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Research Article

Global Optimal Solutions for Proximal Fuzzy Contractions Involving Control Functions

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In this study, we introduce new concepts of $\alpha - \mathcal{F}\mathcal{L}$-contraction and $\alpha - \psi - \mathcal{F}\mathcal{L}$-contraction and we discuss existence results of the best proximity points of such types of non-self-mappings involving control functions in the structure of complete fuzzy metric spaces. Our results extend, generalize, enrich, and improve diverse existing results in the current literature.

1. Introduction

Recent advancements in fixed point theory are one of the central and active research areas of nonlinear functional analysis, which provides a variety of mathematical methods, principles, and techniques for solving a variety of problems arising from various branches of mathematics as well as various fields in science and engineering. The Banach fixed point theorem is considered as one of the most fruitful results in this theory. Due to its vast and significant applicability in pure and applied mathematics, this principle has been generalized and developed in various approaches (see, e.g., [1–22]). In particular, Khojasteh et al. [23] presented an impressive technique to the investigation of fixed point theory by developing the notion of simulation functions, which exhibit a significant unifying power. The idea of simulation functions has been generalized, improved, and extended in different metric spaces (see, e.g., [11, 14, 24, 25]).

The best proximity theory is another expanding and prominent aspect of fixed point theory which plays a fundamental role in the investigation of requirements that guarantee the existence of an optimal approximate fixed point when the functional equation $\mathcal{L}x = x$ has no solution. Indeed, a non-self-mapping $\mathcal{L}: \mathcal{U} \longrightarrow \mathcal{V}$ does not possess necessarily a fixed point, with $\mathcal{U}$ and $\mathcal{V}$ are two nonempty subsets of a classical metric space (Λ, d). Best proximity theory is a remarkable generalization of fixed point theorems. In fact, the best proximity point turned out to be a fixed point in a natural way if the mapping in question is a self-mapping. For more recent developments in best proximity theory and related techniques, refer to [9–11, 19, 26–32].

In the present study, following this line of research interest, we present a simulation function approach to best proximity point problems in fuzzy metric spaces. We initiate new concepts of $\alpha - \psi - \mathcal{F}\mathcal{L}$-contraction, $\alpha - \mathcal{F}\mathcal{L}$-contraction, and generalized $\alpha - \mathcal{F}\mathcal{L}$-contraction, and we discuss existence results of best proximity point of such classes of non-self-mappings involving control functions in the structure of complete fuzzy metric spaces. The furnished results enrich, generalize, and extend various existing findings in the literature.

2. Preliminaries

Throughout this study, $\mathbb{N}$ and $\mathbb{R}$ will represent natural and real numbers, respectively. First, we start with some notions and main properties of fuzzy metric spaces.

Definition 1 (see [33]). A binary operation $*: [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm if it fulfills the following conditions:

(CT1) $*$ is continuous

(CT2) $*$ is commutative and associative

(CT3) $\wp * 1 = \wp$ for all $\wp \in [0,1]$

(CT4) $\wp * \ell \leq \mathcal{J} * \imath$ whenever $\wp \leq \mathcal{J}$ and $\ell \leq \imath$, for all $\wp, \ell, \mathcal{J}, \imath \in [0,1]$

Example 1. Three standard instances are as follows:

(a) $\wp * \ell = \wp.\ell$

(b) $\wp * \ell = \min(\wp, \ell)$

(c) $\wp * \ell = \max[0, \wp + \ell - 1]$

Definition 2 (see George and Veeramani [34]). Let Λ be an arbitrary set, $*$ is a continuous t-norm, and $\mathcal{D}$ is a fuzzy set on $\Lambda \times \Lambda \times (0,\infty)$. The ordered triple $(\Lambda, \mathcal{D}, *)$ is said to be a fuzzy metric space if

$(\mathcal{MS}_1)$ $\mathcal{D}(\vartheta, \theta, \varsigma) > 0$

$(\mathcal{MS}_2)$ $\mathcal{D}(\vartheta, \theta, \varsigma) = 1$ if and only if $\vartheta = \theta$

$(\mathcal{MS}_3)$ $\mathcal{D}(\vartheta, \theta, \varsigma) = \mathcal{D}(\theta, \vartheta, \varsigma)$

$(\mathcal{MS}_4)$ $\mathcal{D}(\vartheta, \theta, \varsigma) * \mathcal{D}(\theta, \omega, \sigma) \leq \mathcal{D}(\vartheta, \omega, \varsigma + \sigma)$

$(\mathcal{MS}_5)$ $\mathcal{D}(\vartheta, \theta, .): (0,\infty) \longrightarrow (0,1]$ is continuous

for all $\vartheta, \theta, \omega \in \Lambda$ and $\varsigma, \sigma > 0$.

For $\varsigma > 0$, the open ball with centre $\vartheta \in \Lambda$ and radius ρ, where $0 < \rho < 1$, is defined by

$$\mathcal{B}(\vartheta, \rho, \varsigma) = \{\theta \in \Lambda: \mathcal{D}(\vartheta, \theta, \varsigma) > 1 - \rho\}. \tag{1}$$

A subset O of a fuzzy metric space $(\Lambda, \mathcal{D}, *)$ is said to be open if given any point $\theta \in O$, there exists $0 < \rho < 1$ and $\varsigma > 0$ such that $\mathcal{B}(\theta, \rho, \varsigma) \subseteq O$. Let τ denote the collection of all open subsets of Λ; hence, τ is a topology on Λ. This topology is Hausdorff and first countable. For further topological results, refer to [2, 34].

Example 2 (see [34]). Let (Λ, d) be a metric space and $*$ be the product t-norm, and define the function $\mathcal{D}: \Lambda^2 \times (0,\infty) \longrightarrow [0,1]$ by

$$\mathcal{D}(\vartheta, \theta, \varsigma) = e^{(-d(\vartheta,\theta)/\varsigma)}, \tag{2}$$

for all $\vartheta, \theta \in \Lambda, \varsigma > 0$. Then, $(\Lambda, \mathcal{D}, *)$ is a fuzzy metric space on Λ.

Lemma 1 (see [1]). *$\mathcal{D}(\vartheta, \theta, .)$ is nondecreasing for all ϑ, θ in Λ.*

Definition 3 (see [34]). Let $(\Lambda, \mathcal{D}, *)$ be a fuzzy metric space.

(1) A sequence $\{\vartheta_n\} \subseteq \Lambda$ is said to be convergent to $\vartheta \in \Lambda$ if an only if $\lim_{n\longrightarrow\infty} \mathcal{D}(\vartheta_n, \vartheta, \varsigma) = 1$ for all $\varsigma > 0$

(2) A sequence $\{\vartheta_n\} \subseteq \Lambda$ is said to be a Cauchy sequence iff for each $\varepsilon \in (0,1)$ and $\varsigma > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{D}(\vartheta_n, \vartheta_m, \varsigma) > 1 - \varepsilon$ for all $n, m \geq n_0$

(3) A fuzzy metric space is called complete if every Cauchy sequence in Λ has a limit in Λ

In [2], Gregori and Sapena initiated the notion of a fuzzy contractive mapping as follows.

Definition 4 (see [2]). Let $(\Lambda, \mathcal{D}, *)$ be a fuzzy metric space. A mapping $\mathcal{L}: \Lambda \longrightarrow \Lambda$ is called a fuzzy contractive mapping if there exists $a \in (0,1)$ such that

$$\frac{1}{\mathcal{D}(\mathcal{L}\vartheta, \mathcal{L}\theta, \varsigma)} - 1 \leq a\left(\frac{1}{\mathcal{D}(\vartheta, \theta, \varsigma)} - 1\right), \tag{3}$$

for each $\vartheta, \theta \in \Lambda$ and $\varsigma > 0$.

Definition 5 (see [4]). Let Ψ be the class of nondecreasing functions $\psi: (0,1] \longrightarrow (0,1]$ fulfilling the following two conditions:

(ψ_1) ψ is continuous

(ψ_2) $\psi(c) > c$ for all $c \in (0,1)$

A self-mapping $\mathcal{L}: \Lambda \longrightarrow \Lambda$ on a fuzzy metric space $(\Lambda, \mathcal{D}, *)$ is called a fuzzy ψ-contractive mapping if

$$\mathcal{D}(\mathcal{L}\vartheta, \mathcal{L}\theta, \varsigma) \geq \psi(\mathcal{D}(\vartheta, \theta, \varsigma)), \quad \text{for all } \vartheta, \theta \in \Lambda, \varsigma > 0. \tag{4}$$

Afterwards, Wardowski [5] proposed the idea of a fuzzy $\mathcal{H}$-contractive mapping as follows.

Definition 6 (see [5]). Let $\mathcal{H}$ be the set of functions $\eta: (0,1] \longrightarrow (0,\infty]$ satisfying the two conditions $(\mathcal{W}_1)$ and $(\mathcal{W}_2)$ given by

$(\mathcal{W}_1)$ η transforms $(0,1]$ onto $[0,\infty)$

$(\mathcal{W}_2)$ η is strictly decreasing

A self-mapping $\mathcal{L}: \Lambda \longrightarrow \Lambda$ on a fuzzy metric space $(\Lambda, \mathcal{D}, *)$ is called a fuzzy $\mathcal{H}$-contractive with respect to the function $\eta \in \mathcal{H}$ if there exists $a \in (0,1)$ such that the following inequality holds:

$$\eta(\mathcal{D}(\mathcal{L}\vartheta, \mathcal{L}\theta, \varsigma)) \leq a\eta(\mathcal{D}(\vartheta, \theta, \varsigma)), \quad \text{for all } \vartheta, \theta \in \Lambda, \varsigma > 0. \tag{5}$$

The following class of control functions has been introduced in [8], where we used the term class $\mathcal{FZ}$ instead of the present $\mathcal{FZ}$-simulation functions.

Definition 7 (see [8]). The function $\xi: (0,1] \times (0,1] \longrightarrow \mathbb{R}$ is said to be a $\mathcal{FZ}$-simulation function if the following properties hold:

(ξ_1) $\xi(1,1) = 0$

(ξ_2) $\xi(\mu, \nu) < (1/\nu) - (1/\mu)$ for each $\mu, \nu \in (0,1)$

(ξ_3) if $\{\mu_n\}, \{\nu_n\}$ are sequences in $(0,1]$ such that $\lim_{n\longrightarrow\infty}\mu_n = \lim_{n\longrightarrow\infty}\nu_n < 1$, then $\lim_{n\longrightarrow\infty}\sup\xi(\mu_n, \nu_n) < 0$

By $\mathcal{FZ}$, we denote the collection of all $\mathcal{FZ}$-simulation functions.

Definition 8 (see [8]). Let $(\Lambda, \mathcal{D}, *)$ be a fuzzy metric space, $\mathcal{L}: \Lambda \longrightarrow \Lambda$ a mapping, and $\xi \in \mathcal{FZ}$. Then, $\mathcal{L}$ is said to be a $\mathcal{FZ}$-contraction with respect to ξ if the following condition is satisfied:

$$\xi(\mathcal{D}(\mathcal{L}\vartheta, \mathcal{L}\theta, \varsigma), \mathcal{D}(\vartheta, \theta, \varsigma)) \geq 0, \quad \text{for all } \vartheta, \theta \in \Lambda, \varsigma > 0. \tag{6}$$

Example 3 (see [8]). The type of fuzzy contractive mappings developed by Gregori and Sapena [2] is a perfect example of $\mathcal{FZ}$-contraction. It can be expressed facilely from the previous definition by taking the $\mathcal{FZ}$-simulation function as

$$\xi(\mu, \nu) = a\left(\frac{1}{\nu} - 1\right) - \frac{1}{\mu} + 1, \quad \text{for all } \mu, \nu \in (0, 1], \tag{7}$$

where $a \in (0, 1)$.

Example 4 (see [8]). The corresponding $\mathcal{FZ}$-simulation function for the fuzzy ψ-contractive mapping is defined by

$$\xi(\mu, \nu) = \frac{1}{\psi(\nu)} - \frac{1}{\mu}, \quad \text{for all } \mu, \nu \in (0, 1] \text{ with } \psi \in \Psi. \tag{8}$$

Definition 9 (see [6]). Let $(\Lambda, \mathcal{D}, *)$ be a fuzzy metric space. We say that a mapping $\mathcal{L}: \Lambda \longrightarrow \Lambda$ is α-admissible if there exists a function $\alpha: \Lambda \times \Lambda \times (0, +\infty) \longrightarrow [0, +\infty)$ such that for all $\vartheta, \theta \in \Lambda$, $\varsigma > 0$,

$$\alpha(\vartheta, \theta, \varsigma) \geq 1 \Longrightarrow \alpha(\mathcal{L}\vartheta, \mathcal{L}\theta, \varsigma) \geq 1. \tag{9}$$

In line with [15] (see also [16]), we use the notion of triangular weak-α-admissible function in the form that is as follows.

Definition 10. Let $\mathcal{L}: \Lambda \longrightarrow \Lambda$ be a mapping and $\alpha: \Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$ be a function. We say that $\mathcal{L}$ is a triangular weak-α-admissible if

$$\begin{aligned} &\alpha(\vartheta, \theta, \varsigma) \geq 1, \\ &\alpha(\theta, \omega, \varsigma) \geq 1 \Longrightarrow \alpha(\vartheta, \omega, \varsigma) \geq 1, \end{aligned} \tag{10}$$

for all $\vartheta, \theta, \omega \in \Lambda$, $\varsigma > 0$.

Definition 11 (see [19]). Let $\mathcal{U}$ and $\mathcal{V}$ be nonempty subsets of a fuzzy metric space $(\Lambda, \mathcal{D}, *)$. Define $\mathcal{U}_0(\varsigma)$ and $\mathcal{V}_0(\varsigma)$ by the following sets:

$$\begin{aligned} \mathcal{U}_0(\varsigma) &= \{u \in \mathcal{U}: \mathcal{D}(u, v, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma), \text{for some } v \in \mathcal{V}\}, \\ \mathcal{V}_0(\varsigma) &= \{v \in \mathcal{V}: \mathcal{D}(u, v, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma), \text{for some } u \in \mathcal{U}\}, \end{aligned} \tag{11}$$

where

$$\mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) = \sup\{\mathcal{D}(u, v, \varsigma): u \in \mathcal{U}, v \in \mathcal{V}\}. \tag{12}$$

Note that, a point $\omega \in \mathcal{U}$ is said to be a fuzzy best proximity point of the mapping $\mathcal{L}$, where $\mathcal{L}: \mathcal{U} \longrightarrow \mathcal{V}$, $\mathcal{U}$ and $\mathcal{V}$ are nonempty subsets of an abstract nonempty set Λ if $\mathcal{D}(\omega, \mathcal{L}\omega, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma)$ for all $\varsigma > 0$.

3. Main Results

Firstly, we define the following concepts.

Definition 12. Let $\mathcal{U}$ and $\mathcal{V}$ be two nonempty subsets of fuzzy metric space $(\Lambda, \mathcal{D}, *)$ and $\alpha: \Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$. We say that $\mathcal{L}: \mathcal{U} \longrightarrow \mathcal{V}$ is an α-proximal admissible if

$$\begin{cases} \alpha(\vartheta, \theta, \varsigma) \geq 1, \\ \mathcal{D}(u, \mathcal{L}\vartheta, \varsigma) = \mathcal{D}(v, \mathcal{L}\theta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) \end{cases} \Longrightarrow \alpha(u, v, \varsigma) \geq 1, \tag{13}$$

for all $u, v, \vartheta, \theta \in \Lambda$ and $\varsigma > 0$.

Remark 1. Note that if $\mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) = 1$, then Definition 12 reduces to Definition 9 of α-admissibility.

Definition 13. Let $\mathcal{U}$ and $\mathcal{V}$ be nonempty subsets of fuzzy metric space $(\Lambda, \mathcal{D}, *)$ and $\alpha: \Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$. We say that $\mathcal{L}: \mathcal{U} \longrightarrow \mathcal{V}$ is an $\alpha - \mathcal{FZ}$-contraction with respect to $\xi \in \mathcal{FZ}$ if $\mathcal{L}$ is an α-proximal admissible such that

$$\begin{cases} \alpha(\vartheta, \theta, \varsigma) \geq 1, \\ \mathcal{D}(u, \mathcal{L}\vartheta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma), \Longrightarrow \xi(\mathcal{D}(u, v, \varsigma), \mathcal{D}(\vartheta, \theta, \varsigma)) \geq 0, \\ \mathcal{D}(v, \mathcal{L}\theta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) \end{cases} \tag{14}$$

for all $u, v, \vartheta, \theta \in \mathcal{U}$ and $\varsigma > 0$.

Definition 14. Let $\mathcal{U}$ and $\mathcal{V}$ be nonempty subsets of fuzzy metric space $(\Lambda, \mathcal{D}, *)$, $\alpha: \Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$, and $\psi \in \Psi$. We say that $\mathcal{L}: \mathcal{U} \longrightarrow \mathcal{V}$ is an $\alpha - \psi - \mathcal{FZ}$-contraction with respect to $\xi \in \mathcal{FZ}$ if $\mathcal{L}$ is an α-proximal admissible such that

$$\begin{cases} \alpha(\vartheta, \theta, \varsigma) \geq 1, \\ \mathcal{D}(u, \mathcal{L}\vartheta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma), \Longrightarrow \xi(\mathcal{D}(u, v, \varsigma), \psi(\mathcal{D}(\vartheta, \theta, \varsigma))) \geq 0, \\ \mathcal{D}(v, \mathcal{L}\theta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) \end{cases} \tag{15}$$

for all $u, v, \vartheta, \theta \in \mathcal{U}$ and $\varsigma > 0$.

Remark 2. Note that Definition 14 cannot be reduced to Definition 13 since $\psi(t) = t$ does not belong to Ψ.

Definition 15. Let $\mathcal{U}$ and $\mathcal{V}$ be nonempty subsets of fuzzy metric space $(\Lambda, \mathcal{D}, *)$ and $\alpha: \Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$. We say that $\mathcal{L}: \mathcal{U} \longrightarrow \mathcal{V}$ is a generalized $\alpha - \mathcal{FZ}$-contraction with respect to $\xi \in \mathcal{FZ}$ if $\mathcal{L}$ is an α-proximal admissible such that

$$\begin{cases} \alpha(\vartheta,\theta,\varsigma)\geq 1, \\ \mathscr{D}(u,\mathscr{L}\vartheta,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma), \Longrightarrow \xi(\mathscr{D}(u,v,\varsigma),\mathscr{R}(\vartheta,\theta,\varsigma))\geq 0, \\ \mathscr{D}(v,\mathscr{L}\theta,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma) \end{cases} \tag{16}$$

for all $u, v, \vartheta, \theta \in \mathscr{U}$ and $\varsigma > 0$, where

$$\mathscr{R}(\vartheta,\theta,\varsigma)=\min\left\{\mathscr{D}(\vartheta,\theta,\varsigma),\frac{\mathscr{D}(\vartheta,u,\varsigma)\mathscr{D}(\theta,v,\varsigma)}{\mathscr{D}(\vartheta,\theta,\varsigma)}\right\}. \tag{17}$$

Next, we give our first main result.

Theorem 1. *Let $\mathscr{U}$ and $\mathscr{V}$ be nonempty subsets of a complete fuzzy metric space $(\Lambda,\mathscr{D},*)$, α: $\Lambda\times\Lambda\times(0,\infty)\longrightarrow[0,\infty)$, $\psi\in\Psi$, and $\xi\in\mathscr{FZ}$ is nonincreasing in its second argument. Assume that $\mathscr{L}$: $\mathscr{U}\longrightarrow\mathscr{V}$ is an $\alpha-\psi-\mathscr{FZ}$-contraction with respect to ξ and*

(i) $\mathscr{L}$ is triangular weak-α-admissible

(ii) $\mathscr{U}$ is closed

(iii) $\mathscr{L}(\mathscr{U}_0)\subseteq\mathscr{V}_0$

(iv) There exists $\vartheta_0,\vartheta_1\in\mathscr{U}$ such that $\mathscr{D}(\vartheta_0,\mathscr{L}\vartheta_0,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)$ and $\alpha(\vartheta_0,\vartheta_1,\varsigma)\geq 1$ for all $\varsigma>0$

(v) $\mathscr{L}$ is continuous.

Then, there exists $\omega\in\mathscr{U}$ such that $\mathscr{D}(\omega,\mathscr{L}\omega,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)$ for all $\varsigma>0$; that is, $\mathscr{L}$ has a best proximity point $\omega\in\mathscr{U}$.

Proof. Due to condition (iv), there exists $\vartheta_0,\vartheta_1\in\mathscr{U}$ such that $\alpha(\vartheta_0,\vartheta_1,\varsigma)\geq 1$ and

$$\mathscr{D}(\vartheta_1,\mathscr{L}\vartheta_0,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma). \tag{18}$$

Regarding (iii), we deduce that $\mathscr{L}\vartheta_1\in\mathscr{V}_0$; hence, there exists $\vartheta_2\in\mathscr{U}$ such that

$$\mathscr{D}(\vartheta_2,\mathscr{L}\vartheta_1,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma). \tag{19}$$

Since $\alpha(\vartheta_0,\vartheta_1,\varsigma)\geq 1$ and $\mathscr{L}$ is an α-proximal admissible, consequently, $\alpha(\vartheta_1,\vartheta_2,\varsigma)\geq 1$. Recursively, a sequence $\{\vartheta_n\}\subset\mathscr{U}_0$ can be defined as follows:

$$\alpha(\vartheta_n,\vartheta_{n+1},\varsigma)\geq 1, \quad \text{for all } n\in\mathbb{N}, \tag{20}$$

$$\mathscr{D}(\vartheta_{n+1},\mathscr{L}\vartheta_n,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma), \quad \text{for all } n\in\mathbb{N}. \tag{21}$$

If there exists $n_0\in\mathbb{N}$ such that $\vartheta_{n_0+1}=\vartheta_{n_0}$, we obtain

$$\mathscr{D}\left(\vartheta_{n_0},\mathscr{L}\vartheta_{n_0},\varsigma\right)=\mathscr{D}\left(\vartheta_{n_0+1},\mathscr{L}\vartheta_{n_0},\varsigma\right)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma), \tag{22}$$

which means that ϑ_{n_0} is a best proximity point of $\mathscr{L}$. Thus, to continue our proof, we suppose that $\vartheta_n\neq\vartheta_{n+1}$ for all $n\in\mathbb{N}$. Making use of (20) and (21), we obtain

$$\begin{aligned}\mathscr{D}(\vartheta_n,\mathscr{L}\vartheta_{n-1},\varsigma)&=\mathscr{D}(\vartheta_{n+1},\mathscr{L}\vartheta_n,\varsigma)\\&=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma), \quad \text{for all } n\in\mathbb{N}.\end{aligned} \tag{23}$$

Regarding that $\mathscr{L}$ is an $\alpha-\psi-\mathscr{FZ}$-contraction with respect to $\xi\in\mathscr{FZ}$, together with (20), (21), and (ξ_2), we obtain

$$\begin{aligned}0&\leq\xi(\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma),\psi(\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma)))\\&<\frac{1}{\psi(\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma))}-\frac{1}{\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma)}.\end{aligned} \tag{24}$$

Consequently, we have

$$\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma)<\psi(\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma))<\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma), \tag{25}$$

which means that $\{\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma)\}$ is a nondecreasing sequence of positive real numbers in $(0,1]$. Then, there exists $c(\varsigma)\leq 1$ such that $\lim_{n\longrightarrow\infty}\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma)=c(\varsigma)\geq 1$ for all $\varsigma>0$. We shall prove that $c(\varsigma)=1$. Reasoning by contradiction, suppose that $c(\varsigma_0)<1$ for some $\varsigma_0>0$. Now, if we take the sequences $\{\tau_n=\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma_0)\}$ and $\{\delta_n=\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma_0)\}$ and considering (ψ_2) and (ξ_3) and that ξ is nonincreasing with respect to its second argument, we obtain

$$\begin{aligned}0&\leq\lim_{n\longrightarrow\infty}\sup\xi(\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma_0),\psi(\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma_0)))\\&\leq\lim_{n\longrightarrow\infty}\sup\xi(\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma_0),\mathscr{D}(\vartheta_{n-1},\vartheta_n,\varsigma_0))\\&<0,\end{aligned} \tag{26}$$

which is a contradiction and yields

$$\lim_{n\longrightarrow\infty}\mathscr{D}(\vartheta_n,\vartheta_{n+1},\varsigma)=1, \quad \text{for all } \varsigma>0. \tag{27}$$

Next, we show that the sequence $\{\vartheta_n\}$ is Cauchy. Reasoning by contradiction, suppose that $\{\vartheta_n\}$ is not a Cauchy sequence. Thus, there exists $\varepsilon\in(0,1)$, $\varsigma_0>0$, and two subsequences $\{\vartheta_{n_k}\}$ and $\{\vartheta_{m_k}\}$ of $\{\vartheta_n\}$ with $n_k>m_k\geq k$ for all $k\in\mathbb{N}$ such that

$$\mathscr{D}\left(\vartheta_{m_k},\vartheta_{n_k},\varsigma_0\right)\leq 1-\varepsilon. \tag{28}$$

Taking into account Lemma 1, we derive

$$\mathscr{D}\left(\vartheta_{m_k},\vartheta_{n_k},\frac{\varsigma_0}{2}\right)\leq 1-\varepsilon. \tag{29}$$

By choosing m_k as the smallest index satisfying (29), we have

$$\mathscr{D}\left(\vartheta_{m_k},\vartheta_{n_k-1},\frac{\varsigma_0}{2}\right)>1-\varepsilon. \tag{30}$$

On account of (28), (30), and $(\mathscr{MS}_4)$, we have

$$\begin{aligned}1-\varepsilon&\geq\mathscr{D}\left(\vartheta_{m_k},\vartheta_{n_k},\varsigma_0\right)\\&\geq\mathscr{D}\left(\vartheta_{m_k},\vartheta_{n_k-1},\frac{\varsigma_0}{2}\right)*\mathscr{D}\left(\vartheta_{n_k-1},\vartheta_{n_k},\frac{\varsigma_0}{2}\right)\\&>(1-\varepsilon)*\mathscr{D}\left(\vartheta_{n_k-1},\vartheta_{n_k},\frac{\varsigma_0}{2}\right).\end{aligned} \tag{31}$$

Taking limit as $k\longrightarrow\infty$ and employing (27), we derive

$$\lim_{n\longrightarrow\infty}\mathscr{D}\left(\vartheta_{m_k},\vartheta_{n_k},\varsigma_0\right)=1-\varepsilon. \tag{32}$$

On the other hand, we have

$$\mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right) \geq \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{m_k}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{n_k}, \vartheta_{n_k-1}, \frac{\varsigma_0}{3}\right),$$
$$\mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) \geq \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{m_k-1}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{n_k-1}, \vartheta_{n_k}, \frac{\varsigma_0}{3}\right), \tag{33}$$

which imply that

$$\lim_{n\longrightarrow\infty} \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right) = 1 - \varepsilon. \tag{34}$$

Furthermore, given that $\mathscr{L}$ is triangular weak-α-admissible and taking into account (20), we deduce that

$$\alpha\left(\vartheta_n, \vartheta_m, \varsigma\right) \geq 1, \quad \text{for all } n, m \in \mathbb{N} \text{ with } n > m. \tag{35}$$

So that

$$\alpha\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) \geq 1, \tag{36}$$

$$\begin{aligned}\mathscr{D}\left(\vartheta_{m_k}, \mathscr{L}\vartheta_{m_k-1}, \varsigma_0\right) &= \mathscr{D}\left(\vartheta_{n_k}, \mathscr{L}\vartheta_{n_k-1}, \varsigma_0\right)\\ &= \mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma_0\right), \quad \text{for all } k \in \mathbb{N}.\end{aligned} \tag{37}$$

Regarding the fact that $\mathscr{L}$ is an $\alpha - \psi - \mathscr{F}\mathscr{Z}$-contraction with respect to $\xi \in \mathscr{F}\mathscr{Z}$ and making use of (35) and (36), we have

$$0 \leq \xi\left(\mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right), \psi\left(\mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right)\right)\right), \quad \text{for all } k \in \mathbb{N}. \tag{38}$$

From (32) and (34), we see that the sequences $\left\{\mu_k = \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right)\right\}$ and $\left\{\nu_k = \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right)\right\}$ have the same limit $1 - \varepsilon < 1$, taking into consideration that ξ is nonincreasing with respect to its second argument; by the property ξ_3, we conclude that

$$\begin{aligned}0 &\leq \lim_{n\longrightarrow\infty} \sup \xi\left(\mu_k, \psi\left(\nu_k\right)\right)\\ &\leq \lim_{n\longrightarrow\infty} \sup \xi\left(\mu_k, \nu_k\right)\\ &< 0,\end{aligned} \tag{39}$$

which is a contradiction. So that $\{\vartheta_n\}$ is a Cauchy sequence in $\mathscr{U}$. As $\mathscr{U}$ is closed subset of a complete fuzzy metric space $(\Lambda, \mathscr{D}, *)$, there exists $\omega \in \mathscr{U}$ such that

$$\lim_{n\longrightarrow\infty} \mathscr{D}\left(\vartheta_n, \omega, \varsigma\right) = 1. \tag{40}$$

As $\mathscr{L}$ is continuous, we conclude that $\mathscr{L}\vartheta_n$ converges to $\mathscr{L}\omega$; thus,

$$\lim_{n\longrightarrow\infty} \mathscr{D}\left(\mathscr{L}\vartheta_n, \mathscr{L}\omega, \varsigma\right) = 1. \tag{41}$$

Due to the continuity of $\mathscr{D}$, we have $\mathscr{D}\left(\vartheta_{n+1}, \mathscr{L}\vartheta_n, \varsigma\right) \longrightarrow \mathscr{D}\left(\omega, \mathscr{L}\omega, \varsigma\right)$. From (21), we deduce

$$\mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma\right) = \lim_{n\longrightarrow\infty} \mathscr{D}\left(\vartheta_{n+1}, \mathscr{L}\vartheta_n, \varsigma\right) = \mathscr{D}\left(\omega, \mathscr{L}\omega, \varsigma\right), \tag{42}$$

which means that $\omega \in \mathscr{U}$ is a best proximity point of $\mathscr{L}$.

In the next theorem, we substitute the continuity of $\mathscr{L}$ in Theorem 1 with the following condition.

(**C**): if $\{\vartheta_n\}$ is a sequence in $\mathscr{U}$ such that $\alpha\left(\vartheta_n, \vartheta_{n+1}, \varsigma\right) \geq 1$ for all $n \in \mathbb{N}$, $\varsigma > 0$, and $\vartheta_n \longrightarrow \vartheta \in \mathscr{U}$ as $n \longrightarrow \infty$, then there exists a subsequence $\left\{\vartheta_{n(k)}\right\}$ of $\{\vartheta_n\}$ such that $\alpha\left(\vartheta_{n(k)}, \vartheta, \varsigma\right) \geq 1$ for all $k \in \mathbb{N}$ and $\varsigma > 0$. □

Theorem 2. *Let $\mathscr{U}$ and $\mathscr{V}$ be nonempty subsets of a complete fuzzy metric space $(\Lambda, \mathscr{D}, *)$ and α: $\Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$, $\psi \in \Psi$, and $\xi \in \mathscr{F}\mathscr{Z}$ is nonincreasing in its second argument. Assume that $\mathscr{L}$: $\mathscr{U} \longrightarrow \mathscr{V}$ is an $\alpha - \psi - \mathscr{F}\mathscr{Z}$-contraction with respect to $\xi \in \mathscr{F}\mathscr{Z}$ and*

(i) $\mathscr{L}$ is triangular weak-α-admissible

(ii) $\mathscr{U}$ is closed

(iii) $\mathscr{L}\left(\mathscr{U}_0\right) \subseteq \mathscr{V}_0$

(iv) There exists $\vartheta_0, \vartheta_1 \in \mathscr{U}$ such that $\mathscr{D}\left(\vartheta_1, \mathscr{L}\vartheta_0, \varsigma\right) = \mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma\right)$ and $\alpha\left(\vartheta_0, \vartheta_1, \varsigma\right) \geq 1$ for all $\varsigma > 0$

(v) If $\{\vartheta_n\}$ is a sequence in $\mathscr{U}$ such that $\alpha\left(\vartheta_n, \vartheta_{n+1}, \varsigma\right) \geq 1$ for all $n \in \mathbb{N}$, $\varsigma > 0$, and $\vartheta_n \longrightarrow \vartheta \in \mathscr{U}$ as $n \longrightarrow \infty$, then there exists a subsequence $\left\{\vartheta_{n(k)}\right\}$ of $\{\vartheta_n\}$ such that $\alpha\left(\vartheta_{n(k)}, \vartheta, \varsigma\right) \geq 1$ for all $k \in \mathbb{N}$ and $\varsigma > 0$

Then, there exists $\omega \in \mathscr{U}$ such that $\mathscr{D}\left(\omega, \mathscr{L}\omega, \varsigma\right) = \mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma\right)$ for all $\varsigma > 0$.

Proof. Following the lines of the proof of Theorem 1, we deduce that there exists a Cauchy sequence $\{\vartheta_n\}$ in $\mathscr{U}_0$ which converges to $\omega \in \mathscr{U}_0$. Since $\mathscr{L}\left(\mathscr{U}_0\right) \subseteq \mathscr{V}_0$, we have $\mathscr{L}\omega \in \mathscr{V}_0$, and then

$$\mathscr{D}\left(a_1, \mathscr{L}\omega, \varsigma\right) = \mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma\right), \quad \text{for some } a_1 \in \mathscr{U}_0. \tag{43}$$

By condition (v), there exists a subsequence $\left\{\vartheta_{n(k)}\right\}$ of $\{\vartheta_n\}$ such that

$$\alpha\left(\vartheta_{n_k}, \omega, \varsigma\right) \geq 1, \quad \text{for all } k \in \mathbb{N}, \varsigma > 0. \tag{44}$$

Regarding that $\mathscr{L}$ is an α-proximal admissible and

$$\mathscr{D}\left(a_1, \mathscr{L}\omega, \varsigma\right) = \mathscr{D}\left(\vartheta_{n_k+1}, \mathscr{L}\vartheta_{n_k}, \varsigma\right) = \mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma\right), \tag{45}$$

we obtain that $\alpha\left(\vartheta_{n_k+1}, a_1, \varsigma\right) \geq 1$. Hence,

$$0 \leq \xi\left(\mathscr{D}\left(a_1, \vartheta_{n_k+1}, \varsigma\right), \psi\left(\mathscr{D}\left(\omega, \vartheta_{n_k}, \varsigma\right)\right)\right), \quad \text{for all } k \in \mathbb{N}. \tag{46}$$

Applying the property (ξ_2), it follows that

$$\mathscr{D}\left(\omega, \vartheta_{n_k}, \varsigma\right) < \psi\left(\mathscr{D}\left(\omega, \vartheta_{n_k}, \varsigma\right)\right) < \mathscr{D}\left(a_1, \vartheta_{n_k+1}, \varsigma\right), \tag{47}$$

which yields $\lim_{k\longrightarrow\infty} \mathscr{D}\left(a_1, \vartheta_{n_k+1}, \varsigma\right) = 1$. Then, $a_1 = \omega$; from (45), we derive that $\mathscr{D}\left(\omega, \mathscr{L}\omega, \varsigma\right) = \mathscr{D}\left(\mathscr{U}, \mathscr{V}, \varsigma\right)$. □

Theorem 3. *Let $\mathscr{U}$ and $\mathscr{V}$ be nonempty subsets of a complete fuzzy metric space $(\Lambda, \mathscr{D}, *)$ and*

α: $\Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$. *Assume that* $\mathscr{L}$: $\mathscr{U} \longrightarrow \mathscr{V}$ *is a generalized* $\alpha - \mathscr{F}\mathscr{Z}$*-contraction with respect to* $\xi \in \mathscr{F}\mathscr{Z}$ *and*

(i) $\mathscr{L}$ *is triangular weak-α-admissible*

(ii) $\mathscr{U}$ *is closed*

(iii) $\mathscr{L}(\mathscr{U}_0) \subseteq \mathscr{V}_0$

(iv) There exists $\vartheta_0, \vartheta_1 \in \mathscr{U}$ *such that* $\mathscr{D}(\vartheta_1, \mathscr{L}\vartheta_0, \varsigma) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma)$ *and* $\alpha(\vartheta_0, \mathscr{L}\vartheta_0, \varsigma) \geq 1$ *for all* $\varsigma > 0$

(v) $\mathscr{L}$ *is continuous*

Then, there exists $\omega \in \mathscr{U}$ *such that* $\mathscr{D}(\omega, \mathscr{L}\omega, \varsigma) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma)$ *for all* $\varsigma > 0$.

Proof. Using condition (iv), there exists $\vartheta_0, \vartheta_1 \in \mathscr{U}$ such that $\alpha(\vartheta_0, \vartheta_1, \varsigma) \geq 1$ and $\mathscr{D}(\vartheta_1, \mathscr{L}\vartheta_0, \varsigma) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma)$. Regarding (iii), we have $\mathscr{L}\vartheta_1 \in \mathscr{V}_0$ which yields that there exists $\vartheta_2 \in \mathscr{U}$ such that

$$\mathscr{D}(\vartheta_2, \mathscr{L}\vartheta_1, \varsigma) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma). \tag{48}$$

Since $\alpha(\vartheta_0, \vartheta_1, \varsigma) \geq 1$ and $\mathscr{L}$ is an α-proximal admissible, it therefore follows that $\alpha(\vartheta_1, \vartheta_2, \varsigma) \geq 1$. We recursively construct the sequence $\{\vartheta_n\} \subset \mathscr{U}_0$ as follows:

$$\begin{aligned} &\alpha(\vartheta_n, \vartheta_{n+1}, \varsigma) \geq 1, \\ &\mathscr{D}(\vartheta_{n+1}, \mathscr{L}\vartheta_n, \varsigma) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma), \quad \text{for all } n \in \mathbb{N}. \end{aligned} \tag{49}$$

Suppose that there exists certain $m_0 \in \mathbb{N}$ such that $\vartheta_{m_0+1} = \vartheta_{m_0}$. Hence,

$$\mathscr{D}\left(\vartheta_{m_0}, \mathscr{L}\vartheta_{m_0}, \varsigma\right) = \mathscr{D}\left(\vartheta_{m_0+1}, \mathscr{L}\vartheta_{m_0}, \varsigma\right) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma), \tag{50}$$

which means that ϑ_{m_0} is a best proximity point of $\mathscr{L}$; thus, the proof is finished. For this reason, to continue our proof, we assume that $\vartheta_n \neq \vartheta_{n+1}$ for all $n \in \mathbb{N}$. Making use of (56) and (60), we obtain

$$\begin{aligned} \mathscr{D}(\vartheta_n, \mathscr{L}\vartheta_{n-1}, \varsigma) &= \mathscr{D}(\vartheta_{n+1}, \mathscr{L}\vartheta_n, \varsigma) \\ &= \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma), \quad \text{for all } n \in \mathbb{N}. \end{aligned} \tag{51}$$

Since $\mathscr{L}$ is a generalized $\alpha - \mathscr{F}\mathscr{Z}$-contraction with respect to $\xi \in \mathscr{F}\mathscr{Z}$, we have

$$0 \leq \xi(\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma), \mathscr{R}(\vartheta_{n-1}, \vartheta_n, \varsigma)), \tag{52}$$

where

$$\begin{aligned} \mathscr{R}(\vartheta_{n-1}, \vartheta_n, \varsigma) &= \min\left\{\mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma), \frac{\mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma)\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma)}{\mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma)}\right\} \\ &= \min\{\mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma), \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma)\}. \end{aligned} \tag{53}$$

Now, if

$$\min\{\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma), \mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma)\} = \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma). \tag{54}$$

applying (ξ_2), we get that

$$0 \leq \xi(\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma), \mathscr{R}(\vartheta_{n-1}, \vartheta_n, \varsigma)) < \frac{1}{\mathscr{R}(\vartheta_{n-1}, \vartheta_n, \varsigma)} - \frac{1}{\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma)}. \tag{55}$$

Thus,

$$\mathscr{R}(\vartheta_{n-1}, \vartheta_n, \varsigma) = \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma) < \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma), \tag{56}$$

which is a contradiction. Consequently,

$$\mathscr{R}(\vartheta_{n-1}, \vartheta_n, \varsigma) = \min\{\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma), \mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma)\} = \mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma). \tag{57}$$

By (55), we obtain that

$$\mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma) < \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma), \quad \text{for all } n \in \mathbb{N}. \tag{58}$$

Hence, we deduce that $\{\mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma)\}$ is a nondecreasing sequence in $(0, 1]$. Thus, there exists $s(\varsigma) \leq 1$ such that $\lim_{n \longrightarrow \infty} \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma) = s(\varsigma)$ for all $\varsigma > 0$. We shall prove that $s(\varsigma) = 1$. Reasoning by contradiction, suppose that $s(\varsigma_0) < 1$ for some $\varsigma_0 > 0$. Now, if we take the sequences $\mathscr{T}_n = \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma_0)$ and $\mathscr{S}_n = \mathscr{D}(\vartheta_{n-1}, \vartheta_n, \varsigma_0)$ and consider (ξ_3), we obtain

$$0 \leq \lim_{n \longrightarrow \infty} \sup \xi(\mathscr{T}_n, \mathscr{S}_n) < 0, \tag{59}$$

which is a contradiction. Therefore,

$$\lim_{n\longrightarrow\infty} \mathscr{D}(\vartheta_n, \vartheta_{n+1}, \varsigma) = 1, \quad \text{for all } \varsigma > 0. \tag{60}$$

Next, we show that $\{\vartheta_n\}$ is Cauchy sequence. On the contrary, assume that $\{\vartheta_n\}$ is not a Cauchy. Hence, there exist $\varepsilon \in (0, 1)$, $\varsigma_0 > 0$, and two subsequences $\{\vartheta_{n_k}\}$ and $\{\vartheta_{m_k}\}$ of $\{\vartheta_n\}$ with $n_k > m_k \geq k$ for all $k \in \mathbb{N}$ such that

$$\mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) \leq 1 - \varepsilon. \tag{61}$$

Taking into account Lemma 1, we derive that

$$\mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \frac{\varsigma_0}{2}\right) \leq 1 - \varepsilon. \tag{62}$$

By choosing m_k as the smallest index satisfying (29), we have

$$\mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k-1}, \frac{\varsigma_0}{2}\right) > 1 - \varepsilon. \tag{63}$$

Making use of (61) and (63) and the triangular inequality, we get

$$\begin{aligned} 1 - \varepsilon &\geq \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) \\ &\geq \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k-1}, \frac{\varsigma_0}{2}\right) * \mathscr{D}\left(\vartheta_{n_k-1}, \vartheta_{n_k}, \frac{\varsigma_0}{2}\right) \\ &> (1 - \varepsilon) * \mathscr{D}\left(\vartheta_{n_k-1}, \vartheta_{n_k}, \frac{\varsigma_0}{2}\right). \end{aligned} \tag{64}$$

Passing to the limit $k \longrightarrow \infty$ and using (60), we derive that

$$\lim_{n\longrightarrow\infty} \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) = 1 - \varepsilon. \tag{65}$$

On the other hand,

$$\begin{aligned} \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right) &\geq \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{m_k}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{n_k}, \vartheta_{n_k-1}, \frac{\varsigma_0}{3}\right), \\ \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) &\geq \mathscr{D}\left(\vartheta_{m_k}, \vartheta_{m_k-1}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \frac{\varsigma_0}{3}\right) * \mathscr{D}\left(\vartheta_{n_k-1}, \vartheta_{n_k}, \frac{\varsigma_0}{3}\right), \end{aligned} \tag{66}$$

which imply that

$$\lim_{n\longrightarrow\infty} \mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right) = 1 - \varepsilon. \tag{67}$$

Furthermore, since $\mathscr{L}$ is triangular weak-α-admissible, we deduce that

$$\alpha(\vartheta_n, \vartheta_m, \varsigma) \geq 1, \quad \text{for all } n, m \in \mathbb{N} \text{ with } n > m. \tag{68}$$

Thus,

$$\alpha\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right) \geq 1, \tag{69}$$

$$\mathscr{D}\left(\vartheta_{m_k}, \mathscr{L}\vartheta_{m_k-1}, \varsigma_0\right) = \mathscr{D}\left(\vartheta_{n_k}, \mathscr{L}\vartheta_{n_k-1}, \varsigma_0\right) = \mathscr{D}(\mathscr{U}, \mathscr{V}, \varsigma_0), \tag{70}$$

for all $k \in \mathbb{N}$. By the fact that $\mathscr{L}$ is a generalized $\alpha - \mathscr{F}\mathscr{Z}$-contraction with respect to $\xi \in \mathscr{F}\mathscr{Z}$ and using (69) and (70), we obtain that

$$0 \leq \xi\left(\mathscr{D}\left(\vartheta_{m_k}, \vartheta_{n_k}, \varsigma_0\right), \mathscr{R}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right)\right), \quad \text{for all } k \in \mathbb{N}, \tag{71}$$

where

$$\mathscr{R}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right) = \min\left\{\mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right), \frac{\mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{m_k}, \varsigma_0\right)\mathscr{D}\left(\vartheta_{n_k-1}, \vartheta_{n_k}, \varsigma_0\right)}{\mathscr{D}\left(\vartheta_{m_k-1}, \vartheta_{n_k-1}, \varsigma_0\right)}\right\}. \tag{72}$$

Letting $k \longrightarrow \infty$ in equality (72) and using (60), we derive

$$\lim_{k\longrightarrow\infty}\mathscr{R}\left(\vartheta_{m_k-1},\vartheta_{n_k-1},\varsigma_0\right) = \min\left\{\frac{1}{1-\varepsilon},1-\varepsilon\right\} = 1-\varepsilon. \tag{73}$$

Take the sequences $\mu_k = \mathscr{D}(\vartheta_{m_k},\vartheta_{n_k},\varsigma_0)$ and $\nu_k = \mathscr{R}(\vartheta_{m_k-1},\vartheta_{n_k-1},\varsigma_0)$ for all $k\in\mathbb{N}$. Applying ξ_3, we derive that

$$0\le \lim_{n\longrightarrow\infty}\sup\xi(\mu_k,\nu_k)<0, \tag{74}$$

which is a contradiction. Then, $\{\vartheta_n\}$ is a Cauchy sequence in $\mathscr{U}$. Given that $\mathscr{U}$ is closed subset of a complete fuzzy metric space $(\Lambda,\mathscr{D},*)$, there exists $\omega\in\mathscr{U}$ such that

$$\lim_{n\longrightarrow\infty}\mathscr{D}(\vartheta_n,\omega,\varsigma)=1. \tag{75}$$

As $\mathscr{L}$ is continuous, we obtain that $\mathscr{L}\vartheta_n$ converges to $\mathscr{L}\omega$; thus,

$$\lim_{n\longrightarrow\infty}\mathscr{D}(\mathscr{L}\vartheta_n,\mathscr{L}\omega,\varsigma)=1. \tag{76}$$

As the metric function $\mathscr{D}$ is continuous, we have $\mathscr{D}(\vartheta_{n+1},\mathscr{L}\vartheta_n,\varsigma)\longrightarrow\mathscr{D}(\omega,\mathscr{L}\omega,\varsigma)$. In view of (51), we get

$$\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)=\lim_{n\longrightarrow\infty}\mathscr{D}(\vartheta_{n+1},\mathscr{L}\vartheta_n,\varsigma)=\mathscr{D}(\omega,\mathscr{L}\omega,\varsigma). \tag{77}$$

Thus, $\omega\in\mathscr{U}$ is a best proximity point of $\mathscr{L}$. □

Theorem 4. *Let $\mathscr{U}$ and $\mathscr{V}$ be nonempty subsets of a complete fuzzy metric space $(\Lambda,\mathscr{D},*)$ and $\alpha\colon\Lambda\times\Lambda\times(0,\infty)\longrightarrow[0,\infty)$. Assuming that $\mathscr{L}\colon\mathscr{U}\longrightarrow\mathscr{V}$ is an $\alpha-\mathscr{F}\mathscr{Z}$-contraction with respect to $\xi\in\mathscr{F}\mathscr{Z}$,*

(i) $\mathscr{L}$ is triangular weak-α-admissible

(ii) $\mathscr{U}$ is closed

(iii) $\mathscr{L}(\mathscr{U}_0)\subseteq\mathscr{V}_0$

(iv) There exists $\vartheta_0,\vartheta_1\in\mathscr{U}$ such that $\mathscr{D}(\vartheta_0,\mathscr{L}\vartheta_0,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)$ and $\alpha(\vartheta_0,\vartheta_1,\varsigma)\ge1$ for all $\varsigma>0$

*(v) $\mathscr{L}$ is continuous or (**C**) holds*

Then, there exists $\omega\in\mathscr{U}$ such that $\mathscr{D}(\omega,\mathscr{L}\omega,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)$ for all $\varsigma>0$.

Proof. Pursuant to the same arguments as those given in the proof of Theorem 3, we know that there exists a Cauchy sequence $\{\vartheta_n\}$ in $\mathscr{U}$ which converges to $\omega\in\mathscr{U}$. Further,

$$\lim_{k\longrightarrow\infty}\mathscr{D}(\vartheta_n,\omega,\varsigma)=1,\quad \text{for all } n\in\mathbb{N},\varsigma>0. \tag{78}$$

If $\mathscr{L}$ is continuous, then

$$\lim_{k\longrightarrow\infty}\mathscr{D}(\mathscr{L}\vartheta_n,\mathscr{L}\omega,\varsigma)=1,\quad \text{for all } n\in\mathbb{N},\varsigma>0. \tag{79}$$

Taking into account (21), we deduce

$$\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)=\lim_{n\longrightarrow\infty}\mathscr{D}(\vartheta_{n+1},\mathscr{L}\vartheta_n,\varsigma)=\mathscr{D}(\omega,\mathscr{L}\omega,\varsigma), \tag{80}$$

which means that $\omega\in\mathscr{U}$ is a best proximity point of $\mathscr{L}$.

Now, suppose that (**C**) holds. Since $\mathscr{L}(\mathscr{U}_0)\subseteq\mathscr{V}_0$, we have $\mathscr{L}\omega\in\mathscr{V}_0$ and then

$$\mathscr{D}(a_1,\mathscr{L}\omega,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma),\quad \text{for some } a_1\in\mathscr{U}_0. \tag{81}$$

By condition (**C**), there exists a subsequence $\{\vartheta_{n(k)}\}$ of $\{\vartheta_n\}$ such that

$$\alpha\left(\vartheta_{n_k},\omega,\varsigma\right)\ge1,\quad \text{for all } k\in\mathbb{N},\varsigma>0. \tag{82}$$

Regarding that $\mathscr{L}$ is an α-proximal admissible and

$$\mathscr{D}(a_1,\mathscr{L}\omega,\varsigma)=\mathscr{D}\left(\vartheta_{n_k+1},\mathscr{L}\vartheta_{n_k},\varsigma\right)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma), \tag{83}$$

we obtain that $\alpha(\vartheta_{n_k+1},a_1,\varsigma)\ge1$. Hence,

$$0\le\xi\left(\mathscr{D}\left(a_1,\vartheta_{n_k+1},\varsigma\right),\mathscr{D}\left(\omega,\vartheta_{n_k},\varsigma\right)\right),\quad \text{for all } k\in\mathbb{N}. \tag{84}$$

Applying the property (ξ_2), it follows that

$$\mathscr{D}\left(\omega,\vartheta_{n_k},\varsigma\right)<\mathscr{D}\left(a_1,\vartheta_{n_k+1},\varsigma\right), \tag{85}$$

which yields $\lim_{k\longrightarrow\infty}\mathscr{D}(a_1,\vartheta_{n_k+1},\varsigma)=1$. Then, $a_1=\omega$; from (45), we derive that $\mathscr{D}(\omega,\mathscr{L}\omega,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)$. This completes the proof.

Note that Theorem 4 cannot be deduced by combining Theorems 1 and 2 since the function $\psi(c)=c$ does not belong to Ψ. Moreover, in Theorems 1 and 2, we have an added condition that ξ is nonincreasing in its second argument. □

4. Consequences

Now, we shall clarify that diverse consequences of the existence results can be easily derived and developed from our main results.

Corollary 1. *Let $\mathscr{U}$ and $\mathscr{V}$ be nonempty subsets of a complete fuzzy metric space $(\Lambda,\mathscr{D},*)$, $\alpha\colon\Lambda\times\Lambda\times(0,\infty)\longrightarrow[0,\infty)$, $\psi\in\Psi$. Assume that $\mathscr{L}\colon\mathscr{U}\longrightarrow\mathscr{V}$ is an α-admissible proximal mapping such that*

$$\begin{cases}\alpha(\vartheta,\theta,\varsigma)\ge1,\\ \mathscr{D}(u,\mathscr{L}\vartheta,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma),\Longrightarrow\mathscr{D}(u,v,\varsigma)\ge\psi(\mathscr{D}(\vartheta,\theta,\varsigma)),\\ \mathscr{D}(v,\mathscr{L}\theta,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)\end{cases} \tag{86}$$

for all $u,v,\vartheta,\theta\in\mathscr{U}$ and $\varsigma>0$. Assume also that

(i) $\mathscr{L}$ is triangular weak-α-admissible

(ii) $\mathscr{U}$ is closed

(iii) $\mathscr{L}(\mathscr{U}_0)\subseteq\mathscr{V}_0$

(iv) There exists $\vartheta_0,\vartheta_1\in\mathscr{U}$ such that $\mathscr{D}(\vartheta_0,\mathscr{L}\vartheta_0,\varsigma)=\mathscr{D}(\mathscr{U},\mathscr{V},\varsigma)$ and $\alpha(\vartheta_0,\vartheta_1,\varsigma)\ge1$ for all $\varsigma>0$

*(v) $\mathscr{L}$ is continuous or (**C**) holds*

Then, there exists $\omega \in \mathcal{U}$ *such that* $\mathcal{D}(\omega, \mathcal{L}\omega, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma)$ *for all* $\varsigma > 0$.

Proof. Define ξ: $(0,1] \times (0,1] \longrightarrow \mathbb{R}$ by

$$\xi(\mu, \nu) = \frac{1}{\psi(\nu)} - \frac{1}{\mu}, \quad \text{for all } \mu, \nu \in (0,1]. \tag{87}$$

Since $\xi \in \mathcal{FZ}$, Theorem 4 leads to the desired results. □

Corollary 2. *Let* $\mathcal{U}$ *and* $\mathcal{V}$ *be nonempty subsets of a complete fuzzy metric space* $(\Lambda, \mathcal{D}, *)$, α: $\Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$, $\eta \in \mathcal{H}$. *Assume that* $\mathcal{L}$: $\mathcal{U} \longrightarrow \mathcal{V}$ *is an* α*-admissible proximal mapping such that*

$$\begin{cases} \alpha(\vartheta, \theta, \varsigma) \geq 1, \\ \mathcal{D}(u, \mathcal{L}\vartheta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma), \Longrightarrow \eta(\mathcal{D}(u, v, \varsigma)) \leq a\eta(\mathcal{D}(\vartheta, \theta, \varsigma)), \\ \mathcal{D}(v, \mathcal{L}\theta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) \end{cases} \tag{88}$$

for all $u, v, \vartheta, \theta \in \mathcal{U}$ *and* $\varsigma > 0$. *Assume also that*

(i) $\mathcal{L}$ *is triangular weak-*α*-admissible*

(ii) $\mathcal{U}$ *is closed*

(iii) $\mathcal{L}(\mathcal{U}_0) \subseteq \mathcal{V}_0$

(iv) *There exists* $\vartheta_0, \vartheta_1 \in \mathcal{U}$ *such that* $\mathcal{D}(\vartheta_0, \mathcal{L}\vartheta_0, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma)$ *and* $\alpha(\vartheta_0, \vartheta_1, \varsigma) \geq 1$ *for all* $\varsigma > 0$

(v) $\mathcal{L}$ *is continuous or* (**C**) *holds*

Then, there exists $\omega \in \mathcal{U}$ *such that* $\mathcal{D}(\omega, \mathcal{L}\omega, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma)$ *for all* $\varsigma > 0$.

Proof. It follows from Theorem 4 using the $\mathcal{FZ}$-simulation function $\xi(\mu, \nu) = (1/\eta^{-1}(a.\eta(\nu))) - (1/\mu)$, for all $\mu, \nu \in (0,1]$. □

Corollary 3. *Let* $\mathcal{U}$ *and* $\mathcal{V}$ *be nonempty subsets of a complete fuzzy metric space* $(\Lambda, \mathcal{D}, *)$, α: $\Lambda \times \Lambda \times (0, \infty) \longrightarrow [0, \infty)$. *Assume that* $\mathcal{L}$: $\mathcal{U} \longrightarrow \mathcal{V}$ *is an* α*-admissible proximal mapping such that*

$$\begin{cases} \alpha(\vartheta, \theta, \varsigma) \geq 1, \\ \mathcal{D}(u, \mathcal{L}\vartheta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma), \Longrightarrow \left(\dfrac{1}{\mathcal{D}(u, v, \varsigma)} - 1\right) \leq \phi\left(\dfrac{1}{\mathcal{D}(\vartheta, \theta, \varsigma)} - 1\right), \\ \mathcal{D}(v, \mathcal{L}\theta, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma) \end{cases} \tag{89}$$

for all $u, v, \vartheta, \theta \in \mathcal{U}$ *and* $\varsigma > 0$, *where* ϕ: $[0, \infty) \longrightarrow [0, \infty)$ *with* $\phi(t) < t$ *for all* $t > 0$ *and* $\phi(0) = 0$. *Assume also*

(i) $\mathcal{L}$ *is triangular weak-*α*-admissible*

(ii) $\mathcal{U}$ *is closed*

(iii) $\mathcal{L}(\mathcal{U}_0) \subseteq \mathcal{V}_0$

(iv) *There exists* $\vartheta_0, \vartheta_1 \in \mathcal{U}$ *such that* $\mathcal{D}(\vartheta_0, \mathcal{L}\vartheta_0, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma)$ *and* $\alpha(\vartheta_0, \vartheta_1, \varsigma) \geq 1$ *for all* $\varsigma > 0$

(v) $\mathcal{L}$ *is continuous or* (**C**) *holds.*

Then, there exists $\omega \in \mathcal{U}$ *such that* $\mathcal{D}(\omega, \mathcal{L}\omega, \varsigma) = \mathcal{D}(\mathcal{U}, \mathcal{V}, \varsigma)$ *for all* $\varsigma > 0$, *i.e.,* $\mathcal{L}$ *has a best proximity point* $\omega \in \mathcal{U}$.

Proof. It follows from Theorem 4 by taking $\xi(\mu, \nu) = \phi((1/\nu) - 1) - (1/\mu) + 1$ for all $\mu, \nu \in (0,1]$. □

Example 5. Let $\Lambda = \mathcal{U} = \mathcal{V} = (0, \infty)$ and $\mathcal{D}$ be a fuzzy set on $\Lambda^2 \times (0, \infty)$ given by $\mathcal{D}(\vartheta, \theta, \varsigma) = (\min\{\vartheta, \theta\}/\max\{\vartheta, \theta\})$, $\varsigma > 0$, and $*$ is a t-norm given by $\mathcal{J} * \imath = \mathcal{J}.\imath$ for all $\mathcal{J}, \imath \in [0,1]$. $(\Lambda, \mathcal{D}, *)$ is a complete fuzzy metric space. Consider the mappings $\mathcal{L}$: $\Lambda \longrightarrow \Lambda$ by $\mathcal{L}(\vartheta) = \sqrt{\vartheta}$. Define $\alpha(\vartheta, \theta, \varsigma) = 1$ for all $\vartheta, \theta \in \Lambda$, $\varsigma > 0$, and ξ: $(0,1] \times (0,1] \longrightarrow \mathbb{R}$ by

$$\xi(\mu, \nu) = \frac{1}{\psi(\nu)} - \frac{1}{\mu}, \quad \text{for all } \mu, \nu \in (0,1], \text{where } \psi(c) = \sqrt{c}. \tag{90}$$

It is easy to see that $\xi \in \mathcal{FZ}$ and $\xi(\mathcal{D}(\mathcal{L}\vartheta, \mathcal{L}\theta, \varsigma), \mathcal{D}(\vartheta, \theta, \varsigma)) \geq 0$. Therefore, all conditions of Theorem 4 are satisfied, and $x = 1$ is a fixed point of $\mathcal{L}$.

We must point to the fact that, by defining the control function ξ and the admissible mapping $\alpha(\vartheta, \theta)$ in a proper way, it is possible to particularize and derive a number of varied consequences of our main results. We skip making such a number of corollaries since they seem clear.

5. Conclusion

This paper has dealt with a $\mathcal{FZ}$-simulation function approach to best proximity point problems in fuzzy metric spaces. We have initiated some classes of non-self-mappings and discussed existence results of the best proximity points of such types of non-self-mappings. Our results can be further extended by replacing the fuzzy metric space by various settings (e.g., partially ordered fuzzy metric spaces and complex valued fuzzy metric spaces), and more generalization can be obtained by the study of optimal coincidence points, optimal best proximity coincidence points, and the setting of cyclic mappings.

Data Availability

The data used to support the findings of this study are included in the references within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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