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Research Article

The Mixed Liu Estimator in Stochastic Restricted Linear Measurement Error Model

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Ghapani and Babdi [1] proposed a mixed Liu estimator in linear measurement error model with stochastic linear restrictions. In this article, we propose an alternative mixed Liu estimator in the linear measurement error model with stochastic linear restrictions. The performance of the new mixed Liu estimator over the mixed estimator, Liu estimator, and mixed Liu estimator proposed by Ghapani and Babdi [1] are discussed in the sense of mean squared error matrix. Finally, a simulation study is given to show the performance of these estimators.

1. Introduction

When we use the linear regression model to deal with the problem, some regression explanatory can not be observed, and the values of the regression explanatory often have measurement. If we direct use these values to set the model, the estimator of regression coefficient may not be a consistent estimator. In order to overcome this problem, the statisticians and econometrician have proposed the linear measurements model. Fuller [2] and Cheng and Van Ness [3] have discussed the model.

It is well known that, in standard linear regression model when there exists collinearity, the ordinary least squares estimator is no longer a good estimator. Some statisticians are discussed how to deal with collinearity. One method is to consider the biased estimator, such as Stein [4]; Hoerl and Kennard [5]; Liu [6]; Yang and Chang [7]; and Wu and Yang [8] et al. Another method is to consider the linear restriction and stochastic linear restrictions [9], such as Li and Yang et al. [10].

In the linear measurement error model, the collinearity problem may also lead the estimator unstable. In order to deal with this problem, Saleh and Shalabh [11] considered the ridge estimator and Ghapani and Babdi [1] considered the Liu estimator. When the linear restrictions or stochastic linear restrictions are satisfied in linear measurement error model, Li et al. [12] proposed some new estimators and discussed the properties of these estimators under Pitman's closeness criterion. Saleh and Shalabh [11] discussed the preliminary test ridge estimator in the linear measurement error model with linear restrictions, Li and Yang [13] consider the weighted mixed estimator. Ghapani et al. [14] have discussed the weighted ridge estimator in linear measurement error model with stochastic linear restrictions. There are many researches on Liu estimator for different models done by various researchers. To mention a few, Arashi et al. [15], Kibria [16], Alheety and Kibria [17], Ghapani [18], and, very recently, Li et al. [19] are among them.

In this paper, we use a different method to propose a new mixed Liu estimator by construct a Lagrange function. Furthermore, we discuss the properties of the new estimator.

The rest of the paper is organized as follows. In Section 2, we propose the mixed Liu in linear measurement error model with stochastic linear restrictions, and the properties of the new estimator are studied in Section 3. A simulation study has been conducted to support the theoretical results in Section 4, and some conclusion remarks are given in Section 5.

2. The Proposed Estimator

In this section, we will introduce the mixed Liu estimator in the linear measurement error model with stochastic linear restrictions.

2.1. The Liu Estimator. Let us study the following linear measurement error model:

$$\begin{cases} y = Z\beta + \varepsilon, \\ X = Z + \Delta, \end{cases} \tag{1}$$

where $y = (y_1, y_2, \ldots, y_n)'$ denotes an $n \times 1$ vector of response variables, β shows a $p \times 1$ vector of unknown parameters, and Z denotes an $n \times p$ matrix of unobservable values of explanatory variables which can be observed through the matrix X with the measurement error $\Delta' = (\delta_1, \delta_2, \ldots, \delta_n)$, where $\delta_i, i = 1, 2, \ldots, n$ are $p \times 1$ uncorrelated random vectors with $E(\delta_i) = 0, \operatorname{Var}(\delta_i) = \sum$. We assume that the common variance $\sum$ of measurement errors associated with the explanatory variables is known. And, we also suppose that ε is an $n \times 1$ vector of unobservable random errors with $E(\varepsilon) = 0, \operatorname{Var}(\varepsilon) = \sigma^2 I$. We let ε and Δ to be mutually independent, and we assume the ith rows of matrices Z and X with z_t' and x_t', respectively.

Fuller [2] has introduced the consistent estimator of β, which is presented as follows:

$$\widehat{\beta} = \left(X'X - n\sum\right)^{-1} X'y, \tag{2}$$

and this estimator is obtained by solving the following function:

$$S_1(\beta) = \arg\min_\beta \left[(y - X\beta)'(y - X\beta) - n\beta' \sum \beta \right]. \tag{3}$$

In order to deal with the collinearity problem, Ghapani and Babdi [1] introduced a Liu estimator (LE), and this estimator can be obtained as follows: based on (3), consider the following objective function:

$$\begin{aligned} S_2(\beta) = \arg\min_\beta \Big[&(y - X\beta)'(y - X\beta) - n\beta' \sum \beta \\ &+ (d\widehat{\beta} - \beta)'(d\widehat{\beta} - \beta) \Big]. \end{aligned} \tag{4}$$

Dealing with (4), we can obtain

$$\widehat{\beta}(d) = \left(X'X - n\sum + I\right)^{-1}\left(X'y + d\widehat{\beta}\right), \quad 0 < d < 1. \tag{5}$$

In Section 2.2, we will present the new estimator.

2.2. The Mixed Liu Estimator. In this article, we suppose that the stochastic linear restrictions on the parametric component are of the following form:

$$h = H\beta + e, e \sim N\left(0, \sigma^2 W\right), \tag{6}$$

where h is a $q \times 1$ observable random vector, H is a $q \times p$ known matrix with rank $(H) = q$ for $q < p$, and e shows a $q \times 1$ error vector with $E(e) = 0$ and $\operatorname{Var}(e) = \sigma^2 W$, and we also assume that W is a known positive definite matrix. Furthermore, we also suppose that e is stochastically independent of ε and Δ.

Based on models (1) and (6), using the mixed method, we can minimize the following equation:

$$\begin{aligned} S_3(\beta) = \arg\min_\beta \Big[&(y - X\beta)'(y - X\beta) - n\beta' \sum \beta \\ &+ (h - H\beta)' W^{-1} (h - H\beta) \Big], \end{aligned} \tag{7}$$

with respect to β. By (7), we obtain the mixed estimator (ME):

$$\widehat{\beta}_{\mathrm{ME}} = \left(X'X - n\sum + H'W^{-1}H\right)^{-1}\left(X'y + H'W^{-1}h\right). \tag{8}$$

Ghapani and Babdi [1] proposed a mixed Liu estimator, which is defined as follows:

$$\begin{aligned} \widehat{\beta}_{\mathrm{MLE}} = &\left(X'X - n\sum + H'W^{-1}H\right)^{-1} \\ &\times \left(\left(X'X - n\sum + I\right)^{-1}\left(X'X - n\sum + dI\right)X'y + H'W^{-1}h\right). \end{aligned} \tag{9}$$

Now, we will propose a new mixed Liu estimator. Consider the following function:

$$\begin{aligned} S_4(\beta) = \arg\min_\beta \Big[&(y - X\beta)'(y - X\beta) - n\beta' \sum \beta \\ &+ (h - H\beta)' W^{-1} (h - H\beta) + \left(d\widehat{\beta}_{\mathrm{ME}} - \beta\right)'\left(d\widehat{\beta}_{\mathrm{ME}} - \beta\right) \Big]. \end{aligned} \tag{10}$$

Dealing with (9), we can obtain

$$\begin{aligned} \widehat{\beta}_{\mathrm{NMLE}}(d) = &\left(X'X - n\sum + H'W^{-1}H + I\right)^{-1} \\ &\cdot \left(X'y + H'W^{-1}h + d\widehat{\beta}_{\mathrm{ME}}\right), \end{aligned} \tag{11}$$

where $0 < d < 1$ denotes the biasing parameter and $\widehat{\beta}_{\mathrm{ME}}$ denotes the mixed estimator, and we call this estimator as a new mixed Liu estimator.

The new estimator can also be written as follows:

$$\begin{aligned} \widehat{\beta}_{\mathrm{NMLE}}(d) = &\left(X'X - n\sum + H'W^{-1}H + I\right)^{-1} \\ &\cdot \left(X'X - n\sum + H'W^{-1}H + dI\right)\widehat{\beta}_{\mathrm{ME}}. \end{aligned} \tag{12}$$

By the definition of the new estimator, we can see that the new estimator is a general estimator which contains $\widehat{\beta}_{\mathrm{ME}}$, $\widehat{\beta}(d)$, and $\widehat{\beta}$ as special cases.

If $d = 1$, $\widehat{\beta}_{\mathrm{NMLE}}(d) = \widehat{\beta}_{\mathrm{ME}}$.

If $H = 0$, $\widehat{\beta}_{\mathrm{NMLE}}(d) = \widehat{\beta}(d)$.

If $d = 1$ and $H = 0$, $\widehat{\beta}_{\mathrm{NMLE}}(d) = \widehat{\beta}$.

In Section 3, we will study the asymptotic properties of these estimators.

3. The Properties of These Estimators

In this section, we will give the comparison of the new estimator with some estimators. Firstly, we give the properties of these estimators.

3.1. Large Sample Properties of These Estimators. Though the exact distribution and small sample properties of these estimators are difficult to obtain, in this paper, we use the large sample asymptotic approximation theory to study the asymptotic distribution of the estimators. We assume that the parameter β is identifiable and we also assume that as n tends to infinity, the limits of $n^{-1}(Z'Z + H'W^{-1}H), n^{-1}(Z'Z + H'W^{-1}H + dI)$, and $n^{-1}(Z'Z + H'W^{-1}H + I)$ exist and E denotes the global expectation taken at the true value β.

Theorem 1. *$\widehat{\beta}_{NMLE}(d)$ is asymptotically normally distributed. The asymptotically mean and variance of $\widehat{\beta}_{NMLE}(d)$ are, respectively, given as $E[\widehat{\beta}_{MLE}(d)] = M_1^{-1}M_d\beta$ and $AVar[\widehat{\beta}_{NMLE}(d)] = M_1^{-1}M_dM_0^{-1}(B+\sigma^2(Z'Z+H'W^{-1}H))M_0^{-1}M_dM_1^{-1}$, where $M_d = n^{-1}(Z'Z+H'W^{-1}H+dI)$ and $B=(n\sigma^2+\beta'Z'Z\beta)\Sigma$.*

Proof. By Fung et al. [20], $E(X'X) = Z'Z + n\sum$, we have

$$X'X = Z'Z + n\sum + O_p\left(n^{(1/2)}\right). \tag{13}$$

Therefore, we may write

$$n^{-1}\left(X'X+H'W^{-1}H+I\right) = n^{-1}\left(Z'Z+H'W^{-1}H+I\right) + \sum + O_p\left(n^{-(1/2)}\right), \tag{14}$$

$$n^{-1}\left(X'X+H'W^{-1}H+dI\right) = n^{-1}\left(Z'Z+H'W^{-1}H+dI\right) + \sum + O_p\left(n^{-(1/2)}\right), \tag{15}$$

$$n^{-1}\left(X'X+H'W^{-1}H\right) = n^{-1}\left(Z'Z+H'W^{-1}H\right) + \sum + O_p\left(n^{-(1/2)}\right). \tag{16}$$

Thus, by (14)–(16), we can obtain that

$$\begin{aligned}
\sqrt{n}\widehat{\beta}_{\mathrm{NMLE}}(d)
&= \left\{n^{-1}\left(Z'Z+H'W^{-1}H+I\right)+O_p\left(n^{-(1/2)}\right)\right\}^{-1}\\
&\quad\times\left\{n^{-1}\left(Z'Z+H'W^{-1}H+dI\right)+O_p\left(n^{-(1/2)}\right)\right\}\\
&\quad\times\left\{n^{-1}\left(Z'Z+H'W^{-1}H\right)+O_p\left(n^{-(1/2)}\right)\right\}^{-1}n^{-(1/2)}\left(X'y+H'W^{-1}h\right)\\
&= \left[I+O_p\left(n^{-(1/2)}\right)\right]^{-1}\left\{n^{-1}\left(Z'Z+H'W^{-1}H+I\right)+O_p\left(n^{-(1/2)}\right)\right\}^{-1}\\
&\quad\times\left\{n^{-1}\left(Z'Z+H'W^{-1}H+dI\right)+O_p\left(n^{-(1/2)}\right)\right\}\\
&\quad\times\left\{n^{-1}\left(Z'Z+H'W^{-1}H\right)+O_p\left(n^{-(1/2)}\right)\right\}^{-1}n^{-(1/2)}\left(X'y+H'W^{-1}h\right)\\
&= \left[I+O_p\left(n^{-(1/2)}\right)\right]\left\{n^{-1}\left(Z'Z+H'W^{-1}H+I\right)+O_p\left(n^{-(1/2)}\right)\right\}^{-1}\\
&\quad\times\left\{n^{-1}\left(Z'Z+H'W^{-1}H+dI\right)+O_p\left(n^{-(1/2)}\right)\right\}\\
&\quad\times\left\{n^{-1}\left(Z'Z+H'W^{-1}H\right)+O_p\left(n^{-(1/2)}\right)\right\}^{-1}n^{-(1/2)}\left(X'y+H'W^{-1}h\right).
\end{aligned} \tag{17}$$

Since the limit of $C_1 = n^{-1}(Z'Z+H'W^{-1}H+I)$, $C_d = n^{-1}(Z'Z+H'W^{-1}H+dI)$, and $C_0 = n^{-1}(Z'Z+H'W^{-1}H)$ exists, then by (17), we may get

$$\sqrt{n}\widehat{\beta}_{\mathrm{NMLE}}(d) = C_1^{-1}C_dC_0^{-1}\xi + O_p\left(n^{-(1/2)}\right), \tag{18}$$

where $\xi = n^{-(1/2)}(X'y+H'W^{-1}h)$. By Fung et al. [20], ξ is asymptotically normal and $E(\xi) = (Z'Z+H'W^{-1}H)\beta = M_0\beta$. Thus, we get

$$\sqrt{n}\left(\widehat{\beta}_{\mathrm{NMLE}}(d) - M_1^{-1}M_d\beta\right) = C_1^{-1}C_dC_0^{-1}[\xi - E(\xi)] + O_p\left(n^{-(1/2)}\right), \tag{19}$$

which indicate that $\sqrt{n}(\widehat{\beta}_{\mathrm{NMLE}}(d) - M_1^{-1}M_d\beta)$ is asymptotically normal with mean zero. By (18), we get $A\mathrm{Var}[\sqrt{n}(\widehat{\beta}_{\mathrm{NMLE}}(d))] = C_1^{-1}C_dC_0^{-1}\mathrm{Var}[\xi]C_0^{-1}C_dC_1^{-1}$. From [1], we get

$$\mathrm{Var}[\xi] = n^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right). \tag{20}$$

Then, we get

$$A\mathrm{Var}\left[\widehat{\beta}_{\mathrm{NMLE}}(d)\right] = M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)\cdot M_0^{-1}M_dM_1^{-1}. \tag{21}$$

Corollary 1. *$\widehat{\beta}_{ME}$ has asymptotically normal distribution with $E[\widehat{\beta}_{ME}] = \beta$ and $AVar[\widehat{\beta}_{ME}] = M_0^{-1}(B+\sigma^2(Z'Z+H'W^{-1}H))M_0^{-1}$.*

Corollary 2. *$\widehat{\beta}(d)$ has asymptotically normal distribution with $E[\widehat{\beta}(d)] = G_d\beta$ and $AVar[\widehat{\beta}(d)] = G_d(Z'Z)^{-1}(B+\sigma^2Z'Z)(Z'Z)^{-1}G_d$, where $G_d = (Z'Z+I)^{-1}(Z'Z+dI)$.*

By [1], we know that $E[\widehat{\beta}_{\mathrm{MLE}}(d)] = M_0^{-1}(G_dZ'Z+H'W^{-1}H)\beta$ and $A\mathrm{Var}[\widehat{\beta}_{\mathrm{MLE}}(d)] = M_0^{-1}[G_dBG_d+\sigma^2(G_dZ'ZG_d+H'W^{-1}H)]M_0^{-1}$.

3.2. Comparisons among Biased Estimators. In this subsection, we will present the comparison of the new estimator to the $\widehat{\beta}_{\mathrm{ME}}$, $\widehat{\beta}(d)$, and $\widehat{\beta}_{\mathrm{MLE}}(d)$ under the mean-squared error matrix. Firstly, we present the mean-squared error matrix of an estimator $\widehat{\theta}$ of θ is defined as

$$\mathrm{MSEM}(\widehat{\theta}) = E(\widehat{\theta}-\theta)'(\widehat{\theta}-\theta) = \mathrm{Var}(\widehat{\theta}) + \mathrm{Bias}(\widehat{\theta})\mathrm{Bias}(\widehat{\theta})', \tag{22}$$

where $\mathrm{Bias}(\widehat{\theta}) = E(\widehat{\theta}) - \theta$ denotes the bias vector. In order to present the main results, we give some lemmas.

Lemma 1 (see [21]). *0 Suppose that M be a positive matrix, namely, $M > 0$ and a be some vector, then $M - aa' \geq 0$ if and only $a'M^{-1}a \leq 1$.*

Lemma 2 (see [9]). *Let $n \times n$ matrices $M > 0$, $N \geq 0$, then $M > N$ if and only if $\lambda_{\max}(NM^{-1}) < 1$.*

Then, we can compute the asymptotic MSEM of the estimators $\widehat{\beta}, \widehat{\beta}(k), \widehat{\beta}(d)$, and $\widehat{\beta}(k,d)$ as follows:

$$\begin{aligned}
\mathrm{AMSEM}\left[\widehat{\beta}_{\mathrm{NMLE}}(d)\right] &= M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1} + b_1b_1',\\
\mathrm{AMSEM}\left[\widehat{\beta}_{\mathrm{ME}}\right] &= M_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1},\\
\mathrm{AMSEM}[\widehat{\beta}(d)] &= G_d(Z'Z)^{-1}\left(B+\sigma^2Z'Z\right)(Z'Z)^{-1}G_d + b_2b_2',\\
\mathrm{AMSEM}\left[\widehat{\beta}_{\mathrm{MLE}}(d)\right] &= M_0^{-1}\left[G_dBG_d + \sigma^2\left(G_dZ'ZG_d + H'W^{-1}H\right)\right]M_0^{-1} + b_3b_3',
\end{aligned} \tag{23}$$

where $b_1 = M_1^{-1}M_d\beta$, $b_2 = (G_d - I)\beta$, and $b_3 = (M_0^{-1}(G_dZ'Z + H'W^{-1}H) - I)\beta$.

In order to compare the $\widehat{\beta}_{\mathrm{NMLE}}(d)$ to $\widehat{\beta}_{\mathrm{MLE}}(d)$, $\widehat{\beta}_{\mathrm{ME}}$, and $\widehat{\beta}(d)$, we consider the asymptotic AMSEM differences:

$$\begin{aligned}
\nabla_1 &= \mathrm{AMSEM}\left(\widehat{\beta}_{\mathrm{ME}}\right) - \mathrm{AMSEM}\left(\widehat{\beta}_{\mathrm{NMLE}}(d)\right)\\
&= D_1 - b_1b_1',\\
\nabla_2 &= \mathrm{AMSEM}\left(\widehat{\beta}_{\mathrm{LE}}\right) - \mathrm{AMSEM}\left(\widehat{\beta}_{\mathrm{NMLE}}(d)\right)\\
&= D_2 + b_2b_2' - b_1b_1',\\
\nabla_3 &= \mathrm{AMSEM}\left(\widehat{\beta}_{\mathrm{MLE}}\right) - \mathrm{AMSEM}\left(\widehat{\beta}_{\mathrm{NMLE}}(d)\right)\\
&= D_3 + b_3b_3' - b_1b_1',
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
D_1 &= M_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}\\
&\quad - M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1},\\
D_2 &= G_d(Z'Z)^{-1}\left(B+\sigma^2Z'Z\right)(ZZ)^{-1}G_d\\
&\quad - M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1},\\
D_3 &= M_0^{-1}\left[G_dBG_d+\sigma^2\left(G_dZ'ZG_d+H'W^{-1}H\right)\right]M_0^{-1}\\
&\quad - M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1}.
\end{aligned} \tag{25}$$

Now, we give the comparison of the estimator $\widehat{\beta}_{\mathrm{NMLE}}(d)$ to the $\widehat{\beta}_{\mathrm{ME}}$ in the MSEM sense.

Theorem 2. *The $\widehat{\beta}_{NMLE}(d)$ is better than the estimator $\widehat{\beta}_{ME}$ in the MSEM sense, if and only if $b_1'D_1^{-1}b_1 \leq 1$.*

Proof. Now we prove

$$\begin{aligned}
D_1 &= M_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}\\
&\quad - M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1} > 0.
\end{aligned} \tag{26}$$

Let $H = M_0^{-1}(B + \sigma^2(Z'Z + H'W^{-1}H))M_0^{-1}$, we have $H > 0$, then we can write D_1 as follows:

$$\begin{aligned}
D_1 &= H - M_1^{-1}M_dHM_dM_1^{-1}\\
&= (1-d)M_1^{-1}M_d\left(HM_d^{-1} + M_d^{-1}H + (1-d)M_d^{-1}H^2M_d^{-1}\right)M_dM_1^{-1}.
\end{aligned} \tag{27}$$

Since $0 < d < 1$, $M_d^{-1} > 0$, and $H > 0$, we have $D_1 > 0$. By Lemma 1, we have $\widehat{\beta}_{\mathrm{NMLE}}(d)$ is better than the estimator $\widehat{\beta}_{\mathrm{ME}}$ in the MSEM sense, if and only if $b_1'D_1^{-1}b_1 \leq 1$. □

Theorem 3. *When*

$$\lambda_{\max}\left[M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1}\cdot\left(G_d(Z'Z)^{-1}\left(B+\sigma^2Z'Z\right)(Z'Z)^{-1}G_d\right)^{-1}\right] \leq 1, \tag{28}$$

the $\widehat{\beta}_{NMLE}(d)$ is better than the estimator $\widehat{\beta}(d)$ in the MSEM sense, if and only if $b_1'(D_2 + b_2b_2')^{-1}b_1 \leq 1$.

Proof. Since

$$\begin{aligned}
G_d(Z'Z)^{-1}\left(B+\sigma^2Z'Z\right)(Z'Z)^{-1}G_d &> 0,\\
M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1} &> 0,
\end{aligned} \tag{29}$$

then by Lemma 2, when

$$\lambda_{\max}\left[M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1}\cdot\left(G_d\left(Z'Z\right)^{-1}\left(B+\sigma^2Z'Z\right)\left(Z'Z\right)^{-1}G_d\right)^{-1}\right]\leq 1, \tag{30}$$

we have $D_2>0$, then by Lemma 1, we get that the new estimator is superior to the $\widehat{\beta}(d)$ in the MSEM sense, if and only if $b_1'(D_2+b_2b_2')^{-1}b_1\leq 1$. □

Theorem 4. *When*

$$\lambda_{\max}\left[M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1}\cdot\left(M_0^{-1}\left[G_dBG_d+\sigma^2\left(G_dZ'ZG_d+H'W^{-1}H\right)\right]M_0^{-1}\right)^{-1}\right]\leq 1. \tag{31}$$

the $\widehat{\beta}_{NMLE}(d)$ is better than the estimator $\widehat{\beta}_{MLE}(d)$ in the MSEM sense, if and only if $b_1'(D_3+b_3b_3')^{-1}b_1\leq 1$.

Proof. Since

$$\begin{gathered}M_0^{-1}\left[G_dBG_d+\sigma^2\left(G_dZ'ZG_d+H'W^{-1}H\right)\right]M_0^{-1}>0,\\ M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1}>0,\end{gathered} \tag{32}$$

then by Lemma 2, when

$$\lambda_{\max}\left[M_1^{-1}M_dM_0^{-1}\left(B+\sigma^2\left(Z'Z+H'W^{-1}H\right)\right)M_0^{-1}M_dM_1^{-1}\cdot\left(M_0^{-1}\left[G_dBG_d+\sigma^2\left(G_dZ'ZG_d+H'W^{-1}H\right)\right]M_0^{-1}\right)^{-1}\right]\leq 1, \tag{33}$$

we have $D_3>0$, then by Lemma 1, we get that the new estimator is superior to the $\widehat{\beta}_{\text{MLE}}(d)$ in the MSEM sense, if and only if $b_1'(D_3+b_3b_3')^{-1}b_1\leq 1$. □

4. Monte Carlo Simulation Experiments

In this section, we will conduct a Monte Carlo simulation experiment is designed to show the performance of these estimators. Following McDonald and Galarneau [22], we may get the explanatory variables by using

$$z_{ij}=\left(1-\rho^2\right)^{1/2}w_{ij}+\rho w_{i(p+1)}, \tag{34}$$

where w_{ij} are got by the standard normal distribution and ρ is chosen so that the correlation between any two variables is ρ^2. Three different values of the correlation are used, namely, 0.9, 0.95, and 0.99. The real values of the parameter vector β are chosen as the eigenvector of the matrix $Z'Z$ corresponding to the largest eigenvalue. Moreover, we have considered the explanatory variable as $p=4$. We also assume that $\Sigma=\text{diag}(0.01,\ldots,0.01)$ and $\sigma=1,5$, and 10. The sample size is taken to be 50, 100, and 150.

The stochastic linear restrictions of H are generated by norm distributions and $e\sim N(0,\sigma^2 I)$.

Note that in this paper we did not introduced any estimators of the shrinkage parameter d; therefore, we only consider some values of d such that $0<d<1$. We generated 5000 data sets containing the explanatory variables and the dependent variable. The simulated mean-squared error (MSE) is used to compare the estimators such that it can be computed as follows:

$$\widehat{\beta}_r=\frac{\sum_{r=1}^{5000}\left(\widehat{\beta}_r-\beta\right)'\left(\widehat{\beta}_r-\beta\right)}{5000}, \tag{35}$$

where $\widehat{\beta}_r$ is any estimator considered in this paper in the r^{th} repetition. All computations are performed using the R Program.

We have summarized the results of the simulation in Tables 1–5. We can conclude the following from the tables.

(1) The new estimator is always superior to the ME and LE.

(2) The new estimator is superior to the MLE in most cases. When the $\rho^2=0.99$, that is the multicollineariy is serve, the new estimator is superior to the MLE.

(3) When the n is small, the new estimator performs well.

Table 1: MSE values of the estimator for different values of d and ρ when $\sigma = 1$ and $n = 50$.

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho = 0.90$									
OME	0.3466	0.3466	0.3466	0.3466	0.3466	0.3466	0.3466	0.3466	0.3466
LE	0.3630	0.3740	0.3851	0.3964	0.4080	0.4196	0.4315	0.4436	0.4558
MLE	0.2849	0.2914	0.2979	0.3046	0.3113	0.3182	0.3251	0.3322	0.3394
NMLE	0.2839	0.2905	0.2972	0.3040	0.3109	0.3179	0.3249	0.3321	0.3393
$\rho = 0.95$									
OME	0.7517	0.7517	0.7517	0.7517	0.7517	0.7517	0.7517	0.7517	0.7517
LE	0.7320	0.7899	0.8503	0.9132	0.9787	1.0467	1.1172	1.1903	1.2659
MLE	0.5077	0.5311	0.5554	0.5806	0.6068	0.6339	0.6620	0.6910	0.7209
NMLE	0.4947	0.5202	0.5464	0.5734	0.6012	0.6297	0.6590	0.6891	0.7200
$\rho = 0.99$									
OME	2.2716	2.2716	2.2716	2.2716	2.2716	2.2716	2.2716	2.2716	2.2716
LE	1.5203	1.9326	2.4041	2.9348	3.5247	4.1739	4.8823	5.6499	6.4768
MLE	0.9794	1.0775	1.1870	1.3078	1.4400	1.5836	1.7386	1.9049	2.0826
NMLE	0.8374	0.9553	1.0835	1.2221	1.3711	1.5305	1.7002	1.8803	2.0708

Table 2: MSE values of the estimator for different values of d and ρ when $\sigma = 1$ and $n = 100$.

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho = 0.90$									
OME	0.1493	0.1493	0.1493	0.1493	0.1493	0.1493	0.1493	0.1493	0.1493
LE	0.1551	0.1567	0.1583	0.1600	0.1616	0.1633	0.1650	0.1667	0.1684
MLE	0.1375	0.1388	0.1401	0.1414	0.1427	0.1440	0.1453	0.1466	0.1479
NMLE	0.1375	0.1388	0.1401	0.1414	0.1427	0.1440	0.1453	0.1466	0.1479
$\rho = 0.95$									
OME	0.2596	0.2596	0.2596	0.2596	0.2596	0.2596	0.2596	0.2596	0.2596
LE	0.2746	0.2801	0.2856	0.2912	0.2969	0.3026	0.3083	0.3141	0.3200
MLE	0.2249	0.2286	0.2323	0.2361	0.2399	0.2438	0.2477	0.2516	0.2556
NMLE	0.2248	0.2285	0.2323	0.2361	0.2399	0.2438	0.2477	0.2516	0.2556
$\rho = 0.99$									
OME	1.2140	1.2140	1.2140	1.2140	1.2140	1.2140	1.2140	1.2140	1.2140
LE	1.1520	1.2763	1.4073	1.5451	1.6897	1.8411	1.9993	2.1642	2.3360
MLE	0.7091	0.7556	0.8045	0.8558	0.9095	0.9656	1.0240	1.0849	1.1482
NMLE	0.6951	0.7447	0.7963	0.8499	0.9056	0.9632	1.0229	1.0846	1.1482

Table 3: MSE values of the estimator for different values of d and ρ when $\sigma = 1$ and$n = 150$.

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho = 0.90$									
OME	0.1009	0.1009	0.1009	0.1009	0.1009	0.1009	0.1009	0.1009	0.1009
LE	0.1040	0.1048	0.1055	0.1063	0.1070	0.1077	0.1085	0.1093	0.1100
MLE	0.0953	0.0959	0.0965	0.0971	0.0977	0.0984	0.0990	0.0996	0.1002
NMLE	0.0953	0.0959	0.0965	0.0971	0.0977	0.0984	0.0990	0.0996	0.1002
$\rho = 0.95$									
OME	0.2121	0.2121	0.2121	0.2121	0.2121	0.2121	0.2121	0.2121	0.2121
LE	0.2228	0.2262	0.2296	0.2330	0.2365	0.2400	0.2435	0.2471	0.2507
MLE	0.1891	0.1916	0.1941	0.1966	0.1992	0.2017	0.2043	0.2069	0.2095
NMLE	0.1891	0.1916	0.1941	0.1967	0.1992	0.2018	0.2043	0.2069	0.2095
$\rho = 0.99$									
OME	0.9328	0.9328	0.9328	0.9328	0.9328	0.9328	0.9328	0.9328	0.9328
LE	0.9467	1.0154	1.0867	1.1606	1.2370	1.3159	1.3975	1.4815	1.5682
MLE	0.6152	0.6461	0.6781	0.7112	0.7454	0.7807	0.8171	0.8545	0.8931
NMLE	0.6132	0.6451	0.6778	0.7115	0.7461	0.7816	0.8180	0.8554	0.8936

Table 4: MSE values of the estimator for different values of d and ρ when $\sigma = 5$ and $n = 50$.

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho = 0.90$									
OME	7.3456	7.3456	7.3456	7.3456	7.3456	7.3456	7.3456	7.3456	7.3456
LE	7.6629	7.8677	8.0756	8.2868	8.5011	8.7187	8.9395	9.1634	9.3906
MLE	6.1826	6.3046	6.4285	6.5541	6.6815	6.8108	6.9418	7.0746	7.2092
NMLE	6.1587	6.2846	6.4119	6.5407	6.6711	6.8029	6.9363	7.0713	7.2077
$\rho = 0.95$									
OME	12.6727	12.6727	12.6727	12.6727	12.6727	12.6727	12.6727	12.6727	12.6727
LE	13.0169	13.6400	14.2788	14.9335	15.6039	16.2900	16.9920	17.7097	18.4432
MLE	9.5570	9.8722	10.1952	10.5259	10.8643	11.2105	11.5644	11.9261	12.2955
NMLE	9.4611	9.7919	10.1291	10.4729	10.8232	11.1801	11.5434	11.9133	12.2897
$\rho = 0.99$									
OME	58.8476	58.8476	58.8476	58.8476	58.8476	58.8476	58.8476	58.8476	58.8476
LE	38.5358	48.9573	60.7964	74.0531	88.7275	104.8194	122.3290	141.2563	161.6011
MLE	24.7887	27.3642	30.2419	33.4217	36.9038	40.6882	44.7747	49.1635	53.8544
NMLE	21.2034	24.2775	27.6287	31.2570	35.1625	39.3452	43.8051	48.5421	53.5563

Table 5: MSE values of the estimator for different values of d and ρ when $\sigma = 10$ and $n = 50$.

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho = 0.90$									
OME	24.0082	24.0082	24.0082	24.0082	24.0082	24.0082	24.0082	24.0082	24.0082
LE	25.2015	25.6559	26.1144	26.5771	27.0441	27.5153	27.9907	28.4703	28.9541
MLE	21.0517	21.3685	21.6882	22.0108	22.3364	22.6649	22.9963	23.3307	23.6680
NMLE	21.0249	21.3460	21.6697	21.9960	22.3249	22.6563	22.9904	23.3271	23.6664
$\rho = 0.95$									
OME	53.3814	53.3814	53.3814	53.3814	53.3814	53.3814	53.3814	53.3814	53.3814
LE	54.0990	56.6023	59.1640	61.7840	64.4622	67.1988	69.9936	72.8467	75.7581
MLE	40.2308	41.5689	42.9377	44.3372	45.7676	47.2288	48.7207	50.2435	51.7970
NMLE	39.8085	41.2095	42.6373	44.0919	45.5732	47.0813	48.6162	50.1778	51.7662
$\rho = 0.99$									
OME	203.6969	203.6969	203.6969	203.6969	203.6969	203.6969	203.6969	203.6969	203.6969
LE	141.8366	176.1374	214.9035	258.1351	305.8320	357.9942	414.6219	475.7149	541.2733
MLE	93.2165	101.8555	111.4037	121.8610	133.2275	145.5031	158.6878	172.7817	187.7847
NMLE	80.6870	91.0673	102.2695	114.2935	127.1394	140.8072	155.2968	170.6083	186.7417

5. Conclusions

In this paper, we use a new method to propose a new mixed estimator in the linear measurement error model and we also discuss the properties of the new estimator. A Monte Carlo simulation experiment is designed to evaluate the performances of the estimators in terms of the simulated mean-squared error criterion. Simulation results indicated that the new estimator performed better than the rest of the estimators when the multicollinearity problem exists in the data. Therefore, the new estimator can be an alternative to the existing estimators especially in the presence of highly correlated data.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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