

Retraction

Retracted: Covering Fuzzy Rough Sets via Variable Precision

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] M. Atef and A. A. Azzam, "Covering Fuzzy Rough Sets via Variable Precision," *Journal of Mathematics*, vol. 2021, Article ID 5525766, 10 pages, 2021.

Research Article

Covering Fuzzy Rough Sets via Variable Precision

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Lately, covering fuzzy rough sets via variable precision according to a fuzzy γ -neighborhood were established by Zhan et al. model. Also, Ma et al. gave the definition of complementary fuzzy γ -neighborhood with reflexivity. In a related context, we used the concepts by Ma et al. to construct three new kinds of covering-based variable precision fuzzy rough sets. Furthermore, we establish the relevant characteristics. Also, we study the relationships between Zhan's model and our three models. Finally, we introduce a MADM approach to make a decision on a real problem.

1. Introduction

Pawlak [1, 2] presented the classical definition of rough sets as a valuable mathematical method to deal with the vagueness and granularity of information systems and data processing. His theory and its generalizations since then have produced applications in different areas [3–13].

One of the most elaborated generalizations of rough sets is potentially covering-based rough sets (CRS). There are several scholars working on CRS with various views in previous years, see, for more information, [14–22]. After that, the definition of a fuzzy β -neighborhood was seen by Ma [23] and the fuzzy complementary β -neighborhood by Yang and Hu [24]. Also, Yang and Hu [25, 26] introduced the concepts of fuzzy β -minimal description and fuzzy β -maximal description. They used these definitions to construct a fuzzy β covering approximation space (F β CAS). D'eer et al. [27] studied fuzzy neighborhoods based on fuzzy coverings.

The definition of rough fuzzy sets and fuzzy rough sets was found by Dubois and Prade [28]. Different research studies on covering-based rough set and fuzzy rough set have recently been investigated [29–33].

Variable precision rough sets' (VPRSs) notion was obtained by Ziarko [34] and variable precision fuzzy rough sets (VPFRSs) were built by Zhao et al. [35]. In addition, the PROMETHEE II approach based on variable precision fuzzy rough sets was also proposed by Jiang et al. [36]. Different kinds of variable precision were further applied in various areas [37–40].

One of the standard decision-making processes is TOPSIS (technique for order preference by similarity to an ideal solution). Yoon and Hwang [41] indicated that TOPSIS will solve the problem of multiattribute decision-making (MADM), where the aim is to obtain an object with the highest effect value (PIS) and the lowest effect value (NIS). There are several papers concerning TOPSIS published in different fields [42–51].

Zhan et al. [52] put the definition of fuzzy γ -neighborhoods and also studied the covering-based variable precision fuzzy rough sets (CVPFRSs). Furthermore, Ma et al. [53] defined the complementary fuzzy γ -neighborhoods and presented another two types of neighborhoods by merging the fuzzy γ -neighborhoods and the complementary fuzzy γ -neighborhoods. Based on these kinds of fuzzy γ -neighborhoods, this paper proposes to introduce three

new kinds of CVPFRSs models as a generalization of the Zhan et al. [52] method. Thus, we discuss some of their properties. The relationships between these methods are also established. Then, we present and explain the methodology to solve MADM problems. The paper structure is as follows. Section 2 gives the basic notions. Section 3 establishes three novel types of CVPFRSs. A decision-making process to explain the theoretical study is advanced in Section 4. We deduced in Section 5.

2. Preliminaries

We extend a short scanning of some concepts utilized over the paper in this section. In this article, we work on \mathcal{R} -implication operator, in particular, $\mathcal{I} = \mathcal{I}_{\mathcal{F}}$, i.e., $\mathcal{I}_{\mathcal{F}}(a, b) = 1 \wedge (1 - a + b)$, and $\mathcal{I} = \mathcal{I}_{\mathcal{F}}$, i.e., $\mathcal{I}_{\mathcal{F}}(a, b) = 0 \vee (a + b - 1)$. To get more information, see [54].

$$\begin{aligned} \mathcal{N}_a^\beta &= \cap \{ \widehat{\mathcal{C}}_i \in t\widehat{\mathcal{C}} : q(\widehat{\mathcal{C}}_i)h \geq \beta \}, \\ \mathcal{M}_a^\beta(b) &= \mathcal{N}_b^\beta(a), \\ \mathcal{M}_{\widehat{\mathcal{C}}}^\beta(a) &= \{ \widehat{\mathcal{C}} \in t\widehat{\mathcal{C}} : q(\widehat{\mathcal{C}}(a) \geq \beta)h \wedge \forall \widehat{\mathcal{D}} \in \widehat{\mathcal{C}} \wedge \widehat{\mathcal{D}}(a) \geq \beta \wedge \widehat{\mathcal{D}} \subseteq \widehat{\mathcal{C}} \longrightarrow \widehat{\mathcal{D}} = \widehat{\mathcal{C}} \}. \end{aligned} \quad (1)$$

Zhan et al. [52] presented a new definition called fuzzy γ -neighborhood with reflexivity. Using these definitions, they describe the notion of a CVPFRSs based on this definition and solve problems in MADM. The $(\Omega, \widehat{\mathcal{C}})$ pair produced by this neighborhood is called a fuzzy γ -covering approximation space (F γ CAS for short) and $\widehat{\mathcal{C}}$ is called a fuzzy γ -covering [51].

Definition 3 (see [52]). Suppose that $(\Omega, \widehat{\mathcal{C}})$ is a F γ CAS and $\widehat{\mathcal{C}} = \{ \widehat{\mathcal{C}}_1, \widehat{\mathcal{C}}_2, \dots, \widehat{\mathcal{C}}_m \}$. For every $a, b \in \Omega$, the fuzzy γ -neighborhood of a is as follows:

$$\mathbb{N}_1^\gamma(a)(b) = \bigwedge_{\widehat{\mathcal{C}} \in \mathcal{M}_{\widehat{\mathcal{C}}}^\gamma(a)} \mathcal{I}(\widehat{\mathcal{C}}(a), \widehat{\mathcal{C}}(b)). \quad (2)$$

According to the above definition, we have the following result.

Assume that $(\Omega, \widehat{\mathcal{C}})$ is a F γ CAS and the variable precision parameter is $\xi \in [0, 1]$. For every $a \in \Omega$ and $\widehat{\mathcal{A}} \in \mathcal{F}(\Omega)$, the first model of a covering-based variable precision fuzzy rough lower and upper approximation which are denoted by 1-CVPFRLA and 1-CVPFRUA, respectively, are given as follows.

Model 1:

$$\begin{aligned} \mathcal{O}^{-1}(\widehat{\mathcal{A}})(a) &= \bigwedge_{b \in \Omega} \mathcal{I}(\mathbb{N}_1^\gamma(a)(b), \xi \vee \widehat{\mathcal{A}}(b)), \\ \mathcal{O}^{+1}(\widehat{\mathcal{A}})(a) &= \bigwedge_{b \in \Omega} \mathcal{I}(\mathbb{N}_1^\gamma(a)(b), \xi \wedge \widehat{\mathcal{A}}(b)). \end{aligned} \quad (3)$$

Definition 1 (see [32, 55]). Suppose that Ω is the universal arbitrary set and $\mathcal{F}(\Omega)$ is the fuzzy power set of Ω . We mean $\widehat{\mathcal{C}} = \{ \widehat{\mathcal{C}}_1, t\widehat{\mathcal{C}}_2n, q \dots h, \widehat{\mathcal{C}}_m \}$, for $\widehat{\mathcal{C}}_i \in \mathcal{F}(\Omega)$ ($i = 1, 2, \dots, m$), a fuzzy covering of Ω if $(\cup_{i=1}^m \widehat{\mathcal{C}}_i)(a) = 1, \forall a \in \Omega$.

The notion of a fuzzy β -covering was considered by Ma [23] via substituting 1 for the threshold β ($0 < \beta \leq 1$), i.e., we mean $\widehat{\mathcal{C}} = \{ \widehat{\mathcal{C}}_1, t\widehat{\mathcal{C}}_2n, q \dots h, \widehat{\mathcal{C}}_m \}$, for $\widehat{\mathcal{C}}_i \in \mathcal{F}(\Omega)$ ($i = 1, 2, \dots, m$), a fuzzy β -covering of Ω if $(\cup_{i=1}^m \widehat{\mathcal{C}}_i)(a) = 1, \forall a \in \Omega$. In addition, $(\Omega, \widehat{\mathcal{C}})$ is referred to as the a fuzzy β -covering approximation space (briefly, F β CAS).

Definition 2 (see [23–26]). Assume that $(\Omega, \widehat{\mathcal{C}})$ is a F β CAS for some $\beta \in (0, 1]$. For each $a \in \Omega$, the fuzzy β -neighborhood (resp., the fuzzy complementary β -neighborhood and the fuzzy β -minimal description) of a is defined by

If $\mathcal{O}^{-1}(\widehat{\mathcal{A}}) \neq \mathcal{O}^{+1}(\widehat{\mathcal{A}})$, then $\widehat{\mathcal{A}}$ is said to be a covering-based variable precision fuzzy rough set (briefly, 1-CVPFRS); otherwise it is definable [52].

Ma et al. [53] generalizes Zhan's model by introducing three kinds of neighborhoods as follows.

Definition 4. Assume that $(\Omega, \widehat{\mathcal{C}})$ is a F γ CAS. For any $a, b \in \Omega$, three types of the fuzzy γ -neighborhoods of x are as follows:

- (1) $\mathbb{N}_2^\gamma(a)(b) = \mathbb{N}_1^\gamma(b)(a)$
- (2) $\mathbb{N}_3^\gamma(a)(b) = \mathbb{N}_1^\gamma(a)(b) \wedge \mathbb{N}_2^\gamma(a)(b)$
- (3) $\mathbb{N}_4^\gamma(a)(b) = \mathbb{N}_1^\gamma(a)(b) \vee \mathbb{N}_2^\gamma(a)(b)$

To explain the comparisons between these four kinds of neighborhoods, we give the next example.

Example 1. If $(\Omega, \widehat{\mathcal{C}})$ is a F γ CAS, $\Omega = \{a_1, a_2, a_3, a_4\}$ and $\widehat{\mathcal{C}} = \{ \widehat{\mathcal{C}}_1, t\widehat{\mathcal{C}}_2n, q\widehat{\mathcal{C}}_3 \}$ is a three fuzzy γ covering on Ω set as follows:

$$\begin{aligned} \widehat{\mathcal{C}}_1 &= \frac{0.89}{a_1} + \frac{0.88}{a_2} + \frac{0.79}{a_3} + \frac{0.56}{a_4}, \\ \widehat{\mathcal{C}}_2 &= \frac{0.77}{a_1} + \frac{0.85}{a_2} + \frac{0.67}{a_3} + \frac{0.84}{a_4}, \\ \widehat{\mathcal{C}}_3 &= \frac{0.69}{a_1} + \frac{0.78}{a_2} + \frac{0.93}{a_3} + \frac{0.63}{a_4}. \end{aligned} \quad (4)$$

Let $\gamma = 0.74$ and $\mathcal{F} = \mathcal{F}_{\mathcal{F}}$. Then, the following values hold for each point on Ω for the three types of neighborhoods which are set in Definition 4.

Firstly, we compute the results for $\mathbb{N}_2^\gamma(a_r) \forall i \in \{1, 2, 3, 4\}$:

$$\begin{aligned} \mathbb{N}_2^\gamma(a_1) &= \frac{1}{a_1} + \frac{0.91}{a_2} + \frac{0.76}{a_3} + \frac{0.93}{a_4}, \\ \mathbb{N}_2^\gamma(a_2) &= \frac{0.99}{a_1} + \frac{1}{a_2} + \frac{0.85}{a_3} + \frac{1}{a_4}, \\ \mathbb{N}_2^\gamma(a_3) &= \frac{0.9}{a_1} + \frac{0.82}{a_2} + \frac{1}{a_3} + \frac{0.83}{a_4}, \\ \mathbb{N}_2^\gamma(a_4) &= \frac{0.67}{a_1} + \frac{0.68}{a_2} + \frac{0.7}{a_3} + \frac{1}{a_4}. \end{aligned} \tag{5}$$

Secondly, we compute the results for $\mathbb{N}_3^\gamma(a_r) \forall i \in \{1, 2, 3, 4\}$:

$$\begin{aligned} \mathbb{N}_3^\gamma(a_1) &= \frac{1}{a_1} + \frac{0.91}{a_2} + \frac{0.76}{a_3} + \frac{0.67}{a_4}, \\ \mathbb{N}_3^\gamma(a_2) &= \frac{0.91}{a_1} + \frac{1}{a_2} + \frac{0.82}{x_3} + \frac{0.68}{a_4}, \\ \mathbb{N}_3^\gamma(a_3) &= \frac{0.76}{a_1} + \frac{0.82}{a_2} + \frac{1}{a_3} + \frac{0.7}{a_4}, \\ \mathbb{N}_3^\gamma(a_4) &= \frac{0.67}{a_1} + \frac{0.68}{a_2} + \frac{0.7}{a_3} + \frac{1}{a_4}. \end{aligned} \tag{6}$$

Finally, we compute the results for $\mathbb{N}_4^\gamma(a_r) \forall i \in \{1, 2, 3, 4\}$:

$$\begin{aligned} \mathbb{N}_4^\gamma(a_1) &= \frac{1}{a_1} + \frac{0.99}{a_2} + \frac{0.9}{a_3} + \frac{0.93}{a_4}, \\ \mathbb{N}_4^\gamma(a_2) &= \frac{0.99}{a_1} + \frac{1}{a_2} + \frac{0.85}{a_3} + \frac{1}{a_4}, \\ \mathbb{N}_4^\gamma(a_3) &= \frac{0.9}{a_1} + \frac{0.85}{a_2} + \frac{1}{a_3} + \frac{0.83}{a_4}, \\ \mathbb{N}_4^\gamma(a_4) &= \frac{0.93}{a_1} + \frac{1}{a_2} + \frac{0.83}{a_3} + \frac{1}{a_4}. \end{aligned} \tag{7}$$

From the above example, you can see the differences between these kinds of neighborhoods. Also, you can conclude that $\mathbb{N}_3^\gamma(a_r)$ is considered as the union between $\mathbb{N}_1^\gamma(a_r)$ and $\mathbb{N}_2^\gamma(a_r)$. Furthermore, $\mathbb{N}_4^\gamma(a_r)$ is considered as the intersection between $\mathbb{N}_1^\gamma(a_r)$ and $\mathbb{N}_2^\gamma(a_r)$. Therefore, it is easy to say that the third neighborhood $\mathbb{N}_3^\gamma(a_r)$ is better than others.

3. Three New Models of Covering Fuzzy Rough Sets via Variable Precision

Now, we are implementing three CVPFRSs' models based on different kinds of a reflexive fuzzy γ -neighborhood.

Assume that (Ω, \mathcal{E}) is a FyCAS and the parameter $\xi \in [0, 1]$. For every $a \in \Omega$ and $\mathcal{A} \in \mathcal{F}(\Omega)$, three models of CVPFRSs are defined as follows.

Model 2:

$$\begin{aligned} \mathcal{O}^{-2}(\widehat{\mathcal{A}})(a) &= \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_2^\gamma(a)(b), \xi \vee \widehat{\mathcal{A}}(b)), \\ \mathcal{O}^{+2}(\widehat{\mathcal{A}})(a) &= \bigvee_{b \in \Omega} \mathcal{F}(\mathbb{N}_2^\gamma(a)(b), \xi \wedge \widehat{\mathcal{A}}(b)). \end{aligned} \tag{8}$$

Model 3:

$$\begin{aligned} \mathcal{O}^{-3}(\widehat{\mathcal{A}})(a) &= \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_3^\gamma(a)(b), \xi \vee \widehat{\mathcal{A}}(b)), \\ \mathcal{O}^{+3}(\widehat{\mathcal{A}})(a) &= \bigvee_{b \in \Omega} \mathcal{F}(\mathbb{N}_3^\gamma(a)(b), \xi \wedge \widehat{\mathcal{A}}(b)). \end{aligned} \tag{9}$$

Model 4:

$$\begin{aligned} \mathcal{O}^{-4}(\widehat{\mathcal{A}})(a) &= \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_4^\gamma(a)(b), \xi \vee \widehat{\mathcal{A}}(b)), \\ \mathcal{O}^{+4}(\widehat{\mathcal{A}})(a) &= \bigvee_{b \in \Omega} \mathcal{F}(\mathbb{N}_4^\gamma(a)(b), \xi \wedge \widehat{\mathcal{A}}(b)), \end{aligned} \tag{10}$$

where the three models are called the 2-CVPFRLA (resp., 3-CVPFRLA and 4-CVPFRLA) and the 2-CVPFRUA (resp., 3-CVPFRUA and 4-CVPFRUA), respectively.

If $\mathcal{O}^{-2}(\widehat{\mathcal{A}})$ (resp., $\mathcal{O}^{-3}(\widehat{\mathcal{A}})$, $\mathcal{O}^{-4}(\widehat{\mathcal{A}})$) $\neq \mathcal{O}^{+2}(\widehat{\mathcal{A}})$ (resp., $\mathcal{O}^{+3}(\widehat{\mathcal{A}})$, $\mathcal{O}^{+4}(\widehat{\mathcal{A}})$), then $\widehat{\mathcal{A}}$ is called a 2-CVPFRS (resp., 3-CVPFRS, 4-CVPFRS), otherwise it is definable.

The next example clarifies the above.

Example 2. (continued from Example 1). Suppose that $\widehat{\mathcal{A}} = (0.58/a_1) + (0.65/a_2) + (0.77/a_3) + (0.76/a_4)$. Then, we have the following results for the above four models (i.e., 1-CVPFRS, 2-CVPFRS, 3-CVPFRS, and 4-CVPFRS).

Model 1:

$$\begin{aligned} \mathcal{O}^{-1}(\widehat{\mathcal{A}}) &= \frac{0.58}{a_1} + \frac{0.65}{a_2} + \frac{0.77}{a_3} + \frac{0.65}{a_4}, \\ \mathcal{O}^{+1}(\widehat{\mathcal{A}}) &= \frac{0.67}{a_1} + \frac{0.65}{a_2} + \frac{0.77}{a_3} + \frac{0.76}{a_4}. \end{aligned} \tag{11}$$

Model 2:

$$\begin{aligned} \mathcal{O}^{-2}(\widehat{\mathcal{A}}) &= \frac{0.58}{a_1} + \frac{0.59}{a_2} + \frac{0.68}{a_3} + \frac{0.76}{a_4}, \\ \mathcal{O}^{+2}(\widehat{\mathcal{A}}) &= \frac{0.69}{a_1} + \frac{0.76}{a_2} + \frac{0.77}{a_3} + \frac{0.76}{a_4}. \end{aligned} \tag{12}$$

Model 3:

$$\begin{aligned} \mathcal{O}^{-3}(\widehat{\mathcal{A}}) &= \frac{0.58}{a_1} + \frac{0.65}{a_2} + \frac{0.77}{a_3} + \frac{0.76}{a_4}, \\ \mathcal{O}^{+3}(\widehat{\mathcal{A}}) &= \frac{0.67}{a_1} + \frac{0.65}{a_2} + \frac{0.77}{a_3} + \frac{0.76}{a_4}. \end{aligned} \tag{13}$$

Model 4:

$$\begin{aligned}\mathcal{O}^{-4}(\widehat{\mathcal{A}}) &= \frac{0.58}{a_1} + \frac{0.59}{a_2} + \frac{0.68}{a_3} + \frac{0.65}{a_4}, \\ \mathcal{O}^{+4}(\widehat{\mathcal{A}}) &= \frac{0.69}{a_1} + \frac{0.76}{a_2} + \frac{0.77}{a_3} + \frac{0.76}{a_4}.\end{aligned}\quad (14)$$

Remark 1. From Example 2, it is easy to see that

- (1) $\mathcal{O}^{-2}(\widehat{\mathcal{A}}) \not\subseteq \mathcal{O}^{-1}(\widehat{\mathcal{A}})$ and $\mathcal{O}^{-1}(\widehat{\mathcal{A}}) \not\subseteq \mathcal{O}^{-2}(\widehat{\mathcal{A}})$
- (2) $\mathcal{O}^{+2}(\widehat{\mathcal{A}}) \subseteq \mathcal{O}^{+1}(\widehat{\mathcal{A}})$ and $\mathcal{O}^{+1}(\widehat{\mathcal{A}}) \not\subseteq \mathcal{O}^{+2}(\widehat{\mathcal{A}})$

The 1-CVPFRS model and the 2-CVPFRS model are clearly not capable of containing each other.

Next, if $r = 1$, we propose Theorem 1, and also, it meets in case $r = 2, 3, 4$.

Theorem 1. Assume that $(\Omega, \widehat{\mathcal{E}})$ is a FyCAS and the parameter is $\xi \in [0, 1]$. For any $\widehat{\mathcal{A}}, \widehat{\mathcal{B}} \in \mathcal{F}(\Omega)$ and $\xi, \varepsilon \in [0, 1]$ ($\forall r \in \{1, 2, 3, 4\}$), the following properties hold:

- (1) $\mathcal{O}^{-r}(\widehat{\mathcal{A}}^c) = (\mathcal{O}^{+r}(\widehat{\mathcal{A}}))^c$
- (2) $\mathcal{O}^{+r}(\widehat{\mathcal{A}}^c) = (\mathcal{O}^{-r}(\widehat{\mathcal{A}}))^c$
- (3) $\mathcal{O}^{-r}(\Omega) = \Omega$
- (4) $\mathcal{O}^{+r}(\emptyset) = \emptyset$
- (5) If $\widehat{\mathcal{A}} \leq \widehat{\mathcal{B}}$, then $\mathcal{O}^{-r}(\widehat{\mathcal{A}}) \leq \mathcal{O}^{-r}(\widehat{\mathcal{B}})$
- (6) If $\widehat{\mathcal{A}} \leq \widehat{\mathcal{B}}$, then $\mathcal{O}^{+r}(\widehat{\mathcal{A}}) \leq \mathcal{O}^{+r}(\widehat{\mathcal{B}})$
- (7) $\mathcal{O}^{-r}(\widehat{\mathcal{A}} \wedge \widehat{\mathcal{B}}) = \mathcal{O}^{-r}(\widehat{\mathcal{A}}) \wedge \mathcal{O}^{-r}(\widehat{\mathcal{B}})$
- (8) $\mathcal{O}^{+r}(\widehat{\mathcal{A}} \wedge \widehat{\mathcal{B}}) \leq \mathcal{O}^{+r}(\widehat{\mathcal{A}}) \wedge \mathcal{O}^{+r}(\widehat{\mathcal{B}})$
- (9) $\mathcal{O}^{-r}(\widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}) \geq \mathcal{O}^{-r}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{-r}(\widehat{\mathcal{B}})$
- (10) $\mathcal{O}^{+r}(\widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}) = \mathcal{O}^{+r}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{+r}(\widehat{\mathcal{B}})$
- (11) If $\xi \leq \varepsilon$, then $\mathcal{O}_\xi^{-r}(\widehat{\mathcal{A}}) \leq \mathcal{O}_\varepsilon^{-r}(\widehat{\mathcal{B}})$
- (12) If $\xi \leq \varepsilon$, then $\mathcal{O}_\varepsilon^{+r}(\widehat{\mathcal{A}}) \leq \mathcal{O}_\xi^{+r}(\widehat{\mathcal{B}})$

Proof. We shall only prove (1), (3), (5), (7), (9), and (11).

- (1) $(\mathcal{O}^{-1}(\widehat{\mathcal{A}}^c)) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee \mathcal{N}(\widehat{\mathcal{A}}(b))) = \bigwedge_{b \in \Omega} \mathcal{N}(\mathcal{F}(\mathbb{N}_1^y(a)(b), \mathcal{N}(\xi) \wedge \widehat{\mathcal{A}}(b))) = \mathcal{N}(\bigwedge_{b \in \Omega} (\mathcal{F}(\mathbb{N}_1^y(a)(b), \mathcal{N}(\xi) \wedge \widehat{\mathcal{A}}(b)))) = (\mathcal{O}^{+1}(\widehat{\mathcal{A}}))^c$.
- (3) As \mathcal{F} is left monotonic and $\Omega(a) = 1$, for every $a \in \Omega$. Then, we have $\mathcal{O}^{-1}(\Omega) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee \Omega(b)) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \Omega(b)) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), 1) = 1 = \Omega(a)$.
- (5) \mathcal{F} is right monotonic and for every $a \in \Omega$. If $\widehat{\mathcal{A}} \leq \widehat{\mathcal{B}}$, then we get the following result. $\mathcal{O}^{-1}(\widehat{\mathcal{A}})(a) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee \widehat{\mathcal{A}}(b)) \leq \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee \widehat{\mathcal{B}}(b)) = \mathcal{O}^{-1}(\widehat{\mathcal{B}})(a)$.
- (7) \mathcal{F} is right monotonic and for all $a \in \Omega$. Then, we have $\mathcal{O}^{-1}(\widehat{\mathcal{A}} \wedge \widehat{\mathcal{B}})(a) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee (\widehat{\mathcal{A}} \wedge \widehat{\mathcal{B}})(b)) = \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee \widehat{\mathcal{A}}(b)) \wedge \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_1^y(a)(b), \xi \vee \widehat{\mathcal{B}}(b)) = \mathcal{O}^{-1}(\widehat{\mathcal{A}})(a) \wedge \mathcal{O}^{-1}(\widehat{\mathcal{B}})(a)$.
- (9) As \mathcal{F} is right monotonic, $\widehat{\mathcal{A}} \leq \widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}$ and $\widehat{\mathcal{B}} \leq \widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}$. Then, by (3), we obtain the following

$$\mathcal{O}^{-1}(\widehat{\mathcal{A}}) \leq \mathcal{O}^{-1}(\widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}) \text{ and } \mathcal{O}^{-1}(\widehat{\mathcal{B}}) \leq \mathcal{O}^{-1}(\widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}).$$

Thus, $\mathcal{O}^{-1}(\widehat{\mathcal{A}} \vee \widehat{\mathcal{B}}) \geq \mathcal{O}^{-1}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{-1}(\widehat{\mathcal{B}})$.

(11) It is obtained directly from Definition of Model 1. \square

The relationships between our models and the Zhan model in [52] are defined as follows. The following characteristics are clear and will be seen without proof.

Proposition 1. Assume that $(\Omega, \widehat{\mathcal{E}})$ is a FyCAS of Ω . For every $\widehat{\mathcal{A}} \in \mathcal{F}(\Omega)$ and $\forall r \in \{1, 2, 3, 4\}$, we have the following properties:

- (1) $\mathcal{O}^{-r}(\widehat{\mathcal{A}}) \subseteq \widehat{\mathcal{A}} \subseteq \mathcal{O}^{+r}(\widehat{\mathcal{A}})$
- (2) $\mathcal{O}^{-3}(\widehat{\mathcal{A}}) = \mathcal{O}^{-1}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{-2}(\widehat{\mathcal{A}})$ and $\mathcal{O}^{-3}(\widehat{\mathcal{A}}) = \mathcal{O}^1(\widehat{\mathcal{A}}) \vee \mathcal{O}^2(\widehat{\mathcal{A}})$
- (3) $\mathcal{O}^{-4}(\widehat{\mathcal{A}}) = \mathcal{O}^{-1}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{-2}(\widehat{\mathcal{A}})$ and $\mathcal{O}^{+4}(\widehat{\mathcal{A}}) = \mathcal{O}^1(\widehat{\mathcal{A}}) \vee \mathcal{O}^2(\widehat{\mathcal{A}})$
- (4) $\mathcal{O}^{-4}(\widehat{\mathcal{A}}) = \mathcal{O}^{-1}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{-3}(\widehat{\mathcal{A}})$ and $\mathcal{O}^{+3}(\widehat{\mathcal{A}}) = \mathcal{O}^1(\widehat{\mathcal{A}}) \vee \mathcal{O}^4(\widehat{\mathcal{A}})$
- (5) $\mathcal{O}^{-4}(\widehat{\mathcal{A}}) = \mathcal{O}^{-2}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{-3}(\widehat{\mathcal{A}})$ and $\mathcal{O}^{+3}(\widehat{\mathcal{A}}) = \mathcal{O}^{+2}(\widehat{\mathcal{A}}) \vee \mathcal{O}^{+4}(\widehat{\mathcal{A}})$

Proposition 2. Suppose that $(\Omega, \widehat{\mathcal{E}})$ is a FyCAS of Ω . For any $\widehat{\mathcal{A}} \in \mathcal{F}(\Omega)$ and $\forall a \in \Omega$.

Then, $\mathbb{N}_1^y(a) = \mathbb{N}_2^y(a) \Leftrightarrow$ either $\mathcal{O}^{-1}(\widehat{\mathcal{A}}) = \mathcal{O}^{-2}(\widehat{\mathcal{A}}) = \mathcal{O}^{-3}(\widehat{\mathcal{A}}) = \mathcal{O}^{-4}(\widehat{\mathcal{A}})$ or $\mathcal{O}^{+1}(\widehat{\mathcal{A}}) = \mathcal{O}^{+2}(\widehat{\mathcal{A}}) = \mathcal{O}^{+3}(\widehat{\mathcal{A}}) = \mathcal{O}^{+4}(\widehat{\mathcal{A}})$.

4. Decision-Making Approach to MADM Based on CVPFRS

This section introduces a new decision-making method to solve MADM problems by using CVPFRS' models.

4.1. Description and Process. In medicine, some types of drugs exist for the treatment of a disease, such as viral fever, dysentery, and chest problems. Assume that $\Omega = \{a_1, a_2, \dots, a_n\}$ is n kinds of drugs (alternatives) and $\widehat{\mathcal{E}} = \{\widehat{\mathcal{E}}_1, \widehat{\mathcal{E}}_2, \dots, \widehat{\mathcal{E}}_m\}$ is m symptoms (attributes). According to the decision assessment, maker's efficacy effect of the drug x_i on the symptoms $\widehat{\mathcal{E}}_r$ ($\forall r = 1, 2, \dots, m$ and $i = 1, 2, \dots, n$) has been determined. Hence, $(\Omega, \widehat{\mathcal{E}})$ establishes an FyCAS. According to the presented work, in the next steps, we introduce a decision-making algorithm that finds the most effective drug.

Step 1 : fuzzy decision matrix \mathcal{F} of medicine evaluations set as below:

$$\mathcal{F} = \begin{pmatrix} \Omega & \widehat{\mathcal{E}}_1 & \widehat{\mathcal{E}}_2 & \dots & \widehat{\mathcal{E}}_m \\ \widehat{\mathcal{E}} & a_1 & d_{11} & d_{12} & \dots & d_{1m} \\ & a_2 & d_{21} & d_{22} & \dots & d_{2m} \\ & \vdots & \vdots & \vdots & \ddots & \vdots \\ & a_n & d_{n1} & d_{n2} & \dots & d_{nm} \end{pmatrix}. \quad (15)$$

Step 2 : calculate the lower and upper approximations of $\widehat{\mathcal{C}}_r$ and evaluate the lower and upper fuzzy decision-making matrix of medicine evaluations:

$$\begin{aligned} \mathcal{O}^{-k}(\widehat{\mathcal{C}}_r)(a) &= \bigwedge_{b \in \Omega} \mathcal{F}(\mathbb{N}_q^y(a)(b), \xi \vee \widehat{\mathcal{C}}_r(b)), \\ \mathcal{O}^{+k}(\widehat{\mathcal{C}}_r)(a) &= \bigvee_{b \in \Omega} \mathcal{F}(\mathbb{N}_q^y(a)(b), \xi \wedge \widehat{\mathcal{C}}_r(b)), \quad (\forall k, q \in \{1, 2, 3, 4\}). \end{aligned} \quad (16)$$

Step 3 : three deflections among the estimations of any two alternatives are called the deflections among drugs $\widehat{\mathcal{D}}_r$, the lower deflections among drugs $\widehat{\mathcal{D}}_r^\ominus$, and the upper deflections among drugs $\widehat{\mathcal{D}}_r^\oplus$, respectively. These three deviations are computed as follows:

$$\begin{aligned} \widehat{\mathcal{D}}_r(a_i, a_j) &= \widehat{\mathcal{C}}_r(a_i) - \widehat{\mathcal{C}}_r(a_j) \\ \widehat{\mathcal{D}}_r^\ominus(a_i, a_j) &= \mathcal{O}^{-k}(\widehat{\mathcal{C}}_r)(a_i) - \mathcal{O}^{-k}(\widehat{\mathcal{C}}_r)(a_j), \\ \widehat{\mathcal{D}}_r^\oplus(a_i, a_j) &= \mathcal{O}^{+k}(\widehat{\mathcal{C}}_r)(a_i) - \mathcal{O}^{+k}(\widehat{\mathcal{C}}_r)(a_j), \end{aligned} \quad (17)$$

where $k \in \{1, 2, 3, 4\}$.

Step 4 : according to the three deviations, three drug preference values are referred to as drug preference values $\widehat{\mathcal{P}}_r$, lower drug preference values $\widehat{\mathcal{P}}_r^\ominus$, and upper drug preference values $\widehat{\mathcal{P}}_r^\oplus$. These three values of choice among alternatives are therefore computed as follows:

$$\begin{aligned} \widehat{\mathcal{P}}_r(a_i, a_j) &= \begin{cases} 0 & \text{if } \widehat{\mathcal{D}}_r(a_i, a_j) < 0, \\ \frac{\widehat{\mathcal{D}}_r(a_i, a_j)}{\alpha} & \text{if } 0 \leq \widehat{\mathcal{D}}_r(a_i, a_j) \leq \alpha, \\ 1 & \text{if } \widehat{\mathcal{D}}_r(a_i, a_j) > \alpha, \end{cases} \\ \widehat{\mathcal{P}}_r^\ominus(a_i, a_j) &= \begin{cases} 0 & \text{if } \widehat{\mathcal{D}}_r^\ominus(a_i, a_j) < 0, \\ \frac{\widehat{\mathcal{D}}_r^\ominus(a_i, a_j)}{\alpha} & \text{if } 0 \leq \widehat{\mathcal{D}}_r^\ominus(a_i, a_j) \leq \alpha, \\ 1 & \text{if } \widehat{\mathcal{D}}_r^\ominus(a_i, a_j) > \alpha, \end{cases} \\ \widehat{\mathcal{P}}_r^\oplus(a_i, a_j) &= \begin{cases} 0 & \text{if } \widehat{\mathcal{D}}_r^\oplus(a_i, a_j) < 0, \\ \frac{\widehat{\mathcal{D}}_r^\oplus(a_i, a_j)}{\alpha} & \text{if } 0 \leq \widehat{\mathcal{D}}_r^\oplus(a_i, a_j) \leq \alpha, \\ 1 & \text{if } \widehat{\mathcal{D}}_r^\oplus(a_i, a_j) > \alpha. \end{cases} \end{aligned} \quad (18)$$

where α denotes the value of preference threshold.

Step 5 : calculate three general drug preference indices, referred to as the overall drug preference indices for alternatives $\widehat{\mathcal{O}}$, the overall lower drug preference indices for alternatives $\widehat{\mathcal{O}}^\ominus$, and the overall upper drug preference indices for alternatives $\widehat{\mathcal{O}}^\oplus$, as follows:

$$\begin{aligned} \widehat{\mathcal{O}}(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{W}}_r \widehat{\mathcal{P}}_r(a_i, a_j), \\ \widehat{\mathcal{O}}^\ominus(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{W}}_r \widehat{\mathcal{P}}_r^\ominus(a_i, a_j), \\ \widehat{\mathcal{O}}^\oplus(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{W}}_r \widehat{\mathcal{P}}_r^\oplus(a_i, a_j), \end{aligned} \quad (19)$$

where $\widehat{\mathcal{W}} = (\widehat{\mathcal{W}}_1, \widehat{\mathcal{W}}_2, \dots, \widehat{\mathcal{W}}_m)$ is the vector of the weight of attributes such that $\sum_{r=1}^m \widehat{\mathcal{W}}_r = 1$ and $\widehat{\mathcal{W}}_r \in [0, 1]$.

Step 6 : three outflows of medicines are referred to as the outflows of alternatives \mathcal{L}_\bullet , the lower outflows of alternatives $\mathcal{L}_\bullet^\ominus$, and the upper outflows of medicines $\mathcal{L}_\bullet^\oplus$. These flows are thus constructed as follows:

$$\begin{aligned} \mathcal{L}_\bullet(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{O}}(a_i, a_j), \\ \mathcal{L}_\bullet^\ominus(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{O}}^\ominus(a_i, a_j), \\ \mathcal{L}_\bullet^\oplus(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{O}}^\oplus(a_i, a_j). \end{aligned} \quad (20)$$

We also create three input flows of drugs called the input flows of drugs \mathcal{L}_\circ , the lower input flows of drugs $\mathcal{L}_\circ^\ominus$, and the upper input flows of drugs \mathcal{L}_\circ^\oplus , respectively, as follows:

$$\begin{aligned} \mathcal{L}_\circ(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{O}}(a_i, a_j), \\ \mathcal{L}_\circ^\ominus(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{O}}^\ominus(a_i, a_j), \\ \mathcal{L}_\circ^\oplus(a_i, a_j) &= \sum_{r=1}^m \widehat{\mathcal{O}}^\oplus(a_i, a_j). \end{aligned} \quad (21)$$

Step 7 : the next formula computes the net flow of alternatives:

$$\mathcal{L}(a_i, a_j) = (\mathcal{L}_\bullet + \mathcal{L}_\bullet^\ominus + \mathcal{L}_\bullet^\oplus) - (\mathcal{L}_\circ + \mathcal{L}_\circ^\ominus + \mathcal{L}_\circ^\oplus), \quad (22)$$

Input: the \mathcal{F} fuzzy decision matrix, the α choice threshold, and the ξ parameter.
 Output: decision-aking.

- (1) Compute the lower and upper approximations by using Model 3
- (2) Compute three deflections among drugs (i.e., $\hat{\mathcal{D}}_r, \hat{\mathcal{D}}_r^{\ominus}$, and $\hat{\mathcal{D}}_r^{\oplus}$)
- (3) Compute three preference values among drugs (i.e., $\hat{\mathcal{P}}_r, \hat{\mathcal{P}}_r^{\ominus}$, and $\hat{\mathcal{P}}_r^{\oplus}$)
- (4) Compute three overall preference indices among drugs (i.e., $\hat{\theta}, \hat{\theta}^{\ominus}$, and $\hat{\theta}^{\oplus}$)
- (5) Compute three leaving flows of drugs (i.e., $\mathcal{L}_\bullet, \mathcal{L}_\bullet^{\ominus}$, and $\mathcal{L}_\bullet^{\oplus}$)
- (6) Compute three entering flows of drugs (i.e., $\mathcal{L}_\circ, \mathcal{L}_\circ^{\ominus}$, and $\mathcal{L}_\circ^{\oplus}$)
- (7) Compute the net flow of alternatives \mathcal{L}
- (8) Ranking the alternatives and obtain the decision

ALGORITHM 1: Algorithm for the presented drug selections.

hence ranking the alternatives.

In accordance with these steps, we include an algorithm based on Model 3 (3-CVPFRS) to solve decision-making issues. Algorithm 1 summarizes the measures leading to it.

4.2. A Numerical Example. The steps aforementioned have been illustrated as follows with a check instance.

Example 3. Alternatives (medicines) construct a set $\Omega = \{a_1, a_2, \dots, a_6\}$ which are treated a diseases $\tilde{\mathcal{A}}$, and their symptoms are gathered by the attribute set $\tilde{\mathcal{E}} =$ fever ($\tilde{\mathcal{E}}_1$), cough ($\tilde{\mathcal{E}}_2$), headache ($\tilde{\mathcal{E}}_3$), stomachaches ($\tilde{\mathcal{E}}_4$), dizzy giddy ($\tilde{\mathcal{E}}_5$). Here, the following steps of the algorithm mentioned are implemented.

Step 1: over the set of symptoms, experts analyze each medication and present its conclusions with acceptable values set out in Table 1.

Step 2: let us fix $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{E}}$. Then, by 2-CVPFRS, we have the following:

$$\begin{aligned}
 \mathbb{N}_3^y(a_1) &= \frac{1}{a_1} + \frac{0.89}{a_2} + \frac{0.82}{a_3} + \frac{0.77}{a_4} + \frac{0.54}{a_5} + \frac{0.49}{a_6}, \\
 \mathbb{N}_3^y(a_2) &= \frac{0.89}{a_1} + \frac{1}{a_2} + \frac{0.83}{a_3} + \frac{0.74}{a_4} + \frac{0.65}{a_5} + \frac{0.55}{a_6}, \\
 \mathbb{N}_3^y(a_3) &= \frac{0.82}{a_1} + \frac{0.83}{a_2} + \frac{1}{a_3} + \frac{0.87}{a_4} + \frac{0.48}{a_5} + \frac{0.64}{a_6}, \\
 \mathbb{N}_3^y(a_4) &= \frac{0.77}{a_1} + \frac{0.74}{a_2} + \frac{0.87}{a_3} + \frac{1}{a_4} + \frac{0.73}{a_5} + \frac{0.51}{a_6}, \\
 \mathbb{N}_3^y(a_5) &= \frac{0.54}{a_1} + \frac{0.65}{a_2} + \frac{0.48}{a_3} + \frac{0.73}{a_4} + \frac{1}{a_5} + \frac{0.51}{a_6}, \\
 \mathbb{N}_3^y(a_6) &= \frac{0.49}{a_1} + \frac{0.55}{a_2} + \frac{0.64}{a_3} + \frac{0.51}{a_4} + \frac{0.51}{a_5} + \frac{1}{a_6}.
 \end{aligned}
 \tag{23}$$

Thus, the 3-CVPFRLA and 3-CVPFRUA are obtained as follows:

$$\begin{aligned}
 \theta^{-3}(\tilde{\mathcal{E}}_1) &= \frac{0.92}{a_1} + \frac{0.91}{a_2} + \frac{0.86}{a_3} + \frac{0.73}{a_4} + \frac{0.56}{a_5} + \frac{1}{a_6}, \\
 \theta^{+3}(\tilde{\mathcal{E}}_1) &= \frac{0.9}{a_1} + \frac{0.9}{a_2} + \frac{0.9}{a_3} + \frac{0.77}{a_4} + \frac{0.56}{a_5} + \frac{1}{a_6}, \\
 \theta^{-3}(\tilde{\mathcal{E}}_2) &= \frac{0.54}{a_1} + \frac{0.65}{a_2} + \frac{0.48}{a_3} + \frac{0.61}{a_4} + \frac{1}{a_5} + \frac{0.51}{a_6}, \\
 \theta^{+3}(\tilde{\mathcal{E}}_2) &= \frac{0.61}{a_1} + \frac{0.65}{a_2} + \frac{0.71}{a_3} + \frac{0.84}{a_4} + \frac{0.9}{a_5} + \frac{0.51}{a_6}, \\
 \theta^{-3}(\tilde{\mathcal{E}}_3) &= \frac{0.95}{a_1} + \frac{0.94}{a_2} + \frac{0.82}{a_3} + \frac{0.95}{a_4} + \frac{0.82}{a_5} + \frac{0.49}{a_6}, \\
 \theta^{+3}(\tilde{\mathcal{E}}_3) &= \frac{0.9}{a_1} + \frac{0.9}{a_2} + \frac{0.82}{a_3} + \frac{0.9}{a_4} + \frac{0.82}{a_5} + \frac{0.49}{a_6}, \\
 \theta^{-3}(\tilde{\mathcal{E}}_4) &= \frac{0.48}{a_1} + \frac{0.52}{a_2} + \frac{0.57}{a_3} + \frac{0.44}{a_4} + \frac{0.52}{a_5} + \frac{0.93}{a_6}, \\
 \theta^{+3}(\tilde{\mathcal{E}}_4) &= \frac{0.48}{a_1} + \frac{0.52}{a_2} + \frac{0.57}{a_3} + \frac{0.44}{a_4} + \frac{0.52}{a_5} + \frac{0.9}{a_6}, \\
 \theta^{-3}(\tilde{\mathcal{E}}_5) &= \frac{0.28}{a_1} + \frac{0.25}{a_2} + \frac{0.42}{a_3} + \frac{0.51}{a_4} + \frac{0.24}{a_5} + \frac{0.28}{a_6}, \\
 \theta^{+3}(\tilde{\mathcal{E}}_5) &= \frac{0.28}{a_1} + \frac{0.29}{a_2} + \frac{0.46}{a_3} + \frac{0.51}{a_4} + \frac{0.24}{a_5} + \frac{0.28}{a_6}.
 \end{aligned}
 \tag{24}$$

Steps 3 and 4: by using the previous data, it is easy to compute the three deflections among the estimations of any two alternatives and the three preference values among drugs.

Step 5: from this information, we construct the values for three overall preference indices among drugs as set in Tables 2–4.

Step 6: the three leaving flows of drugs are calculated as follows:

$$\begin{aligned}
 \mathcal{L}_\bullet(a_i, a_j) &= \frac{0.66895}{a_1} + \frac{0.9367}{a_2} + \frac{0.6456}{a_3} + \frac{1.411}{a_4} + \frac{0.8467}{a_5} + \frac{0.58875}{a_6}, \\
 \mathcal{L}^\circ(a_i, a_j) &= \frac{0.61255}{a_1} + \frac{0.70305}{a_2} + \frac{0.6116}{a_3} + \frac{0.9596}{a_4} + \frac{0.9227}{a_5} + \frac{0.64875}{a_6}, \\
 \mathcal{L}^\#(a_i, a_j) &= \frac{0.45795}{a_1} + \frac{0.5091}{a_2} + \frac{0.7696}{a_3} + \frac{1.04425}{a_4} + \frac{0.5787}{a_5} + \frac{0.58775}{a_6}, \\
 \mathcal{L}_\circ(a_i, a_j) &= \frac{0.6219}{a_1} + \frac{0.3413}{a_2} + \frac{0.975}{a_3} + \frac{0.5367}{a_4} + \frac{1.0578}{a_5} + \frac{1.565}{a_6}, \\
 \mathcal{L}^\ominus(a_i, a_j) &= \frac{0.4894}{a_1} + \frac{0.456}{a_2} + \frac{0.65545}{a_3} + \frac{0.4811}{a_4} + \frac{0.9353}{a_5} + \frac{1.441}{a_6}, \\
 \mathcal{L}^\oplus(a_i, a_j) &= \frac{0.51755}{a_1} + \frac{0.4273}{a_2} + \frac{0.32625}{a_3} + \frac{0.3077}{a_4} + \frac{0.86605}{a_5} + \frac{1.5025}{a_6}.
 \end{aligned}
 \tag{25}$$

Step 7: the values of the net flow of alternatives are computed as follows:

$$\begin{aligned}
 \mathcal{L}(a_1) &= 0.1106, \\
 \mathcal{L}(a_2) &= 0.92425, \\
 \mathcal{L}(a_3) &= 0.0701, \\
 \mathcal{L}(a_4) &= 2.08935, \\
 \mathcal{L}(a_5) &= -0.51105, \\
 \mathcal{L}(a_6) &= -2.68325.
 \end{aligned}
 \tag{26}$$

Thus, the drugs' ranking is as follows:

$$a_4 \geq a_2 \geq a_1 \geq a_3 \geq a_5 \geq a_6.
 \tag{27}$$

4.3. Comparative Analysis. Here, we give the differences between the proposed method (i.e., 2-CVPFRS, 3-CVPFRS, and 4-CVPFRS) and the previous methods (i.e., Jiang's method [36], PROMETHEE II [56], TOPSIS [57], WAA [58], OWA [59], and VIKOR [60]). Based on the sorting values of various decision-making approaches summarized in Table 5, our approach is therefore rational and effective.

According to Table 5, (1) the best position of our presented method, Jiang's method [36], PROMETHEE II [56], TOPSIS [57], WAA [58], OWA [59], and VIKOR [60], is still consistent, that is, a_4 is the best drug. Thus, our suggested approach is rational and efficient from the point of view of the decision outcome (the best option in the decision-making process). (2) Five drug classifications based on various methods are not precisely the same in [36], meaning that the best drug is equal (i.e., the drug a_4). However, operating on the fuzzy γ -neighborhoods without reflexivity in Jiang's [36] process, our methodology relies on the fuzzy γ -neighborhoods with reflexivity,

which makes our approach proposed more rational and effective.

The best way to clarify these results, you can see Figures 1 and 2 which simplify the comparisons between the presented method and others.

Figure 1 explains the comparisons between the lower approximation for the four models (i.e., 1-CVPFRLA, 2-CVPFRLA, 3-CVPFRLA, and 4-CVPFRLA). This figure clarifies that the 3-CVPFRLA is larger than the others.

Figure 2 clarifies the comparisons between the upper approximation for the four models (i.e., 1-CVPFRUA, 2-CVPFRUA, 3-CVPFRUA, and 4-CVPFRUA). This figure shows that 3-CVPFRUA is smaller than the others.

- (1) Two documented issues with fuzzy γ -neighborhoods are conquered by our presented methods. However, not all techniques can escape the obstacles that are not reflexive operators in fuzzy γ -neighborhoods and that the lower approximations they describe are not usually included in the corresponding upper approximation. For this reason, our approach for solving MADM issues is based on the CVPFRS models (i.e., 1-CVPFRS, 2-CVPFRS, and 3-CVPFRS). Moreover, by a comparative study in Section 4.3, by using fuzzy γ -neighborhoods, the proposed models are more freely used than the classical models.
- (2) We can see in Section 4 that our presented models (i.e., Algorithm 1) are elastic and scalable, whereby decision makers can use fuzzy γ -neighborhoods to pick various logical operators and parameters according to current status.
- (3) We can easily observe from a comparative study that our models presented are superior to Jiang's method [36], PROMETHEE II [56], TOPSIS [57], WAA [58], OWA [59], and VIKOR [60]. This implies that the innovative decision-making approaches suggested are rational and feasible.

TABLE 1: Decision-making matrix \mathcal{F} with fuzzy information.

	a_1	a_2	a_3	a_4	a_5	a_6
\mathcal{E}_1	0.92	1	1	0.73	0.56	1
\mathcal{E}_2	0.54	0.65	0.48	0.84	1	0.51
\mathcal{E}_3	1	0.94	0.82	1	0.82	0.49
\mathcal{E}_4	0.48	0.52	0.57	0.44	0.52	0.93
\mathcal{E}_5	0.28	0.25	0.46	0.51	0.24	0.28

TABLE 2: The overall preference indices among drugs.

$\widehat{\mathcal{O}}(a_i, a_j)$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	0.0622	0.1095	0.195	0.1607	0.0945
a_2	0.0425	0	0.0083	0.094	0.116	0.0805
a_3	0.11	0.2035	0	0.4175	0.174	0.07
a_4	0.0547	0.188	0.0775	0	0.067	0.1495
a_5	0.19675	0.212	0.2103	0.2445	0	0.19425
a_6	0.265	0.271	0.24	0.46	0.329	0

TABLE 3: The overall lower preference indices among drugs.

$\widehat{\mathcal{O}}^{\ominus}(a_i, a_j)$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	0.0427	0.0725	0.119	0.1607	0.0945
a_2	0.0431	0	0.0783	0.1126	0.116	0.106
a_3	0.101	0.12845	0	0.1475	0.174	0.1045
a_4	0.0547	0.0714	0.0625	0	0.143	0.1495
a_5	0.17275	0.1895	0.1583	0.2205	0	0.19425
a_6	0.241	0.271	0.24	0.36	0.329	0

TABLE 4: The overall upper preference indices among drugs.

$\widehat{\mathcal{O}}^{\oplus}(a_i, a_j)$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	0.02435	0.124	0.171	0.1027	0.0955
a_2	0	0	0.0983	0.1565	0.084	0.0885
a_3	0.039	0.039	0	0.10375	0.064	0.0805
a_4	0.0397	0.046	0.055	0	0.033	0.134
a_5	0.14125	0.14125	0.1863	0.204	0	0.18925
a_6	0.238	0.2545	0.306	0.409	0.295	0

TABLE 5: Table for the ranking results for different methods.

Different models	Obtain a decision
Our model	$a_4 \geq a_2 \geq a_1 \geq a_3 \geq a_5 \geq a_6$
Jiang model [36]	$a_4 \geq a_2 \geq a_3 \geq a_1 \geq a_5 \geq a_6$
PROMETHEE II [56]	$a_4 \geq a_2 \geq a_1 = a_3 \geq a_5 \geq a_6$
TOPSIS [57]	$a_4 \geq a_2 \geq a_1 \geq a_5 \geq a_3 \geq a_6$
WAA [58]	$a_4 \geq a_2 \geq a_1 = a_3 \geq a_5 \geq a_6$
OWA [59]	$a_4 \geq a_3 \geq a_2 \geq a_1 \geq a_6 \geq a_5$
VIKOR [60]	$a_4 \geq a_3 \geq a_1 \geq a_2 \geq a_5 \geq a_6$

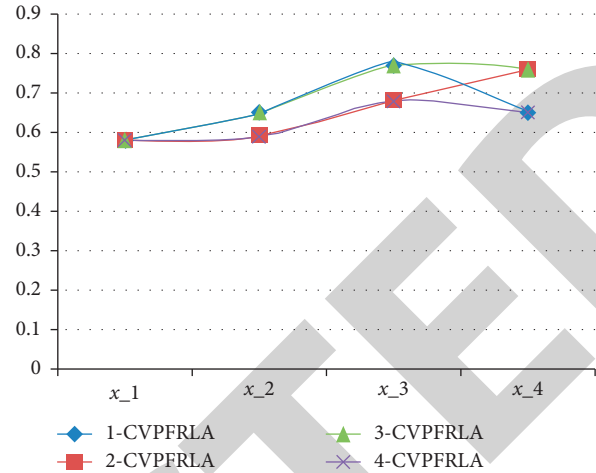


FIGURE 1: The presentation of lower approximations by using our models and the previous model.

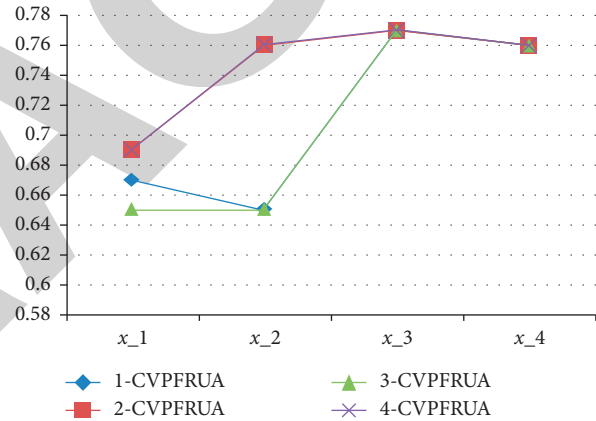


FIGURE 2: The presentation of upper approximations by using our models and the previous model.

5. Conclusion

As an improvement of the Zhan et al. method [52] and by using the concepts of neighborhoods by Ma et al. in [53], we then established new three kinds of covering-based variable precision fuzzy rough sets (i.e., 2-CVPFERS, 3-CVPFERS, and 4-CVPFERS). Relationship between these three paradigms and the paradigm of Zhan is also dealt with. This correlation indicates that the 3-CVPFERS is better than other models (i.e., the lower approximation is greater than others and the upper approximation is lower than others, as can be seen in Figures 1 and 2 based on Example 3). Finally, we set up an application for MADM to solve a problem. In the existing decision-making principles of interval-valued q-rung orthopair fuzzy sets [61] and linguistic interval-valued Pythagorean fuzzy sets [62], we hope this fuzzy rough concept can be incorporated.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

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