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Retraction

Retracted: Hybrid Decision-Making Frameworks under Complex Spherical Fuzzy *N*-Soft Sets

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Akram, M. Shabir, A. N. Al-Kenani, and J. C. R. Alcantud, "Hybrid Decision-Making Frameworks under Complex Spherical Fuzzy *N*-Soft Sets," *Journal of Mathematics*, vol. 2021, Article ID 5563215, 46 pages, 2021.

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Research Article

Hybrid Decision-Making Frameworks under Complex Spherical Fuzzy N-Soft Sets

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This paper presents the novel concept of complex spherical fuzzy N-soft set ($CSFNS_fS$) which is capable of handling two-dimensional vague information with parameterized ranking systems. First, we propose the basic notions for a theoretical development of $CSFNS_fSs$, including ranking functions, comparison rule, and fundamental operations (complement, union, intersection, sum, and product). Furthermore, we look into some properties of $CSFNS_fSs$. We then produce three algorithms for multiattribute decision-making that take advantage of these elements. We demonstrate their applicability with the assistance of a numerical problem (selection of best third-party app of the year). A comparison with the performance of Pythagorean N-soft sets speaks for the superiority of our approach. Moreover, with an aim to expand the range of techniques for multiattribute group decision-making problems, we design a $CSFNS_f$-TOPSIS method. We use a complex spherical fuzzy N-soft weighted average operator in order to aggregate the decisions of all experts according to the power of the attributes and features of alternatives. We present normalized-Euclidean distances (from the alternatives to both the $CSFNS_f$ positive and negative ideal solutions, respectively) and revised closeness index in order to produce a best feasible alternative. As an illustration, we design a mathematical model for the selection of the best physiotherapist doctor of Mayo hospital, Lahore. We conduct a comparison with the existing complex spherical fuzzy TOPSIS method that confirms the stability of the proposed model and the reliability of its results.

1. Introduction

Multiattribute decision-making (MADM) and multiattribute group decision-making (MAGDM) methods are broad sections in the field of decision-making. Researchers and practitioners have resorted to them in order to evaluate optimal solutions among a finite number of choices under several attributes. For the purpose of improving the flexibility of the evaluations that support the decision-making process, Zadeh [1] proposed fuzzy set (FS) theory that reshaped the field of decision-making and related disciplines such as mathematical social sciences [2, 3]. In FS theory, a membership degree belongs to the interval $[0, 1]$; thus, when assigned to an object, it represents its degree of belongingness to a mathematical object (a fuzzy set); in formal logic, it means a degree of truth. This means an extension of binary valuations, which is, henceforth, referred to as crisp evaluations. Concerning its use for solving MADM and MAGDM problems in fuzzy environments, Song et al. [4] gave an algorithm based on arithmetic operators, and Chen [5] built up a theory for a fuzzy-TOPSIS method. No doubt, FS theory produced a turn of direction in the field of decision-making. However, it was not designed to look at the dissatisfaction nature of humans in decision-making. This drawback prompted Atanassov [6] to present intuitionistic fuzzy sets (IFS) in 1986. They allocate both degrees of satisfaction and dissatisfaction to an object.

Other extensions soon followed. Yager [7] further extended the IFS to Pythagorean fuzzy set (P_yFS) in which the sums of the squares of degree of satisfaction and dissatisfaction should be within the closed unit interval. Later on, Cuong [8] introduced picture fuzzy set (PFS) keeping in view the existence of neutral positions under natural circumstances. For example, in case of voting systems a candidate could be either satisfied, remain neutral, and disagree with any given participant [9]. Many researchers chose this environment for solving decision-making problems, but others pointed out that PFS has the limitation that it is not applicable in situations where the sum of the degrees of satisfaction, neutrality, and dissatisfaction exceeds 1. This is the origin of spherical fuzzy sets and the spherical fuzzy TOPSIS method presented by Gundogdu and Kahraman [10] in 2019. Similarly and also motivated by spherical fuzzy sets, Kahraman et al. [11] used a spherical TOPSIS method to find the best location for hospital. Later on, Mahmood et al. [12] proposed T-spherical fuzzy sets, a generalization of spherical fuzzy sets, which are less restrictive. They overcame all the limitations of the existing models except in the presence of 2-dimensional problems. Such 2-dimensional problems in MADM and MAGDM can now be analyzed with the tool developed by Ramot et al. [13], who introduced complex fuzzy set in which the degree of satisfaction belongs to the complex unit circle and consists of a periodic term as well as the amplitude term which belong to the unit closed interval. Akram and Bashir [14] extended the averaging operators in the framework of complex fuzzy sets. Alkouri and Salleh [15] presented the idea of complex intuitionistic fuzzy set (CIFS), which describes both degree of satisfaction and dissatisfaction within the complex unit circle, where the sum of amplitude and periodic terms of the satisfaction and dissatisfaction degrees should be within the unit interval $[0, 1]$.

Recently, Akram et al. [16] introduced the concept of complex spherical fuzzy set and extended the TOPSIS method to that setting. As an application, a model for the selection of best water supply strategy for Nohoor village in Iran was considered. This novel concept contains degrees of satisfaction, neutrality, and dissatisfaction which lie in the complex unit circle. They are further restricted by the condition that the sum of the squares of their amplitude and phase terms should be less than or equal to 1.

There is a widespread handicap in the aforementioned models and methods: they discard the frameworks that are characterized by the satisfaction of certain attributes or the fulfilment of properties. Soft set theory, launched in 1999, accommodates all type of parameters [17]. Alkouri and Salleh [15] introduced some new operators on soft set theory which soon found applications in the fields of operations research, game theory, stability, regularization, medicine, and obviously in decision-making. Following this trend, researchers brought up many models and methods for soft sets and its extensions, inclusive of a new decision-making method for valuation fuzzy soft sets introduced by Alcantud et al. [18]. Despite these improvements there were still problems in real life that could not be solved using the existing MADM and MAGDM methods, for example, because the objects are evaluated using a ranking system or a nonbinary scale. When we check out from hotels, hotel staff ask for our feedback, which we give, for example, in the form of 4 stars, 3 stars, 2 stars, 1 star, and big dot: 4 stars mean "outstanding," 3 stars mean "superb," 2 stars mean "good," 1 star means "satisfactory," and big dot means "unacceptable." Similarly, nonbinary rates are given to third-party apps, whether we use a transportation service (Uber, Cabify, etc.) or online shopping facilities. As technology improved and extended, people have become accostumed to such types of ranking systems due to their ease of use and widespread utilization. For this reason, many researchers have become interested in formal models for nonbinary evaluations. The idea presented by Fatima et al. [19], namely, N-soft set and their decision-making methods, stirred up new decision-making methodologies. Very soon and keeping in view the possible fuzziness of the parameters, Akram et al. [20] combined the concept of N-soft with a fuzzy definition of the attributes thus producing fuzzy N-soft sets (FNS_fS). This novel prescription involves a finite number of ordered grades as well as fuzziness in the conception of the attributes that are used for decision-making. Still another hybrid model called hesitant N-soft set was introduced by Akram et al. [21] in order to allow for hesitancy in the allocation of grades. Hesitant fuzzy N-soft sets [22] combine the features of these two models. Akram et al. [23] extended the idea of fuzzy N-soft set in another direction. They conceived intuitionistic fuzzy N-soft sets ($IFNS_fS$) that describe the dissatisfactory part separately, with the usual constraint that the sums of the degrees of membership and nonmembership always belong to $[0, 1]$. Finally, so far, Zhang et al. [24] extended $IFNS_fS$ to Pythagorean fuzzy N-soft set ($PFNS_fS$) which is more flexible than the existing models.

The motivation of this article depends on the following facts:

(1) The existing models $IFNS_fS$ and $PFNS_fS$ make decisions based on degrees of membership and nonmembership; however, they are unable to incorporate a neutral part of judgement.

(2) The decision-making techniques based on existing models FNS_fS, $IFNS_fS$, and $PFNS_fS$ can solve only problems of the 1-dimensional type. Neither of these models can operate in the presence of a periodic term or 2-dimensional type problems.

(3) Although CSFSs deal with 2-dimensional problems of real life, they are unable to describe parameterized information as well as finitely many ranked grades of association of the alternatives with the pertinent parameters.

(4) These limitations motivated us to put forward a new model called $CSFNS_fS$ which efficiently deals with abstention (together with degrees of satisfaction and dissatisfaction) as well as the periodic term of 2-dimensional decision-making problems. At the same time, $CSFNS_fS$ competently handles the ordered grades of the alternatives according to the different attributes.

The main contributions of this article are as follows:

(1) The proposed model, $CSFNS_fS$, allows for neutral opinions in the framework of 2-dimensional problems. In this way, it can manipulate conditions on amplitude and periodic terms with more flexibility.
(2) This model establishes a modern theory that captures a new perspective of decision-making. It is based on ratings or ranking systems including ordered grades of elements according to related attributes.
(3) The algorithms and $CSFNS_f$-TOPSIS method defined in this article solve MADM and MAGDM problems, respectively. They apply to more general situations than the existing algorithms and TOPSIS Method. These methods for decision-making under the framework of $CSFNS_f$ are illustrated with numerical examples.
(4) The comparative study with $PFNS_f$ algorithms and the CSF-TOPSIS method shows their ability and significance.

The rest of the paper is organized as follows. Section 2 contains some definitions from existing models. In Section 3, we propose the novel concept of $CSFNS_fS$ which is then followed by the operations on $CSFNS_fSs$ and $CSFNS_fNs$. Section 3 describes three algorithms for making decisions and performs a comparison with a $PFNS_f$ method. In Section 4, we develop a theatrical foundation for the $CSFNS_fS$-TOPSIS method. In Section 5, we present the mathematical algorithms of these decision-making mechanisms that are applied to some numerical examples. Section 6 describes the comparison analysis with CSF-TOPSIS method. In Section 7, we conclude the paper and provide future directions of research.

2. Preliminaries

Definition 1 (see [10]). A *spherical fuzzy set* (SFS) Υ on a universe of discourse U has the form

$$\Upsilon = \langle u, \mu_\Upsilon(u), \eta_\Upsilon(u), \nu_\Upsilon(u) | u \in U\rangle, \tag{1}$$

where $\mu_\Upsilon(u)$, $\eta_\Upsilon(u)$, and $\nu_\Upsilon(u)$, which lie within the unit interval, are called the grade of the positive, neutral, and negative membership, respectively; and they are restricted by the condition $\mu_\Upsilon(u)^2 + \eta_\Upsilon(u)^2 + \nu_\Upsilon(u)^2 \leq 1$, for every $u \in U$. The degree of refusal of u in U is defined as

$$\Theta_\Upsilon(u) = \sqrt{1 - \left(p_\Upsilon(u)^2 + \nu_\Upsilon(u)^2 + r_\Upsilon(u)^2\right)}. \tag{2}$$

The triplet $(\mu_\Upsilon(u), \eta_\Upsilon(u), \nu_\Upsilon(u))$ is called spherical fuzzy number (SFN).

Definition 2 (see [25]). A *complex T-spherical fuzzy set* (*CTSFS*) Υ on the universe U is defined as

$$\Upsilon = \langle (u, \mu_\Upsilon(u), \eta_\Upsilon(u), \nu_\Upsilon(u)) | u \in U\rangle, \tag{3}$$

where $\mu_\Upsilon(u) = p_\Upsilon(u)e^{i2\pi\phi_\Upsilon(u)}$, $\eta_\Upsilon(u) = \nu_\Upsilon(u)e^{i2\pi\delta_\Upsilon(u)}$, and $\nu_\Upsilon(u) = r_\Upsilon(u)e^{i2\pi\lambda_\Upsilon(u)}$, which denote the positive, neutral, and negative degree of membership, respectively. They are restricted by the conditions $p_\Upsilon(u)^q + \nu_\Upsilon(u)^q + r_\Upsilon(u)^q \leq 1$ and $\phi_\Upsilon(u)^q + \delta_\Upsilon(u)^q + \lambda_\Upsilon(u)^q \leq 1$, for each $u \in U$, where $i = \sqrt{-1}$, and p_Υ, ν_Υ, r_Υ, ϕ_Υ, δ_Υ, $\lambda_\Upsilon \in [0, 1]$. The degree of refusal of u in U is defined as

$$\begin{aligned}\Pi_\Upsilon(u) = {} & \sqrt{1 - \left(p_\Upsilon(u)^q + \nu_\Upsilon(u)^q + r_\Upsilon(u)^q\right)} \\ & \cdot e^{i2\pi\sqrt{1-\left(\phi_\Upsilon(u)^q+\delta_T(u)^q+\lambda_\Upsilon(u)^q\right)}}.\end{aligned} \tag{4}$$

The triplet $(\mu_\Upsilon, \eta_\Upsilon, \nu_\Upsilon) = (p_\Upsilon e^{i2\pi\phi_\Upsilon}, \nu_\Upsilon e^{i2\pi\delta_\Upsilon}, r_\Upsilon e^{i2\pi\lambda_\Upsilon})$ is called *CTSFN*.

Particular case: When $T = 2$, a *CTSFS* becomes a complex spherical fuzzy set (CSFS).

Definition 3 (see [26]). Let W be a nonempty set and R be a set of attributes and $Z \subseteq R$. A *soft set* S_fS over W is a pair (Γ, Z), where Γ is a set-valued function from Z to the set of all subsets of W, which is denoted as

$$(\Gamma, Z) = \left\{\langle z, \Gamma(z)\rangle | z \in Z, \Gamma(z) \in 2^W\right\}. \tag{5}$$

Definition 4. Let W be a nonempty set and R be a set of attributes, $Z \subseteq R$. A *complex spherical fuzzy soft set* ($CSFS_fS$) over W is a pair (Λ, Z), where Λ is a function from Z to the set of all subsets of CSFSs of W, which is denoted as

$$\begin{aligned}(\Lambda, Z) &= \left\{\langle z, \Lambda(z)\rangle | z \in Z, \Lambda(z) \in CSFS^W\right\} \\ &= \left\{\langle z, (w, (\mu_z(w), \eta_z(w), \nu_z(w)))\rangle\right\} \\ &= \left\{\langle z, \left(w, p_z(w)e^{i2\pi\phi_z(w)}, \nu_z(w)e^{i2\pi\delta_z(w)}, r_z(w)e^{i2\pi\lambda_z(w)}\right)\rangle\right\},\end{aligned} \tag{6}$$

where p_z, ν_z, r_z, $\phi_z, \delta_z, \lambda_z \in [0, 1]$ are restricted by the conditions

$$\begin{aligned}&0 \leq p_z(w)^2 + \nu_z(w)^2 + r_z(w)^2 \leq 1, \\ &0 \leq \phi_z(w)^2 + \delta_z(w)^2 + \lambda_z(w)^2 \leq 1, \\ &\forall w \in W.\end{aligned} \tag{7}$$

Definition 5 (see [19]). Let W be a nonempty set and R be a set of attributes. Let $Z \subseteq R$ and $G = \{0, 1, 2, \ldots, N - 1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple (F, Z, N) is called *N-soft set* (NS_fS) over W if F is a mapping from Z to $2^{U\times G}$, with the property that, for each $z \in Z$ and $w \in W$, there exist a unique $(w, g_z^w) \in W \times G$ such that $(w, g_z^w) \in F(z)$, $w \in W$, $g_z^w \in G$ [27–40].

3. Complex Spherical Fuzzy N-Soft Sets

Definition 6. Let W be a nonempty set and R be a set of attributes. Let $Z \subseteq R$ and $G = \{0, 1, 2, \ldots, N - 1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple (F_J, Z, N) is called a *complex spherical fuzzy N-soft set* ($CSFNS_fS$) on Z, when (F, Z, N) $(F\colon Z \longrightarrow 2^{W\times G})$ is an NS_fS on W, if $F_J\colon Z \longrightarrow 2^{W\times G} \times \text{CSFN}$ is a mapping, which is defined as

$$
\begin{aligned}
\left(F_J, Z, N\right) &= \left\{\langle z, F(z), J(z)\rangle | z \in Z, \left(F(z), J(z)\right) \in 2^{W\times G} \times \text{CSFN}\right\} \\
&= \left\{\langle z, \left((w, g_z^w), \left(\mu_z(w), \eta_z(w), \nu_z(w)\right)\right)\rangle\right\} \\
&= \left\{\langle z, \left((w, g_z^w), p_z(w)e^{i2\pi\phi_z(w)}, \nu_z(w)e^{i2\pi\delta_z(w)}, r_z(w)e^{i2\pi\lambda_z(w)}\right)\rangle\right\},
\end{aligned} \tag{8}
$$

where $J\colon Z \longrightarrow \text{CSFN}$, CSFN denotes the collection of all complex spherical fuzzy numbers of W, g_z^w denotes the level of attribute for the element w and $p_z, \nu_z, r_z, \phi_z, \delta_z, \lambda_z \in [0,1]$, restricted with conditions

$$
\begin{aligned}
&0 \le p_z(w)^2 + \nu_z(w)^2 + r_z(w)^2 \le 1, \\
&0 \le \phi_z(w)^2 + \delta_z(w)^2 + \lambda_z(w)^2 \le 1, \quad \text{for all } w \text{ belongs to } W.
\end{aligned} \tag{9}
$$

Definition 7. Let $F_J(z_k) = ((w_j, g_k^j), p_{kj}e^{i2\pi\phi_{kj}}, \nu_{kj}e^{i2\pi\delta_{kj}} r_{kj}e^{i2\pi\lambda_{kj}})$ be a $CSFNS_fS$. Then, the *complex spherical fuzzy N-soft number* ($CSFNS_fN$) is defined as

$$
\begin{aligned}
\Upsilon_{kj} &= \left(g_k^j, p_{kj}e^{i2\pi\phi_{kj}}, \nu_{kj}e^{i2\pi\delta_{kj}}, r_{kj}e^{i2\pi\lambda_{kj}}\right), \\
\Omega_{\Upsilon_{kj}} &= \sqrt{1-\left(p_{kj}^2 + w_{kj}^2 + r_{kj}^2\right)}\, e^{i2\pi\sqrt{1-\left(\phi_{kj}^2+\delta_{kj}^2+\lambda_{kj}^2\right)}},
\end{aligned} \tag{10}
$$

is the hesitancy degree, where $p_{kj}, \nu_{kj}, r_{kj}, \phi_{kj}, \delta_{kj}$, and λ_{kj} represent $p_{z_k}(w_j), \nu_{z_k}(w_j), r_{z_k}(w_j), \phi_{z_k}(w_j), \delta_{z_k}(w_j)$, and $\lambda_{z_k}(w_j)$, respectively.

Definition 8. Consider a $CSFNS_fN\, \Upsilon_{kj} = (g_k^j, p_{kj}e^{i2\pi\phi_{kj}}, \nu_{kj}e^{i2\pi\delta_{kj}} r_{kj}e^{i2\pi\lambda_{kj}})$. The score function $S(\Upsilon_{kj})$ is

$$
S_{\Upsilon_{kj}} = \left(\frac{g_k^j}{N-1}\right)^2 + \left(p_{kj}^2 - w_{kj}^2 - r_{kj}^2\right) + \left[\phi_{kj}^2 - \delta_{kj}^2 - \lambda_{kj}^2\right], \tag{11}
$$

where $S_{\Upsilon_{kj}} \in [-2, 3]$. The accuracy function $A(\Upsilon_{kj})$ is

$$
A_{\Upsilon_{kj}} = \left(\frac{g_k^j}{N-1}\right)^2 + \left(p_{kj}^2 + w_{kj}^2 + r_{kj}^2\right) + \left[\phi_{kj}^2 + \delta_{kj}^2 + \lambda_{kj}^2\right], \tag{12}
$$

where $A_{\Upsilon_{kj}} \in [0, 3]$, respectively.

Definition 9. Let $\Upsilon_{lj} = (g_l^j, p_{lj}e^{i2\pi\phi_{lj}}, \nu_{lj}e^{i2\pi\delta_{lj}}, r_{lj}e^{i2\pi\lambda_{lj}})$ and $\Upsilon_{kj} = (g_k^j, p_{kj}e^{i2\pi\phi_{kj}}, \nu_{kj}e^{i2\pi\delta_{kj}}, r_{kj}e^{i2\pi\lambda_{kj}})$ be two $CSFNS_fNs$:

(1) If $S_{\Upsilon_{lj}} < S_{\Upsilon_{kj}}$, then $\Upsilon_{lj} \prec \Upsilon_{kj}$ (Υ_{lj} is inferior to Υ_{kj}).

(2) If $S_{\Upsilon_{lj}} > S_{\Upsilon_{kj}}$, then $\Upsilon_{lj} \succ \Upsilon_{kj}$ (Υ_{lj} is superior to Υ_{kj}).

(3) If $S_{\Upsilon_{lj}} = S_{\Upsilon_{kj}}$, then

   (i) $A_{\Upsilon_{lj}} < A_{\Upsilon_{kj}}$, then $\Upsilon_{lj} \prec \Upsilon_{kj}$ (Υ_{lj} is inferior to Υ_{kj})

   (ii) $A_{\Upsilon_{lj}} > A_{\Upsilon_{kj}}$, then $\Upsilon_{lj} \succ \Upsilon_{kj}$ (Υ_{lj} is superior to Υ_{kj})

   (iii) $A_{\Upsilon_{lj}} = A_{\Upsilon_{kj}}$, then $\Upsilon_{lj} \sim \Upsilon_{kj}$ (Υ_{lj} is equivalent to Υ_{kj})

Remark 1. We see that

(1) For $N = 2$, $CSFNS_fS$ becomes complex spherical fuzzy soft set

(2) When $|Z| = 1$, $CSFNS_fS$ becomes complex spherical fuzzy set

(3) When $\phi_z = \delta_z = \lambda_z = 0$, $CSFNS_fS$ becomes spherical fuzzy N-soft set

Example 1. In a city, a parent wants to choose the best school for their child. It is necessary to go after the advice of experts, for the selection of a school based on rankings and ratings. Let $W = \{w_1, w_2, w_3\}$ be the family of three schools under consideration and $Z = \{z_1 = \text{size of school}, z_2 = \text{location}, z_3 = \text{academic performance}, z_4 = \text{services}\}$ be the attributes which are used to assign rankings to schools by the experts. In a relation to these parameters, a 5-soft set is given in Table 1, where

Four diamonds means "Outstanding"

Three diamonds means "Super"

Two diamonds means "Good"

One diamond means "Satisfactory"

Big dot means "Acceptable"

This level assessment by diamonds can be represented by numbers as $G = \{0, 1, 2, 3, 4\}$, where

0 means "•"

1 means "◇"

2 means "◇◇"

3 means "◇◇◇"

4 means "◇◇◇◇"

Table 2 can be adopted as natural convention of 5-soft set model.

By Definition 6, when the data is vague and uncertain, we need $CSFNS_fSs$ which provides us information on how these grades are given to schools. The evaluation of schools by experts follows the following grading:

$$
\begin{aligned}
&\text{when } g_z^w = 0, \quad -2.00 \le S_J < -1.85, \\
&\text{when } g_z^w = 1, \quad -1.85 \le S_J < -1.30, \\
&\text{when } g_z^w = 2, \quad -1.30 \le S_J < 0.15, \\
&\text{when } g_z^w = 3, \quad 0.15 \le S_J < 1.30, \\
&\text{when } g_z^w = 4, \quad 1.30 \le S_J < 2.00.
\end{aligned} \tag{13}
$$

According to the above criteria, we can obtain Table 3. At last, $CSF5S_fS$ is defined as

Table 1: Evaluation data provided by the experts.

W/Z	z_1	z_2	z_3	z_4
w_1	◇◇	◇◇◇	•	◇◇
w_2	•	◇◇◇	◇	◇
w_3	◇◇◇◇	◇	◇◇◇	◇◇

Table 2: Tabular representation of 5-soft set.

W/Z	z_1	z_2	z_3	z_4
w_1	2	3	0	2
w_2	0	3	1	1
w_3	4	1	3	2

Table 3: Grading criteria.

g_z^w/J	Positive membership		Neutral membership		Negative membership	
Grades	p_z	$2\pi\phi_z$	v_z	$2\pi\delta_z$	r_z	$2\pi\lambda_z$
$g_z^w = 0$	$[0, 0.15]$	$[0, 0.3\pi)$	$[0, 0.017)$	$[0, 0.034\pi)$	$[0.98, 1)$	$[1.96\pi, 2\pi]$
$g_z^w = 1$	$(0.15, 0.35)$	$[0.3\pi, 0.7\pi)$	$[0.017, 0.1031)$	$[0.034\pi, 0.2062\pi)$	$(0.88, 0.98)$	$[1.76\pi, 1.96\pi)$
$g_z^w = 2$	$[0.35, 0.65)$	$[0.7\pi, 1.3\pi)$	$[0, 0.017)$	$[0, 0.034\pi)$	$(0.58, 0.88]$	$[1.16\pi, 1.76\pi)$
$g_z^w = 3$	$[0.65, 0.85]$	$[1.3\pi, 1.7\pi]$	$[0.017, 0.1031)$	$[0.034\pi, 0.2062\pi)$	$[0.28, 0.58]$	$[0.56\pi, 1.16\pi)$
$g_z^w = 4$	$(0.85, 1]$	$(1.7\pi, 2\pi]$	$[0, 0.017)$	$[0, 0.034\pi)$	$[0, 0.28)$	$[0, 0.56\pi)$

$$
\begin{aligned}
\left(\mu_{z_1}, \eta_{z_1}, \nu_{z_1}\right) &= \Big\{\left((w_1, 2), \left(0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi}\right)\right), \left((w_2, 0), \left(0.02e^{i0.06\pi}, 0.012e^{i0.026\pi}, 0.98e^{i1.962\pi}\right)\right),\\
&\quad ((w_3, 4), (1, 0, 0))\Big\},\\
\left(\mu_{z_2}, \eta_{z_2}, \nu_{z_2}\right) &= \Big\{\left((w_1, 3), \left(0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi}\right)\right), \left((w_2, 3), \left(0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi}\right)\right),\\
&\quad \left((w_3, 1), \left(0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{1.784\pi}\right)\right)\Big\},\\
\left(\mu_{z_3}, \eta_{z_3}, \nu_{z_3}\right) &= \Big\{\left((w_1, 0), \left(0.1e^{i0.24\pi}, 0.012e^{i0.022\pi}, 0.985e^{i1.964\pi}\right)\right), \left((w_2, 1), \left(0.2e^{i0.36\pi}, 0.027e^{i0.05\pi}, 0.91e^{i1.824\pi}\right)\right),\\
&\quad \left((w_3, 3), \left(0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}\right)\right)\Big\},\\
\left(\mu_{z_4}, \eta_{z_4}, \nu_{z_4}\right) &= \Big\{\left((w_1, 2), \left(0.5e^{i1.1\pi}, 0.1e^{i0.18\pi}, 0.59e^{1.28i\pi}\right)\right), \left((w_2, 1), \left(0.3e^{i0.56\pi}, 0.019e^{i0.042\pi}, 0.885e^{1.72i\pi}\right)\right),\\
&\quad \left((w_3, 2), \left(0.45e^{i0.86\pi}, 0.015e^{i0.022\pi}, 0.78e^{i1.566\pi}\right)\right)\Big\}.
\end{aligned}
\tag{14}
$$

The tabular representation of $CSF5S_fS$ is shown by Table 4.

Definition 10. A $CSFNS_fS$ (F_J, Z, N) over a nonempty set W is said to be *efficient*, where (F, Z, N) is an NS_fS if $F_J(z) = \langle (w, N-1), 1, 0, 0\rangle$ for some $z \in Z$, $w \in W$.

Example 2. Let $(F_J, Z, 5)$ be $CSF5S_fS$, as in Example 1. It is easy to check from Table 4 that $F_J(z_1) = ((w_3, 4), 1, 0, 0)$, i.e., Example 1 is efficient.

Definition 11. Let (F_J, Z, N_1) and (H_A, B, N_2) be two $CSFNS_fSs$ on a universe of discourse W. Then, they are said to be *equal* if and only if $F = H$, $J = A$, $Z = B$, and $N_1 = N_2$.

Definition 12. Let (F_J, Z, N) be $CSFNS_fS$ on W. The *weak complement* of $CSFNS_fS$ is defined as the weak complement of the N-soft set (F, Z, N), that is, any N-soft set such that $F^c(z) \cap F(z) = \varnothing$ for all $z \in Z$. The weak complement of $CSFNS_fS$ of (F_J, Z, N) is represented as (F_J^c, Z, N).

Example 3. Let $(F_J, Z, 5)$ be $CSF5S_fS$, as in Example 1. The weak complement (F_J^c, Z, N) is given in Table 5.

Definition 13. Let (F_J, Z, N) be $CSFNS_fS$ on W. The *complex spherical fuzzy complement* of $CSFNS_fS$ is denoted as (F_{J^c}, Z, N) and is defined as

Table 4: Tabular representation of the $CSF5S_fS$ $(F_J, Z, 5)$.

$(F_J, Z, 5)$	z_1	z_2	z_3	z_4
w_1	$(2, (0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi}))$	$(3, (0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi}))$	$(0, (0.1e^{i0.24\pi}, 0.012e^{i0.022\pi}, 0.985e^{i1.964\pi}))$	$(2, (0.5e^{i1.1\pi}, 0.1e^{i0.18\pi}, 0.59e^{1.28i\pi}))$
w_2	$(0, (0.02e^{i0.06\pi}, 0.012e^{i0.026\pi}, 0.98e^{i1.962\pi}))$	$(3, (0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi}))$	$(1, (0.2e^{i0.36\pi}, 0.027e^{i0.05\pi}, 0.91e^{i1.824\pi}))$	$(1, (0.3e^{i0.56\pi}, 0.019e^{i0.042\pi}, 0.885e^{1.72i\pi}))$
w_3	$(4, (1, 0, 0))$	$(1, (0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{1.784\pi}))$	$(3, (0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}))$	$(2, (0.45e^{i0.86\pi}, 0.015e^{i0.022\pi}, 0.78e^{i1.566\pi}))$

TABLE 5: A weak complement of the $CSF5S_fS$ $(F_J, Z, 5)$ in Example 1.

$(F_J^c, Z, 5)$	z_1	z_2	z_3	z_4
w_1	$(3, (0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi}))$	$(0, (0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi}))$	$(1, (0.1e^{i0.24\pi}, 0.012e^{i0.022\pi}, 0.985e^{i1.964\pi}))$	$(3, (0.5e^{i1.1\pi}, 0.1e^{i0.18\pi}, 0.59e^{1.28i\pi}))$
w_2	$(3, (0.02e^{i0.06\pi}, 0.012e^{i0.026\pi}, 0.98e^{i1.962\pi}))$	$(1, (0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi}))$	$(4, (0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}))$	$(2, (0.3e^{i0.56\pi}, 0.019e^{i0.042\pi}, 0.885e^{1.72i\pi}))$
w_3	$(2, (1, 0, 0))$	$(3, (0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{1.784\pi}))$	$(1, (0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}))$	$(4, (0.45e^{i0.86\pi}, 0.015e^{i0.022\pi}, 0.78e^{i1.566\pi}))$

$$F_{J^c}(z) = \langle \nu_z(w), \eta_z(w), \mu_z(w) \rangle = \langle (w, g_z^w)\left(r_z(w)e^{i2\pi\lambda_z(w)}, \nu_z(w)e^{i2\pi\delta_z(w)}, p_z(w)e^{i2\pi\phi_z(w)}\right)\rangle. \tag{15}$$

Example 4. Let $(F_J, Z, 5)$ be $CSF5S_fS$, as in Example 1. The complex spherical fuzzy complement (F_{J^c}, Z, N) is given in Table 6.

Definition 14. Let (F_J, Z, N) be a $CSFNS_fS$ on W. $(F_{J^c}^c, Z, N)$ is referred to as a *weak complex spherical fuzzy complement* of (F_J, Z, N) if and only if (F_J^c, Z, N) is a weak complement and (F_{J^c}, Z, N) is a complex spherical fuzzy complement of (F_J, Z, N).

Example 5. Let $(F_J, Z, 5)$ be $CSF5S_fS$, as in Example 1. The weak complex spherical fuzzy complement $(F_{J^c}^c, Z, N)$ is given in Table 7.

Definition 15. Let (F_J, Z, N) be a $CSFNS_fS$ on W; then, the *top weak complex spherical fuzzy complement* $(F_J^{>}, Z, N)$ is defined as

$$\left(F_J^{>}, Z, N\right) = \begin{cases} F_J(z_k) = \left\langle \left(w_j, N-1\right), r_{z_k}\left(w_j\right)e^{i2\pi\lambda_{z_k}\left(w_j\right)}, \nu_{z_k}\left(w_j\right)e^{i2\pi\delta_{z_k}\left(w_j\right)} p_{z_k}\left(w_j\right)e^{i2\pi\phi_{z_k}\left(w_j\right)} \right\rangle, & \text{if } g_k^j < N-1, \\ F_J(z_k) = \left\langle \left(w_j, 0\right), r_{z_k}\left(w_j\right)e^{i2\pi\lambda_{z_k}\left(w_j\right)}, \nu_{z_k}\left(w_j\right)e^{i2\pi\delta_{z_k}\left(w_j\right)} p_{z_k}\left(w_j\right)e^{i2\pi\phi_{z_k}\left(w_j\right)} \right\rangle, & \text{if } g_k^j = N-1. \end{cases} \tag{16}$$

Example 6. Let $(F_J, Z, 5)$ be $CSF5S_fS$, as in Example 1. The top weak complex spherical fuzzy complement $(F_J^{>}, Z, N)$, is given in Table 8.

Definition 16. Let (F_J, Z, N) be $CSFNS_fS$ on W; then, the *bottom weak complex spherical fuzzy complement* $(F_J^{<}, Z, N)$ is defined as

$$\left(F_J^{<}, Z, N\right) = \begin{cases} F_J(z_k) = \left\langle \left(w_j, 0\right), r_{z_k}\left(w_j\right)e^{i2\pi\lambda_{z_k}\left(w_j\right)}, \nu_{z_k}\left(w_j\right)e^{i2\pi\delta_{z_k}\left(w_j\right)} p_{z_k}\left(w_j\right)e^{i2\pi\phi_{z_k}\left(w_j\right)} \right\rangle, & \text{if } g_k^j > 0, \\ F_J(z_k) = \left\langle \left(w_j, N-1\right), r_{z_k}\left(w_j\right)e^{i2\pi\lambda_{z_k}\left(w_j\right)}, \nu_{z_k}\left(w_j\right)e^{i2\pi\delta_{z_k}\left(w_j\right)} p_{z_k}\left(w_j\right)e^{i2\pi\phi_{z_k}\left(w_j\right)} \right\rangle, & \text{if } g_k^j = 0. \end{cases} \tag{17}$$

Example 7. Let $(F_J, Z, 5)$ be $CSF5S_fS$, as in Example 1. The bottom weak complex spherical fuzzy complement $(F_J^{<}, Z, N)$ is given in Table 9.

Definition 17. Let W be a nonempty set and (F_J, Z, N_1) and (H_A, B, N_2) be $CSFN_1S_fS$ and $CSFN_2S_fSs$ on W, respectively, and their *restricted intersection* is defined as $(K_L, M, S) = (F_J, Z, N_1) \cap_R (H_A, B, N_2)$, where $K_L = F_J \cap_R H_A$, $M = Z \cap B$, $S = \min(N_1, N_2)$, i.e., $\forall u_k \in M$ and $w_J \in W$, $(g_k^j, (\mu_{ij}, \eta_{ij}, \nu_{ij})) \in K_L(u_j)$, $g_k^j = \min(g_k^1, g_k^2)$, $\mu_{ij}(u_j) = \min(\mu_{ij}^1(u_k^1), \mu_{ij}^2(u_k^2)) = \min(p_{ij}^1(u_k^1), p_{ij}^2(u_k^2)) e^{i2\pi(\min(\phi_{ij}^1(u_k^1), \phi_{ij}^2(u_k^2)))}$, $\eta_{ij}(u_j) = \max(\eta_{ij}^1(u_k^1), \eta_{ij}^2(u_k^2)) = \max(\nu_{ij}^1(u_k^1), \nu_{ij}^2(u_k^2))e^{i2\pi(\max(\delta_{ij}^1(u_k^1), \delta_{ij}^2(u_k^2)))}$, $\nu_{ij}(u_j) = \max(\nu_{ij}^1(u_k^1), \nu_{ij}^2(u_k^2)) = \max(r_{ij}^1(u_k^1), r_{ij}^2(u_k^2))e^{i2\pi(\max(\lambda_{ij}^1(u_k^1), \lambda_{ij}^2(u_k^2)))}$, where, $(g_k^1, (\mu_{ij}^1(u_k^1), \eta_{ij}^1(u_k^1), \nu_{ij}^1(u_k^1))) \in (\mu_{F_J}(u_k^1), \eta_{F_J}(u_k^1), \nu_{F_J}(u_k^1))$, and $(g_k^2, (\mu_{ij}^2(u_k^2), \eta_{ij}^2(u_k^2), \nu_{ij}^2(u_k^2))) \in (\mu_{H_A}(u_k^2), \eta_{H_A}(u_k^2), \nu_{H_A}(u_k^2))$, with $u_k^1 \in Z$ and $u_k^2 \in B$.

Example 8. Let $(E_P, Z, 5)$ and $(H_A, B, 6)$ be two $CSF5S_fS$ and $CSF6S_fS$, given in Tables 10 and 11, respectively. Their restricted intersection $(K_L, M, 5) = (E_P, Z, 5) \cap_R (H_A, B, 6)$ is shown in Table 12.

Definition 18. Let W be a nonempty set and (F_J, Z, N_1) and (H_A, B, N_2) be two $CSFNS_fSs$ on W; their *extended intersection* is defined as $(\mathcal{Q}_D, C, Y) = (F_J, Z, N_1) \cap_E (H_A, B, N_2)$, where $\mathcal{Q}_D = F_J \cap_E H_A$, $C = Z \cup B$, and $Y = \max(N_1, N_2)$, that is, $\forall u_k \in C$ and $w_j \in W$, $(g_k^j, (\mu_{kj}, \eta_{kj}, \nu_{kj})) \in \mathcal{Q}_D(u_k)$, with

Table 6: The complex spherical fuzzy complement (F_{J^c}, Z, N) of the $CSF5S_fS$ in Example 1.

$(F_{J^c}, Z, 5)$	z_1	z_2	z_3	z_4
w_1	$(2, (0.6e^{i1.22\pi}, 0.017e^{i0.0356\pi}, 0.4e^{i0.82\pi}))$	$(3, (0.28e^{i0.58\pi}, 0.018e^{i0.038\pi}, 0.65e^{i1.32\pi}))$	$(0, (0.985e^{i1.964\pi}, 0.012e^{i0.022\pi}, 0.1e^{i0.24\pi}))$	$(2, (0.59e^{1.28i\pi}, 0.1e^{i0.18\pi}, 0.5e^{i1.1\pi}))$
w_2	$(0, (0.98e^{i1.962\pi}, 0.012e^{i0.026\pi}, 0.02e^{i0.06\pi}))$	$(3, (0.3e^{i0.56\pi}, 0.019e^{i0.04\pi}, 0.7e^{i1.42\pi}))$	$(1, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(1, (0.885e^{1.72i\pi}, 0.019e^{i0.042\pi}, 0.3e^{i0.56\pi}))$
w_3	$(4, (0, 0, 1))$	$(1, (0.89e^{1.784\pi}, 0.1e^{i0.204\pi}, 0.16e^{i0.34\pi}))$	$(3, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(2, (0.78e^{i1.566\pi}, 0.015e^{i0.022\pi}, 0.45e^{i0.86\pi}))$

Table 7: The weak complex spherical fuzzy complement $(F^c_{J^c}, Z, N)$ of the $CSF5S_fS$ in Example 1.

$(F^c_{J^c}, Z, 5)$	z_1	z_2	z_3	z_4
w_1	$(3, (0.6e^{i1.22\pi}, 0.017e^{i0.0356\pi}, 0.4e^{i0.82\pi}))$	$(0, (0.28e^{i0.58\pi}, 0.018e^{i0.038\pi}, 0.65e^{i1.32\pi}))$	$(1, (0.985e^{i1.964\pi}, 0.012e^{i0.022\pi}, 0.1e^{i0.24\pi}))$	$(3, (0.59e^{1.28i\pi}, 0.1e^{i0.18\pi}, 0.5e^{i1.1\pi}))$
w_2	$(3, (0.98e^{i1.962\pi}, 0.012e^{i0.026\pi}, 0.02e^{i0.06\pi}))$	$(1, (0.3e^{i0.56\pi}, 0.019e^{i0.04\pi}, 0.7e^{i1.42\pi}))$	$(4, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(2, (0.885e^{1.72i\pi}, 0.019e^{i0.042\pi}, 0.3e^{i0.56\pi}))$
w_3	$(2, (0, 0, 1))$	$(3, (0.89e^{1.784\pi}, 0.1e^{i0.204\pi}, 0.16e^{i0.34\pi}))$	$(1, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(4, (0.78e^{i1.566\pi}, 0.015e^{i0.022\pi}, 0.45e^{i0.86\pi}))$

Table 8: The top weak complex spherical fuzzy complement $(F_J^{>}, Z, N)$ of the $CSF5S_fS$ set in Example 1.

$(F_J^{>}, Z, 5)$	z_1	z_2	z_3	z_4
w_1	$(4, (0.6e^{i1.22\pi}, 0.017e^{i0.0356\pi}, 0.4e^{i0.82\pi}))$	$(4, (0.28e^{i0.58\pi}, 0.018e^{i0.038\pi}, 0.65e^{i1.32\pi}))$	$(4, (0.985e^{i1.964\pi}, 0.012e^{i0.022\pi}, 0.1e^{i0.24\pi}))$	$(4, (0.59e^{1.28i\pi}, 0.1e^{i0.18\pi}, 0.5e^{i1.1\pi}))$
w_2	$(4, (0.98e^{i1.962\pi}, 0.012e^{i0.026\pi}, 0.02e^{i0.06\pi}))$	$(4, (0.3e^{i0.56\pi}, 0.019e^{i0.04\pi}, 0.7e^{i1.42\pi}))$	$(4, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(4, (0.885e^{1.72i\pi}, 0.019e^{i0.042\pi}, 0.3e^{i0.56\pi}))$
w_3	$(0, (0, 0, 1))$	$(4, (0.89e^{1.784\pi}, 0.1e^{i0.204\pi}, 0.16e^{i0.34\pi}))$	$(4, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(4, (0.78e^{i1.566\pi}, 0.015e^{i0.022\pi}, 0.45e^{i0.86\pi}))$

Table 9: The bottom weak complex spherical fuzzy complement $(F_J^{<}, Z, N)$ of the $CSF5S_fS$ in Example 1.

$(F_J^{<}, Z, 5)$	z_1	z_2	z_3	z_4
w_1	$(0, (0.6e^{i1.22\pi}, 0.017e^{i0.0356\pi}, 0.4e^{i0.82\pi}))$	$(0, (0.28e^{i0.58\pi}, 0.018e^{i0.038\pi}, 0.65e^{i1.32\pi}))$	$(4, (0.985e^{i1.964\pi}, 0.012e^{i0.022\pi}, 0.1e^{i0.24\pi}))$	$(0, (0.59e^{1.28i\pi}, 0.1e^{i0.18\pi}, 0.5e^{i1.1\pi}))$
w_2	$(4, (0.98e^{i1.962\pi}, 0.012e^{i0.026\pi}, 0.02e^{i0.06\pi}))$	$(0, (0.3e^{i0.56\pi}, 0.019e^{i0.04\pi}, 0.7e^{i1.42\pi}))$	$(0, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(0, (0.885e^{1.72i\pi}, 0.019e^{i0.042\pi}, 0.3e^{i0.56\pi}))$
w_3	$(0, (0, 0, 1))$	$(0, (0.89e^{1.784\pi}, 0.1e^{i0.204\pi}, 0.16e^{i0.34\pi}))$	$(0, (0.32e^{0.62\pi}, 0.101e^{i0.204\pi}, 0.69e^{i1.384\pi}))$	$(0, (0.78e^{i1.566\pi}, 0.015e^{i0.022\pi}, 0.45e^{i0.86\pi}))$

Table 10: Tabular representation of the $CSF5S_fS$ $(E_P, Z, 5)$.

$(E_P, Z, 5)$	z_1	z_2	z_3
w_1	$(2, (0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi}))$	$(3, (0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi}))$	$(0, (0.1e^{i0.24\pi}, 0.012e^{i0.022\pi}, 0.985e^{i1.964\pi}))$
w_2	$(0, (0.02e^{i0.06\pi}, 0.012e^{i0.026\pi}, 0.98e^{i1.962\pi}))$	$(3, (0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi}))$	$(1, (0.2e^{i0.36\pi}, 0.027e^{i0.05\pi}, 0.91e^{i1.824\pi}))$
w_3	$(4, (1, 0, 0))$	$(1, (0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{1.784\pi}))$	$(3, (0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}))$

Table 11: Tabular representation of $CSF6S_fS$ $(H_A, B, 6)$.

$(H_A, B, 6)$	z_1	z_2	z_6
w_1	$(1, (0.23e^{i0.44\pi}, 0.019e^{i0.036\pi}, 0.92e^{i1.85\pi}))$	$(5, (0.95e^{i1.88\pi}, 0.03e^{i0.062\pi}, 0.14e^{i0.29\pi}))$	$(3, (0.69e^{i1.39\pi}, 0.04e^{i0.084\pi}, 0.65e^{i1.36\pi}))$
w_2	$(0, (0.03e^{i0.1\pi}, 0.015e^{i0.032\pi}, 0.983e^{i1.968\pi}))$	$(2, (0.35e^{i0.66\pi}, 0.014e^{i0.029\pi}, 0.8e^{i1.6\pi}))$	$(4, (0.87e^{1.72\pi}, 0.035e^{i0.068\pi}, 0.93e^{i1.862\pi}))$
w_3	$(2, (0.4e^{i0.82\pi}, 0.012e^{i0.026\pi}, 0.77e^{i1.52\pi}))$	$(3, (0.6e^{1.22\pi}, 0.02e^{i0.06\pi}, 0.49e^{i\pi}))$	$(1, (0.17e^{0.344\pi}, 0.035e^{i0.068\pi}, 0.93e^{i1.862\pi}))$

Table 12: Tabular representation of restricted intersection $(K_L, M, 5)$.

$(K_L, M, 5)$	z_1	z_2
w_1	$(1, (0.23e^{i0.44\pi}, 0.019e^{i0.036\pi}, 0.92e^{i1.85\pi}))$	$(3, (0.65e^{i1.32\pi}, 0.03e^{i0.062\pi}, 0.28e^{i0.58\pi}))$
w_2	$(0, (0.02e^{i0.06\pi}, 0.015e^{i0.032\pi}, 0.983e^{i1.968\pi}))$	$(2, (0.35e^{i0.66\pi}, 0.019e^{i0.04\pi}, 0.8e^{i1.6\pi}))$
w_3	$(2, (0.4e^{i0.82\pi}, 0.012e^{i0.026\pi}, 0.77e^{i1.52\pi}))$	$(1, (0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{i1.784\pi}))$

$$
\mathcal{Q}_D(u_k) = \begin{cases}
\left(g_k^1, \left(\mu_{kj}^1, \eta_{kj}^1, \nu_{kj}^1\right)\right), & \text{if } u_k \in Z - B, \\
\left(g_k^2, \left(\mu_{kj}^2, \eta_{kj}^2, \nu_{kj}^2\right)\right), & \text{if } u_k \in B - Z, \\
\left(g_k^j, \left(\mu_{kj}, \eta_{kj}, \nu_{kj}\right)\right), & \text{such that } g_k^j = \min\left(g_k^1, g_k^2\right), \\
\mu_{kj}(u_k) = \min\left(\mu_{kj}^1\left(u_k^1\right), \mu_{kj}^2\left(u_k^2\right)\right) = \min\left(p_{kj}^1\left(u_k^1\right), p_{kj}^2\left(u_k^2\right)\right) e^{i2\pi\left(\min\left(\phi_{kj}^1\left(u_k^1\right), \phi_{kj}^2\left(u_k^2\right)\right)\right)}, \\
\eta_{kj}(u_k) = \max\left(\eta_{kj}^1\left(u_k^1\right), \eta_{kj}^2\left(u_k^2\right)\right) = \max\left(\nu_{kj}^1\left(u_k^1\right), \nu_{kj}^2\left(u_k^2\right)\right) e^{i2\pi\left(\max\left(\delta_{kj}^1\left(u_k^1\right), \delta_{kj}^2\left(u_k^2\right)\right)\right)}, \\
= \nu_{ij}(u_k) = \max\left(\nu_{kj}^1\left(u_k^1\right), \nu_{kj}^2\left(u_k^2\right)\right) = \max\left(r_{kj}^1\left(u_k^1\right), r_{kj}^2\left(u_k^2\right)\right) e^{i2\pi\left(\min\left(\lambda_{kj}^1\left(u_k^1\right), \lambda_{kj}^2\left(u_k^2\right)\right)\right)}, \\
\text{where } \left(g_k^1, \left(\mu_{kj}^1\left(u_k^1\right), \eta_{kj}^1\left(u_k^1\right), \nu_{kj}^1\left(u_k^1\right)\right)\right) \in \left(\mu_{F_J}\left(u_k^1\right), \eta_{F_J}\left(u_k^1\right), \nu_{F_J}\left(u_k^1\right)\right), \\
\text{and } \left(g_k^2, \left(\mu_{kj}^2\left(u_k^2\right), \eta_{kj}^2\left(u_k^2\right), \nu_{kj}^2\left(u_k^2\right)\right)\right) \in \left(\mu_{H_A}\left(u_k^2\right), \eta_{H_A}\left(u_k^2\right), \nu_{H_A}\left(u_k^2\right)\right), \\
\text{with } u_k^1 \in Z \text{ and } u_k^2 \in B.
\end{cases} \tag{18}
$$

Example 9. Let $(E_P, Z, 5)$ and $(H_A, B, 6)$ be two $CSF5S_fS$ and $CSF6S_fS$, given in Tables 10 and 11, respectively. Their extended intersection $(\mathcal{Q}_D, C, Y) = (E_P, Z, N_1) \cap_E (H_A, B, N_2)$ is shown in Table 13.

Definition 19. Let W be a nonempty set and (F_J, Z, N_1) and (H_A, B, N_2) be two $CSFNS_fSs$ on W; their *restricted union* is defined as $(\mathcal{R}_T, M, S) = (F_J, Z, N_1) \cup_R (H_A, B, N_2)$, where $\mathcal{R}_T = F_J \cup_R H_A$, $M = Z \cap B$, $S = \max(N_1, N_2)$, i.e., $\forall u_k \in M$ and $w_j \in W$, $(g_k^j, (\mu_{kj}, \eta_{kj}, \nu_{kj})) \in \mathcal{R}_T(u_k)$, $\mathbf{g}_k^j = \max(g_k^1, g_k^2)$, $\mu_{kj}(u_k) = \max(\mu_{kj}^1(u_k^1), \mu_{kj}^2(u_k^2)) = \max(p_{kj}^1(u_k^1), p_{kj}^2(u_k^2)) e^{i2\pi(\max(\phi_{kj}^1(u_k^1), \phi_{kj}^2(u_k^2)))}$, $\eta_{kj}(u_k) = \min(\eta_{kj}^1(u_k^1), \eta_{kj}^2(u_k^2)) = \min(\nu_{kj}^1(u_k^1), \nu_{kj}^2(u_k^2)) e^{i2\pi(\min(\delta_{kj}^1(u_k^1), \delta_{kj}^2(u_k^2)))}$, $\nu_{kj}(u_k) = \min(\nu_{kj}^1(u_k^1), \nu_{kj}^2(u_k^2)) = \min(r_{kj}^1(u_k^1), r_{kj}^2(u_k^2)) e^{i2\pi(\min(\lambda_{kj}^1(u_k^1), \lambda_{kj}^2(u_k^2)))}$, where, $(g_k^1, (\mu_{kj}^1(u_k^1), \eta_{kj}^1(u_k^1), \nu_{kj}^1(u_k^1))) \in (\mu_{F_J}(u_k^1), \eta_{F_J}(u_k^1), \nu_{F_J}(u_k^1))$, and $(g_k^2, (\mu_{kj}^2(u_k^2), \eta_{kj}^2(u_k^2), \nu_{kj}^2(u_k^2))) \in (\mu_{H_A}(u_k^2), \eta_{H_A}(u_k^2), \nu_{H_A}(u_k^2))$, with $u_k^1 \in Z$ and $u_k^2 \in B$.

Example 10. Let $(E_P, Z, 5)$ and $(H_A, B, 6)$ be two $CSF5S_fS$ and $CSF6S_fS$, given in Tables 10 and 11, respectively. Their restricted union $(\mathcal{Q}_D, C, Y) = (E_P, Z, N_1) \cap_E (H_A, B, N_2)$ is shown in Table 14.

Definition 20. Let W be a nonempty set (F_J, Z, N_1) and (H_A, B, N_2) be $CSFN_1S_fSs$ and $CSFN_2S_fSs$ on W; their *extended union* is defined as $(\mathfrak{O}_X, C, Y) = (F_J, Z, N_1) \cup_E (H_A, B, N_2)$, where $\mathfrak{O}_X = F_J \cup_E H_A$, $C = Z \cup B$, $Y = \max(N_1, N_2)$, that is, $\forall u_k \in C$ and $w_j \in W$, $(g_k^j, (\mu_{kj}, \eta_{kj}, \nu_{kj})) \in \mathfrak{O}_X(u_k)$, with

Table 13: Tabular representation of extended intersection $(\mathcal{Q}_D, C, 6)$.

$(\mathcal{Q}_D, C, 6)$	z_1	z_2	z_3	z_6
w_1	$(1, (0.23e^{i0.44\pi}, 0.019e^{i0.036\pi}, 0.92e^{i1.85\pi}))$	$(3, (0.65e^{i1.32\pi}, 0.03e^{i0.062\pi}, 0.28e^{i0.58\pi}))$	$(0, (0.1e^{i0.24\pi}, 0.04e^{i0.084\pi}, 0.985e^{i1.964\pi}))$	$(3, (0.69e^{i1.39\pi}, 0.04e^{i0.084\pi}, 0.65e^{i1.36\pi}))$
w_2	$(0, (0.02e^{i0.06\pi}, 0.015e^{i0.032\pi}, 0.983e^{i1.968\pi}))$	$(2, (0.35e^{i0.66\pi}, 0.019e^{i0.04\pi}, 0.8e^{i1.6\pi}))$	$(1, (0.2e^{i0.36\pi}, 0.027e^{i0.05\pi}, 0.91e^{i1.824\pi}))$	$(4, (0.87e^{1.72\pi}, 0.035e^{i0.068\pi}, 0.93e^{i1.862\pi}))$
w_3	$(2, (0.4e^{i0.82\pi}, 0.012e^{i0.026\pi}, 0.77e^{i1.52\pi}))$	$(1, (0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{1.784\pi}))$	$(3, (0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}))$	$(1, (0.17e^{0.344\pi}, 0.035e^{i0.068\pi}, 0.93e^{i1.862\pi}))$

TABLE 14: Tabular representation of restricted union $(\mathscr{R}_T, M, 6)$.

$(\mathscr{R}_T, M, 6)$	z_1	z_2
w_1	$(2, (0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi}))$	$(5, (0.95e^{i1.88\pi}, 0.018e^{i0.038\pi}, 0.14e^{i0.29\pi}))$
w_2	$(0, (0.03e^{i0.1\pi}, 0.012e^{i0.026\pi}, 0.98e^{i1.962\pi}))$	$(3, (0.7e^{i1.42\pi}, 0.014e^{i0.029\pi}, 0.3e^{i0.56\pi}))$
w_3	$(4, (1, 0, 0))$	$(3, (0.6e^{1.22\pi}, 0.02e^{i0.06\pi}, 0.49e^{i\pi}))$

$$\mathfrak{O}_X(u_k) = \begin{cases} \left(g_k^1, \left(\mu_{kj}^1, \eta_{kj}^1, \nu_{kj}^1\right)\right), & \text{if } u_k \in Z - B, \\ \left(g_k^2, \left(\mu_{kj}^2, \eta_{kj}^2, \nu_{kj}^2\right)\right), & \text{if } u_k \in B - Z, \\ \left(g_k^j, \left(\mu_{kj}, \eta_{kj}, \nu_{kj}\right)\right), & \text{such that } g_k^j = \max\left(g_k^1, g_k^2\right), \\ \mu_{kj}(u_k) = \max\left(\mu_{kj}^1\left(u_k^1\right), \mu_{kj}^2\left(u_k^2\right)\right) = \max\left(p_{kj}^1\left(u_k^1\right), p_{kj}^2\left(u_k^2\right)\right)e^{i\left(\max\left(\phi_{kj}^1\left(u_k^1\right), \phi_{kj}^2\left(u_k^2\right)\right)\right)}, \\ \eta_{kj}(u_k) = \min\left(\eta_{kj}^1\left(u_k^1\right), \eta_{kj}^2\left(u_k^2\right)\right) = \min\left(\nu_{kj}^1\left(u_k^1\right), \nu_{kj}^2\left(u_k^2\right)\right)e^{i\left(\min\left(\delta_{kj}^1\left(u_k^1\right), \delta_{kj}^2\left(u_k^2\right)\right)\right)}, \\ \nu_{kj}(u_k) = \min\left(\nu_{kj}^1\left(u_k^1\right), \nu_{kj}^2\left(u_k^2\right)\right) = \min\left(r_{kj}^1\left(u_k^1\right), r_{kj}^2\left(u_k^2\right)\right)e^{i\left(\min\left(\lambda_{kj}^1\left(u_k^1\right), \lambda_{kj}^2\left(u_k^2\right)\right)\right)}, \\ \text{where } \left(g_k^1, \left(\mu_{kj}^1\left(u_k^1\right), \eta_{kj}^1\left(u_k^1\right), \nu_{kj}^1\left(u_k^1\right)\right)\right) \in \left(\mu_{F_J}\left(u_k^1\right), \eta_{F_J}\left(u_k^1\right), \nu_{F_J}\left(u_k^1\right)\right), \\ \text{and } \left(g_k^2, \left(\mu_{kj}^2\left(u_k^2\right), \eta_{kj}^2\left(u_k^2\right), \nu_{kj}^2\left(u_k^2\right)\right)\right) \in \left(\mu_{H_A}\left(u_k^2\right), \eta_{H_A}\left(u_k^2\right), \nu_{H_A}\left(u_k^2\right)\right), \\ \text{with } u_k^1 \in Z \text{ and } u_k^2 \in B. \end{cases} \tag{19}$$

Example 11. Let $(E_P, Z, 5)$ and $(H_A, B, 6)$ be two $CSF5S_fS$ and $CSF6S_fS$, given in Tables 10 and 11, respectively. Their restricted union $(\mathcal{Q}_D, C, Y) = (E_P, Z, N_1) \cap_E (H_A, B, N_2)$ is shown in Table 15.

We state the following properties without their proofs.

Theorem 1. *Let (F_J, Z, N) be $CSFNS_fS$ over a nonempty set W. Then,*

(1) $(F_J, Z, N) \cap_R (F_J, Z, N) = (F_J, Z, N)$
(2) $(F_J, Z, N) \cap_E (F_J, Z, N) = (F_J, Z, N)$
(3) $(F_J, Z, N) \cup_R (F_J, Z, N) = (F_J, Z, N)$
(4) $(F_J, Z, N) \cup_E (F_J, Z, N) = (F_J, Z, N)$

We state the following properties without their proofs.

Theorem 2. *Let (F_J, Z, N_1) and (H_A, B, N_2) be two $CSFNS_fSs$ over the same universe W; then, the absorption properties hold:*

(1) $((F_J, Z, N_1) \cup_E (H_A, B, N_2)) \cap_R (F_J, Z, N_1) = (F_J, Z, N_1)$
(2) $(F_J, Z, N_1) \cup_E ((H_A, B, N_2) \cap_R (F_J, Z, N_1) = (F_J, Z, N_1))$
(3) $((F_J, Z, N_1) \cap_R (H_A, B, N_2)) \cup_E (F_J, Z, N_1) = (F_J, Z, N_1)$
(4) $(F_J, Z, N_1) \cap_R ((H_A, B, N_2) \cup_E (F_J, Z, N_1)) = (F_J, Z, N_1)$

We state the following properties without their proofs.

Theorem 3. *Let (F_J, Z, N_1), (H_A, B, N_2) and $(D_\wp, \mathfrak{R}, N_2)$ be any three $CSFNS_fSs$ over the same universe W; then, the following properties hold:*

(1) $(F_J, Z, N_1) \cup_E (H_A, B, N_2) = (H_A, B, N_2) \cup_E (F_J, Z, N_1)$
(2) $(F_J, Z, N_1) \cup_R (H_A, B, N_2) = (H_A, B, N_2) \cup_R (F_J, Z, N_1)$
(3) $(F_J, Z, N_1) \cap_E (H_A, B, N_2) = (H_A, B, N_2) \cap_E (F_J, Z, N_1)$
(4) $(F_J, Z, N_1) \cap_R (H_A, B, N_2) = (H_A, B, N_2) \cap_R (F_J, Z, N_1)$
(5) $((F_J, Z, N_1) \cup_E (H_A, B, N_2)) \cup_E (D_\wp, \mathfrak{R}, N_3) = (F_J, Z, N_1) \cup_E ((H_A, B, N_2) \cup_E (D_\wp, \mathfrak{R}, N_3))$
(6) $((F_J, Z, N_1) \cup_R (H_A, B, N_2)) \cup_R (D_\wp, \mathfrak{R}, N_3) = (F_J, Z, N_1) \cup_R ((H_A, B, N_2) \cup_R (D_\wp, \mathfrak{R}, N_3))$
(7) $((F_J, Z, N_1) \cap_E (H_A, B, N_2)) \cap_E (D_\wp, \mathfrak{R}, N_3) = (F_J, Z, N_1) \cap_E ((H_A, B, N_2) \cap_E (D_\wp, \mathfrak{R}, N_3))$
(8) $((F_J, Z, N_1) \cap_R (H_A, B, N_2)) \cap_R (D_\wp, \mathfrak{R}, N_3) = (F_J, Z, N_1) \cap_R ((H_A, B, N_2) \cap_R (D_\wp, \mathfrak{R}, N_3))$
(9) $(F_J, Z, N_1) \cup_E ((H_A, B, N_2) \cap_R (D_\wp, \mathfrak{R}, N_3)) = ((F_J, Z, N_1) \cup_E (H_A, B, N_2)) \cap_R ((F_J, Z, N_1) \cup_E (D_\wp, \mathfrak{R}, N_3))$

Table 15: Tabular representation of extended union $(\mathfrak{D}_X, C, 6)$.

$(\mathfrak{D}_X, C, 6)$	z_1	z_2	z_3	z_6
w_1	$(2, (0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi}))$	$(5, (0.95e^{i1.88\pi}, 0.018e^{i0.038\pi}, 0.14e^{i0.29\pi}))$	$(0, (0.1e^{i0.24\pi}, 0.04e^{i0.084\pi}, 0.985e^{i1.964\pi}))$	$(3, (0.69e^{i1.39\pi}, 0.04e^{i0.084\pi}, 0.65e^{i1.36\pi}))$
w_2	$(0, (0.03e^{i0.1\pi}, 0.012e^{i0.026\pi}, 0.98e^{i1.962\pi}))$	$(3, (0.7e^{i1.42\pi}, 0.014e^{i0.029\pi}, 0.3e^{i0.56\pi}))$	$(1, (0.2e^{i0.36\pi}, 0.027e^{i0.05\pi}, 0.91e^{i1.824\pi}))$	$(4, (0.87e^{1.72\pi}, 0.035e^{i0.068\pi}, 0.93e^{i1.862\pi}))$
w_3	$(4, (1, 0, 0))$	$(3, (0.6e^{1.22\pi}, 0.02e^{i0.06\pi}, 0.49e^{i\pi}))$	$(3, (0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi}))$	$(1, (0.17e^{0.344\pi}, 0.035e^{i0.068\pi}, 0.93e^{i1.862\pi}))$

(10) $(F_J, Z, N_1) \cap_E ((H_A, B, N_2) \cup_R (D_\wp, \mathfrak{R}, N_3)) = ((F_J, Z, N_1) \cap_E (H_A, B, N_2)) \cup_R ((F_J, Z, N_1) \cup_E (D_\wp, \mathfrak{R}, N_3))$

(11) $(F_J, Z, N_1) \cup_R ((H_A, B, N_2) \cap_E (D_\wp, \mathfrak{R}, N_3)) = ((F_J, Z, N_1) \cup_R (H_A, B, N_2)) \cap_E ((F_J, Z, N_1) \cup_R (D_\wp, \mathfrak{R}, N_3))$

(12) $(F_J, Z, N_1) \cap_R ((H_A, B, N_2) \cup_E (D_\wp, \mathfrak{R}, N_3)) = ((F_J, Z, N_1) \cap_R (H_A, B, N_2)) \cup_E ((F_J, Z, N_1) \cap_R (D_\wp, \mathfrak{R}, N_3))$

Definition 21. Let (F_J, Z, N) be a $CSFNS_fS$, where (F, Z, N) is NS_fS over the universe W, and $0 < L < N$ be a threshold. $CSFS_fS$ over W associated with (F, Z, N) and L, denoted by (F_J^L, Z), is defined as follows:

$$F_J^L(z) = \begin{cases} \left(\mu_{kj}(z), \eta_{kj}(z), \nu_{kj}(z)\right), & \text{if } (w, g_z^w) \in F(z) \text{ and } g_z^w \geq L, \\ (0, 0, 1), & \text{otherwise.} \end{cases} \tag{20}$$

Example 12. Let (E_J, Z, N) be a $CSF5S_fS$ given in Table 10. Then, $CSFS_fS$ associated with the thresholds 1, 2, 3, and 4 are shown in Tables 16–19, respectively.

Definition 22. Let (F_J, Z, N) be $CSFNS_fS$, where (F, Z, N) is NS_fS over the universe W. Let $0 < L < N$ and $\alpha \in [-2, 2]$ be two thresholds. S_fS over W associated with (F_J, Z, N) and (L, α), denoted by $(F_J^{(L,\alpha)}, Z)$, is defined as follows:

$$F_J^{(L,\alpha)}(z) = \left\{ S_z^{F_J^L}(w) > \alpha \colon w \in W, \forall z \in Z \right\}, \tag{21}$$

where $S_z^{F_J^L}$ represents the score function of $F_J^L(z)$.

Definition 23. Let $T_{lj} = (g_l^j, p_{lj}e^{i2\pi\phi_{lj}}, \nu_{lj}e^{i2\pi\delta_{lj}}, r_{lj}e^{i2\pi\lambda_{lj}})$ and $T_{kj} = (g_k^j, p_{kj}e^{i2\pi\phi_{kj}}, \nu_{kj}e^{i2\pi\delta_{kj}}, r_{kj}e^{i2\pi\lambda_{kj}})$ be two $CSFNS_fNs$ and $\sigma > 0$. Some operation for $CSFNS_fNs$ are

$$\begin{aligned}
\sigma T_{lj} &= \left(g_l^j, \left[1 - \left(1 - p_{lj}^2\right)^\sigma\right] e^{i2\pi\left[1 - \left(1 - \phi_{lj}^2\right)^\sigma\right]}, \nu_{lj}^\sigma e^{i2\pi\delta_{lj}^\sigma}, r_{lj}^\sigma e^{i2\pi\lambda_{lj}^\sigma} \right), \\
T_{lj}^\sigma &= \left(g_l^j, p_{lj}^\sigma e^{i2\pi\phi_{lj}^\sigma}, \left[1 - \left(1 - \nu_{lj}^2\right)^\sigma\right] e^{i2\pi\left[1 - \left(1 - \delta_{lj}^2\right)^\sigma\right]}, \left[1 - \left(1 - r_{lj}^2\right)^\sigma\right] e^{i2\pi\left[1 - \left(1 - \lambda_{lj}^2\right)^\sigma\right]} \right), \\
T_{lj} \oplus T_{kj} &= \left(\max\left(g_l^j, g_k^j\right), \sqrt{p_{lj}^2 + p_{kj}^2 - p_{lj}^2 p_{kj}^2}\, e^{i2\pi\sqrt{\phi_{lj}^2 + \phi_{kj}^2 - \phi_{lj}^2\phi_{kj}^2}}, \nu_{lj}\nu_{kj} e^{i2\pi\delta_{lj}\delta_{kj}}, r_{lj} r_{kj} e^{i2\pi\lambda_{lj}\lambda_{kj}} \right), \\
T_{lj} \otimes T_{kj} &= \left(\min\left(g_l^j, g_k^j\right), p_{lj} p_{kj} e^{i2\pi\phi_{lj}\phi_{kj}}, \sqrt{\nu_{lj}^2 + \nu_{kj}^2 - \nu_{lj}^2 \nu_{kj}^2}\, e^{i2\pi\sqrt{\delta_{lj}^2 + \delta_{kj}^2 - \delta_{lj}^2\delta_{kj}^2}}, \sqrt{r_{lj}^2 + r_{kj}^2 - r_{lj}^2 r_{kj}^2}\, e^{i2\pi\sqrt{\lambda_{lj}^2 + \lambda_{kj}^2 - \lambda_{lj}^2\lambda_{kj}^2}} \right).
\end{aligned} \tag{22}$$

4. $CSFNS_f$-TOPSIS Method for MAGDM

In this section, we combine $CSFNS_fS$ with the TOPSIS method. The main idea of this methodology is the selection of a best alternative using both the positive ideal solution (PIS) and the negative ideal solution (NIS). Therefore, we present the corresponding $CSFNS_f$-TOPSIS method in order to solve MAGDM problems in a $CSFNS_f$ environment under such methodology. The elements and steps of this algorithm for MAGDM are as follows.

Let $W = \{w_1, w_2, w_3, \ldots, w_q\}$ denote the set of alternatives that are evaluated by s experts $\widetilde{E}_1, \widetilde{E}_2, \widetilde{E}_3, \ldots, \widetilde{E}_s$. According to the needs of MAGDM problems, set of m attributes $Z = \{z_1, z_2, z_3, \ldots, z_m\}$ are assigned to these alternatives by the experts. Let $\sigma = (\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_s)^T$ be the weight vector, which represents the weightage of experts such that $\sum_{d=1}^{s} \sigma_d = 1$, where $\sigma_d \in [0, 1]$. The step by step Algorithm 1 of $CSFNS_f$-TOPSIS method is presented in Section 5.1, and its theoretical description is as follows:

Step 1: according to the MAGDM problem and attributes related to the alternatives, each expert assigns ratings to them. There is a linguistic term corresponding with each rating, which could be a number of stars (such as "three stars," "two stars," and "one star" in MAGDM), numerical labels (such as 3 as a label for "high," 2 for "medium," and 0 for "low"). In such a way, NS_fS (F^d, Z, N) is found corresponding to each expert $\widetilde{E}_d$ with $G = \{0, 1, 2, 3, \ldots, N - 1\}$ as the set of grades, where $N \in \{1, 2, 3, \ldots\}$ and $d \in \{1, 2, 3, \ldots, s\}$. Now, $CSFNS_fN$ is assigned by the dth expert $\widetilde{E}_d$, corresponding to each rank in the NS_fS (F^d, Z, N), according to the grading criteria defined for the MAGDM problem. Similarly, we get $s\,CSFNS_fSs$ by s experts, respectively. The complex spherical fuzzy N-soft decision matrix $CSFNS_fDM$ of the dth expert $\widetilde{E}_d$ is as follows:

Table 16: $CSFS_fS$ related with $(E, Z, 5)$ and threshold 1.

$(E^1, Z, 5)$	z_1	z_2	z_3
w_1	$(0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi})$	$(0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi})$	$(0, 0, 1)$
w_2	$(0, 0, 1)$	$(0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi})$	$(0.2e^{i0.36\pi}, 0.027e^{i0.05\pi}, 0.91e^{i1.824\pi})$
w_3	$(1, 0, 0)$	$(0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{1.784\pi})$	$(0.69e^{i1.384\pi}, 0.101e^{i0.204\pi}, 0.32e^{0.62\pi})$

Table 17: $CSFS_fS$ related with $(E, Z, 5)$ and threshold 2.

$(E^2, Z, 5)$	z_1	z_2	z_3
w_1	$(0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i1.22\pi})$	$(0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi})$	$(0, 0, 1)$
w_2	$(0, 0, 1)$	$(0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi})$	$(0, 0, 1)$
w_3	$(1, 0, 0)$	$(0, 0, 1)$	$(0, 0, 1)$

Table 18: $CSFS_fS$ related with $(E_J, Z, 5)$ and threshold 3.

$(E^3, Z, 5)$	z_1	z_2	z_3
w_1	$(0, 0, 1)$	$(0.65e^{i1.32\pi}, 0.018e^{i0.038\pi}, 0.28e^{i0.58\pi})$	$(0, 0, 1)$
w_2	$(0, 0, 1)$	$(0.7e^{i1.42\pi}, 0.019e^{i0.04\pi}, 0.3e^{i0.56\pi})$	$(0, 0, 1)$
w_3	$(1, 0, 0)$	$(0, 0, 1)$	$(0, 0, 1)$

Table 19: $CSFS_fS$ related with $(E, Z, 5)$ and threshold 4.

$(E^4, Z, 5)$	z_1	z_2	z_3
w_1	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
w_2	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
w_3	$(1, 0, 0)$	$(0, 0, 1)$	$(0, 0, 1)$

$$\mathscr{P}^{(d)} = \begin{pmatrix} \left(g_1^{1(d)}, \mu_{11}^{(d)}, \eta_{11}^{(d)}, \nu_{11}^{(d)}\right) & \left(g_2^{1(d)}, \mu_{12}^{(d)}, \eta_{12}^{(d)}, \nu_{12}^{(d)}\right) & \cdots & \left(g_m^{1(d)}, \mu_{1m}^{(d)}, \eta_{1m}^{(d)}, \nu_{1m}^{(d)}\right) \\ \left(g_1^{2(d)}, \mu_{21}^{(d)}, \eta_{21}^{(d)}, \nu_{21}^{(d)}\right) & \left(g_2^{2(d)}, \mu_{22}^{(d)}, \eta_{22}^{(d)}, \nu_{22}^{(d)}\right) & \cdots & \left(g_m^{2(d)}, \mu_{2m}^{(d)}, \eta_{2m}^{(d)}, \nu_{2m}^{(d)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(g_1^{q(d)}, \mu_{q1}^{(d)}, \eta_{q1}^{(d)}, \nu_{q1}^{(d)}\right) & \left(g_2^{q(d)}, \mu_{q2}^{(d)}, \eta_{q2}^{(d)}, \nu_{q2}^{(d)}\right) & \cdots & \left(g_m^{q(d)}, \mu_{qm}^{(d)}, \eta_{qm}^{(d)}, \nu_{qm}^{(d)}\right) \end{pmatrix}, \tag{23}$$

where $\mathscr{P}_{jk}^{(d)} = ((g_k^j)^{(d)}, \mu_{jk}^{(d)}, \eta_{jk}\ (d), \nu_{jk}^{(d)}) = ((g_k^j)^{(d)}, p_{jk}^{(d)} e^{i2\pi\phi_{jk}^{(d)}}, \nu_{jk}^{(d)} e^{i2\pi\delta_{jk}^{(d)}}, r_{jk}^{(d)} e^{i2\pi\lambda_{jk}^{(d)}})$, $j = \{1, 2, 3, \ldots, q\}$, $k = \{1, 2, 3, \ldots, m\}$, and $d = \{1, 2, 3, \ldots, s\}$.

Step 2: to formulate the aggregate complex spherical fuzzy N-soft decision matrix ($ACSFNS_fDM$), $\mathscr{P} = (\mathscr{P}_{jk})_{q\times m}$, $CSFNS_fDM$ of all experts are assembled with the help of $CSFNS_fWA$ operator. For $ACSFNS_fDM$, $CSFNS_fWA$ operator is defined as

$$\begin{aligned} \mathscr{P}_{jk} &= CSFNS_fWA\left(\mathscr{P}_{jk}^{(1)}, \mathscr{P}_{jk}^{(2)}, \ldots, \mathscr{P}_{jk}^{(s)}\right) \\ &= \sigma_1 \mathscr{P}_{jk}^{(1)} \oplus \sigma_{(2)} \mathscr{P}_{jk}^{(2)} \oplus \cdots \oplus \sigma_s \mathscr{P}_{jk}^{(s)} \\ &= \left(\max_1^{d=s}\left(\left(g_k^j\right)^{(d)}\right), \sqrt{1-\prod_1^{d=s}\left(1-\left(p_{jk}^{(d)}\right)^2\right)^{\sigma_d}}\, e^{i2\pi\sqrt{1-\prod_1^{d=s}\left(1-\left(\phi_{jk}^{(d)}\right)^2\right)^{\sigma_d}}}, \prod_1^{d=s} \nu_{jk}^{(d)} e^{i2\pi\prod_1^{d=s}\delta_{jk}^{(d)}}, \prod_1^{d=s} r_{jk}^{(d)} e^{i2\pi\prod_1^{d=s}\lambda_{jk}^{(d)}}\right) \\ &= \left(g_k^j, p_{jk} e^{i2\pi\phi_{jk}}, \nu_{jk} e^{i2\pi\delta_{jk}}, r_{jk} e^{i2\pi\lambda_{jk}}\right). \end{aligned} \tag{24}$$

Using these entities, we can form $ACSFNS_fDM$ as

$$\mathscr{P}=\begin{pmatrix} \left(g_1^1,\mu_{11},\eta_{11},\nu_{11}\right) & \left(g_2^1,\mu_{12},\eta_{12},\nu_{12}\right) & \cdots & \left(g_m^1,\mu_{1m},\eta_{1m},\nu_{1m}\right) \\ \left(g_1^2,\mu_{21},\eta_{21},\nu_{21}\right) & \left(g_2^2,\mu_{22},\eta_{22},\nu_{22}\right) & \cdots & \left(g_m^2,\mu_{2m},\eta_{2m},\nu_{2m}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(g_1^q,\mu_{q1},\eta_{q1},\nu_{q1}\right) & \left(g_2^q,\mu_{q2},\eta_{q2},\nu_{q2}\right) & \cdots & \left(g_m^q,\mu_{qm},\eta_{qm},\nu_{qm}\right) \end{pmatrix}. \tag{25}$$

Step 3: in MAGDM problem, each attribute has it is own worth. Therefore, each expert $\widetilde{E}_d$ assigns rank as weightage of each attribute z_k relative to their importance in MAGDM problem. Furthermore, $CSFNS_fNs$ are assigned to the weights, according to the grading criteria, by the experts. Let $\chi_k^{(d)}=(g_k^{(d)},p_k^{(d)}e^{i2\pi\phi_k^{(d)}},\nu_k^{(d)}e^{i2\pi\delta_k^{(d)}},r_k^{(d)}e^{i2\pi\lambda_k^{(d)}})$ be the weightage of kth attribute given by the dth expert. To find out the weight vector $\chi=(\chi_1,\chi_2,\ldots,\chi_m)^T$, we aggregated them, as follows:

$$\begin{aligned}\chi_k &= CSFNS_fWA\left(\chi_1^{(1)},\chi_2^{(2)},\ldots,\chi_m^{(s)}\right)\\ &=\sigma_1\chi_k^1\oplus\sigma_2\chi_k^2\oplus\cdots\oplus\sigma_s\chi_k^s\\ &=\left(\max_{d=1}^{s}\left(\left(g_k^j\right)^{(d)}\right),\sqrt{1-\prod_1^{d=s}\left(1-\left(p_k^{(d)}\right)^2\right)^{\sigma_d}}\,e^{i2\pi\sqrt{1-\prod_1^{d=s}\left(1-\left(\phi_k^{(d)}\right)^2\right)^{\sigma_d}}},\prod_1^{d=s}\nu_k^{(d)}e^{i2\pi\prod_1^{d=s}\delta_k^{(d)}},\prod_1^{d=s}r_k^{(d)}e^{i2\pi\prod_1^{d=s}\lambda_k^{(d)}}\right)\\ &=\left(g_k,p_ke^{i2\pi\phi_k},\nu_ke^{i2\pi\delta_k},r_ke^{i2\pi\lambda_k}\right).\end{aligned} \tag{26}$$

Step 4: calculate the aggregated weighted complex spherical fuzzy N-soft decision matrix ($AWCSFNS_fDM$) using $ACSFNS_fDM\ \mathscr{Y}_{jk}$ and the weight vector of attribute χ_k as follows:

$$\begin{aligned}\overline{\mathscr{P}_{jk}} &= \mathscr{P}_{jk}\otimes\chi_k\\ &=\left(\min\left(\left(g_k^j\right),g_k\right),p_{jk}p_ke^{i2\pi\phi_{jk}\phi_k},\sqrt{\nu_{jk}^2+\nu_k^2-\nu_{jk}^2\nu_k^2}\,e^{i2\pi\sqrt{\delta_{jk}^2+\delta_k^2-\delta_{jk}^2\delta_k^2}},\sqrt{r_{jk}^2+r_k^2-r_{jk}^2r_k^2}\,e^{i2\pi\sqrt{\lambda_{jk}^2+\lambda_k^2-\lambda_{jk}^2\lambda_k^2}}\right)\\ &=\left(\overline{g}_k^j,\overline{\mu}_{jk},\overline{\eta}_{jk},\overline{\nu}_{jk}\right)\\ &=\left(\overline{g}_k^j,\overline{p}_{jk}e^{i2\pi\overline{\phi}_{jk}},\overline{\nu}_{jk}e^{i2\pi\overline{\delta}_{jk}},\overline{r}_{jk}e^{i2\pi\overline{\lambda}_{jk}}\right).\end{aligned} \tag{27}$$

Using these entities, we can form $AWCSFNS_fDM$ as

$$\overline{\mathscr{P}}=\begin{pmatrix} \left(\overline{g}_1^1,\overline{\mu}_{11},\overline{\eta}_{11},\overline{\nu}_{11}\right) & \left(\overline{g}_{12}^1,\overline{\mu}_{12},\overline{\eta}_{12},\overline{\nu}_{12}\right) & \cdots & \left(\overline{g}_m^1,\overline{\mu}_{1m},\overline{\eta}_{1m},\overline{\nu}_{1m}\right) \\ \left(\overline{g}_1^2,\overline{\mu}_{21},\overline{\eta}_{21},\overline{\nu}_{21}\right) & \left(\overline{g}_2^2,\overline{\mu}_{22},\overline{\eta}_{22},\overline{\nu}_{22}\right) & \cdots & \left(\overline{g}_m^2,\overline{\mu}_{2m},\overline{\eta}_{2m},\overline{\nu}_{2m}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\overline{g}_1^q,\overline{\mu}_{q1},\overline{\eta}_{q1},\overline{\nu}_{q1}\right) & \left(\overline{g}_2^q,\overline{\mu}_{q2},\overline{\eta}_{q2},\overline{\nu}_{q2}\right) & \cdots & \left(\overline{g}_m^q,\overline{\mu}_{qm},\overline{\eta}_{qm},\overline{\nu}_{qm}\right) \end{pmatrix}. \tag{28}$$

Step 5: let $\mathscr{P}_B$ and $\mathscr{P}_C$ be the collection of benefit-type attribute and cost-type attribute, respectively. $CSFNS_f$-PIS related to the attribute z_k can be taken as follows:

$$\widehat{\mathscr{P}}_k=\begin{cases}\max_{j=1}^q\overline{\mathscr{P}}_{jk}, & \text{if } z_k\in\mathscr{P}_B,\\ \min_{j=1}^q\overline{\mathscr{P}}_{jk}, & \text{if } z_k\in\mathscr{P}_C.\end{cases} \tag{29}$$

Now, $CSFNS_f$-NIS related to the attribute z_k can be taken as follows:

$$\check{\mathscr{P}}_k = \begin{cases} \max_{j=1}^{q} \overline{\mathscr{P}}_{jk}, & \text{if } z_k \in \mathscr{P}_C, \\ \min_{j=1}^{q} \overline{\mathscr{P}}_{jk}, & \text{if } z_k \in \mathscr{P}_B. \end{cases} \tag{30}$$

To evaluate $\max \overline{\mathscr{P}}_{jk}$ and $\min \overline{\mathscr{P}}_{jk}$, we use the score value and accuracy value of $CSFNS_f N$. $CSFNS_f$-PIS and $CSFNS_f$-NIS are denoted as follows: $\hat{\mathscr{P}}_k = (\hat{g}_k, \hat{\mu}_k, \hat{\eta}_k, \hat{\nu}_k) = (\hat{g}_k, \hat{p}_k e^{i2\pi\hat{\phi}_k}, \hat{v}_k e^{i2\pi\hat{\delta}_k}, \hat{r}_k e^{i2\pi\hat{\lambda}_k})$ and $\check{\mathscr{P}}_k = (\check{g}_k, \check{\mu}_k, \check{\eta}_k, \check{\nu}_k) = (\check{g}_k, \check{p}_k e^{i2\pi\check{\phi}_k}, \check{v}_k e^{i2\pi\check{\delta}_k}, \check{r}_k e^{i2\pi\check{\lambda}_k})$, respectively.

Step 6: calculate the normalized Euclidean distance of each alternative w_j from $CSFNS_f$-PIS and $CSFNS_f$-NIS. In this way, we get the best alternative that is nearer to $CSFNS_f$-PIS and far from $CSFNS_f$-NIS. The normalized Euclidean distance between $CSFNS_f$-PIS and any of the alternative w_j can be formulated as follows:

$$\begin{aligned} d\left(\hat{\mathscr{P}}_k, w_j\right) = \Bigg(\frac{1}{4k}\sum_{k=1}^{m}\Bigg[&\left(\left(\frac{\hat{g}_k}{N-1}\right)^2 - \left(\frac{\overline{g}_k^j}{N-1}\right)^2\right)^2 + \left(\hat{p}_k^2 - \overline{p}_{jk}^2\right)^2 + \left(\hat{v}_k^2 - \overline{v}_{jk}^2\right)^2 + \left(\hat{r}_k^2 - \overline{r}_{jk}^2\right)^2 \\ &+ \left(\hat{\phi}_k^2 - \overline{\phi}_{jk}^2\right)^2 + \left(\hat{\delta}_k^2 - \overline{\delta}_{jk}^2\right)^2 + \left(\hat{\lambda}_k^2 - \overline{\lambda}_{jk}^2\right)^2\Bigg]\Bigg)^{1/2}. \end{aligned} \tag{31}$$

Similarly, the normalized Euclidean distance between $CSFNS_f$-NIS and any of the alternative w_j, can be formulated as follows:

$$\begin{aligned} d\left(\check{\mathscr{P}}_k, w_j\right) = \Bigg(\frac{1}{4k}\sum_{k=1}^{m}\Bigg[&\left(\left(\frac{\check{g}_k}{N-1}\right)^2 - \left(\frac{\overline{g}_k^j}{N-1}\right)^2\right)^2 + \left(\check{p}_k^2 - \overline{p}_{jk}^2\right)^2 + \left(\check{v}_k^2 - \overline{v}_{jk}^2\right)^2 + \left(\check{r}_k^2 - \overline{r}_{jk}^2\right)^2 \\ &+ \left(\check{\phi}_k^2 - \overline{\phi}_{jk}^2\right)^2 + \left(\check{\delta}_k^2 - \overline{\delta}_{jk}^2\right)^2 + \left(\check{\lambda}_k^2 - \overline{\lambda}_{jk}^2\right)^2\Bigg]\Bigg)^{1/2}. \end{aligned} \tag{32}$$

Step 7: to chose one of the most appropriate alternative, we have to use some ranking index. For this purpose, the revised closeness index corresponding to the alternative w_k is evaluated using the formula [10]

$$\mathfrak{I}\left(w_j\right) = \frac{d\left(\hat{\mathscr{P}}_k, w_j\right)}{\min_j d\left(\hat{\mathscr{P}}_k, w_j\right)} - \frac{d\left(\check{\mathscr{P}}_k, w_j\right)}{\max_j d\left(\check{\mathscr{P}}_k, w_j\right)}, \tag{33}$$

where $k = 1, 2, \ldots, m$.

Step 8: the alternative with the minimum value of revised closeness index would be the best solution for the MAGDM problem. Therefore, the ascending order of the revised closeness index gives the ranking of the alternatives.

5. Development of Algorithms and Numerical Examples

In this section, we describe multiattribute decision-making (MADM) methods that work on models to identify the best alternative. Therefore, we characterize respective algorithms for the MADM problems in $CSFNS_f$ environment, as well as we present Algorithm 1 for $CSFNS_f$-TOPSIS method described in Section 4. Let $W = \{w_1, w_2, w_3, \ldots, w_q\}$ be a set, representing the available alternatives with a set of attributes $Z = \{z_1, z_2, z_3, \ldots, z_m\}$ having weight vector $\sigma = (\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_m)^T$ describing the worth of attributes according to the MADM problem, where $\sum_{k=1}^{m} \sigma_k = 1$ and $\sigma_k \in [0, 1]$.

The algorithm for $CSFNS_f$-TOPSIS method is described in Algorithm 1.

Let us now introduce some explicit MADM and MAGDM problems and solve them using Algorithms 1– 4, respectively. We apply Algorithms 2– 4 to solve the MADM problem defined in Section 5.1 and Algorithm 1 is used to solve the MAGDM problem defined in Section 5.2 which show their importance and feasibility in the field of decision-making.

5.1. Selection of Best Third-Party App of the Year.

A third-party app is a software application made by someone other than the manufacturer of a mobile device or its operating system. This world is full of gadgets and gadgets are full of apps. We can access the world if we have these apps. Therefore, selecting one of the best third-party app of the

Input: $W = \{w_1, w_2, w_3, \ldots, w_q\}$ as universal element.
$Z = \{z_1, z_2, z_3, \ldots, z_m\}$ as set of attributes.
$NS_fS\,(F, Z, N)$ with $G = \{0, 1, 2, 3, \ldots, N-1\}$, $N \in \{1, 2, 3, \ldots\}$, $(\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_s)^T$ as weight vector of experts $\widetilde{E}_d$.
(1) Construct $CSFNS_fDM\ \mathscr{P}^{(d)}$, corresponding to each level of attribute for the element w_j.
(2) Compute $ACSFNS_fDM$ using equation $\mathscr{P}_{jk} = (\max_1^{d=s}((g_k^j)^{(d)}), \sqrt{1-\prod_1^{d=s}(1-(p_{jk}^{(d)})^2)^{\sigma_d}}\ e^{i2\pi\sqrt{1-\prod_1^{d=s}(1-(\phi_{jk}^{(d)})^2)^{\sigma_d}}}, \prod_1^{d=s} v_{jk}^{(d)} e^{i2\pi\prod_1^{d=s}\delta_{jk}^{(d)}}, \prod_1^{d=s} r_{jk}^{(d)} e^{i2\pi\prod_1^{d=s}\lambda_{jk}^{(d)}})$.
(3) Evaluate the weight vector $\chi = (\chi_1, \chi_2, \ldots, \chi_m)^T$ as follows: $\chi_k = (\max_{d=1}^{s}((g_k^j)^{(d)}), \sqrt{1-\prod_1^{d=s}(1-(p_k^{(d)})^2)^{\sigma_d}}\ e^{i2\pi\sqrt{1-\prod_1^{d=s}(1-(\phi_k^{(d)})^2)^{\sigma_d}}}, \prod_1^{d=s} v_k^{(d)} e^{i2\pi\prod_1^{d=s}\delta_k^{(d)}}, \prod_1^{d=s} r_k^{(d)} e^{i2\pi\prod_1^{d=s}\lambda_k^{(d)}})$.
(4) Calculate $AWCSFNS_fDM$ using $ACSFNS_fDM$ and the weight vector of attributes as follows: $\overline{\mathscr{P}_{jk}} = (\min((g_k^j), g_k), p_{jk}p_k e^{i2\pi\phi_{jk}\phi_k}, \sqrt{v_{jk}^2 + v_k^2 - v_{jk}^2 v_k^2}\ e^{i2\pi\sqrt{\delta_{jk}^2+\delta_k^2-\delta_{jk}^2\delta_k^2}}, \sqrt{r_{jk}^2 + r_k^2 - r_{jk}^2 r_k^2}\ e^{i2\pi\sqrt{\lambda_{jk}^2+\lambda_k^2-\lambda_{jk}^2\lambda_k^2}})$
(5) Calculate $CSFNS_f$ PIS and $CSFNS_f$ NIS, using equations (29) and (30).
(6) Calculate the normalized Euclidean distance of $CSFNS_f$ PIS and $CSFNS_f$ NIS from each alternative, by utilizing equations (31) and (32), respectively.
(7) Calculate the revised closeness index corresponding to each alternative using the formula from [10] $\mathfrak{I}(w_j) = (d(\widehat{\mathscr{P}}_k, w_j)/\min_j d(\widehat{\mathscr{P}}_k, w_j)) - (d(\check{\mathscr{P}}_k, w_j)/\max_j d(\check{\mathscr{P}}_k, w_j))$.
(8) Identify the alternative with minimum revised closeness index.

Algorithm 1: The algorithm of $CSFNS_f$-TOPSIS method.

(1) h!
Input: $W = \{w_1, w_2, w_3, \ldots, w_q\}$ as universal element.
$Z = \{z_1, z_2, z_3, \ldots, z_m\}$ as set of attributes.
$NS_fS\,(F, Z, N)$ with $G = \{0, 1, 2, 3, \ldots, N-1\}$, $N \in \{1, 2, 3, \ldots\}$.
(2) Construct the $CSFNS_fN\Upsilon_{kj}$, corresponding to each level of attribute for the element w_j.
(3) Compute $X_j = \oplus_{k=1}^{m}\Upsilon_{kj}$ using equation (22), where $\Upsilon_{kj} = (g_k^j, p_{kj}e^{i2\pi\phi_{kj}}, v_{kj}e^{i2\pi\delta_{kj}}, r_{kj}e^{i2\pi\lambda_{kj}})$, $(w_j, g_k^j) \in F(z)$.
(4) Calculate the score function S_{X_j}, using equation (11) for all $j = \{1, 2, 3, \ldots, q\}$.
(5) **if** $S_j = S_p$, *for some* $j, p \in \{1, 2, 3, \ldots, q\}$, **then**
(6) Use accuracy degree and identify alternative with maximum accuracy value
(7) **else**
(8) Identify the alternative with maximum score value.
(9)

Algorithm 2: The algorithm of choice values of $CSFNS_fSs$.

Input: $W = \{w_1, w_2, w_3, \ldots, w_q\}$ as universal element.
$Z = \{z_1, z_2, z_3, \ldots, z_m\}$ as set of attributes.
$NS_fS\,(F, Z, N)$ with $G = \{0, 1, 2, 3, \ldots, N-1\}$, $N \in \{1, 2, 3, \ldots\}$, $(\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_k)^T$ as weight vector.
(1) Construct $CSFNS_fN\Upsilon_{kj}$, corresponding to each level of attribute for the element w_j.
(2) Compute $X_j^{\sigma} = \oplus_{k=1}^{m}\sigma_k\Upsilon_{kj}$, where $\Upsilon_{kj} = (g_k^j, p_{kj}e^{i2\pi\phi_{kj}}, v_{kj}e^{i2\pi\delta_{kj}}, r_{kj}e^{i2\pi\lambda_{kj}})$, $(w_j, g_k^j) \in F(z)$.
(3) Calculate the score function S_{X_j}, for all $j = \{1, 2, 3, \ldots, q\}$. Calculate all the indices j for which $S_j = \max_j S_{X_j^{\sigma}}$.
(4) **if** $S_j = S_p$, *for some* $j, p \in \{1, 2, 3, \ldots, q\}$, **then**
(5) Use accuracy degree and identify alternative with maximum accuracy value
(6) **else**
(7) Identify the alternative with maximum score value.
(8)

Algorithm 3: The algorithm of weighted choice values of $CSFNS_fSs$.

year and keeping in view the priorties of people is a very difficult task. For this purpose, the data has been collected from the websites http://www.makeawebsitehub.com and http://www.trustraduis.com regarding to each third-party app. To find out the best app of the year, we will use $CSFNS_fS$.

Let $\mathbb{A} = \{a_1 = \text{Facebook}, a_2 = \text{Skype}, a_3 = \text{Viber}, a_4 = \text{Twitter}, a_5 = \text{Whatsapp}\}$ be universe of third-party apps and $Z = \{z_1 = \text{telecom framework}, z_2 = \text{reliability}, z_3 = \text{worldwide contact}, z_4 = \text{data usage}\}$ be the attributes. According to these attributes, a 6-soft set is modeled in Table 20, where

Input: $W = \{w_1, w_2, w_3, \ldots, w_q\}$ as universal element.
$Z = \{z_1, z_2, z_3, \ldots, z_m\}$ as set of attributes.
$NS_fS\,(F, Z, N)$ with $G = \{0, 1, 2, 3, \ldots, N-1\}$,
$N \in \{1, 2, 3, \ldots\}$,
Input $0 < L < N-1$, threshold.

(1) Work out for $F_J^L(z) = \begin{cases} (\mu_{kj}(z), \eta_{kj}(z), \nu_{kj}(z)), & \text{if } (w, g_z^w) \in F(z) \text{ and } g_z^w \geq L, \\ (0, 0, 1), & \text{otherwise.} \end{cases}$
(2) First, compute $X_j^L = \oplus_{k=1}^m \Upsilon_{kj}^L$, where $\Upsilon_{kj}^L = (p_{kj}e^{i2\pi\phi_{kj}}, \nu_{kj}e^{i2\pi\delta_{kj}}, r_{kj}e^{i2\pi\lambda_{kj}})$, $(w_j, g_k^j) \in F(z)$.
(3) Calculate the score function $S_{X_j^L}$, for all $j = \{1, 2, 3, \ldots, q\}$.
(4) Calculate all the indices j for which $S_j^L = \max_j S_{X_j^L}$.
(5) **if** $S_j = S_p$, *for some* $j, p \in \{1, 2, 3, \ldots, q\}$, **then**
(6)   Use accuracy degree and identify alternative with maximum accuracy value
(7) **else**
(8)   Identify the alternative with maximum score value.
(9)

Algorithm 4: The algorithm of L-choice values of $CSFNS_fSs$.

Five diamonds mean "Marvellous"

Four diamonds mean "Outstanding"

Three diamonds mean "Super"

Two diamonds mean "Good"

One diamond means "Satisfactory"

Big dot means "Acceptable."

This level assessment by diamonds can be represented by numbers as $G = \{0, 1, 2, 3, 4, 5\}$, where

0 means "•"

1 means "◇"

2 means "◇◇"

3 means "◇◇◇,"

4 means "◇◇◇◇"

5 means "◇◇◇◇◇"

Thus, tabular representation of 6-soft set is shown in Table 21 and the tabular representation of $CSF6S_fS\,(F_J, Z, 6)$ is shown in Table 22.

5.1.1. Choice Values of $CSF6S_fS$. The choice values of $CSF6S_fS$ is evaluated using the steps defined in Algorithm 2. Table 23 presents the calculated choice values of $CSF6S_fS$ for the selection of the third-party app. We can observe from Table 23 that, according to the choice values, the ranking of third-party apps is as follows: $\mathbb{a}_1 > \mathbb{a}_4 > \mathbb{a}_5 > \mathbb{a}_2 > \mathbb{a}_3$, which shows that $\mathbb{a}_1$ = Facebook has maximum choice value. Therefore, Facebook is selected as best third-party app of the year.

5.1.2. Weighted Choice Values of $CSF6S_fS$. Let $\sigma_1 = 0.4, \sigma_2 = 0.3, \sigma_3 = 0.2$, and $\sigma_4 = 0.1$ be the weights for each attribute z_k, $k = 1, 2, 3, 4$. Using these weights in Algorithm 3, we can compute weighted choice values of $CSF6S_fS$, which are given by Table 24.

It is clear from Table 24 that G_4 has maximum score; therefore, $\mathbb{a}_4$ = Facebook is selected as best third-party app of the year. According to the weighted choice values, ranking of third-party apps is as follows: $\mathbb{a}_1 > \mathbb{a}_4 > \mathbb{a}_5 > \mathbb{a}_2 > \mathbb{a}_3$.

5.1.3. L-Choice Values of $CSF6S_fS$. The L-choice values of $CSF6S_fS$ are evaluated using Algorithm 4 to find out the best alternative for the proposed MADM problem. Let $L = 4$ be threshold; then, 4-choice values of $CSFS_fS$ is shown in Table 25. We can observe that, from Table 25, the ranking of third-party apps according to 4-choice values is as follows: $\mathbb{a}_1 > \mathbb{a}_4 > \mathbb{a}_5 > \mathbb{a}_2 \geq \mathbb{a}_3$, which shows that $\mathbb{a}_1$ = Facebook has maximum choice value so that Facebook is selected as the best third-party app of the year.

5.2. Selection of the Best Physiotherapist Doctor of Mayo Hospital in Lahore. Physiotherapy helps to restore movement and function when people are affected by injury or disability. A physiotherapist treats such kind of people and helps them through exercise, manual therapy, education, and advice. A physiotherapist is very helpful in maintaining the health of people of all ages as well as encourages them for happy life. A physiotherapist must have patience, communication skills, and ability to establish a good relationship with patients and their families. The motive of this study is to select the best physiotherapist doctor in Lahore relative to their attributes under the environment of $CSFNS_f$. For this purpose, the data has been collected from the students of Mayo Hospital, Lahore, enact here as experts $\widetilde{E}_1$, $\widetilde{E}_2$, $\widetilde{E}_3$, and $\widetilde{E}_4$ whose weight vectors are $\sigma = (0.4, 0.2, 0.1, 0.3)^T$. The following physiotherapists of Mayo Hospital are treated as alternatives in this MAGDM problem:

w_1: Dr. Amna

w_2: Dr. Rizwan

w_3: Dr. Akmal

w_4: Dr. Sidra

w_5: Dr. Saleem

Five attributes considered as key factors for a physiotherapist are as follows:

z_1: knowledge and experience.

z_2: behavioral (positivity, patience, and humbleness).

z_3: availability and flexibility.

Table 20: Evaluation of data from the websites.

$\mathbb{A}/Z$	z_1	z_2	z_3	z_4
$\mathbb{a}_1$	◇◇◇◇◇	◇◇◇◇	◇◇◇◇	◇◇
$\mathbb{a}_2$	◇◇◇	◇◇	◇◇◇	◇◇◇
$\mathbb{a}_3$	◇◇	◇◇	◇◇	◇◇◇
$\mathbb{a}_4$	◇◇	◇◇◇	◇◇◇◇◇	◇◇◇◇
$\mathbb{a}_5$	◇◇◇	◇◇◇◇	◇◇◇	◇◇

Table 21: Tabular representation of 6-soft set.

$\mathbb{A}/Z$	z_1	z_2	z_3	z_4
$\mathbb{a}_1$	5	4	4	2
$\mathbb{a}_2$	3	2	3	3
$\mathbb{a}_3$	2	2	2	3
$\mathbb{a}_4$	2	3	5	4
$\mathbb{a}_5$	3	4	3	2

z_4: master of skills (communication, organizational, or problem-solving skills).

z_5: session fee.

We solve this MAGDM problem by following the $CSFNS_f$-TOPSIS method.

Step 1: according to these attributes, each expert model 6-soft set is in Table 26, where

Five stars mean "Marvellous"
Four stars mean "Outstanding"
Three stars mean "Super"
Two stars mean "Good"
One star mean "Satisfactory"
Big dot means "Acceptable"

Table 3 represents the grading criteria, used for assigning the $CSFNS_fN$ corresponding to each rank by the expert $\widetilde{E}_1$, $\widetilde{E}_2$, $\widetilde{E}_3$, and $\widetilde{E}_4$ tabulated in Tables 27–30, respectively.

Step 2: using equation (24), we can put together the opinions of all experts. $ACSFNS_fDM$ formed by aggregation is given in Table 31.

Step 3: to demonstrate the importance of attributes in the MAGDM problem, experts rank them and associate $CSFNS_fN$ to each attribute which are arranged in Table 32. We cumulated the weights given by experts using equation (26) to form $CSFNS_f$ weight vector χ of attributes, i.e.,

$$\chi = \begin{pmatrix} \left(4, \left(0.89e^{i1.72\pi}, 0.017e^{i0.034\pi}, 0.23e^{0.5i\pi}\right)\right) \\ \left(4, \left(0.91e^{i1.86\pi}, 0.016e^{i0.034\pi}, 0.09e^{0.2i\pi}\right)\right) \\ \left(3, \left(0.62e^{i1.24\pi}, 0.016e^{i0.028\pi}, 0.53e^{1.1i\pi}\right)\right) \\ \left(3, \left(0.53e^{i1.06\pi}, 0.02e^{i0.04\pi}, 0.73e^{1.42i\pi}\right)\right) \\ \left(2, \left(0.55e^{i1.14\pi}, 0.019e^{i0.042\pi}, 0.67e^{1.34i\pi}\right)\right) \end{pmatrix}. \tag{34}$$

Step 4: by utilizing $ACSFNS_fDM$ and weight vector χ of attribute in equation (34), $AWCSFNS_fDM$ is evaluated and summarized in Table 33.

Step 5: in the MAGDM problem, all the attributes' knowledge and experience, behavior, availability and flexibility, and master of skills are benefit-type attributes except the session fee, which is a cost-type attribute. According to the nature of attributes and applying equation (29) and (30), $CSFNS_f$-PIS and $CSFNS_f$-NIS are evaluated and arranged in Table 34

Step 6: Table 35 represents the normalized Euclidean distance from each alternative to $CSFNS_f$-PIS and $CSFNS_f$-NIS using equations (31) and (32), respectively.

Step 7: the revised closeness index of each alternative is calculated by utilizing equation (33) and given in Table 36.

Step 8: since w_1 has least revised closeness index, therefore, Dr. Amna is the best physiotherapist in Mayo Hospital, Lahore. The ranking of alternatives is shown in Table 37.

6. Comparative Analysis

We now compare our proposed model with Pythagorean fuzzy N-soft set ($PFNS_fS$) that was discussed by Zhang et al. [24].

(1) Table 38 represents the ratings of MADM problem as shown in Section5.1. in $PFNS_fNs$.

(2) Table 39 presents the calculated choice values of $PF6S_fS$ using the algorithm defined in [24] for the selection of the third-party app. Clearly, from Table 39, $\mathbb{a}_1$ is the best choice, and the ranking of third-party apps is as follows: $\mathbb{a}_1 > \mathbb{a}_4 > \mathbb{a}_5 > \mathbb{a}_2 > \mathbb{a}_3$.

(3) Let $L = 4$ be threshold; then, 4-choice values of PFS_fS are shown in Table 40. The ranking of third-party apps according to 4-choice values is as follows:

Table 22: Tabular representation of the $CSF6S_fS$ $(F_J, Z, 6)$.

$(F_J, Z, 6)$	z_1	z_2	z_3	z_4
$\mathbb{a}_1$	$(5, (0.95e^{i1.88\pi}, 0.0172e^{i0.0346\pi}, 0.13e^{i0.2\pi}))$	$(4, (0.87e^{i1.76\pi}, 0.01e^{i0.024\pi}, 0.21e^{i0.38\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.012e^{i0.026\pi}, 0.2e^{i0.394\pi}))$	$(2, (0.48e^{i0.98\pi}, 0.012e^{i0.022\pi}, 0.77e^{1.52i\pi}))$
$\mathbb{a}_2$	$(3, (0.7e^{i1.34\pi}, 0.0173e^{i0.0344\pi}, 0.49e^{i\pi}))$	$(2, (0.32e^{i0.66\pi}, 0.015e^{i0.03\pi}, 0.9e^{i1.78\pi}))$	$(3, (0.6e^{i1.24\pi}, 0.081e^{i0.16\pi}, 0.69e^{1.39\pi}))$	$(3, (0.62e^{i1.23\pi}, 0.079e^{i0.15\pi}, 0.73e^{1.48i\pi}))$
$\mathbb{a}_3$	$(2, (0.41e^{i0.84\pi}, 0.014e^{i0.026\pi}, 0.77e^{i1.52\pi}))$	$(2, (0.33e^{i0.64\pi}, 0.016e^{i0.028\pi}, 0.89e^{1.8\pi}))$	$(2, (0.35e^{i0.8\pi}, 0.016e^{i0.033\pi}, 0.84e^{1.7\pi}))$	$(3, (0.53e^{i1.1\pi}, 0.063e^{i0.13\pi}, 0.67e^{i1.38\pi}))$
$\mathbb{a}_4$	$(2, (0.47e^{i0.92\pi}, 0.0169e^{i0.0338\pi}, 0.9e^{i1.8\pi}))$	$(3, (0.72e^{i1.4\pi}, 0.05e^{i0.88\pi}, 0.59e^{i1.22\pi}))$	$(5, (0.9e^{i1.82\pi}, 0.08e^{i0.158\pi}, 0.18e^{0.358\pi}))$	$(4, (0.77e^{i1.52\pi}, 0.017e^{i0.032\pi}, 0.35e^{0.66i\pi}))$
$\mathbb{a}_5$	$(3, (0.71e^{i1.44\pi}, 0.0174e^{i0.0342\pi}, 0.51e^{i0.98\pi}))$	$(4, (0.89e^{i1.784\pi}, 0.013e^{i0.0278\pi}, 0.145e^{i0.4\pi}))$	$(3, (0.55e^{i1.2\pi}, 0.06e^{i0.118\pi}, 0.7e^{i1.42\pi}))$	$(2, (0.49e^{i0.96\pi}, 0.011e^{i0.022\pi}, 0.76e^{1.54i\pi}))$

Table 23: Tabular representation of the choice values of $(F_J, Z, 6)$.

$(F_J, Z, 6)$	z_1	z_2	z_3	z_4	X_j	S_{X_j}
$\mathbb{a}_1$	$(5, (0.95e^{i1.88\pi}, 0.0172e^{i0.0346\pi}, 0.13e^{i0.2\pi}))$	$(4, (0.87e^{i1.76\pi}, 0.01e^{i0.024\pi}, 0.21e^{i0.38\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.012e^{i0.026\pi}, 0.2e^{i0.394\pi}))$	$(2, (0.48e^{i0.98\pi}, 0.012e^{i0.022\pi}, 0.77e^{1.52i\pi}))$	$(5, (0.9979e^{i1.9958\pi}, 2.47 \times 10^{-8} e^{i6.34\times10^{-8}\pi}, 4.21 \times 10^{-3} e^{5.62\times10^{-3} i\pi}))$	2.6860
$\mathbb{a}_2$	$(3, (0.7e^{i1.34\pi}, 0.0173e^{i0.0344\pi}, 0.49e^{i\pi}))$	$(2, (0.32e^{i0.66\pi}, 0.015e^{i0.03\pi}, 0.9e^{i1.78\pi}))$	$(3, (0.6e^{i1.24\pi}, 0.081e^{i0.16\pi}, 0.69e^{1.39\pi}))$	$(3, ((0.62e^{i1.23\pi}, 0.079e^{i0.15\pi}, 0.73e^{1.48i\pi}))$	$(3, (0.9053e^{i1.8024\pi}, 1.66 \times 10^{-6} e^{i5.02\times10^{-7}\pi}, 0.22e^{0.44i\pi}))$	1.78227
$\mathbb{a}_3$	$(2, (0.41e^{i0.84\pi}, 0.014e^{i0.026\pi}, 0.77e^{i1.52\pi}))$	$(2, (0.33e^{i0.64\pi}, 0.016e^{i0.028\pi}, 0.89e^{1.8\pi}))$	$(2, (0.35e^{i0.8\pi}, 0.016e^{i0.033\pi}, 0.84e^{1.7\pi}))$	$(3, (0.53e^{i1.1\pi}, 0.063e^{i0.13\pi}, 0.67e^{1.38i\pi}))$	$(3, (0.7295e^{i1.53\pi}, 2.25 \times 10^{-7} e^{i18.93\times10^{-7}\pi}, 0.3857e^{0.802i\pi}))$	1.0606
$\mathbb{a}_4$	$(2, (0.47e^{i0.92\pi}, 0.0169e^{i0.0338\pi}, 0.9e^{i1.8\pi}))$	$(3, (0.72e^{i1.4\pi}, 0.05e^{i0.88\pi}, 0.59e^{i1.22\pi}))$	$(5, (0.9e^{i1.82\pi}, 0.08e^{i0.158\pi}, 0.18e^{0.358\pi}))$	$(4, (0.77e^{i1.52\pi}, 0.017e^{i0.032\pi}, 0.35e^{0.66i\pi}))$	$(5, (0.9853e^{i1.9704\pi}, 1.149 \times 10^{-6} e^{i18.78\times10^{-6}\pi}, 0.033e^{0.064i\pi}))$	2.6337
$\mathbb{a}_5$	$(3, (0.71e^{i1.44\pi}, 0.0174e^{i0.0342\pi}, 0.51e^{i0.98\pi}))$	$(4, (0.89e^{i1.784\pi}, 0.013e^{i0.0278\pi}, 0.145e^{i0.4\pi}))$	$(3, (0.55e^{i1.2\pi}, 0.06e^{i0.118\pi}, 0.7e^{i1.42\pi}))$.	$(2, (0.49e^{i0.96\pi}, 0.011e^{i0.022\pi}, 0.76e^{1.54i\pi}))$	$(4, (0.9723e^{i1.9506\pi}, 1.493 \times 10^{-7} e^{i3.08\times10^{-7}\pi}, 0.053e^{0.106i\pi}))$	2.3353

Table 24: Tabular representation of the weighted choice values of $(F_J, Z, 6)$.

$(F_J, Z, 6)$	z_1	z_2	z_3	z_4	X_j^{σ}	$S_{X_j^{\sigma}}$
$\mathbb{a}_1$	$(5, (0.95e^{i1.88\pi}, 0.0172e^{i0.0346\pi}, 0.13e^{i0.2\pi}))$	$(4, (0.87e^{i1.76\pi}, 0.01e^{i0.024\pi}, 0.21e^{i0.38\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.012e^{i0.026\pi}, 0.2e^{i0.394\pi}))$	$(2, (0.48e^{i0.98\pi}, 0.012e^{i0.022\pi}, 0.77e^{1.52i\pi}))$	$(5, (0.9019e^{i1.7974\pi}, 0.0131e^{i0.0278\pi}, 0.195e^{0.3402i\pi}))$	2.2482
$\mathbb{a}_2$	$(3, (0.7e^{i1.34\pi}, 0.0173e^{i0.0344\pi}, 0.49e^{i\pi}))$	$(2, (0.32e^{i0.66\pi}, 0.015e^{i0.03\pi}, 0.9e^{i1.78\pi}))$	$(3, (0.6e^{i1.24\pi}, 0.081e^{i0.16\pi}, 0.69e^{1.39\pi}))$	$(3, (0.62e^{i1.23\pi}, 0.079e^{i0.15\pi}, 0.73e^{1.48i\pi}))$	$(3, (0.5963e^{i1.168\pi}, 0.0263e^{i0.052\pi}, 0.655e^{1.319i\pi}))$	0.081
$\mathbb{a}_3$	$(2, (0.41e^{i0.84\pi}, 0.014e^{i0.026\pi}, 0.77e^{i1.52\pi}))$	$(2, (0.33e^{i0.64\pi}, 0.016e^{i0.028\pi}, 0.89e^{1.8\pi}))$	$(2, (0.35e^{i0.8\pi}, 0.016e^{i0.033\pi}, 0.84e^{1.7\pi}))$	$(3, (0.53e^{i1.1\pi}, 0.063e^{i0.13\pi}, 0.67e^{1.38i\pi}))$	$(3, (0.3925e^{i0.813\pi}, 0.015e^{i0.032\pi}, 0.807e^{1.61i\pi}))$	−0.7378
$\mathbb{a}_4$	$(2, (0.47e^{i0.92\pi}, 0.0169e^{i0.0338\pi}, 0.9e^{i1.8\pi}))$	$(3, (0.72e^{i1.4\pi}, 0.05e^{i0.88\pi}, 0.59e^{i1.22\pi}))$	$(5, (0.9e^{i1.82\pi}, 0.08e^{i0.158\pi}, 0.18e^{0.358\pi}))$	$(4, (0.77e^{i1.52\pi}, 0.017e^{i0.032\pi}, 0.35e^{0.66i\pi}))$	$(5, (0.7235e^{i1.443\pi}, 0.0319e^{i0.121\pi}, 0.5229e^{1.049i\pi}))$	1.185
$\mathbb{a}_5$	$(3, (0.71e^{i1.44\pi}, 0.0174e^{i0.0342\pi}, 0.51e^{i0.98\pi}))$	$(4, (0.89e^{i1.784\pi}, 0.013e^{i0.0278\pi}, 0.145e^{i0.4\pi}))$	$(3, (0.55e^{i1.2\pi}, 0.06e^{i0.118\pi}, 0.7e^{i1.42\pi}))$	$(2, (0.49e^{i0.96\pi}, 0.011e^{i0.022\pi}, 0.76e^{1.54i\pi}))$	$(4, (0.7570e^{i1.532\pi}, 0.0195e^{i0.0394\pi}, 0.5503e^{0.843i\pi}))$	1.123

Table 25: Tabular representation of the 4-choice values of $(F_J, Z, 6)$.

$(F_J^4, Z, 6)$	z_1	z_2	z_3	z_4	X_j	S_{X_j}
$\mathbb{a}_1$	$(0.95e^{i1.88\pi}, 0.0172e^{i0.0346\pi}, 0.13e^{i0.2\pi}))$	$(0.87e^{i1.76\pi}, 0.01e^{i0.024\pi}, 0.21e^{i0.38\pi})$	$(0.88e^{i1.78\pi}, 0.012e^{i0.026\pi}, 0.2e^{i0.394\pi})$	$(0, 0, 1)$	$(0.9973e^{i1.9944\pi}, 2.064 \times 10^{-6}e^{i1.45\times10^{-11}\pi}, 5.46 \times 10^{-3}e^{2.8\times10^{-5}i\pi})$	1.989
$\mathbb{a}_2$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	−1
$\mathbb{a}_3$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	−1
$\mathbb{a}_4$	$(0, 0, 1)$	$(0, 0, 1)$	$(0.9e^{i1.82\pi}, 0.08e^{i0.158\pi}, 0.18e^{0.358\pi})$	$(0.77e^{i1.52\pi}, 0.017e^{i0.032\pi}, 0.35e^{0.66i\pi})$	$(0.9605e^{i1.926\pi}, 1.36 \times 10^{-3}e^{i3.18\times10^{-7}\pi}, 0.063e^{i6.978\times10^{-3}\pi})$	1.8459
$\mathbb{a}_5$	$(0, 0, 1)$	$(0.89e^{i1.784\pi}, 0.013e^{i0.0278\pi}, 0.145e^{i0.4\pi})$	$(0, 0, 1)$	$(0, 0, 1)$	$(0.89e^{i1.78\pi}, 0.013e^{i0.0278\pi}, 0.145e^{0.4i\pi})$	1.5263

Table 26: Experts' rating according to attributes.

Attributes	Alternatives	$\widetilde{E}_1$	$\widetilde{E}_2$	$\widetilde{E}_3$	$\widetilde{E}_4$
	w_1	**** = 4	*** = 3	*** = 3	**** = 4
	w_2	*** = 3	*** = 3	*** = 3	*** = 3
z_1	w_3	* = 1	• = 0	** = 2	* = 1
	w_4	** = 2	*** = 3	**** = 4	** = 2
	w_5	** = 2	* = 1	** = 2	• = 0
	w_1	**** = 4	**** = 4	**** = 4	*** = 3
	w_2	** = 2	*** = 3	** = 2	* = 1
z_2	w_3	* = 1	* = 1	* = 1	** = 2
	w_4	*** = 3	** = 2	*** = 3	**** = 4
	w_5	* = 1	** = 2	** = 2	* = 1
	w_1	**** = 4	**** = 4	**** = 4	**** = 4
	w_2	* = 1	** = 2	** = 2	• = 0
z_3	w_3	** = 2	** = 2	** = 2	** = 2
	w_4	*** = 3	* = 1	• = 0	** = 2
	w_5	*** = 3	** = 2	**** = 4	* = 1
	w_1	*** = 3	*** = 3	** = 2	** = 2
	w_2	**** = 4	**** = 4	*** = 3	*** = 3
z_4	w_3	* = 1	*** = 3	**** = 4	** = 2
	w_4	** = 2	*** = 3	*** = 3	*** = 3
	w_5	* = 1	* = 1	** = 2	** = 2
	w_1	**** = 4	*** = 3	*** = 3	** = 2
	w_2	** = 2	** = 2	*** = 3	** = 2
z_5	w_3	* = 1	** = 2	* = 1	* = 1
	w_4	*** = 3	** = 2	* = 1	• = 0
	w_5	** = 2	• = 0	*** = 3	* = 1

$\mathbb{a}_1 > \mathbb{a}_4 > \mathbb{a}_5 > \mathbb{a}_2 \geq \mathbb{a}_3$, which further shows that $\mathbb{a}_1 =$ Facebook has maximum choice value.

(4) We conclude the same results from both choice values and L-choice values of $PFNS_fS$ [24], which shows the reliability of our proposed method, and it can be applied to any MADM problem.

(5) The data arranged in Table 23 is able to handle more real-life problems compared to Pythagorean N-soft set and intuitionistic N-soft set as it includes the neutral membership degree as well as it could deal with 2-dimensional data.

(6) The proposed model would provide the same results under spherical fuzzy N-soft environment by taking the periodic terms equal to zero.

6.1. Comparison with Complex Spherical Fuzzy TOPSIS Method. In this section, we solve the MAGDM problem "selection of best physiotherapist doctor of Mayo Hospital in Lahore" by complex spherical fuzzy TOPSIS method, proposed by Akram et al. [16], to demonstrate the importance and superiority of the proposed model. The solution by the complex spherical fuzzy TOPSIS method is as follows:

Step 1: the linguistic term corresponding to each rank assessed by the experts are the same as given in Table 26. To apply the CSF TOPSIS method, the grading part is excluded from $CSFNS_fN$ and CSFNs are assigned by each expert $\widetilde{E}_1, \widetilde{E}_2, \widetilde{E}_3$, and $\widetilde{E}_4$, which are arranged in Tables 41–44, respectively, according to the grading criteria defined in Table 3.

Step 2: using the weight vector of experts $\sigma = \{0.4, 0.2, 0.1, 0.3\}^T$ and complex spherical fuzzy weighted average (CSFWA) operator [16], we can calculate the aggregated complex spherical fuzzy decision matrix (ACSFDM), whose entries are evaluated by the formula defined as follows [16]:

$$\mathscr{P}_{jk} = \left(\sqrt{1-\prod_{1}^{d=s}\left(1-\left(p_{jk}^{(d)}\right)^2\right)^{\sigma_d}}\, e^{i2\pi\sqrt{1-\prod_1^{d=s}\left(1-\left(\phi_{jk}^{(d)}\right)^2\right)^{\sigma_d}}}, \prod_1^{d=s} v_{jk}^{(d)} e^{i2\pi\prod_1^{d=s}\delta_{jk}^{(d)}}, \prod_1^{d=s} r_{jk}^{(d)} e^{i2\pi\prod_1^{d=s}\lambda_{jk}^{(d)}} \right). \tag{35}$$

ACSFDM is summarized in Table 45.

Step 3: the experts' opinion about the importance of attributes are given in Table 46. The experts' opinion are combined using (CSFWA) operator [16], to formulate

Table 27: Tabular representation of $CSFNS_f DM$ of expert $\widetilde{E}_1$.

$(F_J^{(1)}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
w_1	$(4, (0.98e^{i1.98\pi}, 0.01e^{i0.024\pi}, 0.02e^{i0.06\pi}))$	$(4, (0.99e^{i1.94\pi}, 0.012e^{i0.02\pi}, 0.1e^{i0.26\pi}))$	$(4, (0.97e^{i1.96\pi}, 0.013e^{i0.02\pi}, 0.05e^{i0.08\pi}))$	$(3, (0.84e^{i1.66\pi}, 0.019e^{i0.04\pi}, 0.29e^{0.6i\pi}))$	$(4, (0.98e^{i1.92\pi}, 0.012e^{i0.026\pi}, 0.03e^{0.01i\pi}))$
w_2	$(3, (0.84e^{i1.66\pi}, 0.02e^{i0.042\pi}, 0.29e^{i0.62\pi}))$	$(2, (0.37e^{i0.78\pi}, 0.015e^{i0.026\pi}, 0.87e^{i1.72\pi}))$	$(1, (0.16e^{i0.34\pi}, 0.018e^{i0.038\pi}, 0.89e^{1.78\pi}))$	$(4, (0.96e^{i1.94\pi}, 0.013e^{i0.024\pi}, 0.04e^{0.06i\pi}))$	$(2, (0.36e^{i0.74\pi}, 0.016e^{i0.03\pi}, 0.59e^{1.2i\pi}))$
w_3	$(1, (0.17e^{i0.36\pi}, 0.02e^{i0.038\pi}, 0.91e^{i1.84\pi}))$	$(1, (0.19e^{i0.36\pi}, 0.021e^{i0.04\pi}, 0.93e^{1.82\pi}))$	$(2, (0.59e^{i0.12\pi}, 0.0155e^{i0.032\pi}, 0.82e^{1.6\pi}))$	$(1, (0.2e^{i0.38\pi}, 0.022e^{i0.042\pi}, 0.92e^{1.86i\pi}))$	$(1, (0.21e^{i0.44\pi}, 0.024e^{i0.05\pi}, 0.93e^{1.88i\pi}))$
w_4	$(2, (0.58e^{i1.12\pi}, 0.015e^{i0.032\pi}, 0.82e^{i1.66\pi}))$	$(3, (0.66e^{i1.34\pi}, 0.1e^{i0.18\pi}, 0.56e^{i1.14\pi}))$	$(3, (0.67e^{i1.36\pi}, 0.021e^{i0.044\pi}, 0.31e^{0.64\pi}))$	$(2, (0.55e^{i1.14\pi}, 0.014e^{i0.028\pi}, 0.8e^{1.62i\pi}))$	$(3, (0.67e^{i1.38\pi}, 0.025e^{i0.048\pi}, 0.33e^{0.68i\pi}))$
w_5	$(2, (0.52e^{i1.02\pi}, 0.012e^{i0.026\pi}, 0.74e^{i1.5\pi}))$	$(1, (0.21e^{i0.4\pi}, 0.021e^{i0.046\pi}, 0.95e^{i1.88\pi}))$	$(1, (0.7e^{i1.36\pi}, 0.026e^{i0.05\pi}, 0.35e^{i0.68\pi}))$	$(1, (0.16e^{i0.42\pi}, 0.018e^{i0.038\pi}, 0.89e^{1.88i\pi}))$	$(2, (0.58e^{i1.14\pi}, 0.01e^{i0.024\pi}, 0.82e^{1.62i\pi}))$

Table 28: Tabular representation of $CSFNS_f DM$ of expert $\widetilde{E}_2$.

$(F_J^{(2)}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
w_1	$(3, (0.83e^{i1.64\pi}, 0.019e^{i0.036\pi}, 0.3e^{i0.58\pi}))$	$(4, (0.98e^{i1.94\pi}, 0.013e^{i0.024\pi}, 0.1e^{i0.26\pi}))$	$(4, (0.98e^{i1.98\pi}, 0.01e^{i0.024\pi}, 0.02e^{i0.06\pi}))$	$(3, (0.84e^{i1.66\pi}, 0.02e^{i0.036\pi}, 0.31e^{0.63i\pi}))$	$(3, (0.8e^{i1.62\pi}, 0.022e^{i0.042\pi}, 0.32e^{0.6i\pi}))$
w_2	$(3, (0.71e^{i1.38\pi}, 0.028e^{i0.058\pi}, 0.37e^{i0.72\pi}))$	$(3, (0.72e^{i1.42\pi}, 0.03e^{i0.062\pi}, 0.38e^{i0.76\pi}))$	$(2, (0.57e^{i1.1\pi}, 0.012e^{i0.028\pi}, 0.81e^{i1.6\pi}))$	$(4, (0.86e^{i1.74\pi}, 0.0169e^{i0.032\pi}, 0.027e^{0.052i\pi}))$	$(2, (0.56e^{i1.1\pi}, 0.015e^{i0.07\pi}, 0.8e^{1.58i\pi}))$
w_3	$(0, (0.05e^{i0.012\pi}, 0.01e^{i0.06\pi}, 0.985e^{i1.972\pi}))$	$(1, (0.21e^{i0.46\pi}, 0.024e^{i0.52\pi}, 0.94e^{1.86\pi}))$	$(2, (0.5e^{i1.02\pi}, 0.01e^{i0.02\pi}, 0.7e^{1.4\pi}))$	$(3, (0.73e^{i1.48\pi}, 0.032e^{i0.062\pi}, 0.39e^{0.76i\pi}))$	$(2, (0.55e^{i1.08\pi}, 0.01e^{i0.02\pi}, 0.79e^{1.56i\pi}))$
w_4	$(3, (0.74e^{i1.5\pi}, 0.035e^{i0.068\pi}, 0.4e^{i0.78\pi}))$	$(2, (0.54e^{i1.04\pi}, 0.015e^{i0.026\pi}, 0.78e^{i1.54\pi}))$	$(1, (0.24e^{i0.44\pi}, 0.03e^{i0.056\pi}, 0.94e^{1.84\pi}))$	$(3, (0.71e^{i1.38\pi}, 0.028e^{i0.058\pi}, 0.37e^{0.72i\pi}))$	$(2, (0.53e^{i1.04\pi}, 0.013e^{i0.026\pi}, 0.77e^{1.52i\pi}))$
w_5	$(1, (0.24e^{i0.42\pi}, 0.028e^{i0.062\pi}, 0.91e^{i1.84\pi}))$	$(2, (0.51e^{i1.04\pi}, 0.013e^{i0.06\pi}, 0.76e^{i1.5\pi}))$	$(2, (0.52e^{i1.02\pi}, 0.013e^{i0.06\pi}, 0.75e^{i1.52\pi}))$	$(1, (0.23e^{i0.5\pi}, 0.035e^{i0.066\pi}, 0.89e^{1.82i\pi}))$	$(0, (0.1e^{i0.24\pi}, 0.012e^{i0.026\pi}, 0.99e^{1.972i\pi}))$

Table 29: Tabular representation of $CSFNS_fDM$ of expert $\widetilde{E}_3$.

$(F_J^{(3)}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
w_1	$(3, (0.84e^{i1.66\pi}, 0.019e^{i0.04\pi}, 0.29e^{i0.6\pi}))$	$(4, (0.97e^{i1.98\pi}, 0.013e^{i0.028\pi}, 0.14e^{i0.26\pi}))$	$(4, (0.96e^{i1.98\pi}, 0.01e^{i0.04\pi}, 0.04e^{i0.04\pi}))$	$(2, (0.5e^{i1.02\pi}, 0.014e^{i0.026\pi}, 0.74e^{1.46i\pi}))$	$(3, (0.8e^{i1.62\pi}, 0.018e^{i0.038\pi}, 0.29e^{0.6i\pi}))$
w_2	$(3, (0.72e^{i1.46\pi}, 0.036e^{i0.068\pi}, 0.41e^{i0.8\pi}))$	$(2, (0.49e^{i\pi}, 0.013e^{i0.028\pi}, 0.71e^{i1.44\pi}))$	$(2, (0.48e^{i0.98\pi}, 0.015e^{i0.032\pi}, 0.7e^{1.36\pi}))$	$(3, (0.74e^{i1.44\pi}, 0.034e^{i0.07\pi}, 0.42e^{0.82i\pi}))$	$(3, (0.75e^{i1.48\pi}, 0.035e^{i0.072\pi}, 0.43e^{0.82i\pi}))$
w_3	$(2, (0.47e^{i0.96\pi}, 0.015e^{i0.032\pi}, 0.7e^{i1.36\pi}))$	$(1, (0.27e^{i0.5\pi}, 0.034e^{i0.066\pi}, 0.94e^{1.92\pi}))$	$(2, (0.46e^{i0.94\pi}, 0.014e^{i0.03\pi}, 0.68e^{1.38\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.015e^{i0.032\pi}, 0.25e^{0.46i\pi}))$	$(1, (0.29e^{i0.56\pi}, 0.04e^{i0.078\pi}, 0.96e^{1.9i\pi}))$
w_4	$(4, (0.98e^{i1.94\pi}, 0.013e^{i0.06\pi}, 0.1e^{i0.26\pi}))$	$(3, (0.76e^{i1.5\pi}, 0.036e^{i0.074\pi}, 0.44e^{i0.86\pi}))$	$(0, (0.13e^{i0.6\pi}, 0.014e^{i0.03\pi}, 0.986e^{1.97\pi}))$	$(3, (0.77e^{i1.52\pi}, 0.038e^{i0.078\pi}, 0.45e^{0.88i\pi}))$	$(1, (0.3e^{i0.58\pi}, 0.042e^{i0.082\pi}, 0.925e^{1.86i\pi}))$
w_5	$(2, (0.44e^{i0.92\pi}, 0.012e^{i0.028\pi}, 0.66e^{i1.34\pi}))$	$(2, (0.43e^{i0.88\pi}, 0.015e^{i0.028\pi}, 0.66e^{i1.34\pi}))$	$(4, (0.87e^{i1.78\pi}, 0.016e^{i0.028\pi}, 0.21e^{i0.43\pi}))$	$(2, (0.42e^{i0.86\pi}, 0.012e^{i0.02\pi}, 0.65e^{1.32i\pi}))$	$(3, (0.78e^{i1.54\pi}, 0.04e^{i0.076\pi}, 0.46e^{0.9i\pi}))$

Table 30: Tabular representation of $CSFNS_f DM$ of expert $\widetilde{E}_4$.

$(F_J^{(4)}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
w_1	$(4, (0.95e^{i1.94\pi}, 0.012e^{i0.02\pi}, 0.13e^{i0.2\pi}))$	$(3, (0.8e^{i1.62\pi}, 0.018e^{i0.038\pi}, 0.31e^{i0.6\pi}))$	$(4, (0.97e^{i1.96\pi}, 0.01e^{i0.02\pi}, 0.15e^{i0.32\pi}))$	$(2, (0.63e^{i1.28\pi}, 0.012e^{i0.02\pi}, 0.6e^{1.18i\pi}))$	$(2, (0.64e^{i1.26\pi}, 0.01e^{i0.024pi}, 0.59e^{1.2i\pi}))$
w_2	$(3, (0.77e^{i1.56\pi}, 0.041e^{i0.08\pi}, 0.47e^{i0.92\pi}))$	$(1, (0.31e^{i0.64\pi}, 0.022e^{i0.04\pi}, 0.89e^{i1.8\pi}))$	$(0, (0.06e^{i0.14\pi}, 0.012e^{i0.02\pi}, 0.986e^{1.97\pi}))$	$(3, (0.8e^{i1.58\pi}, 0.05e^{i0.102\pi}, 0.48e^{0.94i\pi}))$	$(2, (0.37e^{i1.78\pi}, 0.014e^{i0.024\pi}, 0.86e^{1.74i\pi}))$
w_3	$(1, (0.3e^{i0.64\pi}, 0.05e^{i0.012\pi}, 0.91e^{i1.84\pi}))$	$(2, (0.41e^{i0.84\pi}, 0.01e^{i0.06\pi}, 0.64e^{1.3\pi}))$	$(2, (0.4e^{i0.84\pi}, 0.015e^{i0.032\pi}, 0.63e^{1.28\pi}))$	$(2, (0.39e^{i0.8\pi}, 0.014e^{i0.03\pi}, 0.62e^{1.26i\pi}))$	$(1, (0.33e^{i0.64\pi}, 0.07e^{i0.1\pi}, 0.93e^{1.88i\pi}))$
w_4	$(2, (0.38e^{i0.78\pi}, 0.013e^{i0.07\pi}, 0.61e^{i1.24\pi}))$	$(4, (0.9e^{i1.84\pi}, 0.014e^{i0.03\pi}, 0.19e^{i0.36\pi}))$	$(2, (0.37e^{i0.76\pi}, 0.012e^{i0.026\pi}, 0.6e^{1.22\pi}))$	$(3, (0.81e^{i1.6\pi}, 0.051e^{i0.104\pi}, 0.49e^{i\pi}))$	$(0, (0.13e^{i0.28\pi}, 0.014e^{i0.026\pi}, 0.982e^{1.966i\pi}))$
w_5	$(0, (0.07e^{i0.012\pi}, 0.016e^{i0.03\pi}, 0.991e^{i1.984\pi}))$	$(1, (0.34e^{i0.64\pi}, 0.065e^{i0.132\pi}, 0.95e^{i1.92\pi}))$	$(1, (0.3e^{i0.58\pi}, 0.014e^{i0.03\pi}, 0.93e^{i1.88\pi}))$	$(2, (0.36e^{i0.74\pi}, 0.01e^{i0.024\pi}, 0.59e^{1.2i\pi}))$	$(1, (0.21e^{i0.46\pi}, 0.024e^{i0.052\pi}, 0.94e^{1.86i\pi}))$

Table 31: Tabular representation of $ACSFNS_f DM$.

$(F_J^{(1)}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
w_1	$(4, (0.95e^{i1.94\pi}, 0.013e^{i0.026\pi}, 0.07e^{i0.18\pi}))$	$(4, (0.97e^{i1.9\pi}, 0.013e^{i0.026\pi}, 0.15e^{i0.34\pi}))$	$(4, (0.97e^{i1.96\pi}, 0.01e^{i0.02\pi}, 0.05e^{i0.1\pi}))$	$(3, (0.77e^{i1.54\pi}, 0.016e^{i0.03\pi}, 0.4e^{0.82i\pi}))$	$(4, (0.91e^{i1.76\pi}, 0.013e^{i0.03\pi}, 0.15e^{0.36i\pi}))$
w_2	$(3, (0.79e^{i1.56\pi}, 0.028e^{i0.058\pi}, 0.36e^{i0.74\pi}))$	$(3, (0.48e^{i0.96\pi}, 0.019e^{i0.034\pi}, 0.73e^{i1.46\pi}))$	$(2, (0.33e^{i0.66\pi}, 0.01e^{i0.02\pi}, 0.88e^{1.76\pi}))$	$(4, (0.9e^{i1.82\pi}, 0.02e^{i0.04\pi}, 0.15e^{0.26i\pi}))$	$(3, (0.47e^{i0.96\pi}, 0.016e^{i0.03\pi}, 0.68e^{1.36i\pi}))$
w_3	$(2, (0.25e^{i0.52\pi}, 0.022e^{i0.048\pi}, 0.9e^{i1.74\pi}))$	$(2, (0.29e^{i0.58\pi}, 0.018e^{i0.038\pi}, 0.83e^{1.66\pi}))$	$(2, (0.5e^{i1.04\pi}, 0.013e^{i0.028\pi}, 0.72e^{1.42\pi}))$	$(4, (0.55e^{i1.12\pi}, 0.029e^{i0.038\pi}, 0.6e^{1.2i\pi}))$	$(2, (0.35e^{i0.7\pi}, 0.029e^{i0.054\pi}, 0.9e^{1.82i\pi}))$
w_4	$(4, (0.7e^{i1.18\pi}, 0.016e^{i0.034\pi}, 0.63e^{i1.28\pi}))$	$(4, (0.77e^{i1.56\pi}, 0.034e^{i0.064\pi}, 0.42e^{i0.84\pi}))$	$(3, (0.51e^{i1.02\pi}, 0.01e^{i0.02\pi}, 0.53e^{1.06\pi}))$	$(3, (0.71e^{i1.4\pi}, 0.026e^{i0.054\pi}, 0.55e^{1.12i\pi}))$	$(3, (0.52e^{i1.06\pi}, 0.019e^{i0.038\pi}, 0.6e^{1.22i\pi}))$
w_5	$(2, (0.39e^{i0.74\pi}, 0.015e^{i0.032\pi}, 0.83e^{i1.68\pi}))$	$(2, (0.35e^{i0.7\pi}, 0.025e^{i0.052\pi}, 0.87e^{i1.74\pi}))$	$(4, (0.63e^{i1.26\pi}, 0.02e^{i0.034\pi}, 0.51e^{i1.02\pi}))$	$(2, (0.28e^{i0.6\pi}, 0.016e^{i0.034\pi}, 0.76e^{1.58i\pi}))$	$(3, (0.49e^{i0.96\pi}, 0.015e^{i0.034\pi}, 0.83e^{1.6i\pi}))$

Table 32: Experts opinion related to each attribute.

$(F_J, Z, 6)$	$\widetilde{E}_1$	$\widetilde{E}_2$	$\widetilde{E}_3$	$\widetilde{E}_4$
z_1	$(3, (0.85e^{i1.66\pi}, 0.019e^{i0.042\pi}, 0.29e^{i0.6\pi}))$	$(4, (0.99e^{i1.94\pi}, 0.012e^{i0.02\pi}, 0.1e^{i0.26\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.015e^{i0.032\pi}, 0.25e^{i0.46\pi}))$	$(3, (0.67e^{i1.36\pi}, 0.021e^{i0.044\pi}, 0.31e^{0.64i\pi}))$
z_2	$(3, (0.66e^{i1.34\pi}, 0.021e^{i0.044\pi}, 0.31e^{i0.64\pi}))$	$(4, (0.96e^{i1.94\pi}, 0.013e^{i0.024\pi}, 0.04e^{i0.06\pi}))$	$(3, (0.78e^{i1.54\pi}, 0.04e^{i0.076\pi}, 0.46e^{0.9\pi}))$	$(4, (0.98e^{i1.98\pi}, 0.01e^{i0.024\pi}, 0.02e^{0.06i\pi}))$
z_3	$(2, (0.63e^{i1.28\pi}, 0.012e^{i0.02\pi}, 0.59e^{i1.2\pi}))$	$(3, (0.72e^{i1.42\pi}, 0.03e^{i0.062\pi}, 0.38e^{0.78\pi}))$	$(3, (0.75e^{i1.48\pi}, 0.035e^{i0.072\pi}, 0.43e^{0.82\pi}))$	$(2, (0.42e^{i0.86\pi}, 0.012e^{i0.02\pi}, 0.65e^{i1.32\pi}))$
z_4	$(1, (0.29e^{i0.6\pi}, 0.014e^{i0.03\pi}, 0.94e^{i1.86\pi}))$	$(1, (0.21e^{i0.46\pi}, 0.024e^{i0.052\pi}, 0.94e^{i1.86\pi}))$	$(2, (0.37e^{i0.76\pi}, 0.012e^{i0.026\pi}, 0.6e^{1.22\pi}))$	$(3, (0.78e^{i1.54\pi}, 0.04e^{i0.076\pi}, 0.46e^{0.9i\pi}))$
z_5	$(2, (0.62e^{i1.28\pi}, 0.01e^{i0.024\pi}, 0.58e^{i1.2\pi}))$	$(2, (0.66e^{i1.34\pi}, 0.1e^{i0.18\pi}, 0.56e^{i1.14\pi}))$	$(2, (0.64e^{i1.26\pi}, 0.01e^{i0.024\pi}, 0.59e^{i1.2\pi}))$	$(1, (0.21e^{i0.4\pi}, 0.021e^{i0.046\pi}, 0.95e^{1.88i\pi}))$

Table 33: Tabular representation of $AWCSFNS_fDM$.

	z_1	z_2	z_3	z_4	z_5
w_1	$(4, (0.85e^{i1.66\pi}, 0.021e^{i0.042\pi}, 0.23e^{i0.52\pi}))$	$(4, (0.88e^{i1.76\pi}, 0.021e^{i0.042\pi}, 0.174e^{i0.38\pi}))$	$(3, (0.601e^{i1.22\pi}, 0.019e^{i0.034\pi}, 0.053e^{i1.1\pi}))$	$(3, (0.41e^{i0,82\pi}, 0.025e^{i0.05\pi}, 0.77e^{1.54i\pi}))$	$(2, (0.5e^{i\pi}, 0.023e^{i0.05\pi}, 0.67e^{1.36i\pi}))$
w_2	$(3, (0.7e^{i1.34\pi}, 0.03e^{i0.066\pi}, 0.41e^{i0.86\pi}))$	$(3, (0.43e^{i0.9\pi}, 0.025e^{i0.048\pi}, 0.73e^{i1.46\pi}))$	$(2, (0.2e^{i0.4\pi}, 0.019e^{i0.034\pi}, 0.91e^{1.82\pi}))$	$(3, (0.47e^{i0.96\pi}, 0.028e^{i0.056\pi}, 0.73e^{1.42i\pi}))$	$(2, (0.25e^{i0.54\pi}, 0.025e^{i0.05\pi}, 0.83e^{1.66i\pi}))$
w_3	$(2, (0.22e^{i0.44\pi}, 0.027e^{i0.058\pi}, 0.91e^{i1.74\pi}))$	$(2, (0.26e^{i0.54\pi}, 0.024e^{i0.025\pi}, 0.83e^{1.66\pi}))$	$(2, (0.32e^{i0.64\pi}, 0.021e^{i0.038\pi}, 0.8e^{1.62\pi}))$	$(3, (0.29e^{i0.58\pi}, 0.035e^{i0.068\pi}, 0.73e^{1.42i\pi}))$	$(2, (0.19e^{i0.34\pi}, 0.035e^{i0.068\pi}, 0.95e^{1.9i\pi}))$
w_4	$(4, (0.62e^{i1.02\pi}, 0.023e^{i0.048\pi}, 0.65e^{i1.34\pi}))$	$(4, (0.70e^{i1.46\pi}, 0.037e^{i0.072\pi}, 0.43e^{i0.86\pi}))$	$(3, (0.32e^{i0.64\pi}, 0.019e^{i0.034\pi}, 0.69e^{1.42\pi}))$	$(3, (0.37e^{i0.74\pi}, 0.033e^{i0.066\pi}, 0.82e^{1.62i\pi}))$	$(2, (0.29e^{i0.6\pi}, 0.027e^{i0.056\pi}, 0.8e^{1.62i\pi}))$
w_5	$(2, (0.35e^{i0.62\pi}, 0.022e^{i0.046\pi}, 0.83e^{i1.7\pi}))$	$(2, (0.31e^{i0.66\pi}, 0.029e^{i0.06\pi}, 0.87e^{i1.74\pi}))$	$(3, (0.39e^{i0.78\pi}, 0.025e^{i0.044\pi}, 0.68e^{i1.38\pi}))$	$(2, (0.15e^{i0.318\pi}, 0.025e^{i0.052\pi}, 0.89e^{1.8i\pi}))$	$(2, (0.27e^{i0.54\pi}, 0.024e^{i0.042\pi}, 0.91e^{1.82i\pi}))$

Table 34: Tabular representation of $CSFNS_f$-PIS and $CSFNS_f$-NIS.

Attribute	$CSFNS_f$-PIS	$CSFNS_f$-NIS
z_1	$(4, (0.85e^{i1.66\pi}, 0.021e^{i0.042\pi}, 0.23e^{i0.52\pi}))$	$(2, (0.22e^{i0.44\pi}, 0.027e^{i0.058\pi}, 0.91e^{i1.74\pi}))$
z_2	$(4, (0.88e^{i1.76\pi}, 0.021e^{i0.042\pi}, 0.174e^{i0.038\pi}))$	$(2, (0.31e^{i0.66\pi}, 0.029e^{i0.06\pi}, 0.87e^{i1.74\pi}))$
z_3	$(3, (0.601e^{i1.22\pi}, 0.019e^{i0.034\pi}, 0.53e^{i1.1\pi}))$	$(2, (0.2e^{i0.4\pi}, 0.019e^{i0.034\pi}, 0.91e^{1.82\pi}))$
z_4	$(3, (0.47e^{i0.96\pi}, 0.028e^{i0.056\pi}, 0.73e^{1.42\pi}))$	$(2, (0.15e^{i0.318\pi}, 0.025e^{i0.052\pi}, 0.89e^{1.8\pi}))$
z_5	$(2, (0.19e^{i0.38\pi}, 0.035e^{i0.068\pi}, 0.95e^{i1.9\pi}))$	$(2, (0.5e^{i\pi}, 0.023e^{i0.046\pi}, 0.67e^{i1.36\pi}))$

Table 35: Tabular representation of normalized Euclidean distance from ideal solution.

Alternative	$d(\widehat{\mathscr{P}}_k, w_j)$	$d(\check{\mathscr{P}}_k, w_j)$
w_1	0.20559	0.55439
w_2	0.36677	0.311911
w_3	0.528106	0.18778
w_4	0.254649	0.396793
w_5	0.50649	0.1936

Table 36: Tabular representation of revised closeness index of each alternative.

Alternative	$\mathfrak{I}(w_j)$
w_1	0
w_2	1.22136
w_3	2.236816
w_4	0.485238
w_5	2.1143

Table 37: Tabular representation of revised closeness index of each alternative.

Alternative	Ranking
w_1	1
w_2	3
w_3	5
w_4	2
w_5	4

the weight vector χ for the attributes, and are defined as follows:

$$\chi_k = \left(\sqrt{1 - \prod_1^{d=s} \left(1 - \left(p_k^{(d)}\right)^2\right)^{\sigma_d}}\, e^{i2\pi \sqrt{1 - \prod_1^{d=s} \left(1 - \left(\phi_k^{(d)}\right)^2\right)^{\sigma_d}}}, \prod_1^{d=s} \nu_k^{(d)} e^{i2\pi \prod_1^{d=s} \delta_k^{(d)}}, \prod_1^{d=s} r_k^{(d)} e^{i2\pi \prod_1^{d=s} \lambda_k^{(d)}} \right). \tag{36}$$

Thus, we have

$$\chi = \begin{pmatrix} \left(0.89e^{i1.72\pi}, 0.017e^{i0.034\pi}, 0.23e^{0.5i\pi}\right) \\ \left(0.91e^{i1.86\pi}, 0.016e^{i0.034\pi}, 0.09e^{0.2i\pi}\right) \\ \left(0.62e^{i1.24\pi}, 0.016e^{i0.028\pi}, 0.53e^{1.1i\pi}\right) \\ \left(0.53e^{i1.06\pi}, 0.02e^{i0.04\pi}, 0.73e^{1.42i\pi}\right) \\ \left(0.55e^{i1.14\pi}, 0.019e^{i0.042\pi}, 0.67e^{1.34i\pi}\right) \end{pmatrix}. \tag{37}$$

Step 4: the aggregated weighted complex spherical decision matrix (AWCSFDM) is arranged in Table 47, where the entries of AWCSFDM are calculated using the formula [16]

$$\overline{\mathscr{P}_{jk}} = \left(p_{jk} p_k e^{i2\pi\phi_{jk}\phi_k}, \sqrt{\nu_{jk}^2 + \nu_k^2 - \nu_{jk}^2 \nu_k^2}\, e^{i2\pi\sqrt{\delta_{jk}^2 + \delta_k^2 - \delta_{jk}^2\delta_k^2}}, \sqrt{r_{jk}^2 + r_k^2 - r_{jk}^2 r_k^2}\, e^{i2\pi\sqrt{\lambda_{jk}^2 + \lambda_k^2 - \lambda_{jk}^2\lambda_k^2}} \right). \tag{38}$$

Step 5: to compute the complex spherical fuzzy positive ideal solution (CSF-PIS) and negative ideal solution (CSF-NIS), we evaluate the score degree of all CSFNs in AWCSFDM, using the formula

$$Sc\left(\overline{\mathscr{P}_{jk}}\right) = \left(p_{kj}^2 - w_{kj}^2 - r_{kj}^2\right) + \left[\phi_{kj}^2 - \delta_{kj}^2 - \lambda_{kj}^2\right]. \tag{39}$$

Table 48 represents the CSF-PIS and CSF-NIS with the help of equation (26).

Step 6: the complex spherical fuzzy normalized Euclidean distance of each alternative is given in Table 49 and computed by the formula [16], from CSF-PIS:

Table 38: Tabular representation of the $PF6S_fS$ defined from the problem proposed in Section 5.1.

	z_1	z_2	z_3	z_4
$\mathbb{a}_1$	$(5, (0.95, 0.13))$	$(4, (0.87, 0.21))$	$(4, (0.88, 0.2))$	$(2, (0.48, 0.77))$
$\mathbb{a}_2$	$(3, (0.7, 0.49))$	$(2, (0.32, 0.9))$	$(3, (0.6, 0.69))$	$(3, (0.62, 0.73))$
$\mathbb{a}_3$	$(2, (0.41, 0.77))$	$(2, (0.33, 0.89))$	$(2, (0.35, 0.84))$	$(3, (0.53, 0.67))$
$\mathbb{a}_4$	$(2, (0.4, 0.9))$	$(3, (0.72, 0.59))$	$(5, (0.9, 0.18))$	$(4, (0.77, 0.35))$
$\mathbb{a}_5$	$(3, (0.71, 0.51))$	$(4, (0.89, 0.145))$	$(3, (0.55, 0.7))$	$(2, (0.49, 0.76))$

Table 39: Tabular representation of Choice value of $PF6S_fS$ in Section 5.1.

	z_1	z_2	z_3	z_4	H_i
$\mathbb{a}_1$	$(5, (0.95, 0.13))$	$(4, (0.87, 0.21))$	$(4, (0.88, 0.2))$	$(2, (0.48, 0.77))$	$(15, 2.83162)$
$\mathbb{a}_2$	$(3, (0.7, 0.49))$	$(2, (0.32, 0.9))$	$(3, (0.6, 0.69))$	$(3, (0.62, 0.73))$	$(11, 1.6754702)$
$\mathbb{a}_3$	$(2, (0.41, 0.77))$	$(2, (0.33, 0.89))$	$(2, (0.35, 0.84))$	$(3, (0.53, 0.67))$	$(9, 1.3498273)$
$\mathbb{a}_4$	$(2, (0.4, 0.9))$	$(3, (0.72, 0.59))$	$(5, (0.9, 0.18))$	$(4, (0.77, 0.35))$	$(14, 2.30656384)$
$\mathbb{a}_5$	$(3, (0.71, 0.51))$	$(4, (0.89, 0.145))$	$(3, (0.55, 0.7))$	$(2, (0.49, 0.76))$	$(12, 2.231012)$

Table 40: Tabular representation of 4 choice value of $PF6S_fS$ Section 5.1.

	z_1	z_2	z_3	z_4	H_i^4
$\mathbb{a}_1$	$(0.95, 0.13)$	$(0.87, 0.21)$	$(0.88, 0.2)$	$(0, 1)$	0.33532
$\mathbb{a}_2$	$(0, 0.5)$	$(0, 1)$	$(0, 0.5)$	$(0, 0.5)$	−0.4375
$\mathbb{a}_3$	$(0, 1)$	$(0, 1)$	$(0, 1)$	$(0, 0.5)$	−0.8125
$\mathbb{a}_4$	$(0, 1)$	$(0, 0.5)$	$(0.9, 0.18)$	$(0.77, 0.35)$	−0.0005
$\mathbb{a}_5$	$(0, 0.5)$	$(0.89, 0.145)$	$(0, 0.5)$	$(0, 1)$	−0.18223125

$$d\left(\widehat{\mathscr{P}}_k, w_j\right) = \left(\frac{1}{3k}\sum_{k=1}^{m}\left[\left(\widehat{p}_k^2 - \overline{p}_{jk}^2\right)^2 + \left(\widehat{v}_k^2 - \overline{v}_{jk}^2\right)^2 + \left(\widehat{r}_k^2 - \overline{r}_{jk}^2\right)^2 + \left(\widehat{\phi}_k^2 - \overline{\phi}_{jk}^2\right)^2 + \left(\widehat{\delta}_k^2 - \overline{\delta}_{jk}^2\right)^2 + \left(\widehat{\lambda}_k^2 - \overline{\lambda}_{jk}^2\right)^2\right]\right)^{1/2}. \tag{40}$$

Similarly, the normalized Euclidean distance between the CSF-NIS and any of the alternative w_j can be formulated as follows:

$$d\left(\check{\mathscr{P}}_k, w_j\right) = \left(\frac{1}{3k}\sum_{k=1}^{m}\left[\left(\check{p}_k^2 - \overline{p}_{jk}^2\right)^2 + \left(\check{v}_k^2 - \overline{v}_{jk}^2\right)^2 + \left(\check{r}_k^2 - \overline{r}_{jk}^2\right)^2 + \left(\check{\phi}_k^2 - \overline{\phi}_{jk}^2\right)^2 + \left(\check{\delta}_k^2 - \overline{\delta}_{jk}^2\right)^2 + \left(\check{\lambda}_k^2 - \overline{\lambda}_{jk}^2\right)^2\right]\right)^{1/2}. \tag{41}$$

Step 7: Equation (33) is used to calculate the revised closeness index of each alternative, as given in Table 50.

Step 8: revised closeness index in Table 50 reveals that w_1 is the best alternative within the ranking $w_1 > w_4 > w_2 > w_5 > w_3$.

6.1.1. Discussion

(1) We now compare the proposed model $CSFNS_f$-TOPSIS method with the existing technique CSF-TOPSIS method to evaluate the accuracy of the result. The same result concludes from both methods as well as the ranking of the alternatives also same.

(2) We also apply technique on SF-TOPSIS methods [10, 11], to select the most appropriate physiotherapist. The same results including the ranking and best solution are organized in Table 51, which enhance the credibility of the proposed method.

(3) The proposed model deals not only with 2-dimensional uncertainties but also with the level of attribute for the alternative. The existing models are unable to handle MAGDM problems, but the proposed model has the ability to tackle those real-life DM problems having ranking system and parameterized information.

(4) The proposed model, $CSFNS_f$-TOPSIS method, could be efficiently applied to the environment of

TABLE 41: Tabular representation of CSFDM of expert $\tilde{E}_1$.

	z_1	z_2	z_3	z_4	z_5
w_1	$(0.98e^{i1.98\pi}, 0.01e^{i0.024\pi}, 0.02e^{i0.06\pi})$	$(0.99e^{i1.94\pi}, 0.012e^{i0.02\pi}, 0.1e^{i0.26\pi})$	$(0.97e^{i1.96\pi}, 0.013e^{i0.02\pi}, 0.05e^{i0.08\pi})$	$(0.84e^{i1.66\pi}, 0.019e^{i0.04\pi}, 0.29e^{0.6i\pi})$	$(0.98e^{i1.92\pi}, 0.012e^{i0.026\pi}, 0.03e^{0.01i\pi})$
w_2	$(0.84e^{i1.66\pi}, 0.02e^{i0.042\pi}, 0.29e^{i0.62\pi})$	$(0.37e^{i0.78\pi}, 0.015e^{i0.026\pi}, 0.87e^{i1.72\pi})$	$(0.16e^{i0.34\pi}, 0.018e^{i0.038\pi}, 0.89e^{1.78\pi})$	$(0.96e^{i1.94\pi}, 0.013e^{i0.024\pi}, 0.04e^{0.06i\pi})$	$(0.36e^{i0.74\pi}, 0.016e^{i0.03\pi}, 0.59e^{1.2i\pi})$
w_3	$(0.17e^{i0.36\pi}, 0.02e^{i0.038\pi}, 0.91e^{i1.84\pi})$	$(0.19e^{i0.36\pi}, 0.021e^{i0.04\pi}, 0.93e^{1.82\pi})$	$(0.59e^{i0.12\pi}, 0.0155e^{i0.032\pi}, 0.82e^{1.6\pi})$	$(0.2e^{i0.38\pi}, 0.022e^{i0.042\pi}, 0.92e^{1.86i\pi})$	$(0.21e^{i0.44\pi}, 0.024e^{i0.05\pi}, 0.93e^{1.88i\pi})$
w_4	$(0.58e^{i1.12\pi}, 0.015e^{i0.032\pi}, 0.82e^{i1.66\pi})$	$(0.66e^{i1.34\pi}, 0.1e^{i0.18\pi}, 0.56e^{i1.14\pi})$	$(0.67e^{i1.36\pi}, 0.021e^{i0.044\pi}, 0.31e^{i0.64\pi})$	$(0.55e^{i1.14\pi}, 0.014e^{i0.028\pi}, 0.8e^{1.62i\pi})$	$(0.67e^{i1.38\pi}, 0.025e^{i0.048\pi}, 0.33e^{0.68i\pi})$
w_5	$(0.52e^{i1.02\pi}, 0.012e^{i0.026\pi}, 0.74e^{i1.5\pi})$	$(0.21e^{i0.4\pi}, 0.021e^{i0.046\pi}, 0.95e^{i1.88\pi})$	$(0.7e^{i1.36\pi}, 0.026e^{i0.05\pi}, 0.35e^{i0.68\pi})$	$(0.16e^{i0.42\pi}, 0.018e^{i0.038\pi}, 0.89e^{1.88i\pi})$	$(0.58e^{i1.14\pi}, 0.01e^{i0.024\pi}, 0.82e^{1.62i\pi})$

Table 42: Tabular representation of CSFDM of expert $\widetilde{E}_2$.

	z_1	z_2	z_3	z_4	z_5
w_1	$(0.83e^{i1.64\pi}, 0.019e^{i0.036\pi}, 0.3e^{i0.58\pi})$	$(0.98e^{i1.94\pi}, 0.013e^{i0.024\pi}, 0.1e^{i0.26\pi})$	$(0.98e^{i1.98\pi}, 0.01e^{i0.024\pi}, 0.02e^{i0.06\pi})$	$(0.84e^{i1.66\pi}, 0.02e^{i0.036\pi}, 0.31e^{0.63i\pi})$	$(0.8e^{i1.62\pi}, 0.022e^{i0.042\pi}, 0.32e^{0.6i\pi})$
w_2	$(0.71e^{i1.38\pi}, 0.028e^{i0.058\pi}, 0.37e^{i0.72\pi})$	$(0.72e^{i1.42\pi}, 0.03e^{i0.062\pi}, 0.38e^{i0.76\pi})$	$(0.57e^{i1.1\pi}, 0.012e^{i0.028\pi}, 0.81e^{1.6\pi})$	$(0.86e^{i1.74\pi}, 0.0169e^{i0.032\pi}, 0.027e^{0.052i\pi})$	$(2, (0.56e^{i1.1\pi}, 0.015e^{i0.07\pi}, 0.8e^{1.58i\pi}))$
w_3	$(0.05e^{i0.012\pi}, 0.01e^{i0.06\pi}, 0.985e^{i1.972\pi})$	$(0.21e^{i0.46\pi}, 0.024e^{i0.52\pi}, 0.94e^{1.86\pi})$	$(0.5e^{i1.02\pi}, 0.01e^{i0.02\pi}, 0.7e^{1.4\pi})$	$(0.73e^{i1.48\pi}, 0.032e^{i0.062\pi}, 0.39e^{0.76i\pi})$	$(2, (0.55e^{i1.08\pi}, 0.01e^{i0.02\pi}, 0.79e^{1.56i\pi}))$
w_4	$(0.74e^{i1.5\pi}, 0.035e^{i0.068\pi}, 0.4e^{i0.78\pi})$	$(0.54e^{i1.04\pi}, 0.015e^{i0.026\pi}, 0.78e^{i1.54\pi})$	$(1, (0.24e^{i0.44\pi}, 0.03e^{i0.056\pi}, 0.94e^{1.84\pi}))$	$(0.71e^{i1.38\pi}, 0.028e^{i0.058\pi}, 0.37e^{0.72i\pi})$	$(2, (0.53e^{i1.04\pi}, 0.013e^{i0.026\pi}, 0.77e^{1.52i\pi}))$
w_5	$(0.24e^{i0.42\pi}, 0.028e^{i0.062\pi}, 0.91e^{i1.84\pi})$	$(0.51e^{i1.04\pi}, 0.013e^{i0.06\pi}, 0.76e^{i1.5\pi})$	$(0.52e^{i1.02\pi}, 0.013e^{i0.06\pi}, 0.75e^{i1.52\pi})$	$(0.23e^{i0.5\pi}, 0.035e^{i0.066\pi}, 0.89e^{1.82i\pi})$	$(0.1e^{i0.24\pi}, 0.012e^{i0.026\pi}, 0.99e^{1.972i\pi})$

Table 43: Tabular representation of CSFDM of expert $\widetilde{E}_3$.

	z_1	z_2	z_3	z_4	z_5
w_1	$(0.84e^{i1.66\pi}, 0.019e^{i0.04\pi}, 0.29e^{i0.6\pi})$	$(0.97e^{i1.98\pi}, 0.013e^{i0.028\pi}, 0.14e^{i0.26\pi})$	$(0.96e^{i1.98\pi}, 0.01e^{i0.04\pi}, 0.04e^{i0.04\pi})$	$(0.5e^{i1.02\pi}, 0.014e^{i0.026\pi}, 0.74e^{1.46i\pi})$	$(0.8e^{i1.62\pi}, 0.018e^{i0.038\pi}, 0.29e^{0.6i\pi})$
w_2	$(0.72e^{i1.46\pi}, 0.036e^{i0.068\pi}, 0.41e^{i0.8\pi})$	$(0.49e^{i\pi}, 0.013e^{i0.028\pi}, 0.71e^{i1.44\pi})$	$(0.48e^{i0.98\pi}, 0.015e^{i0.032\pi}, 0.7e^{1.36\pi})$	$(0.74e^{i1.44\pi}, 0.034e^{i0.07\pi}, 0.42e^{0.82i\pi})$	$(0.75e^{i1.48\pi}, 0.035e^{i0.072\pi}, 0.43e^{0.82i\pi})$
w_3	$(0.47e^{i0.96\pi}, 0.015e^{i0.032\pi}, 0.7e^{i1.36\pi})$	$(0.27e^{i0.5\pi}, 0.034e^{i0.066\pi}, 0.94e^{1.92\pi})$	$(0.46e^{i0.94\pi}, 0.014e^{i0.03\pi}, 0.68e^{1.38\pi})$	$(0.88e^{i1.78\pi}, 0.015e^{i0.032\pi}, 0.25e^{0.46i\pi})$	$(0.29e^{i0.56\pi}, 0.04e^{i0.078\pi}, 0.96e^{1.9i\pi})$
w_4	$(0.98e^{i1.94\pi}, 0.013e^{i0.06\pi}, 0.1e^{i0.26\pi})$	$(0.76e^{i1.5\pi}, 0.036e^{i0.074\pi}, 0.44e^{i0.86\pi})$	$(0.13e^{i0.6\pi}, 0.014e^{i0.03\pi}, 0.986e^{1.97\pi})$	$(0.77e^{i1.52\pi}, 0.038e^{i0.078\pi}, 0.45e^{0.88i\pi})$	$(0.3e^{i0.58\pi}, 0.042e^{i0.082\pi}, 0.925e^{1.86i\pi})$
w_5	$(0.44e^{i0.92\pi}, 0.012e^{i0.028\pi}, 0.66e^{i1.34\pi})$	$(0.43e^{i0.88\pi}, 0.015e^{i0.028\pi}, 0.66e^{i1.34\pi})$	$(0.87e^{i1.78\pi}, 0.016e^{i0.028\pi}, 0.21e^{i0.43\pi})$	$(0.42e^{i0.86\pi}, 0.012e^{i0.02\pi}, 0.65e^{1.32i\pi})$	$(0.78e^{i1.54\pi}, 0.04e^{i0.076\pi}, 0.46e^{0.9i\pi})$

Table 44: Tabular representation of CSFDM of expert $\widetilde{E}_4$.

	z_1	z_2	z_3	z_4	z_5
w_1	$(0.95e^{i1.94\pi}, 0.012e^{i0.02\pi}, 0.13e^{i0.2\pi})$	$(0.8e^{i1.62\pi}, 0.018e^{i0.038\pi}, 0.31e^{i0.6\pi})$	$(0.97e^{i1.96\pi}, 0.01e^{i0.02\pi}, 0.15e^{i0.32\pi})$	$(0.63e^{i1.28\pi}, 0.012e^{i0.02\pi}, 0.6e^{1.18i\pi})$	$(0.64e^{i1.26\pi}, 0.01e^{i0.024pi}, 0.59e^{1.2i\pi})$
w_2	$(0.77e^{i1.56\pi}, 0.041e^{i0.08\pi}, 0.47e^{i0.92\pi})$	$(0.31e^{i0.64\pi}, 0.022e^{i0.04\pi}, 0.89e^{i1.8\pi})$	$(0.06e^{i0.14\pi}, 0.012e^{i0.02\pi}, 0.986e^{1.97\pi})$	$(0.8e^{i1.58\pi}, 0.05e^{i0.102\pi}, 0.48e^{0.94i\pi})$	$(0.37e^{i1.78\pi}, 0.014e^{i0.024\pi}, 0.86e^{1.74i\pi})$
w_3	$(0.3e^{i0.64\pi}, 0.05e^{i0.012\pi}, 0.91e^{i1.84\pi})$	$(0.41e^{i0.84\pi}, 0.01e^{i0.06\pi}, 0.64e^{1.3\pi})$	$(0.4e^{i0.84\pi}, 0.015e^{i0.032\pi}, 0.63e^{1.28\pi})$	$(0.39e^{i0.8\pi}, 0.014e^{i0.03\pi}, 0.62e^{1.26i\pi})$	$(1, (0.33e^{i0.64\pi}, 0.07e^{i0.1\pi}, 0.93e^{1.88i\pi}))$
w_4	$(0.38e^{i0.78\pi}, 0.013e^{i0.07\pi}, 0.61e^{i1.24\pi})$	$(0.9e^{i1.84\pi}, 0.014e^{i0.03\pi}, 0.19e^{i0.36\pi})$	$(0.37e^{i0.76\pi}, 0.012e^{i0.026\pi}, 0.6e^{1.22\pi})$	$(0.81e^{i1.6\pi}, 0.051e^{i0.104\pi}, 0.49e^{i\pi})$	$(0.13e^{i0.28\pi}, 0.014e^{i0.026\pi}, 0.982e^{1.966i\pi})$
w_5	$(0.07e^{i0.012\pi}, 0.016e^{i0.03\pi}, 0.991e^{i1.984\pi})$	$(0.34e^{i0.64\pi}, 0.065e^{i0.132\pi}, 0.95e^{i1.92\pi})$	$(0.3e^{i0.58\pi}, 0.014e^{i0.03\pi}, 0.93e^{i1.88\pi})$	$(0.36e^{i0.74\pi}, 0.01e^{i0.024\pi}, 0.59e^{1.2i\pi})$	$(0.21e^{i0.46\pi}, 0.024e^{i0.052\pi}, 0.94e^{1.86i\pi})$

Table 45: Tabular representation of ACSFDM.

	z_1	z_2	z_3	z_4	z_5
w_1	$(0.95e^{i1.94\pi}, 0.013e^{i0.026\pi}, 0.07e^{i0.18\pi})$	$(0.97e^{i1.9\pi}, 0.013e^{i0.026\pi}, 0.15e^{i0.34\pi})$	$(0.97e^{i1.96\pi}, 0.01e^{i0.02\pi}, 0.05e^{i0.1\pi})$	$(0.77e^{i1.54\pi}, 0.016e^{i0.03\pi}, 0.4e^{0.82i\pi})$	$(0.91e^{i1.76\pi}, 0.013e^{i0.03\pi}, 0.15e^{0.36i\pi})$
w_2	$(0.79e^{i1.56\pi}, 0.028e^{i0.058\pi}, 0.36e^{i0.74\pi})$	$(0.48e^{i0.96\pi}, 0.019e^{i0.034\pi}, 0.73e^{i1.46\pi})$	$(0.33e^{i0.66\pi}, 0.01e^{i0.02\pi}, 0.88e^{1.76\pi})$	$(0.9e^{i1.82\pi}, 0.02e^{i0.04\pi}, 0.15e^{0.26i\pi})$	$(0.47e^{i0.96\pi}, 0.016e^{i0.03\pi}, 0.68e^{1.36i\pi})$
w_3	$(0.25e^{i0.52\pi}, 0.022e^{i0.048\pi}, 0.9e^{i1.74\pi})$	$(0.29e^{i0.58\pi}, 0.018e^{i0.038\pi}, 0.83e^{1.66\pi})$	$(0.5e^{i1.04\pi}, 0.013e^{i0.028\pi}, 0.72e^{1.42\pi})$	$(0.55e^{i1.12\pi}, 0.029e^{i0.038\pi}, 0.6e^{1.2i\pi})$	$(0.35e^{i0.7\pi}, 0.029e^{i0.054\pi}, 0.9e^{1.82i\pi})$
w_4	$(0.7e^{i1.18\pi}, 0.016e^{i0.034\pi}, 0.63e^{i1.28\pi})$	$(0.77e^{i1.56\pi}, 0.034e^{i0.064\pi}, 0.42e^{i0.84\pi})$	$(0.51e^{i1.02\pi}, 0.01e^{i0.02\pi}, 0.53e^{1.06\pi})$	$(0.71e^{i1.4\pi}, 0.026e^{i0.054\pi}, 0.55e^{1.12i\pi})$	$(0.52e^{i1.06\pi}, 0.019e^{i0.038\pi}, 0.6e^{1.22i\pi})$
w_5	$(0.39e^{i0.74\pi}, 0.015e^{i0.032\pi}, 0.83e^{i1.68\pi})$	$(0.35e^{i0.7\pi}, 0.025e^{i0.052\pi}, 0.87e^{i1.74\pi})$	$(0.63e^{i1.26\pi}, 0.02e^{i0.034\pi}, 0.51e^{i1.02\pi})$	$(0.28e^{i0.6\pi}, 0.016e^{i0.034\pi}, 0.76e^{1.58i\pi})$	$(0.49e^{i0.96\pi}, 0.015e^{i0.034\pi}, 0.83e^{1.6i\pi})$

Table 46: Experts opinion related to each attribute.

	$\widetilde{E}_1$	$\widetilde{E}_2$	$\widetilde{E}_3$	$\widetilde{E}_4$
z_1	$(0.85e^{i1.66\pi}, 0.019e^{i0.042\pi}, 0.29e^{i0.6\pi})$	$(0.99e^{i1.94\pi}, 0.012e^{i0.02\pi}, 0.1e^{i0.26\pi})$	$(0.88e^{i1.78\pi}, 0.015e^{i0.032\pi}, 0.25e^{i0.46\pi})$	$(0.67e^{i1.36\pi}, 0.021e^{i0.044\pi}, 0.31e^{0.64i\pi})$
z_2	$(0.66e^{i1.34\pi}, 0.021e^{i0.044\pi}, 0.31e^{i0.64\pi})$	$(0.96e^{i1.94\pi}, 0.013e^{i0.024\pi}, 0.04e^{i0.06\pi})$	$(0.78e^{i1.54\pi}, 0.04e^{i0.076\pi}, 0.46e^{0.9\pi})$	$(0.98e^{i1.98\pi}, 0.01e^{i0.024\pi}, 0.02e^{0.06i\pi})$
z_3	$(0.63e^{i1.28\pi}, 0.012e^{i0.02\pi}, 0.59e^{i1.2\pi})$	$(0.72e^{i1.42\pi}, 0.03e^{i0.062\pi}, 0.38e^{0.78\pi})$	$(0.75e^{i1.48\pi}, 0.035e^{i0.072\pi}, 0.43e^{0.82\pi})$	$(0.42e^{i0.86\pi}, 0.012e^{i0.02\pi}, 0.65e^{i1.32\pi})$
z_4	$(0.29e^{i0.6\pi}, 0.014e^{i0.03\pi}, 0.94e^{i1.86\pi})$	$(0.21e^{i0.46\pi}, 0.024e^{i0.052\pi}, 0.94e^{i1.86\pi})$	$(0.37e^{i0.76\pi}, 0.012e^{i0.026\pi}, 0.6e^{1.22\pi})$	$(0.78e^{i1.54\pi}, 0.04e^{i0.076\pi}, 0.46e^{0.9i\pi})$
z_5	$(0.62e^{i1.28\pi}, 0.01e^{i0.024\pi}, 0.58e^{i1.2\pi})$	$(0.66e^{i1.34\pi}, 0.1e^{i0.18\pi}, 0.56e^{i1.14\pi})$	$(0.64e^{i1.26\pi}, 0.01e^{i0.024\pi}, 0.59e^{i1.2\pi})$	$(0.21e^{i0.4\pi}, 0.021e^{i0.046\pi}, 0.95e^{1.88i\pi})$

Table 47: Tabular representation of AWCSFDM.

	z_1	z_2	z_3	z_4	z_5
w_1	$(0.85e^{i1.66\pi}, 0.021e^{i0.042\pi}, 0.23e^{i0.52\pi})$	$(0.88e^{i1.76\pi}, 0.021e^{i0.042\pi}, 0.174e^{i0.38\pi})$	$(0.601e^{i1.22\pi}, 0.019e^{i0.034\pi}, 0.053e^{i1.1\pi})$	$(0.41e^{i0,82\pi}, 0.025e^{i0.05\pi}, 0.77e^{1.54i\pi})$	$(0.5e^{i\pi}, 0.023e^{i0.05\pi}, 0.67e^{1.36i\pi})$
w_2	$(0.7e^{i1.34\pi}, 0.03e^{i0.066\pi}, 0.41e^{i0.86\pi})$	$(0.43e^{i0.9\pi}, 0.025e^{i0.048\pi}, 0.73e^{i1.46\pi})$	$(0.2e^{i0.4\pi}, 0.019e^{i0.034\pi}, 0.91e^{1.82\pi})$	$(0.47e^{i0.96\pi}, 0.028e^{i0.056\pi}, 0.73e^{1.42i\pi})$	$(0.25e^{i0.54\pi}, 0.025e^{i0.05\pi}, 0.83e^{1.66i\pi})$
w_3	$(0.22e^{i0.44\pi}, 0.027e^{i0.058\pi}, 0.91e^{i1.74\pi})$	$(0.26e^{i0.54\pi}, 0.024e^{i0.025\pi}, 0.83e^{1.66\pi})$	$(0.32e^{i0.64\pi}, 0.021e^{i0.038\pi}, 0.8e^{1.62\pi})$	$(0.29e^{i0.58\pi}, 0.035e^{i0.068\pi}, 0.73e^{1.42i\pi})$	$(0.19e^{i0.34\pi}, 0.035e^{i0.068\pi}, 0.95e^{1.9i\pi})$
w_4	$(0.62e^{i1.02\pi}, 0.023e^{i0.048\pi}, 0.65e^{i1.34\pi})$	$(0.70e^{i1.46\pi}, 0.037e^{i0.072\pi}, 0.43e^{i0.86\pi})$	$(0.32e^{i0.64\pi}, 0.019e^{i0.034\pi}, 0.69e^{1.42\pi})$	$(0.37e^{i0.74\pi}, 0.033e^{i0.066\pi}, 0.82e^{1.62i\pi})$	$(0.29e^{i0.6\pi}, 0.027e^{i0.056\pi}, 0.8e^{1.62i\pi})$
w_5	$(2, (0.35e^{i0.62\pi}, 0.022e^{i0.046\pi}, 0.83e^{i1.7\pi}))$	$(0.31e^{i0.66\pi}, 0.029e^{i0.06\pi}, 0.87e^{i1.74\pi})$	$(0.39e^{i0.78\pi}, 0.025e^{i0.044\pi}, 0.68e^{i1.38\pi})$	$(0.15e^{i0.318\pi}, 0.025e^{i0.052\pi}, 0.89e^{1.8i\pi})$	$(0.27e^{i0.54\pi}, 0.024e^{i0.042\pi}, 0.91e^{1.82i\pi})$

Table 48: Tabular representation of CSF-PIS and CSF-NIS.

Attribute	CSF-PIS	CSFN-NIS
z_1	$(0.85e^{i1.66\pi}, 0.021e^{i0.042\pi}, 0.23e^{i0.52\pi})$	$(0.22e^{i0.44\pi}, 0.027e^{i0.058\pi}, 0.91e^{i1.74\pi})$
z_2	$(0.88e^{i1.76\pi}, 0.021e^{i0.042\pi}, 0.174e^{i0.038\pi})$	$(0.31e^{i0.66\pi}, 0.029e^{i0.06\pi}, 0.87e^{i1.74\pi})$
z_3	$(0.601e^{i1.22\pi}, 0.019e^{i0.034\pi}, 0.53e^{i1.1\pi})$	$(0.2e^{i0.4\pi}, 0.019e^{i0.034\pi}, 0.91e^{1.82\pi})$
z_4	$(0.47e^{i0.96\pi}, 0.028e^{i0.056\pi}, 0.73e^{1.42\pi})$	$(0.15e^{i0.318\pi}, 0.025e^{i0.052\pi}, 0.89e^{1.8\pi})$
z_5	$(0.19e^{i0.38\pi}, 0.035e^{i0.068\pi}, 0.95e^{i1.9\pi})$	$(0.5e^{i\pi}, 0.023e^{i0.046\pi}, 0.67e^{i1.36\pi})$

Table 49: Normalized Euclidean distance from ideal solution.

Alternative	$d(\widehat{\mathscr{P}}_k, w_j)$	$d(\check{\mathscr{P}}_k, w_j)$
w_1	0.182947	0.565764
w_2	0.383839	0.330875
w_3	0.5388422	0.200388
w_4	0.266370	0.349150
w_5	0.510455	0.208933

Table 50: Revised closeness index of each alternative.

Alternative	$\mathfrak{I}(w_j)$
w_1	0
w_2	1.5132
w_3	2.5911
w_4	0.838853
w_5	2.42088

Table 51: Comparison.

Method	Ranking	Best physiotherapist
$CSFNS_f$-TOPSIS	$w_1 > w_4 > w_2 > w_5 > w_3$	w_1
CSF-TOPSIS [16]	$w_1 > w_4 > w_2 > w_5 > w_3$	w_1
SF-TOPSIS [10]	$w_1 > w_4 > w_2 > w_5 > w_3$	w_1
SF-TOPSIS [11]	$w_1 > w_4 > w_2 > w_5 > w_3$	w_1

$SFNS_f$, $CSFS_f$, and SFS_f, by substituting periodic terms equal to zero and $N = 2$.

7. Conclusions

Complex spherical fuzzy N-soft sets ($CSFNS_fS$) broaden the families of both fuzzy sets and N-soft sets. This novel concept has allowed us to propose techniques in a wide environment that have a large ability of solving real-life MADM and MAGDM problems. The model of $CSFNS_fS$ described in this paper copes with 2-dimensional fuzziness, parameterized information, and ordinal ranking systems. In addition to the notion of $CSFNS_fS$, we have defined score and accuracy functions for the purpose of comparing two $CSFNS_fNs$. We have defined useful operations on $CSFNS_fS$ and given relevant examples. We developed three direct algorithms and, furthermore, a $CSFNS_f$-TOPSIS Method to solve decision-making problems. We compared them with existing methods for Pythagorean fuzzy N-soft sets and with the complex spherical fuzzy TOPSIS Method, respectively. For the purpose of extending the theoretical background of TOPSIS methods to the new $CSFNS_f$-TOPSIS method, we have defined complex spherical fuzzy N-soft weighted averaging operator $CSFNS_fWA$ which produces an aggregate complex spherical fuzzy N-soft decision matrix $ACSFNS_fDM$ and aggregates the weight vectors of attributes given by experts. Similarly, we have defined a normalized Euclidean distance in $CSFNS_f$ environment that simultaneously evaluates the distances of alternatives from $CSFNS_f$-PIS and $CSFNS_f$-NIS. This is required to find a revised closeness index. The ascending order of such an index gives us a ranking of the alternatives, where the smallest revised closeness index indicates a best solution. In the future, we intend to pursue the formalization of other methods (ELECTRE I, II, and III and VIKOR methodologies), under the framework of $CSFNS_f$. We can also extend this theory to accommodate T-spherical fuzzy soft sets, T-spherical fuzzy N-soft sets, and complex T-spherical fuzzy N-soft sets.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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