Research Article

On a Discrete-Time Risk Model with Random Income and a Constant Dividend Barrier

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In this paper, a discrete-time risk model with random income and a constant dividend barrier is considered. Under such a dividend policy, once the insurer's reserve hits the level b ($b > 0$), the excess of the reserve over b is paid off as dividends. We derive a homogeneous difference equation for the expected present value of dividend payments. Corresponding solution procedures for the difference equation are investigated. Finally, we give a numerical example to illustrate the applicability of the results obtained.

1. Introduction

In the actuarial literature, many authors focus their research interests on discrete-time risk models, which can be used as an approximation to continuous time models. Li et al. [1] present a review of results for discrete-time risk models, including the compound binomial risk model and some of its extensions. In recent years, discrete-time risk models with dependent structure have received increasing attention, and the readers are referred to Liu and Bao [2] for the related studies on different kinds of dependent models.

Additionally, problems related to dividends have been considered extensively in the discrete-time setting. Because of the certainty of ruin for a risk model with a constant dividend barrier, the calculation of the expected discounted dividend payment is a major problem of interest in the context. Among the class of discrete-time risk models, Tan and Yang [3] derive a recursive algorithm to compute a particular class of Gerber–Shiu penalty functions in the framework of the compound binomial model with randomized dividend payments. Landriault [4] then generalize Tan and Yang's model to consider the compound binomial model with a multithreshold dividend structure and randomized dividend payments. For the compound binomial risk model with possible delay of claims, Xie and Zou [5] investigate the expected present value of total dividends

under stochastic interest rates. Wu and Li [6] further consider the discrete-time risk model with time-delayed claims and a constant dividend barrier, but the main claim is dependent on its associated by-claim. In the framework of a discrete semi-Markov risk model, a randomized dividend policy is studied by Yuen et al. [7]. Zhang and Liu [8] consider a discrete-time risk model with a mathematically tractable dependence structure between interclaim time and claim size in the presence of an impulsive dividend strategy.

All the risk models discussed above are based on a common assumption that the premium is collected with a positive deterministic constant. However, this assumption can be unrealistic and inappropriate in practical contexts because the insurance company may have lump sums of income. Therefore, many authors consider the risk models with stochastic income to capture the uncertainty of the customers' arrivals, for example, Boikov [9], Yang and Zhang [10], Labbé and Sendova [11], Karnaukh [12], Bao and Liu [13], and Zhou et al. [14].

In the present paper, we propose a discrete-time risk model with random income and a constant dividend barrier. A similar model has been discussed by Zhou et al. [14]. However, we analyze the model with a general premium rate c , while Zhou et al. [14] investigate the model with time-delayed claims and unit premium assumption. As stated in Landriault [4], unlike the classical compound Poisson risk

model for which a unit premium rate can be assumed without loss of generality, such reasoning does not hold for the discrete-time risk model. On the contrary, the technique used in the present paper is to solve a high-order difference equation that often arises in the discrete-time risk model with general premium c .

The rest of the paper is organized as follows. A brief description of the discrete-time model and the introduction of the expected present value of total dividends are considered in Section 2. In Section 3, we obtain and solve a homogeneous difference equation satisfied by the expected present value of dividend payments. When the claim sizes follow the geometrical distribution, a closed-form solution to the expected present value of dividends and the corresponding numerical example are provided in Section 4. Finally, we end our works with conclusion in Section 5.

2. The Model

Throughout, denote by $\mathbb{N}$ the set of nature numbers and $\mathbb{N}^+ = \mathbb{N} \setminus \{0\}$. Similar to Bao and Liu [13], we assume that the premium income is a binomial process $\{M_k, k \in \mathbb{N}\}$ with parameter p_0 ($0 < p_0 \leq 1$). More precisely, we denote by p_0 , the probability that a premium of size c ($c \in \mathbb{N}^+$) is received, and $q_0 = 1 - p_0$, the probability that no premium is received in each period. The number of claims is assumed to be another binomial process $\{N_k, k \in \mathbb{N}\}$ with parameter p ($0 < p < 1$). It means that the probability of having a claim is p and the probability of no claim is $q = 1 - p$ in each period. The claims $\{X_j, k \in \mathbb{N}^+\}$ are independent and identically distributed (i.i.d.) positive and integer-valued random variables with the same distribution as the generic random variable X . The probability mass function (p.m.f.) and mean of X are denoted by $f_X(x)$ and μ , respectively. Let $S_k = \sum_{i=1}^k X_i$ be the total amount of settled claims up to the end of the k th time period with $S_0 = 0$. The independence among $\{N_k, k \in \mathbb{N}\}$, $\{M_k, k \in \mathbb{N}\}$, and $\{X_j, k \in \mathbb{N}^+\}$ are also assumed.

Suppose that premiums are received at the beginning of each period, and claims are paid out at the end of each period. We introduce a dividend policy to the company that

a certain amount of dividends will be paid to the policyholder instantly, as long as the surplus of the company at time k is higher than a constant dividend barrier b ($b > 0$). It implies that the dividend payments will only possibly occur at the beginning of each period, right after receiving the premium payment. Then, the corresponding surplus $U_b(k)$ of the insurer at the end of the k th time period can be described as

$$U_b(k) = u + cM_k - S_k - D(k), \quad u = 0, 1, 2, \dots, b, \quad (1)$$

where $U_b(0) = u$ is the initial surplus and $D(k) = D_1 + D_2 + \dots + D_k$ is the sum of the total dividend payments in the first k period with $D(0) = 0$. The amount of dividends D_k paid out in the k th time period is defined as

$$D_k = \max\{U_b(k-1) + c\eta_k - b, 0\}, \quad k \in \mathbb{N}^+, \quad (2)$$

where $\{\eta_k, k \in \mathbb{N}^+\}$ is a sequence of i.i.d. Bernoulli random variables with $\mathbb{P}(\eta_k = 1) = p_0$ and $\mathbb{P}(\eta_k = 0) = q_0$.

Note that the positive safety loading condition holds if $cp_0 > p\mu$. The time of ultimate ruin for model (1) is defined as $\tau_b = \min\{k, U_b(k) < 0\}$. Let v ($0 < v \leq 1$) be a constant annual discount rate for each period. Then, the expected present value of the dividend payments due till ruin is

$$V(u; b) = \mathbb{E} \left[\sum_{k=1}^{\tau_b} (D_k) v^{k-1} | U(0) = u \right]. \quad (3)$$

3. Expected Present Value of Dividend Payments

By considering the occurrence (or not) of the premium income and claims in the next period, we separate the four possible cases as follows: no premium arrival and no claim occurs, a random premium arrival and no claim occurs, a random premium arrival and a claim occurs, and no premium arrival and a claim occurs. For $u = 0, 1, \dots, b - c$, we can obtain the following result by using the total probability formula:

$$\begin{aligned} V(u; b) &= vq_0qV(u; b) + vp_0qV(u + c; b) + vp_0p \sum_{x \leq u+c} V(u + c - x; b)f(x) + vq_0p \sum_{x=1}^u V(u - x; b)f(x) \\ &= vq_0qV(u; b) + vp_0qV(u + c; b) + vp_0p(V * f)(u + c) + vq_0p(V * f)(u), \end{aligned} \quad (4)$$

where $V * f$ holds for the convolution product of V and f .

Now, we show that (4) is a homogeneous difference equation of order c . The (forward) difference operator Δ is defined as

$$\Delta V(u; b) = V(u + 1; b) - V(u; b). \quad (5)$$

Using the property of the forward difference operator Δ , we obtain

$$V(u + c; b) = \sum_{j=0}^c \binom{c}{j} \Delta^j V(u; b). \quad (6)$$

Substituting (6) into (4) implies that

$$V(u; b) = vq_0qV(u; b) + vp_0q \sum_{j=0}^c \binom{c}{j} \Delta^j V(u; b) + vp_0p \sum_{j=0}^c \binom{c}{j} \Delta^j (V * f)(u) + vq_0p(V * f)(u), \quad (7)$$

or, equivalently

$$\sum_{j=0}^c a_j \binom{c}{j} \Delta^j V(u; b) = \sum_{j=0}^c b_j \binom{c}{j} \Delta^j (V * f)(u), \quad (8)$$

where

$$\begin{aligned} a_j &= 1_{\{j=0\}} - vq_0q1_{\{j=0\}} - vp_0q, \\ b_j &= vq_0p1_{\{j=0\}} + vp_0p. \end{aligned} \quad (9)$$

Letting $A(z) = \sum_{j=0}^c a_j z^j$ and $B(z) = \sum_{j=0}^c b_j z^j$, which are two polynomials of degree c in z . Then, (8) can be reformulated as

$$A(\Delta)V(u; b) = B(\Delta)(V * f)(u). \quad (10)$$

Equation (10) implies that $V(u; b)$ satisfies a homogeneous difference equation of order c . From the general theory of difference equations, the solution to (10) can be expressed as

$$V(u; b) = \sum_{j=0}^{c-1} \alpha_j y_j(u; b), \quad (11)$$

where $\{y_j(u; b)\}_{u=0}^{\infty}$ are linearly independent and satisfy

$$A(\Delta)y(u; b) = B(\Delta)(y * f)(u). \quad (12)$$

Multiplying (12) by z^{u+c} and then summing over u from 0 to ∞ lead to

$$\sum_{u=0}^{\infty} z^{u+c} y(u; b) = vq_0q \sum_{u=0}^{\infty} z^{u+c} y(u; b) + vp_0q \sum_{u=0}^{\infty} z^{u+c} y(u+c; b) + vp_0p \sum_{u=0}^{\infty} z^{u+c} (y * f)(u+c) + vq_0p \sum_{u=0}^{\infty} z^{u+c} (y * f)(u), \quad (13)$$

which can be reformulated as

$$\tilde{y}(z; b) = \frac{-vp_0\{q \sum_{u=0}^{c-1} z^u y(u; b) + p \sum_{u=0}^{c-1} z^u (y * f)(u)\}}{(1 - vq_0q)z^c - v(p_0q + p\tilde{g}(z)\tilde{f}(z))}. \quad (14)$$

To find a set of fundamental solutions to (14), we choose $y_j(u; b) = 1_{\{j=u\}}$ for $j, u \in \{0, 1, \dots, c-1\}$. An application of Theorem 3.4 in [15] implies that $\{y_j(u; b)\}_{u=0}^{\infty}$, for $j = 0, 1, \dots, c-1$, are linearly independent. From (14), the generating functions associated with $\{y_j(u; b)\}_{u=0}^{\infty}$ satisfy

$$\tilde{y}_j(z; b) = \frac{-R_j(z)}{\tilde{h}_1(z) - \tilde{h}_2(z)}, \quad (15)$$

where

$$\begin{aligned} \tilde{h}_1(z) &= (1 - vq_0q)z^c, \\ \tilde{h}_2(z) &= v(p_0q + p\tilde{g}(z)\tilde{f}(z)), \\ \tilde{g}(z) &= \sum_{k=0}^{\infty} g(z)z^k = p_0 + q_0z^c, \\ R_j(z) &= vp_0 \left\{ qz^j + p \sum_{u=j+1}^{c-1} z^u f(u-j) \right\}. \end{aligned} \quad (16)$$

We can easily identify the number of zeros on the denominator in (15) by using Rouché's theorem and its generalization. The proof is omitted, and similar discussions can be found in Bao and Liu [16].

Lemma 1. When $0 < v < 1$, the denominator in (15) has exactly c solutions, say $\{z_i\}_{i=1}^c$, inside the unit circle $\mathcal{C} = \{z: |z| = 1\}$.

Lemma 2. When $v = 1$, the denominator in (15) has exactly $c-1$ solutions, say $\{z_i\}_{i=1}^{c-1}$, inside the unit circle $\mathcal{C} = \{z: |z| = 1\}$ and another trivial root $z_c = 1$.

In what follows, we assume these zeros $\{z_i\}_{i=1}^c$ are distinct for simplicity, as the analysis of the multiple z_i 's leads to tedious derivations. We are ready to give alternative expressions for both the numerator and the denominator in (15) with the help of $\{z_i\}_{i=1}^c$. Since the numerator in (15) is analytic for all $\text{Re}(z) \in [-1, 1]$, the zeros of the denominator in (15) have to be zeros of the numerator.

Let $\pi_c(z) = \prod_{j=1}^c (z - z_j)$ and $\pi'_c(z_k) = \prod_{j=1, j \neq k}^c (z_k - z_j)$. Since $\tilde{h}_1(z)$ is a polynomial of degree c in z , using the Lagrange interpolating theorem, one knows

$$\tilde{h}_1(z) = \pi_c(z) \left(\frac{\tilde{h}_1(0)}{\pi_c(0)} + \sum_{i=1}^c \frac{\tilde{h}_2(z_i)}{z_i} \frac{1}{\pi'_c(z_i)} + \sum_{i=1}^c \frac{\tilde{h}_2(z_i)}{z - z_i} \frac{1}{\pi'_c(z_i)} \right). \quad (17)$$

It is easy to see that $\lim_{z \rightarrow \infty} \tilde{h}_1(z)/\pi_c(z) = 1 - vq_0q$, and (17) can be simplified as

$$\frac{\tilde{h}_1(z)}{\pi_c(z)} = 1 - vq_0q + \sum_{i=1}^c \frac{\tilde{h}_2(z_i)}{z - z_i} \frac{1}{\pi'_c(z_i)}, \quad (18)$$

which yields

$$\frac{\tilde{h}_1(z) - \tilde{h}_2(z)}{\pi_c(z)} = 1 - vq_0q - \left(\frac{\tilde{h}_2(z)}{\pi_c(z)} - \sum_{i=1}^c \frac{\tilde{h}_2(z_i)}{z - z_i} \frac{1}{\pi'_c(z_i)} \right). \quad (19)$$

Now, we use a discrete operator $\mathcal{T}_z$ defined as $\mathcal{T}_z y(c) = \sum_{u=0}^{\infty} z^u y(u+c)$, and the corresponding properties of $\mathcal{T}_z$ can be found in Li [17]. Then, $\tilde{h}_2(z)$ can be rewritten as

$$\tilde{h}_2(z) = vpz^c \mathcal{T}_z(g * f)(c) + \eta(z), \quad (20)$$

where $\eta(z) = vp_0q + vp \sum_{u=0}^{c-1} z^u (g * f)(u)$ is a polynomial of degree $c-1$ in z . By imitating the same steps discussed above, we can prove that

$$\frac{\eta(z)}{\pi_c(z)} - \sum_{i=1}^c \frac{\eta(z_i)}{z - z_i} \frac{1}{\pi'_c(z_i)} = 0. \quad (21)$$

Therefore, substituting (20) into (19) yields

$$\begin{aligned} \frac{\tilde{h}_1(z) - \tilde{h}_2(z)}{\pi_c(z)} &= 1 - vq_0q - vp \left(\frac{z^c \mathcal{T}_z(g * f)(c)}{\pi_c(z)} - \sum_{i=1}^c \frac{(z_i)^c \mathcal{T}_{z_i}(g * f)(c)}{z - z_i} \frac{1}{\pi'_c(z_i)} \right) \\ &= 1 - vq_0q - vp \mathcal{T}_z \mathcal{T}_{z_c} \dots \mathcal{T}_{z_2} \mathcal{T}_{z_1}(g * f)(c). \end{aligned} \quad (22)$$

Note that (15) can be modified as

$$\tilde{y}_j(z; b) = \frac{-R_j(z)/\pi_c(z)}{(\tilde{h}_1(z) - \tilde{h}_2(z))/\pi_c(z)}. \quad (23)$$

For the numerator in (23), partial fractions lead to an equivalent representation

$$\frac{-R_j(z)}{\pi_c(z)} = \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \frac{1}{z_k - z}. \quad (24)$$

By inserting (22) and (24) into (23), we obtain

$$\tilde{y}_j(z; b) = \frac{1}{1 - vq_0q} \left[vp \tilde{y}_j(z; b) \mathcal{T}_z \mathcal{T}_{z_c} \dots \mathcal{T}_{z_2} \mathcal{T}_{z_1}(g * f)(c) + \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \frac{1}{z_k - z} \right]. \quad (25)$$

The direct inversion of the generating functions in (25) gives the following theorem immediately.

Theorem 1. For $j = 0, 1, \dots, c-1$, $y_j(u, b)$ satisfies the following defective renewal equation:

$$y_j(u; b) = \varsigma \sum_{n=0}^u y_j(u-n; b) \chi(n) + \zeta(u), \quad u \in \mathbb{N}, \quad (26)$$

where

$$\begin{aligned} \zeta(u) &= \frac{1}{1 - vq_0q} \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \left(\frac{1}{z_k} \right)^{u+1}, \\ \varsigma &= \frac{vp}{1 - vq_0q} \mathcal{T}_1 \mathcal{T}_{z_c} \dots \mathcal{T}_{z_2} \mathcal{T}_{z_1}(g * f)(c), \\ \chi(n) &= \frac{\mathcal{T}_{z_c} \dots \mathcal{T}_{z_2} \mathcal{T}_{z_1}(g * f)(c+n)}{\mathcal{T}_1 \mathcal{T}_{z_c} \dots \mathcal{T}_{z_2} \mathcal{T}_{z_1}(g * f)(c)}. \end{aligned} \quad (27)$$

Proof. To complete the proof, it remains to show that $\varsigma < 1$. In the case of $v \in (0, 1)$, it follows from (22) that

$$\varsigma = \frac{1}{1 - vq_0q} \left(1 - vq_0q - \frac{\tilde{h}_1(1) - \tilde{h}_2(1)}{\pi_c(1)} \right) = 1 - \frac{1 - v}{(1 - vq_0q)\pi_c(1)} < 1. \quad (28)$$

In the case of $v = 1$, since z_c is a root of the denominator in (15), we have

$$\tilde{h}_1(z_c) - \tilde{h}_2(z_c) = 0. \quad (29)$$

Differentiating (29) with respect to v and taking the limit $v \rightarrow 1^-$, from the positive safety loading condition, we eventually find

$$\lim_{v \rightarrow 1^-} z'_c = \frac{1}{cp_0 - p\mu} > 0. \quad (30)$$

Thus, taking the limit $v \rightarrow 1^-$ in (28), one deduces

$$\lim_{v \rightarrow 1^-} \varsigma = 1 - \frac{1}{(1 - q_0q)\pi'_c(1)} \lim_{v \rightarrow 1^-} \frac{1 - v}{1 - z_c} < 1, \quad (31)$$

which completes the proof. $\square$

Starting from $y_j(u; b) = 1_{\{j=u\}}$ for $u = 0, 1, \dots, b-c$, (26) allows for a recursive calculation of the fundamental solutions to the homogeneous difference (14). Regarding the constants α_j ($j = 0, 1, \dots, c-1$) in the structural form of

(12), they are determined such that the remaining c equations are satisfied with initial surplus $u = b - c + 1, \dots, b$, namely,

$$\begin{aligned} V(u; b) = & vq_0qV(u; b) + vp_0q(u + c - b + V(b; b)) \\ & + vp_0p \left\{ 1_{\{x \leq u+c-b\}} f(x) [(u + c - b - x) + V(b; b)] + \sum_{x=u+c-b}^{u+c} V(u + c - x; b) f(x) \right\} \\ & + vq_0p \sum_{x=1}^u V(u - x; b) f(x). \end{aligned} \quad (32)$$

4. Numerical Example

In this section, we explain the solution procedure to the difference equation $V(u; b)$ through a concrete example. In what follows, it is assumed that f is a geometrical distribution with $f(n) = (1 - \theta)\theta^{n-1}$, $n \in \mathbb{N}^+$, and the corresponding probability generating function is $\tilde{f}(z) = ((1 - \theta)z / (1 - \theta z))$. Thus, equation (15) becomes

$$\tilde{y}_j(z; b) = \frac{-R_j(z)(1 - \theta z)}{\Lambda(z)}, \quad (33)$$

where $\Lambda(z) = z^c(1 - \theta z) - vq_0qz^c(1 - \theta z) - vp_0q(1 - \theta z) - vp_0p(1 - \theta)z - vq_0p(1 - \theta)z^{c+1}$. Since $\Lambda(z)$

is a polynomial of degree $c + 1$ with the leading coefficient $a = -\theta + vq_0q\theta - vq_0p(1 - \theta)$, it can be factored as

$$\Lambda(z) = a\pi_c(z)(z - \xi), \quad (34)$$

where ξ is the zero of $\Lambda(z)$ on the complex plane. Note that ξ has a module larger than 1 and z_i ($i = 1, 2, \dots, c$) has a module less than 1. Moreover, routine calculations lead to

$$\frac{\pi_c(z)(1 - \theta z)}{\Lambda(z)} = \frac{\pi_c(z)(1 - \theta z)}{a\pi_c(z)(z - \xi)} = -\frac{\theta}{a} \left(1 + \frac{(1/\theta) - \xi}{\xi - z} \right). \quad (35)$$

By substituting (24) and (35) into (33), we obtain

$$\tilde{y}_j(z; b) = \frac{(1 - \theta z)}{\Lambda(z)} \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \frac{\pi_c(z)}{z_k - z} = -\frac{\theta}{a} \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \left(1 + \frac{(1/\theta) - \xi}{\xi - z} \right) \frac{1}{z_k - z}. \quad (36)$$

Upon inversion, we obtain from (36) that

$$\begin{aligned} y_j(u; b) = & -\frac{\theta}{a} \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \left[z_k^{-(u+1)} + \left(\frac{1}{\theta} - \xi \right) \sum_{l=0}^u \xi^{-(u+1-l)} z_k^{-(l+1)} \right] \\ = & -\frac{\theta}{a} \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \left(1 - \frac{(1/\theta) - \xi}{z_k - \xi} \right) z_k^{-(u+1)} - \frac{\theta}{a} \sum_{k=1}^c \frac{R_j(z_k)}{\pi'_c(z_k)} \cdot \frac{(1/\theta) - \xi}{z_k - \xi} \xi^{-(u+1)}, \end{aligned} \quad (37)$$

where $R_j(z_k)$ is calculated through

$$R_j(z) = vp_0 \left\{ qz^j + \frac{p(1 - \theta)z^{j+1}}{1 - \theta z} [1 - (\theta z)^{c-j-1}] 1_{\{c-1 \geq j+1\}} \right\}. \quad (38)$$

As the explicit expression for $y_j(u; b)$ is given by (37), the constants $\{\alpha_j\}_{j=0}^{c-1}$ can be achieved such that (32) holds for $u = b - c + 1, \dots, b$, which completes the determination of $V(u; b)$. We provide a numerical example to illustrate the theoretical results.

Example 1. Suppose $c = 2$, and from (33), we have

$$\begin{aligned} \Lambda(z) = & z^2(1 - \theta z) - vq_0qz^2(1 - \theta z) - vp_0q(1 - \theta z) \\ & - vp_0p(1 - \theta)z - vq_0p(1 - \theta)z^3. \end{aligned} \quad (39)$$

Let $p = 0.6$, $p_0 = 0.7$, and $v = 0.85$, and the relative safety loading condition $cp_0 > p\mu$ holds for all $\theta \in (0, 4/7)$. By solving $\Lambda(z) = 0$, we obtain the values of z_i 's and ξ , see Table 1.

Explicit expressions for $y_j(u; b)$ are determined by (37). For example, one has, for $\theta = 0.3$,

TABLE 1: Numerical results of z_i and ξ for $p = 0.6$, $p_0 = 0.7$, $\theta = 0.3$, and $\nu = 0.85$.

	ξ	z_1	z_2
$\theta = 0.1$	3.48604534795459	-0.363766275505389	0.824973674803551
$\theta = 0.2$	2.54139402542121	-0.381251310060523	0.813367218414144
$\theta = 0.3$	1.98618491341006	-0.39890721086863	0.797848459477162

TABLE 2: Numerical results of $y_j(u; b)$ for $b = 15$.

u	0	1	2	3
$y_0(u; b)$	1	-5.52900×10^{-17}	3.45811	-4.17551
$y_1(u; b)$	3.31740×10^{-17}	1	-1.05	4.56061
u	4	5	6	7
$y_0(u; b)$	16.1795	-33.3598	92.6721	-220.969
$y_1(u; b)$	-8.96415	25.5918	-60.2312	155.915
u	8	9	10	11
$y_0(u; b)$	568.161	-1406.46	3548.14	-8866.63
$y_1(u; b)$	-384.679	972.075	-2427.14	6096.64
u	12	13	14	15
$y_0(u; b)$	22262.4	-55764.5	139848	-350509
$y_1(u; b)$	-15268.1	38293.9	-95973.1	240620

TABLE 3: Numerical results of α_i for $\theta = 0.3$.

b	15	16	17	18	19
α_0	0.07130	0.05689	0.04539	0.03621	0.02889
α_1	0.10387	0.08287	0.06612	0.05275	0.04209

TABLE 4: Numerical results of α_i for $b = 15$.

θ	0.1	0.15	0.2	0.25	0.3
α_0	0.19716	0.16288	0.12968	0.09878	0.07130
α_1	0.27582	0.22961	0.18453	0.14212	0.10387

FIGURE 1: Impact of θ and b for $V(u; b)$. (a) Numerical results for $V(u; b)$ when $\theta = 0.3$. (b) Numerical results for $V(u; b)$ when $b = 15$.

$$\begin{aligned}
y_0(u; b) &= 0.36125 \times (-2.50685)^u + 0.77878 \\
&\quad \times 1.25337^u - 0.14003 \times 0.50348^u, \\
y_1(u; b) &= -0.24797 \times (-2.50685)^u + 0.33810 \\
&\quad \times 1.25337^u - 0.09014 \times 0.50348^u.
\end{aligned} \tag{40}$$

Then, we obtain the values of $y_j(u; b)$, see Table 2.

To finish the calculation of $V(u; b)$, we solve the system of linear equations (32) satisfied by α_i , see Tables 3 and 4, respectively.

We depict $V(u; b)$ in Figure 1, and it is not surprising that $V(u; b)$ is an increasing function of u . When $\theta = 0.3$ is fixed, the first part of Figure 1 shows that a larger value of b corresponds to a smaller amount of dividend payments. On the contrary, for a fixed $b = 15$, the second part of Figure 1 illustrates that a smaller θ leads to a larger number of dividend payments.

5. Concluding Remarks

In this paper, we consider the compound binomial model with random income and a constant dividend barrier. Furthermore, we analyze the model with a general premium rate c . Using the roots of a generalization of Lundberg's fundamental equation and the general theory on difference equations, we derive an explicit expression for the expected present value of dividend payments up to the time of ruin. In particular, a numerical example is provided to show that the formulae are readily programmable in practice. From the numerical example given above, we can see that the barrier level has a negative effect on the total expected present value of dividends.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors have equal contributions. All authors have read and approved the final manuscript.

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