Research Article

Observer-Based H^∞ Fuzzy Synchronization and Output Tracking Control of Time-Varying Delayed Chaotic Systems

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Tracking control for the output using an observer-based H^∞ fuzzy synchronization of time-varying delayed discrete- and continuous-time chaotic systems is proposed in this paper. First, from a practical point of view, the chaotic systems here consider the influence of time-varying delays, disturbances, and immeasurable states. Then, to facilitate a uniform control design approach for both discrete- and continuous-time chaotic systems, the dynamic models along with time-varying delays and disturbances are reformulated using the T-S (Takagi–Sugeno) fuzzy representation. For control design considering immeasurable states, a fuzzy observer achieves master-slave synchronization. Third, combining both a fuzzy observer for state estimation and a controller (solved from generalized kinematic constraints) output tracking can be achieved. To make the design more practical, we also consider differences of antecedent variables between the plant, observer, and controller. Finally, using Lyapunov's stability approach, the results are sufficient conditions represented as LMIs (linear matrix inequalities). The contributions of the method proposed are threefold: (i) systemic and unified problem formulation of master-slave synchronization and tracking control for both discrete and continuous chaotic systems; (ii) practical consideration of time-varying delay, immeasurable state, different antecedent variables (of plant, observer, and controller), and disturbance in the control problem; and (iii) sufficient conditions from Lyapunov's stability analysis represented as LMIs which are numerically solvable observer and controller gains from LMIs. We carry out numerical simulations on a chaotic three-dimensional discrete-time system and continuous-time Chua's circuit. Satisfactory numerical results further show the validity of the theoretical derivations.

1. Introduction

Chaotic behavior exists in many systems, in some cases being problematic where control is needed. In other cases, chaotic behavior is to be induced, whereas synchronization is the goal. The results [1, 2] represent the synchronization as a T-S (Takagi–Sugeno) fuzzy representation. The result [3] achieves robust state estimation influenced by time delay. The result [4] represents that synchronization is achieved by solving LMIs (linear matrix inequalities). However, from a practical point of view, a unified approach to synchronization and control of both discrete-time and continuous chaotic systems considering immeasurable state, time-varying delay, different antecedent variables, and disturbance is needed.

The T-S fuzzy representation of dynamic models [5, 6] renders nonlinear systems as a parametric combination of several linear subsystems. Linear system control theory can then be applied to each subsystem, where once combined provides a total control solution to the nonlinear system. Finally, control criteria are represented as linear matrix inequality problems (LMIPs) [7–11]. For states immeasurable, results [12, 13] use observers to estimate states.

For tracking, the result [14] uses a robust approach where the command signal is induced into the closed-loop system as disturbance with tracking error residual attenuated. For regulation, the results [15, 16] use feed-forward compensator from linear control theory. The results [17] use linear regulation theory for T-S represents linear

subsystems. The results [18–20] have goals of static or varying output. Note that [21] mentions the challenge of coupled plant and controller rules that ultimately leads to complex controller design. The aforementioned problem has been overcome by fuzzy gain scheduling [22] and integral control schemes [23] for regulating output of DC-DC converters as an example. While the result [23] achieves a satisfactory performance, the gain design relies on the pole assignment technique.

Time delay exists in many practical systems leading to an unstable and suboptimal performance posing a complex control problem. Previous results in control can be categorized as feedback with delay [24–28] or without delay [29, 30]. Note that feedback without delay methods do not need any information of the delay where approaches are more suitable for practical applications. Both stability criteria are then represented into linear matrix inequality problems (LMIPs) [7]. However, for time-delay systems applications, most literature deals with stability to an equilibrium.

Here, we introduce a robust observer-based synchronization and control for chaotic systems with time-varying delay and disturbances in both continuous-time and discrete-time domain. First, the chaotic system with time-varying delay is represented as a T-S fuzzy model. Next, a set of reference variables is formulated based on generalized kinematic constraints. Third, combining the synthesis of the reference variable controller and strategy of the observer-based fuzzy control, the overall output tracking controller is developed. Using Lyapunov's method, stability sufficient conditions are derived and are represented as LMIPs. As of the result, the tracking achieves guaranteed H_∞ performance. The contributions of the proposed approach are (i) systemic and unified problem formulation of master-slave synchronization and tracking control for both discrete and continuous chaotic systems; (ii) practical consideration of time-varying delay, immeasurable state, mismatched antecedent variable, disturbance in control problem; and (iii) numerically solvable observer and controller gains from LMIs of sufficient conditions derived using Lyapunov's stability analysis. The proposed controller is finally validated by satisfactory numerical results.

The remainder of this paper is organized as follows. In the problem statement, we formulate the overall control problem. In the master-slave synchronization, we design the observer for master-slave synchronization. In the output tracking control, we design the output tracking controller. In the guaranteed H_∞ performance, we discuss H_∞ performance of the controller. In the tracking controller realization, we realize the tracking controller based on generalized kinematic constraints. In the simulation results, we carry out numerical simulation to verify the validity of the proposed scheme. Finally, some concluding remarks are made in the conclusions.

2. Problem Formulation

Based on previous modeling methods [5, 31], chaotic systems can be exactly represented by Takagi–Sugeno (T-S) IF-THEN

fuzzy rules. Consider a class of chaotic systems with time-varying delay and disturbance which are described by the following T-S fuzzy rules:

Plant Rule i :

IF $z_1(t)$ is F_{1i} and $\dots$ and $z_f(t)$ is F_{fi} THEN

$$sx(t) = A_i x(t) + A_{di} x(t - \tau(t)) + Bu(t) + \Gamma + w(t),$$

$$y(t) = Cx(t) + v(t),$$

$$y_c(t) = Dx(t),$$

$$x(t) = \varphi(t), \quad t \in [-\tau_0, 0],$$

(1)

where $sx(t)$ denotes $x(t+1)$ for discrete-time system (DTS) or $\dot{x}(t)$ for continuous-time system (CTS); $x(t) \in R^n$, $u(t) \in R$, $y(t) \in R$, and $y_c(t) \in R$ are accordingly the state, control input, measured output, and controlled output; A_i , A_{di} , B , C , and D are matrices with proper dimensions; $z_1(t) \sim z_f(t)$ are the antecedent variables as a combination of states; F_{ji} ($j = 1, 2, \dots, f$) are fuzzy sets; $i = 1, 2, \dots, r$, with r as the number of fuzzy rules; $\varphi(t)$ is the initial condition; $\tau(t)$ is the time-varying delay; $w(t)$ and $v(t)$ are accordingly the modeling error (or external disturbance) and measured disturbance; and Γ is a constant term.

The inferred output (using the singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier) of the fuzzy system is

$$\begin{aligned} sx(t) &= \sum_{i=1}^r \mu_i(z(t)) A_i x(t) + A_{di} x(t - \tau(t)) \\ &\quad + Bu(t) + \Gamma + w(t) \\ y(t) &= Cx(t) + v(t), \end{aligned} \quad (2)$$

where $z(t) = [z_1(t) z_2(t) \dots z_n(t)]^T$; and $\mu_i(z(t)) = w_i(z(t)) / \sum_{i=1}^r w_i(z(t))$ with $w_i(z(t)) = \prod_{j=1}^f F_{ji}(z_j(t))$. Here $\sum_{i=1}^r \mu_i(z(t)) = 1$ for all t , where $\mu_i(z(t)) \geq 0$, for $i = 1, 2, \dots, r$, are the weights normalized. For system (21), we straightforwardly assume $B = [b_0 \ 0 \ \dots \ 0]^T$ with scalar $b_0 \neq 0$ and $x \in \Omega$ with a region of interest Ω .

To consider immeasurable states for output tracking control, define $\hat{x}(t)$ as state estimation from the measured output. To transform the output tracking problem to a stabilization one, we introduce a set of target variables $x_d(t)$ governed by generalized kinematic constraints. We will discuss the analysis and synthesis of the observer and target variables in Sections 4 and 6, respectively. The following assumptions are held for the remainder of the paper.

Assumption 1. Time-varying delay $\tau(t)$ is unknown but bounded by τ_0 , i.e., $0 < \tau(t) \leq \tau_0$, with average constant time delay $\bar{\tau}$ with $0 < \bar{\tau} \leq \tau_0$.

Assumption 2. There exists a known $\bar{q} > 1$, such that $\|x(t - \tau(t))\| \leq \bar{q} \|x(t)\|$ for $\tau(t) \in (0, \tau_0]$.

The output synchronization achieves an H_∞ performance in accordance to the error $e(t) = x(t) - \hat{x}(t)$ as follows:

(i) DTS:

$$\sum_{t=0}^{t_f} e^T(t) \bar{Q} e(t) \leq e^T(0) \bar{P} e(0) + \sum_{q=1}^{\bar{\tau}} e^T(-q) \bar{S} e(-q) + \rho^2 \sum_{t=0}^{t_f} \|w_s(t)\|^2. \quad (3)$$

(ii) CTS:

$$\int_0^{t_f} e^T(t) \bar{Q} e(t) dt \leq e^T(0) \bar{P} e(0) + \int_{-\bar{\tau}}^0 e^T(v) \bar{S} e(v) dv + \rho^2 \int_0^{t_f} \|w_s(t)\|^2 dt, \quad (4)$$

where matrices $\bar{Q} = \bar{Q}^T > 0$, $\bar{P} = \bar{P}^T > 0$, $\bar{S} = \bar{S}^T > 0$; $e(0)$ are the initial values; ρ is the given attenuation level; t_f is terminal time; and $w_s(t)$ is the vector due to $w(t)$, $v(t)$ and residual errors considered as disturbance.

Define the tracking error $x_h(t) = \hat{x}(t) - x_d(t)$. The control goal is to let $e(t)$ and $x_h(t)$ achieve zero, i.e., state $x(t)$ tracks $x_d(t)$; meanwhile, the controlled output $y_c(t)$ tracks $y_d(t)$. The tracking performance achieves H_∞ related to the error $x_e(t) = \begin{bmatrix} e^T(t) & x_h^T(t) \end{bmatrix}^T$:

(i) DTS:

$$\sum_{t=0}^{t_f} x_e^T(t) Q x_e(t) \leq x_e^T(0) P x_e(0) + \sum_{q=1}^{\bar{\tau}} x_e^T(-q) S x_e(-q) + \rho^2 \sum_{t=0}^{t_f} \|w_e(t)\|^2. \quad (5)$$

(ii) CTS:

$$\int_0^{t_f} x_e^T(t) Q x_e(t) dt \leq x_e^T(0) P x_e(0) + \int_{-\bar{\tau}}^0 x_e^T(v) S x_e(v) dv + \rho^2 \int_0^{t_f} \|w_e(t)\|^2 dt, \quad (6)$$

where matrices $Q = Q^T > 0$, $P = P^T > 0$, $S = S^T > 0$; $x_e(0)$ are the initial values; ρ is the given attenuation level; t_f is terminal time; and $w_e(t)$ is the vector due to $w(t)$, $v(t)$ and residual errors considered as disturbance. From a physical perspective, relation (45) or (56) represents that the effect of $w_e(t)$ on tracking error $x_e(t)$ must be below a given level ρ .

We will need the following result for upcoming proofs.

Lemma 1. For any $\bar{x}, \bar{y} \in R^n$ and matrix $\bar{D}$ with appropriate dimension, the relationship

$$2\bar{x}^T \bar{D} \bar{y} \leq \bar{x}^T \bar{x} + \bar{y}^T \bar{D}^T \bar{D} \bar{y}, \quad (7)$$

is satisfied.

3. Master-Slave Synchronization

In this section, we consider master-slave synchronization with immeasurable partial states and antecedent variables. Here, the chaotic system (12) is the master system. We then establish a fuzzy synchronizer to reconstruct the states and

antecedent variables. The IF-THEN rules of the fuzzy synchronizer can be written as

Fuzzy Synchronizer Rule i :

$$\begin{aligned} &\text{IF } \hat{z}_1(t) \text{ is } F_{1i} \text{ and } \dots \text{ and } \hat{z}_f(t) \text{ is } F_{fi} \text{ THEN} \\ &s\hat{x}(t) = A_i \hat{x}(t) + A_{di} \hat{x}(t - \bar{\tau}) + L_i(y(t) - \hat{y}(t)) + \Gamma, \\ &\hat{y}(t) = C_i \hat{x}(t), \end{aligned} \quad (8)$$

where $\hat{y}(t)$ estimates $y(t)$; L_i are the gains of the observer; and $\hat{z}_1(t) \sim \hat{z}_f(t)$ are the antecedent variables dependent on $\hat{x}(t)$. In the fuzzy synchronizer design, we require the average time delay $\bar{\tau}$ in Assumption 1. The inferred output

$$\begin{aligned} s\hat{x}(t) &= \sum_{i=1}^r \mu_i(\hat{z}(t)) [A_i \hat{x}(t) + A_{di} \hat{x}(t - \bar{\tau}) + L_i(y(t) - \hat{y}(t))] + \Gamma, \\ \hat{y}(t) &= \sum_{i=1}^r \mu_i(\hat{z}(t)) C_i \hat{x}(t). \end{aligned} \quad (9)$$

Therefore, the estimation error

$$\begin{aligned}
se(t) &= \sum_{i=1}^r \mu_i(\hat{z}(t)) [(A_i - L_i C)e(t) + A_{di}e(t - \bar{\tau}) - L_i v(t)] + \bar{w}(t) + h_1(t) \\
&= \sum_{i=1}^r \mu_i(\hat{z}(t)) [(A_i - L_i C)e(t) + A_{di}e(t - \bar{\tau}) + E_i w_s(t)] + h_1(t),
\end{aligned} \tag{10}$$

where $\bar{w}(t) = w(t) + h_0(t)$ and unknown time delay lead to $h_0(t) = \sum_{i=1}^r \mu_i(\hat{z}(t)) A_{di} [x(t - \tau(t)) - x(t - \bar{\tau})]$; immeasurable antecedent variables lead to $h_1(t) = \sum_{i=1}^r (\mu_i(z(t)) - \mu_i(\hat{z}(t))) [A_i x(t) + A_{di} x(t - \tau(t))]$; and $\bar{E}_i = [-L_i I]$, $w_s(t) = [v^T(t) \bar{w}^T(t)]^T$.

Note that Assumption 2 is satisfied when the state $x(t)$ is bounded in the interest region. Although mismatched antecedent variables lead to the residue terms $h_1(t)$, the membership function satisfies Lipchitz-like conditions [31], where

$$h_1^T(t) h_1(t) \leq e^T(t) U_1^T U_1 e(t), \tag{11}$$

with a known matrix function U_1 which is $x(t)$ dependent and $x(t - \tau(t))$ (note that $x_d(t)$ and $x_d(t - \tau(t))$ are bounded in the region of interest).

Properly chosen observer gains L_i can attenuate the undesired terms $h_1(t)$ affecting performance. Note that in practical implementation, the time delay τ may be hard to determine. We may therefore use a reasonable U_1 and combine resulting errors to $w_s(t)$.

Consider a Lyapunov-Krasovskii function candidate as follows:

(i) DTS:

$$V(t) = e^T(t) P e(t) + \sum_{q=1}^{\bar{\tau}} e^T(t - q) \bar{S} e^T(t - q). \tag{12}$$

(ii) CTS:

$$V(t) = e^T(t) \bar{P} e(t) + \int_{t-\bar{\tau}}^t e^T(v) \bar{S} e^T(v) dv. \tag{13}$$

Theorem 1. For DTS (10), if there exist symmetric and positive definite matrices $\bar{P}$, and $\bar{S}$, solved from matrix inequalities in (14) for all i , system (10) is therefore stable with guaranteed H_∞ synchronization performance in (3) for a given attenuation level ρ^2 .

$$\begin{bmatrix} \bar{G}_i^T \bar{P} \bar{G}_i + \bar{S} - \bar{P} & (*) & (*) \\ \bar{M}_i^T \bar{P} \bar{G}_i & \bar{M}_i^T \bar{P} \bar{M}_i - \bar{S} & (*) \\ \bar{E}_i^T \bar{P} \bar{G}_i & \bar{E}_i^T \bar{P} \bar{M}_i & \bar{E}_i^T \bar{P} \bar{E}_i - \rho^2 I \end{bmatrix} > 0, \tag{14}$$

where $(*)$ denotes the transposed elements in the symmetric position and $\bar{G}_i = A_i - L_i C$, $\bar{M}_i = A_{di}$, $\bar{H}_{i1} = 2\bar{G}_i^T \bar{P} \bar{G}_i + \bar{S} - \bar{P} - \bar{Q} + 4U_1^T \bar{P} U_1$, $\bar{H}_{i2} = 2\bar{M}_i^T \bar{P} \bar{M}_i - \bar{S}$, and $\bar{H}_{i3} = 2\bar{E}_i^T \bar{P} \bar{E}_i - \rho^2 I$.

Proof. The proof is given in Appendix A. $\square$

Theorem 2. For the CTS (10), if there exist a symmetric and positive definite matrix $\bar{P}$, and $\bar{S}$, solved from matrix inequalities in (15) for all i , system (10) is therefore stable with guaranteed H_∞ synchronization performance in (4) for a given attenuation level ρ^2 .

$$\begin{bmatrix} \bar{G}_i^T \bar{P} + \bar{P} \bar{G}_i + \bar{Q} + \bar{S} + \bar{P} \bar{P} + 4U_1^T U_1 & (*) & (*) \\ \bar{M}_i^T \bar{P} & -\bar{S} & (*) \\ \bar{E}_i^T \bar{P} & 0 & -\rho^2 I \end{bmatrix} < 0. \tag{15}$$

Proof. The proof is given in Appendix B.

In light of the aforementioned analysis, the important task of the synchronization problem is to find the common $\bar{P}$ and $\bar{S}$ from Theorems 1 and 2. However, analytically determining $\bar{P}$ and $\bar{S}$ is nontrivial, where the details of formulating the LMIs will be given in the upcoming sections. $\square$

4. Output Tracking Control

Here, we consider immeasurable states and antecedent variables. Along with time-varying delay, estimated error dynamics are then coupled with tracking error dynamics. First, we formulate an observer to estimate the states and antecedent variables with IF-THEN rules:

Observer Rule i :

IF $\hat{z}(t)$ is F_{li} and $\dots$ and $\hat{z}_f(t)$ is F_{fi} THEN

$$s\hat{x}(t) = A_i \hat{x}(t) + A_{di} \hat{x}(t - \bar{\tau}) + Bu(t) + L_i (y(t) - \hat{y}(t)) + \Gamma,$$

$$\hat{y}(t) = C_i \hat{x}(t).$$

(16)

In the observer design, we require the average time delay $\bar{\tau}$ in Assumption 1. The inferred output

$$\begin{aligned}
s\hat{x}(t) &= \sum_{i=1}^r \mu_i(\hat{z}(t)) \text{sum} [A_i \hat{x}(t) + A_{di} \hat{x}(t - \bar{\tau}) + Bu(t) + L_i(y(t) - \hat{y}(t))] + \Gamma \\
\hat{y}(t) &= \sum_{i=1}^r \mu_i(\hat{z}(t)) C_i \hat{x}(t).
\end{aligned} \tag{17}$$

The IF-THEN rules of the target variables can be written as

Desired variables Rule i :

IF $z_{d1}(t)$ is F_{1i} and $\dots$ and $z_{df}(t)$ is F_{fi} THEN

$$sx_d(t) = A_i x_d(t) + A_{di} x_d(t - \bar{\tau}) + Bu_r(t) + \Gamma \tag{18}$$

$$y_d(t) = C_i x_d(t),$$

where $z_{d1}(t) \sim z_{df}(t)$ are the antecedent variables dependent on $x_d(t)$ and $u_r(t)$ is the desired control force. The inferred output

$$\begin{aligned}
sx_d(t) &= \sum_{i=1}^r \mu_i(z_d(t)) [A_i x_d(t) + A_{di} x_d(t - \bar{\tau}) + Bu_r(t)] + \Gamma \\
y_d(t) &= \sum_{i=1}^r \mu_i(z_d(t)) C_i x_d(t),
\end{aligned} \tag{19}$$

where $z_d(t) = [z_{d1}(t) z_{d2}(t) \dots z_{dn}(t)]^T$. The overall controller

$$u(t) = u_r(t) + u_k(t), \tag{20}$$

where $u_k(t)$ is the combined controller for the linear sub-systems as follows:

Controller Rule i :

IF $\hat{z}(t)$ is F_{1i} and $\dots$ and $\hat{z}_f(t)$ is F_{fi} THEN

$$u_k(t) = -K_i(\hat{x}(t) - x_d(t)), \tag{21}$$

where K_i are the gains of control. The controller inferred output

$$u_k(t) = -\sum_{i=1}^r \mu_i(\hat{z}(t)) K_i(\hat{x}(t) - x_d(t)). \tag{22}$$

The above is the controller analysis whereas the controller realization will be discussed in Section 6.

The tracking error dynamics:

$$\begin{aligned}
sx_h(t) &= \sum_{i=1}^r \mu_i(\hat{z}(t)) [(A_i - BK_i)x_h(t) + A_{di}x_h(t - \bar{\tau}) \\
&\quad + L_i C e(t) + L_i v(t) + h_2(t)],
\end{aligned} \tag{23}$$

where the mismatched antecedent variables between observer and controller are

$$h_2(t) = \sum_{i=1}^r (\mu_i(\hat{z}(t)) - \mu_i(z_d(t))) [A_i x_d(t) + A_{di} x_d(t - \bar{\tau})]. \tag{24}$$

We can now express the system

$$sx_e(t) = \sum_{i=1}^r \mu_i(\hat{z}(t)) [G_i x_e(t) + M_i x_e(t - \bar{\tau}) + E_i w_e(t) + \bar{h}(t)], \tag{25}$$

where

$$\begin{aligned}
x_e(t) &= \begin{bmatrix} e(t) \\ x_h(t) \end{bmatrix}, \\
G_i &= \begin{bmatrix} A_i - L_i C & 0 \\ L_i C & A_i - BK_i \end{bmatrix}, \\
M_i &= \begin{bmatrix} A_{di} & 0 \\ 0 & A_{di} \end{bmatrix}, \\
E_i &= \begin{bmatrix} -L_i & I \\ L_i & 0 \end{bmatrix}, \\
w_e(t) &= \begin{bmatrix} v(t) \\ \bar{w}(t) \end{bmatrix}, \\
\bar{h}(t) &= \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix}.
\end{aligned} \tag{26}$$

Remark 1. If all antecedent variables are measurable, i.e., $\mu_i(\hat{z}(t)) = \mu_i(z(t))$, the target variable

$$\begin{aligned}
sx_d(t) &= \sum_{i=1}^r \mu_i(z(t)) [A_i x_d(t) + A_{di} x_d(t - \bar{\tau}) \\
&\quad + B(u(t) - u_k(t))] + \Gamma,
\end{aligned} \tag{27}$$

and controller

$$u_k(t) = -\sum_{i=1}^r \mu_i(z(t)) K_i(\hat{x}(t) - x_d(t)), \tag{28}$$

and system (25) therefore becomes

$$sx_e(t) = \sum_{i=1}^r \mu_i(z(t)) [G_i x_e(t) + M_i x_e(t - \bar{\tau}) + E_i w_e(t)], \tag{29}$$

with $\bar{h}(t) = 0$.

Remark 2. If all antecedent variables are measurable and disturbance free, i.e., $\mu_i(\hat{z}(t)) = \mu_i(z(t))$, $w(t) = 0$ and $v(t) = 0$, the target variables are (27) and (28) with system

$$sx_e(t) = \sum_{i=1}^r \mu_i(z(t)) [G_i x_e(t) + M_i x_e(t - \bar{\tau}) + E_i w_e(t)], \quad (30)$$

where

$$E_i = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad (31)$$

$$w_e(t) = \begin{bmatrix} 0 \\ \bar{w}(t) \end{bmatrix},$$

with $\bar{w}(t) = h_0(t)$.

Remark 3. Consider $\tau(t) = \bar{\tau}$ for error system (30); we conclude that the biased term $\bar{w}(t)$ will vanish at $\tau(t) = \bar{\tau}$ reducing the system to

$$sx_e(t) = \sum_{i=1}^r \mu_i(z(t)) [G_i x_e(t) + M_i x_e(t - \bar{\tau})]. \quad (32)$$

This means that known time delay leads to error (32).

From the upcoming controller design, Assumption 2 is satisfied when the state $x(t)$ is bounded in the interest region. Although mismatched antecedent variables lead to the residue terms $h_1(t)$ and $h_2(t)$, the membership functions assume Lipchitz-like conditions [31], where

$$h_1^T(t) h_1(t) \leq e^T(t) U_1^T U_1 e(t), \quad (33)$$

with known matrix function U_1 related to $x(t)$ and $x(t - \tau(t))$; and

$$h_2^T(t) h_2(t) \leq x_h^T(t) U_2^T U_2 x_h(t), \quad (34)$$

with existing matrix function U_2 (note that $x_d(t)$ and $x_d(t - \tau(t))$ are bounded in the region of interest).

Summarizing the above, the inequality

$$\bar{h}^T(t) \bar{h}(t) \leq x_e^T(t) (t) \bar{U}^T \bar{U} x_e(t), \quad (35)$$

where $\bar{h}(t) = [h_1^T(t) h_2^T(t)]^T$ and $\bar{U} = \text{block-diag}(U_1, U_2)$.

Properly chosen controller gains K_i and observer gains L_i can attenuate the undesired terms $h_1(t)$ and $h_2(t)$ affecting control performance. Note that in practical implementation, the time delay τ may be hard to determine. We may therefore choose a reasonable large $\bar{U}$ and merge resulting errors to the term $E_i w_e(t)$.

5. Guaranteed H_∞ Performance

For the error system (25), mismatched antecedent variables lead to the residues of $h_1(t)$ and $h_2(t)$. Assuming that the membership functions satisfy a Lipchitz-like condition, we carry out further stability analysis on these bias as follows.

Consider a Lyapunov-Krasovskii function candidate as follows:

(i) DTS:

$$V(t) = x_e^T(t) P x_e(t) + \sum_{q=1}^{\bar{\tau}} x_e^T(t - q) S x_e(t - q). \quad (36)$$

(ii) CTS:

$$V(t) = x_e^T(t) P x_e(t) + \int_{t-\bar{\tau}}^t x_e^T(v) S x_e(v) dv. \quad (37)$$

Theorem 3. For DTS (25), if there exist symmetric and positive definite matrices P , S , and gain matrices K_i ($i = 1, 2, \dots, r$) solved from matrix inequalities in (38) for all i , the system (25) is therefore stable with guaranteed H_∞ tracking control performance in (5) for a given attenuation level ρ^2 .

$$\begin{bmatrix} H_{i1} & (*) & (*) \\ M_i^T P G_i & H_{i2} & (*) \\ E_i^T P G_i & E_i^T P M_i & H_{i3} \end{bmatrix} > 0, \quad (38)$$

where $(*)$ denotes the transposed symmetric position elements and $H_{i1} = 2G_i^T P G_i + S - P - Q + 4\bar{U}^T P \bar{U}$, $H_{i2} = 2M_i^T P M_i - S$, and $H_{i3} = 2E_i^T P E_i - \rho^2 I$.

Proof. The proof is given in Appendix C. $\square$

Theorem 4. For the CTS (25), if there exist a symmetric and positive definite matrix P , S , and gain matrices K_i ($i = 1, 2, \dots, r$) solved from matrix inequalities in (39) for all i , the system (25) is therefore stable with guaranteed H_∞ tracking control performance in (6) for a given attenuation level ρ^2 .

$$\begin{bmatrix} G_i^T P + P G_i + Q + S + P P + 4\bar{U}^T \bar{U} & (*) & (*) \\ M_i^T P & -S & (*) \\ E_i^T P & 0 & -\rho^2 I \end{bmatrix} < 0. \quad (39)$$

Proof. The proof is given in Appendix D. $\square$

Corollary 1. Considering all antecedent variables are measurable, Remark 1 leads to $\mu_i(\hat{z}(t)) = \mu_i(z(t))$.

For DTS (29), if matrix inequalities in (40) for all i

$$\begin{bmatrix} G_i^T P G_i + S - P - Q & (*) & (*) \\ M_i^T P G_i & M_i^T P M_i - S & (*) \\ E_i^T P G_i & E_i^T P M_i & E_i^T P E_i - \rho^2 I \end{bmatrix} > 0, \quad (40)$$

have common solutions $P > 0$ and $S > 0$, system (29) is therefore stable with guaranteed H_∞ tracking control performance (5) for a given attenuation level ρ^2 .

For the CTS system (29), if matrix inequalities (41) for all i

$$\begin{bmatrix} G_i^T P + PG_i + Q + S & (*) & (*) \\ M_i^T P & -S & (*) \\ E_i^T P & 0 & -\rho^2 I \end{bmatrix} < 0, \quad (41)$$

have common solutions $P > 0$ and $S > 0$, system (29) is therefore stable with guaranteed H_∞ tracking control performance (6) for a given attenuation level ρ^2 .

Corollary 2. *Considering all antecedent variables are measurable without disturbance, according to Remark 2, the terms $\mu_i(\hat{z}(t)) = \mu_i(z(t))$, $w(t) = 0$, and $v(t) = 0$. For DTS (30), if matrix inequalities in (42) for all i*

$$\begin{bmatrix} G_i^T PG_i + S - P - Q & (*) & (*) \\ M_i^T PG_i & M_i^T PM_i - S & (*) \\ E_i^T PG_i & E_i^T PM_i & E_i^T PE_i - \rho^2 I \end{bmatrix} > 0, \quad (42)$$

have common solutions $P > 0$ and $S > 0$, system (29) achieves guaranteed H_∞ tracking control performance (5) for a given attenuation level ρ^2 .

In system (30) for CTS, the matrix inequalities in (43) for all i

$$\begin{bmatrix} G_i^T P + PG_i + Q + S & (*) & (*) \\ M_i^T P & -S & (*) \\ E_i^T P & 0 & -\rho^2 I \end{bmatrix} < 0, \quad (43)$$

have common solutions $P = P^T > 0$ and $S = S^T > 0$, then guaranteed H_∞ tracking control performance (6) is achieved for a given attenuation level ρ^2 .

Corollary 3. *Considering all antecedent variables are measurable, without disturbance and exactly known time delay, then according to Remark 3, the terms $\mu_i(\hat{z}(t)) = \mu_i(z(t))$, $w(t) = 0$, $v(t) = 0$, and $\tau(t) = \bar{\tau}$.*

In system (32) for DTS, if P and $S > 0$ are the common solution of the matrix inequalities in (44) for all i

$$\begin{bmatrix} G_i^T PG_i + S - P - Q & (*) \\ M_i^T PG_i & M_i^T PM_i - S \end{bmatrix} > 0, \quad (44)$$

then system (32) is asymptotically stable. In system (32) for DTS, if $P = P^T > 0$ and $S = S^T > 0$ are the common solution of the matrix inequalities in (44) for all i

$$\begin{bmatrix} G_i^T P + PG_i + Q + S & (*) \\ M_i^T P & -S \end{bmatrix} < 0, \quad (45)$$

then system (32) is asymptotically stable.

Improved tracking performance can be achieved by formulating the following minimization problem:

(i) DTS:

$$\min_{P,S} \rho^2, \quad (46)$$

subject to $P > 0, S > 0$ and (38).

(ii) CTS:

$$\min_{P,S} \rho^2, \quad (47)$$

subject to $P > 0, S > 0$ and (39).

In light of the above, we must solve the common P and S solution from the minimization problem (46) or (47). However, to analytically determine that P and S are nontrivial, we therefore discuss the procedure in detail.

First, we consider DTS. From Schur complement, the inequality

$$\begin{bmatrix} P - Q - S - 4\bar{U}^T P \bar{U} & (*) & (*) & (*) \\ 0 & S & (*) & (*) \\ 0 & 0 & \rho^2 I & (*) \\ PG_i & PM_i & PE_i & \frac{1}{2}P \end{bmatrix} > 0. \quad (48)$$

Given $\rho > 0$, $Q = Q^T > 0$, and $\bar{U}$, we must find the observer gain L_i , controller gain K_i , for common $P > 0$, and $S > 0$ satisfying (48). In summary, a three-step procedure is given to solve (48). First, we assume P and Λ are in diagonal block form; i.e., $P = \text{block-diag}(P_1, P_2)$ and $S = \text{block-diag}(S_1, S_2)$. We expand the matrix inequalities (48) to arrive with the inequalities

$$\begin{bmatrix} P_1 - Q_1 - S_1 - 4U_1^T P_1 U_1 & (*) & (*) & (*) \\ 0 & S_1 & (*) & (*) \\ 0 & 0 & \rho^2 I & (*) \\ P_1 A_i - N_i C & P_1 A_{di} & -N_i & \frac{1}{2}P_1 \end{bmatrix} > 0, \quad (49)$$

$$\begin{bmatrix} P_2 - Q_2 - S_2 - 4U_2^T P_2 U_2 & (*) & (*) & (*) \\ 0 & S_2 & (*) & (*) \\ 0 & 0 & \rho^2 I & (*) \\ P_2 A_i - P_2 B K_i & P_2 A_{di} & 0 & \frac{1}{2}P_2 \end{bmatrix} > 0, \quad (50)$$

where $N_i = P_1 L_i$. We summarize the three-step procedure for DTS as follows:

Step D1: given ρ , Q_1 and U_1 , solve (49) to obtain P_1 , S_1 , and $L_i = P_1^{-1} N_i$.

Step D2: given ρ , Q_2 and U_2 , inequalities (50) are equivalent to

$$\begin{bmatrix} X_2 - \Lambda_2 & (*) & (*) & (*) & (*) & (*) \\ 0 & \Lambda_2 & (*) & (*) & (*) & (*) \\ 0 & 0 & \rho^2 I & (*) & (*) & (*) \\ A_i X_2 - B F_i & A_{di} X_2 & 0 & \frac{1}{2} X_2 & (*) & (*) \\ 2U_2 X_2 & 0 & 0 & 0 & X_2 & (*) \\ Q_2 X_2 & 0 & 0 & 0 & 0 & Q_2 \end{bmatrix} > 0. \quad (51)$$

where $X_2 = P_2^{-1}$, $F_i = K_i X_2$ and $\Lambda_2 = X_2 S_2 X_2$. Solve (51) to obtain $K_i = F_i P_2$.

Step D3: solve (48) with matrices satisfying $P > 0$ and $S > 0$. Notice that, in the three-step procedure, the dimensions of matrices P_1 , P_2 , S_1 , and $S_2 \in R^{n \times n}$ in Step

D1 and Step D2 and the matrices $P \in R^{2n \times 2n}$ and $S \in R^{2n \times 2n}$ in Step D3, respectively.

In an analogous manner, considering CTS, we assume P , S , and Q are diagonal block form. According to (48), the inequality

$$\begin{bmatrix} \Delta_i & (*) & (*) & (*) & (*) & (*) \\ P_2 L_i C & \Psi_i & (*) & (*) & (*) & (*) \\ A_{di}^T P_1 & 0 & -S_1 & (*) & (*) & (*) \\ 0 & A_{di}^T P_2 & 0 & -S_2 & (*) & (*) \\ -L_i^T P_1 & L_i^T P_2 & 0 & 0 & -\rho^2 I & (*) \\ P_1 & 0 & 0 & 0 & 0 & -\rho^2 I \end{bmatrix} < 0, \quad (52)$$

where $\Delta_i = (A_i - L_i C)^T P_1 + P_1 (A_i - L_i C) + Q_1 + \Lambda_1 + P_1 P_1 + U_1^T U_1$ and $\Psi_i = (A_i - B K_i)^T P_2 + P_2 (A_i - B K_i) + P_2 P_2 + Q_2 + \Lambda_2 + U_2^T U_2$. We summarize the three-step procedure for CTS as follows:

Step C1: the matrix inequalities (52) imply that $\Psi_i + P_2 A_{di} \Lambda_2^{-1} A_{di}^T P_2 < 0$. By Schur complement, we solve the observer gain L_i from the inequality

$$\begin{bmatrix} A_i^T P_1 + P_1 A_i - N_i C - C^T N_i^T + Q_1 + \Lambda_1 + U_1^T U_1 & (*) & (*) & (*) \\ & A_{di}^T P_1 & -\Lambda_2 & (*) & (*) \\ & -L_i^T P_1 & 0 & -\rho^2 I & (*) \\ & P_1 & 0 & 0 & -I \end{bmatrix} < 0. \quad (53)$$

where $N_i = P_1 L_i$.

Step C2: determine controller gains K_i from solving the inequality

$$\begin{bmatrix} \hat{\Psi}_i & (*) & (*) & (*) \\ A_{di}^T X_2 & -\Lambda_2 & (*) & (*) \\ X_2 & 0 & -Q_2^{-1} & (*) \\ U_2 X_2 & 0 & 0 & -I \end{bmatrix} < 0. \quad (54)$$

where $\hat{\Psi}_i = A_i X_2 + X_2 A_i^T - B F_i - F_i^T B^T + Q_2 + I$, $F_i = K_i X_2$, $\Lambda_2 = X_2 S_2 X_2$, and $X_2 = P_2^{-1}$.

Step C3: once the gains K_i and L_i are available from Step C1 and Step C2, then according to (48) there exist matrices $P > 0$ and $\Lambda > 0$ satisfying the following LMIs:

$$\begin{bmatrix} G_i^T P + P G_i + Q + S + \bar{U}^T \bar{U} & (*) & (*) & (*) \\ & M_i^T P & -\Lambda & (*) & (*) \\ & E_i^T P & 0 & -\rho^2 I & (*) \\ & P & 0 & 0 & -I \end{bmatrix} < 0, \quad (55)$$

where $P \in R^{2n \times 2n}$ and $\Lambda \in R^{2n \times 2n}$.

We can arrive at the analogous results for error systems stated in Remarks 1–3 and omitted here for brevity.

Remark 4. According to the above LMI formulation procedure, we derive from (14) for DTS and (15) for CTS accordingly the LMIs:

$$\begin{bmatrix} \bar{P} - \bar{Q} - \bar{S} - 4U_1^T \bar{P} U_1 & (*) & (*) & (*) \\ 0 & \bar{S} & (*) & (*) \\ 0 & 0 & \rho^2 I & (*) \\ \bar{P} A_i - N_i C & \bar{P} A_{di} & -N_i & \frac{1}{2} \bar{P} \end{bmatrix} > 0, \quad (56)$$

$$\begin{bmatrix} A_i^T \bar{P} + \bar{P} A_i + \bar{Q} - N_i C - C^T N_i^T + \bar{S} + U_1^T U_1 & (*) & (*) & (*) \\ A_{di}^T \bar{P} & -\Lambda_2 & (*) & (*) \\ -L_i^T \bar{P} & 0 & -\rho^2 I & (*) \\ \bar{P} & 0 & 0 & -I \end{bmatrix} < 0,$$

where $N_i = \bar{P} L_i$.

6. Tracking Controller Realization

We now address in detail the design of target variables $x_d(t)$. According to the above discussion, we consider the worst

case mismatched antecedent variables with time-varying delay. The target variable design for systems stated in Remarks 1–3 can be obtained in an analogous manner.

We determine the target variables from decomposing variables (19). We then arrive at

$$s x_{d\ell}(t) = \sum_{i=1}^r \mu_i(z_d(t)) [A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t - \bar{\tau})], \quad \ell = 2, \dots, n, \quad (57)$$

where $x_{d\ell}(t)$ is the ℓ th (for $\ell = 2, \dots, n$) element of $x_d(t)$ and $A_{i,\ell}$ and $A_{di,\ell}$ are accordingly the ℓ th rows of A_i and A_{di} matrices. We can therefore synthesize the control

$$\begin{aligned} u_r(t) &= b_0^{-1} \left(s x_{d1}(t) - \sum_{i=1}^r \mu_i(z_d(t)) [A_{i,1} x_d(t) + A_{di,1} x_d(t - \bar{\tau})] \right), \\ u_k(t) &= - \sum_{i=1}^r \mu_i(\hat{z}(t)) K_i (\hat{x}(t) - x_d(t)). \end{aligned} \quad (58)$$

Note that there exist $n - 1$ equations in (57). We assign a specific state:

$$y_d(t) = D x_d(t). \quad (59)$$

In most practical applications, we can treat $y_d(t)$ as a constraint for solving target variables.

To implement control input (58), we need to determine (i) the average time delay $\bar{\tau}$ and (ii) predictive signal $x_{d1}(t + 1)$ for DTS or derivative signal $\dot{x}_{d1}(t)$ for CTS. We use the average of $\tau(t)$ as the average time delay. The predictive (or derivative) signal is determined as follows.

In the following, we discuss two cases of the observer-based output tracking: (i) output regulation and (ii) time-varying output tracking (where either $y_d(t) = x_{d1}(t)$, and $y_d(t) \neq x_{d1}(t)$). Note that the desired state $x_d(t)$ should be properly chosen such that $x_d(t) \in \Omega$.

6.1. Output Regulation. For the ideal condition where the desired state $\bar{x}_d$ is constant vector and $x(t) = \bar{x}_d$ such that $x_d(t - \bar{\tau}) = \bar{x}_d$ and $x(t - \tau(t)) = \bar{x}_d$, we assign the desired output as $y_d(t) = \bar{x}_{d1}$ ($\bar{x}_{d1}$ is constant). This leads to $x_{d1}(t + 1) = \bar{x}_{d1}$ or $\dot{x}_{d1}(t) = 0$, where we solve the constant desired state from equations.

(i) DTS:

$$\bar{x}_d = \sum_{i=1}^r (\mu_i(z_d)) [A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t - \bar{\tau})], \quad \ell = 2, \dots, n. \quad (60)$$

(ii) CTS:

$$\sum_{i=1}^r (\mu_i(z_d)) [A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t - \bar{\tau})] = 0, \quad \ell = 2, \dots, n, \quad (61)$$

where $z_d(t) = \bar{x}_d$.

6.2. *Time-Varying Output Tracking.* Consider two cases:

- (i) $y_d(t) = x_{d1}(t)$. Consider a smooth desired output $y_d(t) = x_{d1}(t)$; we can then solve the target variables from (57). Once $x_{d1}(t+1)$ or $\dot{x}_{d1}(t)$ is available, we can implement the controller as (58).
- (ii) $y_d(t) = x_{di}(t)$, $i \neq 1$. Since the desired output $y_d(t)$ is not $x_{d1}(t)$, we may derive $x_{d1}(t+1)$ or $\dot{x}_{d1}(t)$ from (57). Note that direct application of signal $x_{d1}(t+1)$ and $\dot{x}_{d1}(t)$ should be avoided, since the control force $u(t)$ appear in $x_{d1}(t+1)$ and $\dot{x}_{d1}(t)$. To cope with this problem, we can use an approximate signal for CTS $\dot{x}_{d1}(t) \approx (x_{d1}(t) - x_{d1}(t - \Delta T))/\Delta T$, where ΔT is the sampling period and for DTS $x_{d1}(t+1) \approx (x_{d1}(t) - tx_{d1}n(t-1))$. Note that we assume the desired output is slow time varying when applying the approximation.

We summarize the overall design procedure for output tracking control as follows:

Step G1: construct T-S fuzzy model for the nonlinear time-delay system as (2)

Step G2: given an attenuation level ρ and matrix Q and $\bar{U}$, then follow the three-step procedure to obtain controller and observer gains

Step G3: synthesize the target variables from (57) and (59) based on a specified form of desired output

Step G4: implement the observer (17) and controller (58)

7. Simulation Results

To verify the theoretical derivations, we consider both discrete and continuous chaotic systems as examples. We consider here only the most practical case where immeasurable states are considered with mismatched antecedent variables.

Example 1. Consider a discrete-time 3-dimensional chaotic system as follows:

$$\begin{aligned} x_1(t+1) &= ax_1^2(t) + x_2(t) + c + ex_1(t - \tau(t)) + u(t) + \omega_1(t), \\ x_2(t+1) &= bx_1(t) + x_3(t) + \omega_2(t), \\ x_3(t+1) &= -bx_1(t) + \omega_2(t), \\ y(t) &= x_1(t) + v(t), \end{aligned} \quad (62)$$

where $x_1(t) \sim x_3(t)$ and $u(t)$ are accordingly the states and control input; $\tau(t)$ is time-varying delay with an upper bound $\tau_o = 15$; $\omega_1(t)$, $\omega_2(t)$, $\omega_3(t)$, $v(t)$ are uniformly distributed external disturbances, all with the amplitude of 0.01; and parameters $a = -1$, $b = 0.33$, $c = 1$, $e = 0.05$.

Choose state $x_1(t)$ as the antecedent variable. According to the fuzzy modeling method [31], the membership functions $F_1 = 0.5(1 + (x_1(t)/d))$, $F_2 = 1 - F_1$ with $|x_1(t)| \leq d$, $d = 1.7$. System (62) can then be exactly represented by the T-S fuzzy model with system matrices

$$\begin{aligned} A_1 &= \begin{bmatrix} a & d & 1 & 0 \\ b & 0 & 1 \\ b & 0 & 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -a & d & 1 & 0 \\ b & 0 & 1 \\ -b & 0 & 0 \end{bmatrix}, \\ A_{d1} = A_{d2} &= \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ \Gamma &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (63)$$

First, for $u(t) = 0$, we show the $x_1(t) - x_2(t)$ phase portrait, $x_2(t) - x_3(t)$ phase portrait, and time-varying delay $\tau(t)$ and $x_1(t)$ of the chaotic system (62) in Figures 1(a)–1(d).

7.1. Synchronization. The synchronization parameters $\rho = 0.9$, $\bar{Q} = 0.1I$, and $U_1 = 0.2I$. We obtain the observer gains from solving LMI (14) in Theorem 1. Let initial conditions $x(0) = [1 \ 0 \ 0]^T$ and $\hat{x}(0) = [0 \ 0 \ 0]^T$ for $t_0 \in [-\tau_0 \ 0]$. The transmitter sent an output $y(t)$ to the receiver; then the receiver estimated the all states. The results are shown in Figures 2(a)–2(d) ($x_1(t)$, $e_1(t)$, $e_2(t)$, and $e_3(t)$).

FIGURE 1: (a) $x_1(t) - x_2(t)$ phase portrait, (b) $x_2(t) - x_3(t)$ phase portrait, (c) time-varying delay $\tau(t)$, and (d) $x_1(t)$ of the discrete-time chaotic system example.

FIGURE 2: Continued.

FIGURE 2: Synchronization of discrete-time chaotic system with (a) $x_1(t)$, (b) $e_1(t)$, (c) $e_2(t)$, and (d) $e_3(t)$.

FIGURE 3: Tracking control of discrete-time chaotic system with trajectories (a) $x_{d1}(t)$ (solid) and $y_r(t)$ (dash), (b) $e_1(t)$, and (c) $x_{h1}(t)$.

7.2. Time-Varying Output Tracking. The control parameters $\rho = 0.9$, $Q = 0.1I$, $U_1 = 0.2I$, and $U_2 = 0.2I$. From solving the LMIs (49) ~ (51) in Theorem 3, we obtain the control gains K_i , observer gains L_i , P , and S . Let initial conditions $x(0) = [1 \ 0 \ 0]^T$ and $\hat{x}(0) = [0 \ 0 \ 0]^T$ for $t_0 \in [-\tau_0 \ 0]$. Consider the controlled output $y_c(t) = x_1(t)$ and assume $x_{d1}(t+1) = 0.4x_{d1}(t) + y_r(t)$ and $y_r(t)$ is a unit square wave. The reference states $x_{d2}(t)$ and $x_{d3}(t)$ therefore are obtained from solving the equation $x_{d2}(t+1) = bx_{d1}(t) + x_{d3}(t)$ and $x_{d3}(t+1) = -bx_{d1}(t)$. The results are shown in Figures 3–5. The trajectories of $x_{d1}(t)$ (solid) and $y_r(t)$ (dash), $e_1(t)$, and $x_{h1}(t)$ are shown in Figures 3(a)–3(c). Figures 4(a)–4(c) show the trajectories of $x_{d2}(t)$, $e_2(t)$, and $x_{h2}(t)$. Figures 5(a)–5(c) show the trajectories of $x_{d3}(t)$, $e_3(t)$, and $x_{h3}(t)$.

Example 2. Consider a 3-dimensional continuous chaotic system described as follows:

$$\begin{aligned}\dot{x}_1(t) &= a[x_2(t) - f_1(x_1(t))] + c + x_1(t - \tau(t)) + u(t) + \omega_1(t), \\ \dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t) + \omega_2(t), \\ \dot{x}_3(t) &= -bx_2(t) + \omega_3(t), \\ y(t) &= x_1(t) + v(t),\end{aligned}\tag{64}$$

where $x_1(t) \sim x_3(t)$ and $u(t)$ are accordingly the states and control; the nonlinear function $f_1(x_1(t)) = (2/7)x_1(t) - (3/14)(|x_1(t) + 1| - |x_1(t) - 1|)$; $\tau(t)$ is time-varying delay with an upper bound $\tau_o = 3.54$; $\omega_1(t)$, $\omega_2(t)$, $\omega_3(t)$, $v(t)$ are uniformly distributed external disturbances all with the amplitude of 0.01; and parameters $a = 9$, $b = 14.286$, $c = 0.1$.

Choose state $x_1(t)$ as the antecedent variable. According to the fuzzy modeling method [31], the following membership functions $F_1 = 0.5(1 + (\phi(t)/d))$, $F_2 = 1 - F_1$ with $\phi(t) = f_1(x_1(t))/x_1(t)$ where $|\phi(t)| \leq d$, $d = 0.26$. System

FIGURE 4: Tracking control of discrete-time chaotic system with trajectories (a) $x_{d2}(t)$, (b) $e_2(t)$, and (c) $x_{h2}(t)$.

FIGURE 5: Tracking control of discrete-time chaotic system with trajectories (a) $x_{d3}(t)$, (b) $e_3(t)$, and (c) $x_{h3}(t)$.

FIGURE 6: (a) $x_1(t) - x_2(t)$ phase portrait, (b) $x_2(t) - x_3(t)$ phase portrait, and (c) time-varying delay $\tau(t)$ and (d) $x_1(t)$ of system (64) of the continuous-time chaotic system example.

FIGURE 7: Synchronization of continuous-time chaotic system with (a) $x_1(t)$, (b) $e_1(t)$, (c) $e_2(t)$, and (d) $e_3(t)$.

FIGURE 8: Tracking control of continuous-time chaotic system with trajectories (a) $x_{d1}(t)$, (b) $e_1(t)$, and (c) $x_{h1}(t)$.

FIGURE 9: Tracking control of continuous-time chaotic system with trajectories (a) $x_{d2}(t)$, (b) $e_2(t)$, and (c) $x_{h2}(t)$.

FIGURE 10: Tracking control of continuous-time chaotic system with trajectories (a) $x_{d3}(t)$, (b) $e_3(t)$, and (c) $x_{h3}(t)$.

(64) can be exactly represented by the T-S fuzzy model with system matrices

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -a & d & a & 0 \\ 1 & -1 & 1 & \\ 0 & -b & 0 & \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} a & d & a & 0 \\ 1 & -1 & 1 & \\ 0 & -b & 0 & \end{bmatrix}, \\
 A_{d1} = A_{d2} &= \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
 B &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.
 \end{aligned} \tag{65}$$

Let initial conditions $x(0) = [1 \ 0 \ 0]^T$ and $\hat{x}(0) = [0 \ 0 \ 0]^T$ for $t_0 \in [-\tau_0 \ 0]$. Synchronization and output tracking are discussed. Figures 6(a)–6(d) show the $x_1(t) - x_2(t)$ phase portrait, $x_2(t) - x_3(t)$ phase portrait, and time-varying delay $\tau(t)$ and $x_1(t)$ of system (64) with $u(t) = 0$.

7.3. Synchronization. The transmitter sent an output $y(t)$ to receiver; then the receiver estimated the all states. Choose parameters $\rho = 0.9$, $\bar{Q} = 0.1I$, $U_1 = 0.3I$. From Theorem 2

solving LMI (15), we obtain the observer gains L_i . Figures 7(a)–7(d) show the trajectories $x_1(t)$, $e_1(t)$, $e_2(t)$, and $e_3(t)$, respectively.

7.4. Time-Varying Output Tracking. Consider controlled output is $y_c(t) = x_1(t)$ and assume $x_{d1}(t) = \sin(2t)$. The control parameters $\rho = 0.9$, $Q = 0.1I$, $U_1 = 0.3I$, $U_2 = 0.2I$. From solving the LMIs (53) ~ (55) in Theorem 4, we obtain the control gains K_i , observer gains L_i , P , and S . In this case, $x_{d2}(t)$ and $x_{d3}(t)$ may solve the equation $\dot{x}_{d2}(t) = x_{d1}(t) - x_{d2}(t) + x_{d3}(t)$ and $\dot{x}_3(t) = -bx_2(t)$. The results are shown in Figures 8–10. The trajectories of $x_{d1}(t)$, $e_1(t)$, and $x_{h1}(t)$ are shown in Figures 8(a)–8(c). Figures 9(a)–9(c) show the trajectories of $x_{d2}(t)$, $e_2(t)$, and $x_{h2}(t)$. Figures 10(a)–10(c) show the trajectories of $x_{d3}(t)$, $e_3(t)$, and $x_{h3}(t)$.

It can be seen that the proposed control method can achieve H_∞ synchronization and output tracking control performance for chaotic time-delay systems.

8. Conclusions and Future Work

Chaotic systems with time-varying delays pose a practical yet challenging control problem. In this paper, we have proposed a unified T-S fuzzy representation of the chaotic system control and synchronization problem with time-varying delays for both discrete and continuous systems. An observer-based output tracking controller was designed considering different antecedent variables between the controller and observer. This is an important consideration

for complex system due to uncertainties from estimation and controller mismatches. Numerical simulations for all cases of continuous, discrete with delays disturbances have been carried out. Robustness is shown to achieve H-infinity performance. It is worthwhile to note that asymptotic and exponential tracking can also be achieved for more ideal conditions. The method can also be applied to general nonlinear system control with limited restrictions. Observer and controller gains are found numerically through LMI toolboxes which further extends the possibility of actual implementations. For future work, error criteria can further evaluate and quantify the performance of the proposed method and provide reference for implementation.

Appendix

A. Proof of Theorem 1

Consider a Lyapunov–Krasovskii function candidate as

$$V(t) = e^T(t) \bar{P} e(t) + \sum_{q=1}^{\bar{\tau}} e^T(t-q) \bar{S} e^T(t-q). \quad (\text{A.1})$$

From (10), we have

$$\begin{aligned} \Delta V(t) &= e^T(t+1) \bar{P} e(t+1) - e^T(t) \bar{P} e(t) + e^T(t) \bar{S} e(t) - e^T(t-\bar{\tau}) \bar{S} e(t-\bar{\tau}) \\ &= \left[\sum_{i=1}^r \mu_i(\hat{z}(t)) [\bar{G} e(t) + \bar{M}_i e(t-\bar{\tau}) + \bar{E}_i w_s(t)] + h_1(t) \right]^T \bar{P} \\ &\quad \times \left[\sum_{i=1}^r \mu_i(\hat{z}(t)) [\bar{G} e(t) + \bar{M}_i e(t-\bar{\tau}) + \bar{E}_i w_s(t)] + h_1(t) \right] \\ &\quad - e^T(t) \bar{P} e(t) + e^T(t) \bar{S} e(t) - e^T(t-\bar{\tau}) \bar{S} e(t-\bar{\tau}) \leq \sum_{i=1}^r \mu_i(\hat{z}(t)) \bar{x}_s^T(t) \\ &\quad \left[\bar{G}_i^T \bar{P} \bar{G}_i + \bar{S} - \bar{P} (*) (*) \bar{M}_i^T \bar{P} \bar{G}_i \bar{M}_i^T \bar{P} \bar{M}_i - S(*) \bar{E}_i^T \bar{P} \bar{G}_i \bar{E}_i^T \bar{P} \bar{M}_i \bar{E}_i^T \bar{P} \bar{E}_i - \rho^2 I \right] \bar{x}_s(t) \\ &\quad + \sum_{i=1}^r \mu_i(\hat{z}(t)) 2h_1^T(t) \bar{P} [G_i e(t) + \bar{M}_i e(t-\bar{\tau}) + \bar{E}_i w_s(t)] \\ &\quad + h_1^T(t) \bar{P} h_1(t) + \rho^2 w_s^T(t) w_s(t), \end{aligned} \quad (\text{A.2})$$

where $\bar{x}_s(t) = [e^T(t) e^T(t-\bar{\tau}) w_s^T(t)]^T$. According to Lemma 1 and the fact $e^T(t) U_1^T U_1 e(t)$, we have

$$\Delta V(t) \leq \sum_{i=1}^r \mu_i(\hat{z}(t)) \bar{x}_s^T(t) \begin{bmatrix} \bar{H}_{i1} & * & * \\ \bar{M}_i^T \bar{P} \bar{G}_i & \bar{H}_{i2} & * \\ \bar{E}_i^T \bar{P} \bar{G}_i & \bar{E}_i^T \bar{P} \bar{M}_i & \bar{H}_{i3} \end{bmatrix} \bar{x}_s(t) + \rho^2 w_s^T(t) w_s(t). \quad (\text{A.3})$$

From this, we obtain

$$\begin{aligned}
& \sum_{t=0}^{t_f} e^T(t) \bar{Q} e(t) \\
&= \sum_{t=0}^{t_f} \left\{ e^T(t) \bar{Q} e(t) + e^T(t+1) \bar{P} e(t+1) - e^T(t) \bar{P} e(t) + e^T(t) \bar{S} e(t) \right. \\
&\quad \left. - e^T(t - \bar{\tau}) \bar{S} e(t - \bar{\tau}) \right\} - x_e^T(t_f + 1) \bar{P} x_e(t_f + 1) + e^T(0) \bar{P} x_e(0) \\
&\quad - \sum_{q=1}^{\bar{\tau}} e^T(t_f + 1 - q) \bar{S} e(t_f + 1 - q) + \sum_{q=1}^{\bar{\tau}} e^T(-q) \bar{S} e(-q) + e^T(0) \bar{P} e(0) \\
&\quad - e^T(t_f + 1) \bar{P} e(t_f + 1) - \sum_{q=1}^{\bar{\tau}} e^T(t_f + 1 - q) \bar{S} e(t_f + 1 - q) + \sum_{q=1}^{\bar{\tau}} e^T(-q) \bar{S} e(-q) \\
&\leq e^T(0) \bar{P} e(0) + \sum_{q=1}^{\bar{\tau}} e^T(-q) \bar{S} e(-q) + \sum_{t=0}^{t_f} [e^T(t) \bar{Q} e(t) + \Delta V(t)] \\
&\leq e^T(0) \bar{P} e(0) + \sum_{q=1}^{\bar{\tau}} e^T(-q) \bar{S} e(-q) + \rho^2 \sum_{t=0}^{t_f} w_s^T(t) w_s(t).
\end{aligned} \tag{A.4}$$

According to (38), we have (14). Therefore, the H_∞ tracking control performance in (3) is achieved with a given ρ^2 .

B. Proof of Theorem 2

Consider a Lyapunov–Krasovskii function candidate as

$$V = e^T(t) \bar{P} e(t) + \int_{t-\bar{\tau}}^t e^T(\lambda) \bar{S} e(\lambda) d\lambda. \tag{B.1}$$

The control objective is required to satisfy

$$\int_0^{t_f} e^T(t) \bar{R} e(t) dt \leq \rho^2 \int_0^{t_f} \|w_s(t)\|^2 dt, \tag{B.2}$$

where t_f is terminal time of control, ρ is a given value that denotes the effect of $w_s(t)$ on $e(t)$, and $\bar{R}$ is a positive definite matrix. Taking the derivative of Lyapunov function (B.1) and applying (B.2) along with (10), it is concluded that

$$\begin{aligned}
J &= e^T(t) \bar{R} e(t) - \rho^2 w_s^T(t) w_s(t) + \dot{V} \\
&= e^T(t) \bar{R} e(t) - \rho^2 w_s^T(t) w_s(t) + 2e^T(t) \bar{P} h_1(t) \\
&\quad + \sum_{i=1}^r \mu_i(\hat{z}(t)) e^T(t) \left\{ \left(\bar{G}_i^T \bar{P} + \bar{P} \bar{G}_i \right) e^T(t) + 2e^T(t) \bar{P} \bar{M}_i e^T(t - \bar{\tau}) + 2e^T(t) \bar{P} \bar{E}_i w_s(t) \right\} \\
&\leq \sum_{i=1}^r \mu_i(\hat{z}(t)) e^T(t) \left[\bar{G}_i^* + \bar{P} \bar{M}_i^T \bar{S}^{-1} \bar{M}_i \bar{P} + \rho^2 \bar{P} \bar{E}_i \bar{E}_i^T \bar{P} \right] e(t),
\end{aligned} \tag{B.3}$$

where $\bar{G}_i^* = \bar{G}_i^T \bar{P} + \bar{P} \bar{G}_i + \bar{R} + \bar{S} + \bar{P} \bar{P} + U_1^T U_1$ since $e^T(t) \bar{P} \bar{M}_i e(t - \bar{\tau}) \leq e^T(t) \bar{P} \bar{M}_i^{-1} \bar{S} \bar{M}_i^T \bar{P} e(t) + e^T(t - \bar{\tau}) \bar{P} e(t - \bar{\tau})$ and $2e^T(t) \bar{P} \bar{E}_i w_s(t) \leq e^T(t) \bar{P} \bar{P} e(t) + e^T(t) U_1^T U_1 e(t)$.

C. Proof of Theorem 3

Proof. Consider a Lyapunov–Krasovskii function candidate as

$$V(t) = x_e^T(t)Px_e(t) + \sum_{q=1}^{\bar{\tau}} x_e^T(t-q)Sx_e(t-q). \quad (C.1)$$

From (25), we have

$$\begin{aligned} \Delta V(t) &= x_e^T(t+1)Px_e(t+1) - x_e^T(t)Px_e(t) + x_e^T(t)Sx_e(t) - x_e^T(t-\bar{\tau})Sx_e(t-\bar{\tau}) \\ &= \left[\sum_{i=1}^r \mu_i(\hat{z}(t)) [G_i x_e(t) + M_i x_e(t-\bar{\tau}) + E_i w_e(t)] + \bar{h}(t) \right]^T P \\ &\quad \times \left[\sum_{j=1}^r \mu_j(\hat{z}(t)) G_j x_e(t) + M_j x_e(t-\bar{\tau}) + E_j w_e(t) + \bar{h}(t) \right] \\ &\quad - x_e^T(t)Px_e(t) + x_e^T(t)Sx_e(t) - x_e^T(t-\bar{\tau})Sx_e(t-\bar{\tau}) \\ &\leq \sum_{i=1}^r \mu_i(\hat{z}(t)) \bar{x}_e^T(t) \begin{bmatrix} G_i^T PG_i + S - P & (*) & (*) \\ M_i^T PG_i & M_i^T PM_i - S & (*) \\ E_i^T PG_i & E_i^T PM_i & E_i^T PE_i - \rho^2 I \end{bmatrix} \bar{x}_e(t) \\ &\quad + \sum_{i=1}^r \mu_i(\hat{z}(t)) 2\bar{h}^T(t)P[G_i x_e(t) + M_i x_e(t-\bar{\tau}) + E_i w_e(t)] \\ &\quad + \bar{h}^T(t)P\bar{h} + \rho^2 w_e^T(t)w_e(t), \end{aligned} \quad (C.2)$$

where $\bar{x}_e(t) = [x_e^T(t) x_{e,T}^T(t-\bar{\tau}) w_e^T(t)]^T$. According to Lemma 1 and the fact $\bar{h}^T(t)P\bar{h}(t) \leq x_e^T(t)\bar{U}^T P \bar{U} x_e(t)$, we have

From this, we obtain

$$\begin{aligned} \Delta V(t) &\leq \sum_{i=1}^r \mu_i(\hat{z}(t)) \bar{x}_e^T(t) \begin{bmatrix} H_{i1} & * & * \\ M_i^T PG_i & H_{i2} & * \\ E_i^T PG_i & E_i^T PM_i & H_{i3} \end{bmatrix} \bar{x}_e(t) \\ &\quad + \rho^2 w_e^T(t)w_e(t). \end{aligned} \quad (C.3)$$

$$\begin{aligned} &\sum_{t=0}^{tf} x_e^T(t)Qx_e(t) \\ &= \sum_{t=0}^{tf} \{x_e^T(t)Qx_e(t) + x_e^T(t+1)Px_e(t+1) - x_e^T(t)Px_e(t) + x_e^T(t)Sx_e(t) \\ &\quad - x_e^T(t-\bar{\tau})Sx_e(t-\bar{\tau})\} - x_e^T(t_f+1)Px_e(t_f+1) + x_e^T(0)Px_e(0) \\ &\quad - \sum_{q=1}^{\bar{\tau}} x_e^T(t_f+1-q)Sx_e(t_f+1-q) + \sum_{q=1}^{\bar{\tau}} x_e^T(-q)Sx_e(-q) + x_e^T(0)Px_e(0) \\ &\quad - x_e^T(t_f+1)Px_e(t_f+1) - \sum_{q=1}^{\bar{\tau}} x_e^T(t_f+1-q)Sx_e(t_f+1-q) + \sum_{q=1}^{\bar{\tau}} x_e^T(-q)Sx_e(-q) \\ &\leq x_e^T(0)Px_e(0) + \sum_{q=1}^{\bar{\tau}} x_e^T(-q)Sx_e(-q) + \sum_{t=0}^{tf} [x_e^T(t)Qx_e(t) + \Delta V(t)] \\ &\leq x_e^T(0)Px_e(0) + \sum_{q=1}^{\bar{\tau}} x_e^T(-q)Sx_e(-q) + \rho^2 \sum_{t=0}^{tf} w_e^T(t)w_e(t). \end{aligned} \quad (C.4)$$

According to (38), we have (5). Therefore, the H_∞ tracking control performance in (5) is achieved with a given ρ^2 . $\square$

D. Proof of Theorem 4

Proof. Consider a Lyapunov–Krasovskii function candidate as

$$V = x_e^T(t)Px_e(t) + \int_{t-\bar{\tau}}^t x_e^T(\lambda)Sx_e(\lambda)d\lambda. \quad (D.1)$$

$$\begin{aligned} J &= x_e^T(t)Rx_e(t) - \rho^2 w_e^T(t)w_e(t) + \dot{V} \\ &= x_e^T(t)Rx_e(t) - \rho^2 w_e^T(t)w_e(t) + 2x_e^T(t)P\bar{h}(t) \\ &\quad + \sum_{i=1}^r \mu_i(\hat{z}(t))x_e^T(t)\{[G_i^T P + PG_i]x_e^T(t) + 2x_e^T(t)PM_i x_e(t - \bar{\tau}) + 2x_e^T(t)PE_i w_e(t)\} \\ &\leq \sum_{i=1}^r \mu_i(\hat{z}(t))\bar{x}_e^T(t)[\bar{G}_i + PM_i^T S^{-1}M_i P + \rho^2 PE_i E_i^T P]\bar{x}_e(t), \end{aligned} \quad (D.3)$$

where $\bar{G}_i = G_i^T P + PG_i + R + S + PP + \bar{U}^T \bar{U}$ since $x_e^T(t)PM_i x_e(t - \bar{\tau}) \leq x_e^T(t)PM_i^{-1}SM_i^T Px_e(t) + x_e^T(t - \bar{\tau})Sx_e(t - \bar{\tau})$ and $2x_e^T(t)PE_i w_e(t) \leq x_e^T(t)PPx_e(t) + x_e^T(t)\bar{U}^T \bar{U}x_e(t)$.

The control objective is required to satisfy

$$\int_0^{t_f} \|x_e^T(t)Rx_e(t)dt\|^2 \leq \rho^2 \int_0^{t_f} \|w_e(t)\|^2 dt, \quad (D.2)$$

where t_f is terminal time of control, ρ is a given value that denotes the effect of $w_e(t)$ on $x_e(t)$, and R is a positive definite matrix. Taking the derivative of Lyapunov function (D.1) and applying (D.2) along with (25), it is concluded that

If the condition of (48) holds, then $J < 0$. By integrating (D.3) from 0 to t_f , we obtain

$$\int_0^{t_f} x_e^T(t)Rx_e(t)dt \leq x_e^T(0)Px_e(0) + \int_{-\bar{\tau}}^0 x_e^T(v)Sx_e(v)dv + \rho^2 \int_0^{t_f} \|w_e(t)\|^2 dt. \quad (D.4)$$

This means that the overall system has H_∞ performance. $\square$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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