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Research Article

On α-Multiplier on Almost Distributive Lattices

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In this paper, we initiate the concept of α-multiplier on almost distributive lattices. We prove some useful results by using the notion of α-multiplier and generalize the idea of multiplier on almost distributive lattices.

1. Introduction

A lattice is an advanced abstract structure that has been studied in abstract algebra during last few decades. Birkhoff introduced the concept of lattice theory in 1940 [1]. Lattice is generalization of Boolean and fuzzy algebras. Latter on Gratzer and Schmidt worked together and showed their interest in the development of lattice theory [2]. In 1955, Helgason introduced the concept of multiplier in Banach Algebra [3]. The idea of multiplier in lattice was given by Larsen [4] in 1971, and Cornish extended this concept of multiplier in distributive lattice [5].

In 1981, the idea of ADLs was initiated by Swamy and Rao [6]. An almost distributive lattice satisfies all the properties of distributive lattice except commutativity of $\wedge$ and $\vee$ and right distributivity of $\vee$ over $\wedge$. Recently, Kim has introduced the idea of a multiplier in ADLs [7] and discussed some fundamental properties of this notion. For detailed study of the subject, we refer to readers [8–10].

Now, we have generalized certain properties of α-multiplier. The notion of α-multiplier for an almost distributive lattice is introduced, and some related properties are investigated. Moreover, we introduced principle α-multiplier and isotone α-multiplier on almost distributive lattices.

2. Preliminaries

Definition 1 (see [6]). An algebra $(G, \wedge, \vee, 0)$ is said to be an almost distributive lattice if it satisfies the following:

(i) $e_1 \vee 0 = e_1$
(ii) $0 \wedge e_1 = 0$
(iii) $(e_1 \vee h_1) \wedge \omega_1 = (e_1 \wedge \omega_1) \vee (h_1 \wedge \omega_1)$
(iv) $e_1 \wedge (h_1 \vee \omega_1) = (e_1 \wedge h_1) \vee (e_1 \wedge \omega_1)$
(v) $e_1 \vee (h_1 \wedge \omega_1) = (e_1 \vee h_1) \wedge (e_1 \vee \omega_1)$
(vi) $(e_1 \vee h_1) \wedge h_1 = h_1$
(vii) $(e_1 \vee h_1) \wedge e_1 = e_1$
(viii) $e_1 \vee (h_1 \wedge e_1) = e_1, \forall e_1, h_1, \omega_1 \in G$

Lemma 1 (see [6]). *Let G be an almost distributive lattice. For any* $e_1, h_1 \in G$, *we have*

(i) $e_1 \vee e_1 = e_1$
(ii) $e_1 \vee e_1 = e_1$
(iii) $(e_1 \wedge h_1) \vee h_1 = h_1$
(iv) $e_1 \wedge (e_1 \vee h_1) = e_1$
(v) $e_1 \vee (h_1 \wedge e_1) = e_1$
(vi) $e_1 \vee h_1 = e_1 \Leftrightarrow e_1 \wedge h_1 = h_1$

(vii) $e_1 \vee h_1 = h_1 \Leftrightarrow e_1 \wedge h_1 = e_1$

Definition 2 (see [6]). For any $e_1, h_1 \in G$, we say that $e_1 \leq h_1$ if $e_1 \wedge h_1 = e_1$ or equivalently, $e_1 \vee h_1 = h_1$.

Lemma 2 *(see [6]). Let G be an almost distributive lattice. For any $e_1, h_1, \omega_1, h_3 \in G$, then the following identities hold:*

(i) $e_1 \wedge h_1 \leq h_1$ and $e_1 \leq e_1 \vee h_1$

(ii) $e_1 \wedge h_1 = h_1 \wedge e_1$ whenever $e_1 \leq h_1$

(iii) $[e_1 \vee (h_1 \vee \omega_1)] \wedge h_3 = [(e_1 \vee h_1) \vee \omega_1] \wedge h_3$

(iv) $e_1 \leq h_1 \Rightarrow e_1 \wedge \omega_1 \leq h_1 \wedge \omega_1, \omega_1 \wedge e_1 \leq \omega_1 \wedge h_1$ and $\omega_1 \vee e_1 \leq \omega_1 \vee h_1$

Definition 3 (see [6]). Let G be a lattice, and 0 is known as a zero element of a lattice G if $0 \wedge e_1 = 0, \forall e_1 \in G$.

Lemma 3 (see [6]). *Let G be an almost distributive lattice. If G has 0, then for any $e_1, h_1 \in G$, the following identities hold:*

(i) $e_1 \vee 0 = e_1$ and $0 \vee e_1 = e_1$

(ii) $e_1 \wedge 0 = 0$

(iii) $e_1 \wedge h_1 = 0$ if and only if $h_1 \wedge e_1 = 0$

Definition 4 (see [6]). Let I be a nonempty subset of G which is called an ideal of G if $h_3 \vee \omega_3 \in I$ and $h_3 \wedge e_3 \in I$ whenever $h_3, \omega_3 \in I$ and $e_3 \in G$.

If I is an ideal of G and $e_3, h_3 \in G$, then $e_3 \wedge h_3 \in I$ if and only if $h_3 \wedge e_3 \in I$.

Lemma 4 (see [6]). *For any $e_1, h_1 \in G$, we have*

(i) $(e_1 \wedge h_1) \vee h_1 = h_1$

(ii) $e_1 \vee (e_1 \wedge h_1) = e_1 = e_1 \wedge (e_1 \vee h_1)$

(iii) $e_1 \vee (h_1 \wedge e_1) = e_1 = (e_1 \vee h_1) \wedge e_1$

Definition 5 (see [7]). Let G be an almost distributive lattice and ζ_1 and ζ_2 be two self maps. We define $\zeta_1 \vee \zeta_2 \colon G \longrightarrow G$ by $(\zeta_1 \vee \zeta_2)(e_1) = (\zeta_1(e_1)) \vee (\zeta_2(e_1)), \forall e_1 \in G$.

Definition 6 (see [7]). Let G_1 and G_2 be two almost distributive lattices. Then, $G_1 \times G_2$ is also an ADL with respect to the pointwise operation given by $(e_1, h_1) \wedge (\omega_1, x) = (e_1 \wedge \omega_1,\ h_1 \wedge x)$ and $(e_1, h_1) \vee (\omega_1, x) = (e_1 \vee \omega_1,\ h_1 \vee x), \forall e_1, h_1 \in G_1, \omega_1, x \in G_2$.

Definition 7 (see [7]). Let G be an almost distributive lattice and ζ be a multiplier of G. Define a set $\mathrm{fix}_\zeta(G)$ by $\mathrm{fix}_\zeta(G) = \{e_1 \in G \colon \zeta(e_1) = e_1\}$.

Definition 8 (see [7]). Let $(G, \wedge, \vee, 0)$ be an almost distributive lattice. For any $u \in G$, define $\Gamma_u = \{(e_1, h_1) \in G \times G | f_u(e_1) = f_u(h_1)\}$, where f_u is a principle multiplier induced by $u \in G$.

3. α-Multiplier on Almost Distributive Lattices

Definition 9. Let G be an almost distributive lattice. A function $\zeta \colon G \longrightarrow G$ is called α-multiplier if $\zeta(e_1 \wedge h_1) = \zeta(e_1) \wedge \alpha(h_1)\ \forall e_1, h_1 \in G$, where α is a mapping on G.

Example 1. Let G be an almost distributive lattice with $0 \in G$. A function ζ defined by $\zeta(e_1) = 0\ \forall e_1 \in G$ is called zero α-multiplier.

Proof. Let G be an almost distributive lattice with $0 \in G$; then, we have to prove that ζ is a zero α-multiplier.

Let $e_1, h_1 \in G$, $\zeta(e_1 \wedge h_1) = \zeta(e_1) \wedge \alpha(h_1) = 0 \wedge \alpha(h_1) = 0$. Hence, ζ is a zero α-multiplier. □

Lemma 5. *Let ζ be α-multiplier of G. If $\alpha \colon G \longrightarrow G$ is homomorphism, then following conditions hold:*

(i) $\zeta(e_1) \leq \alpha(e_1)$

(ii) $\zeta(e_1) \wedge \zeta(h_1) \leq \zeta(e_1 \wedge h_1), \forall e_1, h_1 \in G$

Proof

(i) Since $\zeta(e_1) = \zeta(e_1 \wedge e_1) = \zeta(e_1) \wedge \alpha(e_1)$, it implies that $\zeta(e_1) \leq \alpha(e_1)$.

(ii) Let $e_1, h_1 \in G$. We have to show that $\zeta(e_1) \wedge \zeta(h_1) \leq \zeta(e_1 \wedge h_1)$.

$$\begin{aligned} \zeta(e_1) &\leq \alpha(e_1), \\ \zeta(h_1) &\leq \alpha(h_1), \end{aligned} \tag{1}$$

$\zeta(e_1) \wedge \zeta(h_1) \leq \zeta(e_1) \wedge \alpha(h_1) = \zeta(e_1 \wedge h_1)$. It implies $\zeta(e_1) \wedge \zeta(h_1) \leq \zeta(e_1 \wedge h_1)$. □

Definition 10. Let G be an almost distributive lattice. A function K_e is defined by $K_e(g) = e \wedge a(g), \forall g \in G$, where $\alpha \colon G \longrightarrow G$ is a homomorphism, then K_e is α–multiplier of G, and such α–multiplier of G is called a principle α–multiplier of G.

Definition 11. Let $(G, \wedge, \vee, 0)$ be an almost distributive lattice and K be α-multiplier on G. For any $u \in G$, define $\Gamma_u = \{(e_1, h_1) \in G \times G | k_u(e_1) = k_u(h_1)\}$, where K_u is a principle α-multiplier induced by $u \in G$.

Lemma 6. *Let G be an almost distributive lattice. A function K_{e_3} is defined by $K_{e_3}(a) = e_3 \wedge \alpha(a)$, where $\alpha \colon G \longrightarrow G$ is a homomorphism, then K_{e_3} is α-multiplier of G, and such α-multiplier of G is called a principle α-multiplier of G.*

Proof. Let $h_3, \omega_3, e_3 \in G$, and K be an α-multiplier; then, we have to prove that K_{e_3} is an α-multiplier.

$$K_e(h_3) = e \wedge \alpha(h_3), \quad \forall h_3 \in G, \tag{2}$$

$K_e(h_3 \wedge \omega_3) = e_3 \wedge \alpha(h_3 \wedge \omega_3) = e_3 \wedge (\alpha(h_3) \wedge \alpha(\omega_3)) = (e_3 \wedge \alpha(h_3)) \wedge \alpha(\omega_3) = K_{e_3}(h_3) \wedge \alpha(\omega_3)$. This implies that K_{e_3} is an α-multiplier. □

Definition 12. Let G be an almost distributive lattice and ζ be α−multiplier on G, where α is a mapping on G. If for $e_1 \leq h_1$ implies $\zeta(e_1) \leq \zeta(h_1)$, then ζ is an isotone α−multiplier.

Proposition 1. *Let G be an almost distributive lattice. If $\alpha: G \longrightarrow G$ is an increasing homomorphism, then $\zeta_a(h_2) = a \wedge \alpha(h_2)$, and $\forall h_2 \in G$ is an isotone α-multiplier of G.*

Proof. Let G be an ADL and $h_2, \omega_2 \in G$, with $h_2 \leq \omega_2$ such that $\alpha(h_2) \leq \alpha(\omega_2)$, then we have $\zeta_a(h_2 \wedge \omega_2) = a \wedge (\alpha(h_2) \wedge \alpha(\omega_2)) = (a \wedge \alpha(h_2)) \wedge (a \wedge \alpha(\omega_2)) = \zeta_a(h_2) \wedge \zeta_a(\omega_2)$. It implies $\zeta_a(h_2) \leq \zeta_a(\omega_2)$. Hence, ζ_a is an isotone α-multiplier. □

Lemma 7. *Let G be an almost distributive lattice and ζ be an α-multiplier of G and α be an increasing homomorphism on G. If $h_3 \leq \omega_3$ and $\zeta(\omega_3) = \alpha(\omega_3)$, then $\zeta(h_3) = \alpha(h_3)$.*

Proof. Let h_3, $\omega_3 \in G$ for $h_3 \leq \omega_3$. Since α is an increasing homomorphism, so

$$\begin{aligned} \alpha(h_3) &\leq \alpha(\omega_3), \\ \zeta(\omega_3) &= \alpha(\omega_3). \end{aligned} \tag{3}$$

By using equation (3), we have $\zeta(h_3) = \zeta(\omega_3 \wedge h_3) = \zeta(\omega_3) \wedge \alpha(h_3) = \alpha(\omega_3) \wedge \alpha(h_3)$ since α is an increasing homomorphism. It implies that $\zeta(h_3) = \alpha(h_3)$. □

Theorem 1. *Let G be an almost distributive lattice and ζ be an α-multiplier of G and α be an increasing homomorphism on G. Then, ζ is an isotone α-multiplier.*

Proof. Suppose h_3, $\omega_3 \in G$. By using Lemma 5 (i), we have

$$\zeta(\omega_3) \leq \alpha(\omega_3), \tag{4}$$

$$\begin{aligned} &\text{for } h_3 \leq \omega_3, \\ &\alpha(h_3) \leq \alpha(\omega_3). \end{aligned} \tag{5}$$

Since $h_3 \leq \omega_3$, therefore, we have $\zeta(h_3) = \zeta(\omega_3 \wedge h_3) = \zeta(\omega_3) \wedge \alpha(h_3)$. By equations (4) and (5), we have $\zeta(h_3) \leq \zeta(\omega_3) \wedge \alpha(\omega_3) = \zeta(\omega_3)$. It implies $\zeta(h_3) \leq \zeta(\omega_3)$. Hence, ζ is an isotone α-multiplier. □

Proposition 2. *Let G be an almost distributive lattice, ζ be an α-multiplier of G, and α be homomorphism on G. Then, $\zeta(\omega_1 \vee e_1) = \zeta(\omega_1) \vee \zeta(e_1)$, $\forall \omega_1$, $e_1 \in G$.*

Proof. Let $\omega_1, e_1 \in G$ and ζ be an α-multiplier of G; then, we have to show that $\zeta(\omega_1 \vee e_1) = \zeta(\omega_1) \vee \zeta(e_1)$. By Definition 1, we have $\zeta(\omega_1) \vee \zeta(e_1) = \zeta((\omega_1 \vee e_1) \wedge \omega_1) \vee \zeta((\omega_1 \vee e_1) \wedge e_1) = (\zeta(\omega_1 \vee e_1) \wedge \alpha(\omega_1)) \vee (\zeta(\omega_1 \vee e_1) \wedge \alpha(e_1))$. By Definition 1, we have $\zeta(\omega_1) \vee \zeta(e_1) = \zeta(\omega_1 \vee e_1) \wedge (\alpha(\omega_1) \vee \alpha(e_1)) = \zeta(\omega_1 \vee e_1) \wedge \alpha(\omega_1 \vee e_1) = \zeta(\omega_1 \vee e_1)$. It implies that $\zeta(\omega_1) \vee \zeta(e_1) = \zeta(\omega_1 \vee e_1)$. □

Proposition 3. *Let G be an almost distributive lattice and ζ_1 and ζ_2 be two α-multipliers of G. Then, $\zeta_1 \vee \zeta_2$ is also an α-multiplier of G.*

Proof. Let G be an ADL and ζ_1 and ζ_2 be α-multiplier such that

$$\begin{aligned} \zeta_1(h_3 \wedge \omega_3) &= \zeta_1(h_3) \wedge \alpha(\omega_3), \\ \zeta_2(h_3 \wedge \omega_3) &= \zeta_2(h_3) \wedge \alpha(\omega_3). \end{aligned} \tag{6}$$

Let h_3, $\omega_3 \in G$, ζ_1, and ζ_2 be α-multipliers of G. Now, by Definition 5, we have $\zeta_1 \vee \zeta_2(h_3 \wedge \omega_3) = \zeta_1(h_3 \wedge \omega_3) \vee \zeta_2(h_3 \wedge \omega_3) = (\zeta_1(h_3) \wedge \alpha(\omega_3)) \vee (\zeta_2(h_3) \wedge \alpha(\omega_3))$. By Definitions 1 and 5, we have $\zeta_1 \vee \zeta_2(h_3 \wedge \omega_3) = (\zeta_1(h_3)) \vee (\zeta_2(h_3)) \wedge \alpha(\omega_3) = (\zeta_1 \vee \zeta_2)(h_3) \wedge \alpha(\omega_3)$ which along with equation (6) implies that $((\zeta_1 \vee \zeta_2)(h_3 \wedge \omega_3) = (\zeta_1 \vee \zeta_2)(h_3) \wedge \alpha(\omega_3)$. Hence, $\zeta_1 \vee \zeta_2$ is called α-multiplier of G. □

Proposition 4. *Let G_1 and G_2 be two almost distributive lattices with 0. A function $\zeta: G_1 \times G_2 \longrightarrow G_1 \times G_2$ defined by $\zeta(s, \omega_3) = (0, \alpha(\omega_3)) \forall (s, \omega_3) \in G_1 \times G_2$ and α is a homomorphism. Then, ζ is α-multiplier of $G_1 \times G_2$ with pointwise operation.*

Proof. Let G_1 and G_2 be two ADLs with 0. We define a mapping $\zeta: G_1 \times G_2 \longrightarrow G_1 \times G_2$ by

$$\zeta(s_1, t_1) = (0, \alpha(t_1)). \tag{7}$$

Then, we have to show that ζ is an α-multiplier with pointwise operation such that

$$\zeta((s_1, t_1) \wedge (s_2, t_2)) = \zeta(s_1, t_1) \wedge (\alpha(s_2), \alpha(t_2)). \tag{8}$$

Let $(s_1, t_1), (s_2, t_2) \in G_1 \times G_2$. By Definition 6, $\zeta((s_1, t_1) \wedge (s_2, t_2)) = \zeta((s_1 \wedge s_2), (t_1 \wedge t_2))$. By using equation (7) and Definition 1, we have $\zeta((s_1, t_1) \wedge (s_2, t_2)) = (0, (\alpha \quad (t_1) \wedge \alpha(t_1))) = (0 \wedge \alpha(s_2), \alpha(t_1) \wedge \alpha(t_2))$. By Definition 6 and equation (7), we have $\zeta((s_1, t_1) \wedge (s_2, t_2)) = (0, \alpha(t_1)) \wedge (\alpha(s_2), \alpha(t_2)) = \zeta(s_1, t_1) \wedge (\alpha(s_2), \alpha(t_2))$. This implies that ζ is an α-multiplier with pointwise operation on $G_1 \times G_2$. □

Theorem 2. *Let G be an almost distributive lattice and $B(G)$ be the set of all α-multipliers of G. Then, $B(G)$ under binary operations $\vee$ and $\wedge$ is an almost distributive lattice, where for any ζ_1, $\zeta_2 \in B(G)$, $h_3 \in G$.*

$$\begin{aligned} (\zeta_1 \wedge \zeta_2)(h_3) &= \zeta_1(h_3) \wedge \zeta_2(h_3), \\ (\zeta_1 \vee \zeta_2)(h_3) &= \zeta_1(h_3) \vee \zeta_2(h_3). \end{aligned} \tag{9}$$

Proof. Let ζ_1, $\zeta_2 \in B(G)$. Then, by equation (9), we have $((\zeta_1 \wedge \zeta_2)(h_3 \wedge \omega_3) = \zeta_1(h_3 \wedge \omega_3) \wedge \zeta_2(h_3 \wedge \omega_3) = (\zeta_1(h_3) \wedge \alpha(\omega_3)) \wedge (\zeta_2(h_3) \wedge \alpha(\omega_3)) = (\zeta_1(h_3) \wedge \zeta_2(h_3)) \wedge \alpha(\omega_3) = (\zeta_1 \wedge \zeta_2)(h_3) \wedge \alpha(\omega_3)$. This implies that $(\zeta_1 \wedge \zeta_2)$ is an α-multiplier. Let ζ_1, $\zeta_2 \in B(G)$, and along with equation (9), we have $(\zeta_1 \vee \zeta_2)(h_3 \wedge \omega_3) = \zeta_1(h_3 \wedge \omega_3) \vee \zeta_2(h_3 \wedge \omega_3) = (\zeta_1(h_3) \wedge \alpha(\omega_3)) \vee (\zeta_2(h_3) \wedge \alpha(\omega_3)) = (\zeta_1(h_3) \vee \zeta_2(h_3)) \wedge$

$\alpha(\omega_3) = (\zeta_1 \vee \zeta_2)(h_3) \wedge \alpha(\omega_3)$. This implies that $(\zeta_1 \vee \zeta_2)$ is an α-multiplier. Hence, $(B(G), \vee, \wedge)$ is closed under $\vee$, $\wedge$. Hence, $(B(G), \vee, \wedge)$ is an ADL. □

Theorem 3. *Let G be an almost distributive lattice and $B(G)$ be the set of all α-multiplier on G. Then, set of all principal α-multiplier $P(G)$ $\{\zeta_x | x \in G\}$ is distributive lattice with the following operation $\zeta_{h_2} \vee \zeta_{\omega_3} = \zeta_{h_2 \vee \omega_3}$ and $\zeta_{h_2} \wedge \zeta_{\omega_3} = \zeta_{h_2 \wedge \omega_3}$ for all h_2, $\omega_3 \in G$.*

Proof. Let h_2, $\omega_3 \in G$. Then, $(\zeta_{h_2} \vee \zeta_{\omega_3})(e_1) = \zeta_{h_2}(e_1) \vee \zeta_{\omega_3}(e_1) = (h_2 \wedge \alpha(e_1)) \vee (\omega_3 \wedge \alpha(e_1)) = (h_2 \vee \omega_3) \wedge \alpha(e_1) = \zeta_{h_2 \vee \omega_3}(e_1)$. For some $e_1 \in G$, it implies that $\zeta_{h_2} \vee \zeta_{\omega_3} = \zeta_{h_2 \vee \omega_3} \in P(G)$. Also $(\zeta_{h_2} \wedge \zeta_{\omega_3})(e_1) = \zeta_{h_2}(e_1) \wedge \zeta_{\omega_3}(e_1) = (h_2 \wedge \alpha(e_1)) \wedge (\omega_3 \wedge \alpha(e_1)) = (h_2 \wedge \omega_3) \wedge \alpha(e_1) = \zeta_{h_2 \wedge \omega_3}(e_1)$. For any $e_1 \in G$, it implies $\zeta_{h_2} \wedge \zeta_{\omega_3} = \zeta_{h_2 \wedge \omega_3} \in P(G)$. Hence, $(P(G), \vee, \wedge)$ is closed, and so $P(G)$ is sub almost distributive lattice. Moreover, for any $e_1 \in G$, $\zeta_{h_2 \wedge \omega_3}(e_1) = (h_2 \wedge \omega_3) \wedge \alpha(e_1) = (\omega_3 \wedge h_2) \wedge \alpha(e_1) = \zeta_{\omega_3} \wedge \zeta_{h_2}$. Thus, $\zeta_{h_2} \wedge \zeta_{\omega_3} = \zeta_{\omega_3} \wedge \zeta_{h_2}$. Hence, $P(G)$ is a distributive lattice. □

4. Conclusion

In this paper, we have generalized the idea of multiplier to α-multiplier in almost distributive lattices and investigated some properties of ADLs. We have also explored some results by using the notion of principal α-multiplier and isotone α-multiplier. This generalized concept played a vital role in exploring different properties of almost distributive lattices.

Data Availability

The data used to support this study are included within this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Ying Wang analyzed the results, drafted the final version of the paper, and arranged funding for this paper. Abdul Rauf Khan and Zafar Ullah proved the results. Zahid Karim and Abid Mahmoob approved the results and supervised this work. Mamoona Karim wrote the first version of the paper.

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