $\mathbb{R}$, $(x_i^j)_{i=1}^n$ in X^j , and $(y_i^*)_{i=1}^n$ in F^* . Then,

$$\begin{aligned} \left| \sum_{i=1}^n \lambda_i y_i^*(T(x_i^1, \dots, x_i^m)) \right| &= |\phi_T(z)| \leq \|\phi_T\| \lambda_p^{m+} \left(\sum_{i=1}^n \lambda_i x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i^* \right) \\ &\leq \|\phi_T\| \|(\lambda_i)_i\|_\infty^n \left\| \left(\|x_i^1\| \cdots \|x_i^m\| \right)_{i=1}^n \right\|_{\ell_p^n} \| (y_i^*)_{i=1}^n \|_{\ell_{p^*}^{[weak]}(F^*)}. \end{aligned} \quad (88)$$

$\square$

For each z , in $X_1 \otimes \cdots \otimes X_m \otimes F$, we define

$$\Lambda_p^{m+}(z) = \inf \left\{ \left\| (\lambda_i)_{i=1}^n \right\|_{\infty} \left\| (x_i^1 \otimes \cdots \otimes x_i^m)_{i=1}^n \right\|_{\ell_p^n} w_p((y_i)_{i=1}^n) \right\}, \quad (89)$$

where the infimum is selected over any representation $z = \sum_{i=1}^n \lambda_i x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i$ such that $y_i \geq 0$, for all $1 \leq i \leq n$. Then, for any z in $X_1 \otimes \cdots \otimes X_m \otimes F$, we have

$$\lambda_p^{m+}(z) \leq \Lambda_p^{m+}(z) \leq 2\lambda_p^{m+}(z). \quad (90)$$

Having Λ_p^{m+} at our reach and knowing it defines a norm, we present the respective uniform property it verifies.

Proposition 14 (uniform property). *Let $R_i: X_i \rightarrow Y_i$ be bounded linear operators and $S: F \rightarrow G$ be positive operator between Banach lattices. Then, the operator,*

$$\begin{aligned} R_1 \otimes \cdots \otimes R_m \otimes S: (X_1 \otimes \cdots \otimes X_m \otimes F, \Lambda_p^{m+}) &\rightarrow (Y_1 \otimes \cdots \otimes Y_m \otimes G, \Lambda_p^{m+}), \\ x^1 \otimes \cdots \otimes x^m \otimes y &\mapsto R_1(x^1) \otimes \cdots \otimes R_m(x^m) \otimes S(y), \end{aligned} \quad (91)$$

is bounded and $\|R_1 \otimes \cdots \otimes R_m \otimes S\| \leq \|R_1\| \cdots \|R_m\| \|S\|$.

Proof. Given $z = \sum_{i=1}^n \lambda_i x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i$ with $y_i \geq 0$, we have

$$\begin{aligned} \Lambda_p^{m+}(R_1 \otimes \cdots \otimes R_m \otimes S(z)) &\leq \left\| (\lambda_i)_{i=1}^n \right\|_{\infty} \left\| (R_1 x_i^1 \otimes \cdots \otimes R_m x_i^m)_{i=1}^n \right\|_{\ell_p^n} w_p((y_i)_{i=1}^n) \\ &\leq \|R_1\| \cdots \|R_m\| \|S\| \left\| (\lambda_i)_{i=1}^n \right\|_{\infty} \left\| (x_i^1 \otimes \cdots \otimes x_i^m)_{i=1}^n \right\|_{\ell_p^n} w_p((y_i)_{i=1}^n). \end{aligned} \quad (92)$$

As a consequence, $\Lambda_p^{m+}(R_1 \otimes \cdots \otimes R_m \otimes S(z)) \leq \|R_1\| \cdots \|R_m\| \|S\| \Lambda_p^{m+}(z)$. $\square$

5.3. Positive p -Dominated m -Linear Operators and Polynomials Form a Tensor Viewpoint. As before, throughout this section, $E, E_1, \dots, E_m$ are Banach lattices and Y is a Banach space. In this case, we pay more attention on m -linear operators than polynomials.

Proposition 15. *For each z in $E_1 \otimes \cdots \otimes E_m \otimes Y$, we define*

$$\eta_p^{m+}(z) = \inf \left\{ \left(\prod_{j=1}^m \left\| (x_i^j)_{i=1}^n \right\|_{\ell_{p_j}^n} \right) \left\| (y_i)_{i=1}^n \right\|_{\ell_{p/(p-m)}^n} \right\}, \quad (93)$$

where the infimum is selected over any representations, $z = \sum_{i=1}^n x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i$.

For each z in $(\otimes_s^m E) \otimes Y$, we define

$$\eta_p^+(z) = \inf \left\{ w_p((x_i)_{i=1}^n)^m \left\| (y_i)_{i=1}^n \right\|_{\ell_{p/(p-m)}^n} \right\}, \quad (94)$$

where the infimum is taken over all representations $z = \sum_{i=1}^n x_i \otimes \cdots \otimes x_i \otimes y_i$ with $x_i \geq 0$, for all $1 \leq i \leq n$. Then, the assignment $z \mapsto \eta_p^+(z)$ is a norm on $(\otimes_s^m E) \otimes Y$.

The proof of this proposition is an adaptation of Proposition 10. The next result concerns the characterization of positive p -dominated m -linear operators and polynomial in tensor terms.

Proposition 16 (duality). *Let $T: E_1 \times \cdots \times E_m \rightarrow Y$ be an m -linear operator and $P: E \rightarrow Y$ be an m -homogeneous polynomial. Then,*

- (i) T is positive p -dominated if and only if ϕ_T is bounded on $(E_1 \otimes \cdots \otimes E_m \otimes Y^*, \eta_p^{m+})$
- (ii) P is positive p -dominated if and only if ϕ_P is bounded on $((\otimes_s^m E) \otimes Y^*, \eta_p^+)$

Under this circumstances, $\pi_p^{d+}(T) = \|\phi_T\|$ and $\delta_p^+(P) = \|\phi_P\|$.

Proof. We only prove (i) since (ii) is a straightforward adaptation. First, suppose T is positive p -dominated. Then, for any $z = \sum_{i=1}^n x_i^1 \otimes \cdots \otimes x_i^m \otimes y_i^*$, in $E_1 \otimes \cdots \otimes E_m \otimes Y^*$, we have

$$\begin{aligned} |\phi_T(z)| &\leq \sum_{i=1}^n \|y_i^*\| \|T(x_i^1, \dots, x_i^m)\| \\ &\leq \left(\sum_{i=1}^n \|y_i^*\|^{p/(p-m)} \right)^{(p-m)/p} \left(\sum_{i=1}^n \|T(x_i^1, \dots, x_i^m)\|^{p/m} \right)^{m/p} \\ &\leq \pi_p^{d+}(T) \left(\prod_{j=1}^m \left\| (x_i^j)_{i=1}^n \right\|_{\ell_{p_j}^n} \right) \left\| (y_i^*)_{i=1}^n \right\|_{\ell_{p/(p-m)}^n}. \end{aligned} \quad (95)$$

Hence, $|\phi_T(z)| \leq \pi_p^{d^+}(T) \eta_p^{m+}(z)$ which means that ϕ_T is bounded and $\|\phi_T\| \leq \pi_p^{d^+}(T)$, for the opposite direction. Let $(x_i^j)_{i=1}^n$ in E_j $1 \leq j \leq m$ and $\epsilon > 0$. Then, for each $1 \leq i \leq n$, there

exists y_i^* in Y_* such that $y_i^*(T(x_i^1, \dots, x_i^m)) = \|T(x_i^1, \dots, x_i^m)\|^{p/m}$ and $\|y_i^*\| \leq (1 + \epsilon)\|T(x_i^1, \dots, x_i^m)\|^{(p/m)-1}$. Therefore,

$$\begin{aligned} \sum_{i=1}^n \|T(x_i^1, \dots, x_i^m)\|^{p/m} &= \sum_{i=1}^n y_i^*(T(x_i^1, \dots, x_i^m)) \\ &= \phi_T \left(\sum_{i=1}^n x_i^1 \otimes \dots \otimes x_i^m \otimes y_i^* \right) \\ &\leq \|\phi_T\| \left(\prod_{j=1}^m \|(x_i^j)_{i=1}^n\|_{\ell_{p_j|\text{weak}|}^n(E_j)}^n \right) \|(y_i)_{i=1}^n\|_{\ell_{p/(p-m)}^n} \\ &\leq \|\phi_T\| (1 + \epsilon) \left(\prod_{j=1}^m \|(x_i^j)_{i=1}^n\|_{\ell_{p_j|\text{weak}|}^n(E_j)}^n \right) \left(\sum_{i=1}^n \|T(x_i^1, \dots, x_i^m)\|^{p/m} \right)^{(p-m)/p}. \end{aligned} \quad (96)$$

As a consequence, we obtain

$$\left(\sum_{i=1}^n \|T(x_i^1, \dots, x_i^m)\|^{p/m} \right)^{m/p} \leq \|\phi_T\| (1 + \epsilon) \prod_{j=1}^m \|(x_i^j)_{i=1}^n\|_{\ell_{p_j|\text{weak}|}^n(E_j)}, \quad (97)$$

which implies that T is positive p -dominated and $\pi_p^{d^+}(T) \leq \|\phi_T\|$. $\square$

The proof of next propositions is adaptations of those of Propositions 9 and 11.

Proposition 17. Define, for each z , in $E_1 \otimes \dots \otimes E_m \otimes Y$,

$$E_p^{m+} = \inf \left\{ \prod_{i=1}^n w_p((x_i^j)_{i=1}^n) \|(y_i)_{i=1}^n\|_{\ell_{p/(p-m)}^n} \right\}, \quad (98)$$

where the infimum is taken over all representation $z = \sum_{i=1}^n x_i^1 \otimes \dots \otimes x_i^m \otimes y_i$ with $x_i^j \geq 0$, for all $1 \leq i \leq n$ and $1 \leq j \leq m$. Then,

$$\eta_p^{m+}(z) \leq E_p^{m+}(z) \leq 2\eta_p^{m+}(z), \quad (99)$$

Holds, for all z , in $E_1 \otimes \dots \otimes E_m \otimes Y$.

Proposition 18 (uniform property).

Let $R_i: E_i \longrightarrow M_i$ be a positive operators and $S: Y \longrightarrow Z$ be a bounded linear operator. Then, the operator,

$$\begin{aligned} R_1 \otimes \dots \otimes R_m \otimes S: (E_1 \otimes \dots \otimes E_m \otimes Y, E_p^{m+}) &\longrightarrow (M_1 \otimes \dots \otimes M_m \otimes Z, E_p^{m+}), \\ x^1 \otimes \dots \otimes x^m \otimes y &\mapsto R_1(x^1) \otimes \dots \otimes R_m(x^m) \otimes S(y), \end{aligned} \quad (100)$$

is bounded and $\|R_1 \otimes \dots \otimes R_m \otimes S\| \leq \|R_1\| \dots \|R_m\| \|S\|$.

Let $R: E \longrightarrow M$ be a positive operator and $S: Y \longrightarrow Z$ be a bounded linear operator. Then, the operator defined by

$$\begin{aligned} R \otimes S: ((\otimes_s^m E) \otimes Y, \eta_p^+) &\longrightarrow ((\otimes_s^m M) \otimes Z, \eta_p^+), \\ x \otimes \dots \otimes x \otimes y &\mapsto R(x) \otimes \dots \otimes R(x) \otimes S(y), \end{aligned} \quad (101)$$

is bounded and $\|R \otimes S\| \leq \|R\|^m \|S\|$.

6. Conclusion

In this work, by giving the concepts of Cohen positive strongly p -summing and positive p -dominated m -homogeneous polynomials. We prove a version of Pietsch's domination theorem for the first class among other results and a Bu-type theorem, as well as some inclusions with other known spaces. Moreover, we present a characterization of these classes in tensor terms. In the future work, we will try to extend the same study with other higher polynomial order.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] J. S. Cohen, "Absolutely p -summing, p -nuclear operators and their conjugates," *Mathematische Annalen*, vol. 201, no. 3, pp. 177–200, 1973.
- [2] D. Achour and A. Belacel, "Domination and factorization theorems for positive strongly p -summing operators," *Positivity*, vol. 18, no. 4, pp. 785–804, 2014.
- [3] D. Achour and K. Saadi, "A polynomial characterization of Hilbert spaces," *Collectanea Mathematica*, vol. 61, no. 3, pp. 291–301, 2010.
- [4] A. Bougoutaia and A. Belacel, "Cohen positive strongly p -summing and p -convex multilinear operators," *Positivity*, vol. 23, no. 2, pp. 379–395, 2019.
- [5] J. R. Campos, "Cohen and multiple Cohen strongly summing multilinear operators," *Linear and Multilinear Algebra*, vol. 62, no. 3, pp. 322–346, 2014.
- [6] R. Cilia and J. M. Gutierrez, "Polynomial factorization through $L_p(\mu)$ spaces," *Journal of the Korean Mathematical Society*, vol. 46, no. 6, pp. 1293–1307, 2009.
- [7] Y. Meléndez and A. Tonge, "Polynomials and the Pietsch domination theorem," *Mathematical Proceedings of the Royal Irish Academy*, vol. 99A, no. 2, pp. 195–212, 1999.
- [8] O. Blasco, "Positive p -summing operators on L_p -spaces," *Proceedings of the American Mathematical Society*, vol. 100, no. 2, p. 275, 1987.
- [9] D. Pellegrino, J. Santos, and J. B. Seoane-Sepúlveda, "Some techniques on nonlinear analysis and applications," *Advances in Mathematics*, vol. 229, no. 2, pp. 1235–1265, 2012.
- [10] R. M. Aron and M. Schottenloher, "Compact holomorphic mappings on banach spaces and the approximation property," *Bulletin of the American Mathematical Society*, vol. 80, no. 6, pp. 1245–1250, 1974.
- [11] D. Popa, "Cohen-Kwapień type theorems for multiple summing operators," *Quaestiones Mathematicae*, vol. 36, no. 3, pp. 399–410, 2013.
- [12] Q. Bu, "Some mapping properties of p -summing operators with Hilbertian domain," in *Contemporary Mathematics*, K. Jarosz, Ed., vol. 328, pp. 145–149, American Mathematical Society, Providence, RI, USA, 2003.
- [13] D. Achour and L. Mezrag, "On the cohen strongly p -summing multilinear operators," *Journal of Mathematical Analysis and Applications*, vol. 327, pp. 550–563, 2007.
- [14] A. Defant and K. Floret, "Tensor norms and operator ideals," *Number 176 in North-Holland Mathematics Studies*, North-Holland, Amsterdam, Netherlands, 1993.