

Research Article

T-S Fuzzy Control of Uncertain Fractional-Order Systems with Time Delay

Yilin Hao and Xiulan Zhang 

School of Mathematics and Physics, Guangxi University for Nationalities, Nanning, China

Correspondence should be addressed to Xiulan Zhang; xlzhang@gxun.edu.cn

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In this article, the adaptive control of uncertain fractional-order time-delay systems (FOTDSs) with external disturbances is discussed. A Takagi-Sugeno (T-S) fuzzy model with if-then rules is adopted to characterize the dynamic equation of the FOTDS. Besides, a fuzzy adaptive method is proposed to stabilize the model. By utilizing the Lyapunov functions, a robust controller is constructed to stabilize the FOTDS. Due to the uncertainty of system parameters, some fractional-order adaptation laws are designed to update these parameters. At the same time, some if-then rules with linear structure based on the fuzzy T-S adaption concept are established. The designed method not only guarantees that the state of closed-loop system asymptotically converges to origin but also keeps the signal in the FOTDS bounded. Finally, the applicability of the control method is proved by simulation examples.

1. Introduction

In the last century, fractional-order calculus has been widely used in many electrical systems, robot systems, engineering systems, and thermodynamic systems, which has attracted extensive attention [1–5]. The fractional-order system (FOS) has more advantages than classical integer-order system [6–8]. Firstly, FOSs focus on the characteristics of the whole time domain. Secondly, FOSs show the properties of the whole space. Based on the above two characteristics, FOSs have played a significant role in the control field. Actually, the description of FOS is more consistent with the real dynamic system. It can be used to characterize more complex systems, and it also provides a guarantee for the genetic and memory properties of different substances. So, the application and theoretical research of FOSs will become more and more popular [9, 10].

The traditional integer-order theory is not suitable for FOSs because of its unique definition, so the researchers adopt two new methods to stabilize the FOS. The first solution is Lyapunov function, and the other is Laplace transformation to stabilize FOS [11–15]. In control analysis,

it is necessary to make the state trajectory of the system follow the desired command. In this study, many control methods are proposed to achieve good performance for noninteger-order systems. In addition, the researchers are also working to develop various control methods, such as sliding mode control [16–18], pinning control [19, 20], adaptive fuzzy control [21, 22], PID control [23, 24], and backstepping control [25, 26], to achieve the effective control. For example, an adaptive sliding mode control for FOS stabilization was studied in [27]. In [28], according to Lyapunov theory, the stability of the adaptive control method for the synchronous error system was analyzed. In [29], a class of global Mittag-Leffler stability problems with coupled FOSs was proposed. However, in modeling dynamics, the existence of nonlinearity makes it more difficult to deal with stability and other performance. To resolve the difficulties, many methods have been adopted to deal with these obstacles, among which the Takagi-Sugeno (T-S) model method is the most effective tool to describe the dynamic equations [30–32]. The most obvious advantage of this method is that it uses less fuzzy rules, and then the linear subsystems are fuzzy combined to get the overall model of

the system. The results of control analysis and stability analysis with the T-S control method are also reported in the literature [33–35]. In [33], under the condition of quantization, a dynamic output feedback controller was used to control the fuzzy T-S fractional-order neural network. A static feedback controller with fuzzy T-S structure was designed in [34]. The regularization and control of singular rectangular FOSs were considered in [35]. Although there are many reports about the stability of T-S systems, few studies on time-delay systems were published.

In fact, FOSs often contain uncertainty and time delays, which are two common phenomena [36–38]. Their existence will increase the instability of the model and make it more complex. Therefore, it will be more important to solve such problems in theoretical application. Even so, researchers have developed lots of methods to solve the uncertainties and time delay. Moreover, many studies focus on the stability of fractional-order time-delay systems (FOTDSs) with uncertainties, such as [37, 39, 40]. The recently developed immersion boundary method was used to model the complex solid boundary in [39]. A single input and output delay model was given in [37]. In [40], the control problem of the first-order dynamic discrete system was discussed. Among these methods, some of them are easier to control by using their unique characteristics [41, 42]. At the same time, the external disturbance in the time-delay system is inevitable; in other words, such system will become more complex. However, for our known knowledge, there are still some studies on the control of FOSs with disturbances [43–48].

Based on the inspiration of the above literature, this article focuses on T-S fuzzy control for uncertain FOTDSs. Aiming at uncertain FOSs, a fuzzy adaptive control strategy is established. At the same time, a fuzzy T-S system is constructed to characterize the system equation. Next, according to the adaptive control theory and Lyapunov function, simple linear control rules are designed to guarantee the asymptotic stability of the unknown system. Some adaptive laws for estimating unknown parameters of the system are given. Lastly, simulation results show the superiority of the T-S method. The purpose of this paper is to obtain the stability conditions by fractional Lyapunov functions, and the main contributions are as follows. (1) A suitable Lyapunov function is constructed, and the Lyapunov stability theory is used to ensure that the model is stable. (2) The controller obtained by using the linear structure can improve the stability of the FOTDS. (3) The adaptive laws are given to update the uncertainty of the model. These can enhance the control effect of the system and cause lower complexity.

Other structures of the article are as follows. The modeling of the T-S fuzzy system and some related concepts of fractional calculus are introduced in Section 2. Section 3 gives the stability analysis results of FOTDSs. Numerical simulations are shown in Section 4. Section 5 summarizes this full text.

2. Basic Knowledge and System Introduction

In the control process, some definitions of fractional derivative are stated. Since the solutions of differential equations are related to the initial conditions given by traditional

methods, Caputo derivative is useful in practical dynamics. In the article, we utilize Caputo's fractional derivative.

2.1. Preliminaries γ

Definition 1 (see [49]). The γ th Caputo derivative of a function $h(t)$ is

$${}^C_{t_0} \mathbf{D}_t^\gamma h(t) = \frac{1}{\Gamma(n-\gamma)} \int_{t_0}^t \frac{h^{(n)}(t)}{(t-t)^\gamma} dt, \quad (1)$$

where $0 < \gamma < 1$.

Definition 2 (see [49]). The γ th Riemann Liouville integral of function $h(t)$ is

$${}_{t_0} \mathbf{I}_t^\gamma h(t) = {}_{t_0} \mathbf{D}_t^{-\gamma} h(t) = \frac{1}{\Gamma(\gamma)} \int_{t_0}^t \frac{h(t)}{(t-t)^{1-\gamma}} dt, \quad (2)$$

where $\Gamma(\cdot)$ represents Euler's function which is denoted by

$$\Gamma(\ell) = \int_0^\infty e^{-t} t^{\ell-1} dt. \quad (3)$$

Definition 3 (see [49]). The γ th Riemann–Liouville fractional derivative is

$${}^{RL}_{t_0} \mathbf{D}_t^\gamma h(t) = \frac{1}{\Gamma(n-\gamma)} \frac{d^n}{dt^n} \int_{t_0}^t \frac{h(t)}{(t-t)^{\gamma-1}} dt, \quad (4)$$

where $\gamma \leq n \in \mathbf{Z}_+$.

Lemma 1 (see [50]). Let $\mathbf{F}(t) \in \mathbf{R}^{n \times n}$ be a differentiable matrix. So, for any $t \geq t_0$, one obtains

$${}^C_{t_0} \mathbf{D}_t^\gamma [\text{tr}\{\mathbf{F}^T(t)\mathbf{B}\mathbf{F}(t)\}] \leq 2\text{tr}\{\mathbf{F}^T(t)\mathbf{B} {}^C_{t_0} \mathbf{D}_t^\gamma \mathbf{F}(t)\}. \quad (5)$$

Lemma 2 (see [50]). Let $\mathbf{x}(t) \in \mathbf{R}^n$ be a derivative vector. Then, for any $t \geq t_0$,

$$\frac{1}{2} {}^C_{t_0} \mathbf{D}_t^\gamma [\mathbf{x}^T(t)\mathbf{B}\mathbf{x}(t)] \leq \mathbf{x}^T(t)\mathbf{B} {}^C_{t_0} \mathbf{D}_t^\gamma \mathbf{x}(t), \quad (6)$$

where $\mathbf{B} \in \mathbf{R}^{n \times n}$ is a positive definite symmetric constant matrix.

Defining an uncertain FOS as follows:

$${}^C_{t_0} \mathbf{D}_t^\gamma \mathbf{x}(t) = \mathbf{h}(t, \mathbf{x}, \mathbf{u}, \nabla), \quad (7)$$

where $\gamma \in (0, 1)$, t is the time, \mathbf{x} is state vector, \mathbf{u} represents control input, and ∇ denotes uncertain variable.

Lemma 3 (see [51]). If FOS (7) is Mittag-Leffler stable, then it will be asymptotically stable, i.e.,

$$\lim_{t \rightarrow \infty} \mathbf{x}(t) = 0. \quad (8)$$

Theorem 1 (see [51]). Let $\mathbf{x} = 0$ be an equilibrium point of system (7). Suppose there is a Lyapunov function $V(\mathbf{x}, t)$ and class- k function Θ_ι , $\iota = 1, 2, 3$, meeting

$$\begin{cases} \Theta_1(\|\mathbf{x}(t)\|), \leq V(\mathbf{x}, t) \leq \Theta_2(\|\mathbf{x}(t)\|), \\ {}^C_{t_0} \mathbf{D}_t^\gamma V(\mathbf{x}, t), \leq -\Theta_3(\|\mathbf{x}(t)\|), \end{cases} \quad (9)$$

where $\gamma \in (0, 1)$, so the original equilibrium point of system (7) is asymptotically stable.

2.2. Generalized T-S Fuzzy Model. Based on the expansion of the traditional T-S system, the generalized T-S system of FOS is derived. Then, the system dynamics can be expressed by the local fractional linear model. Finally, the linear composition is fuzzy mixed to get the whole model of the system. Consider the following generalized T-S fuzzy structure.

Rule 1: if $\kappa_1(t)$ is \mathfrak{R}_{11} , $\kappa_2(t)$ is \mathfrak{R}_{12}, \dots , and $\kappa_k(t)$ is \mathfrak{R}_{1k} , then

$$\mathbf{D}^\gamma \mathbf{x}(t) = \mathbf{A}_i \mathbf{x}(t), i = 1, 2, \dots, r, \quad (10)$$

where \mathfrak{R}_{ij} , $j = 1, 2, \dots, k$, denotes the fuzzy set, r represents the number of fuzzy rules, $\mathbf{A}_i \in \mathbf{R}^{n \times n}$ is a constant matrix, and $\mathbf{x}(t) \in \mathbf{R}^n$ and $\kappa(t) = [\kappa_1(t), \kappa_2(t), \dots, \kappa_k(t)]^T$ represent the state vector and premise variable, respectively.

The system output is inferred as

$$\mathbf{D}^\gamma \mathbf{x}(t) = \sum_{i=1}^r \vartheta_i(\kappa(t)) \cdot \mathbf{A}_i \mathbf{x}(t), \quad (11)$$

where

$$\vartheta_i(\kappa(t)) = \frac{v_i(\kappa(t))}{\sum_{i=1}^r v_i(\kappa(t))} = \frac{\prod_{j=1}^k \mathfrak{R}_{ij}(\kappa_j(t))}{\sum_{i=1}^r \prod_{j=1}^k \mathfrak{R}_{ij}(\kappa_j(t))}, \quad (12)$$

where $\mathfrak{R}_{ij}(\kappa_j(t))$ is a fuzzy membership function, satisfying

$$\begin{cases} v_i(\kappa(t)), & \geq 0, \\ \sum_{i=1}^r v_i(\kappa(t)), & > 0, \\ \vartheta_i(\kappa(t)), & \geq 0, \\ \sum_{i=1}^r \vartheta_i(\kappa(t)), & = 1. \end{cases} \quad (13)$$

3. Main Results

In the part, the stability of T-S fuzzy FOTDS with uncertainty is studied. And a fuzzy control method is given to control uncertain FOTDS.

3.1. System Description. Suppose that model (7) with unknown terms and time delay is designed as follows:

Rule 1: if $\kappa_1(t)$ is \mathfrak{R}_{11} , $\kappa_2(t)$ is \mathfrak{R}_{12}, \dots , and $\kappa_k(t)$ is \mathfrak{R}_{1k} , then

$$\mathbf{D}^\gamma \mathbf{x}(t) = \mathbf{Q}_i \mathbf{x}(t) + \mathbf{J}_i \mathbf{x}(t - \iota) + \mathbf{Q}_{si} \mathbf{x}(t) + \mathbf{J}_{si} \boldsymbol{\eta}(t) + \mathbf{u}(t), \quad (14)$$

where $\mathbf{x}(t) \in \mathbf{R}^n$, $\kappa(t) = [\kappa_1(t), \kappa_2(t), \dots, \kappa_k(t)]^T$, and \mathfrak{R}_{ij} are the same as the above definitions. $\mathbf{x}(t - \iota) \in \mathbf{R}^n$ denotes the time-delay term, \mathbf{Q}_i and \mathbf{J}_i are some constant matrices, \mathbf{Q}_{si} and \mathbf{J}_{si} are unknown parameters, and $\boldsymbol{\eta}(t) \in \mathbf{R}^m$ and $\mathbf{u}(t) \in \mathbf{R}^n$ are disturbance and control input, respectively.

So, the final output is

$$\mathbf{D}^\gamma \mathbf{x}(t) = \sum_{i=1}^r \vartheta_i(\kappa(t)) \{ \mathbf{Q}_i \mathbf{x}(t) + \mathbf{J}_i \mathbf{x}(t - \iota) + \mathbf{Q}_{si} \mathbf{x}(t) + \mathbf{J}_{si} \boldsymbol{\eta}(t) \} + \mathbf{u}(t). \quad (15)$$

To achieve the asymptotic stability of system (15), a fuzzy adaptive controller is given as follows.

Control Rule 1: if $\kappa_1(t)$ is \mathfrak{R}_{11} , $\kappa_2(t)$ is \mathfrak{R}_{12}, \dots , and $\kappa_k(t)$ is \mathfrak{R}_{1k} , then

$$\mathbf{u}_i(t) = -(\mathbf{E}_i \mathbf{x}(t) + \mathbf{F}_i \mathbf{x}(t - \iota) + \widehat{\mathbf{Q}}_{si} \mathbf{x}(t) + \widehat{\mathbf{J}}_{si} \boldsymbol{\eta}(t)), \quad (16)$$

where $\widehat{\mathbf{Q}}_{si}$ and $\widehat{\mathbf{J}}_{si}$ are estimations of \mathbf{Q}_{si} and \mathbf{J}_{si} , which can be obtained by some adaptive laws, \mathbf{E}_i and \mathbf{F}_i are feedback gain matrices that can be given.

From equations (15) and (16), the fuzzy controller is obtained as

$$\mathbf{u}(t) = -\sum_{i=1}^r \vartheta_i(\kappa(t)) (\mathbf{E}_i \mathbf{x}(t) + \mathbf{F}_i \mathbf{x}(t - \iota) + \widehat{\mathbf{Q}}_{si} \mathbf{x}(t) + \widehat{\mathbf{J}}_{si} \boldsymbol{\eta}(t)). \quad (17)$$

The adaptive laws are

$$\mathbf{D}^\gamma \widehat{\mathbf{Q}}_{si} = \varrho_{si} \vartheta_i(\kappa(t)) \mathbf{x}(t) \mathbf{x}^T(t), \quad (18)$$

$$\mathbf{D}^\gamma \widehat{\mathbf{J}}_{si} = \varrho_{fi} \vartheta_i(\kappa(t)) \mathbf{x}(t) \boldsymbol{\eta}^T(t), \quad (19)$$

where ϱ_{si} and ϱ_{fi} are positive constant adaption gains.

So, system (15) with the control law (17) is inferred as

$$\begin{aligned} \mathbf{D}^{\nu} \mathbf{x}(t) &= \sum_{i=1}^r \vartheta_1(k(t)) \{ \mathbf{Q}_i \mathbf{x}(t) + \mathbf{J}_i \mathbf{x}(t-\iota) + \mathbf{Q}_{s_i} \mathbf{x}(t) + \mathbf{J}_{s_i} \boldsymbol{\eta}(t) \} \\ &\quad - \sum_{i=1}^r \vartheta_1(\boldsymbol{\kappa}(t)) \{ \mathbf{E}_i \mathbf{x}(t) + \mathbf{F}_i \mathbf{x}(t-\iota) + \widehat{\mathbf{Q}}_{s_i} \mathbf{x}(t) + \widehat{\mathbf{J}}_{s_i} \boldsymbol{\eta}(t) \} \\ &= \sum_{i=1}^r \vartheta_1(\boldsymbol{\kappa}(t)) \{ (\mathbf{Q}_i - \mathbf{E}_i) \mathbf{x}(t) + (\mathbf{J}_i - \mathbf{F}_i) \mathbf{x}(t-\iota) + (\mathbf{Q}_{s_i} - \widehat{\mathbf{Q}}_{s_i}) \mathbf{x}(t) + (\mathbf{J}_{s_i} - \widehat{\mathbf{J}}_{s_i}) \boldsymbol{\eta}(t) \}. \end{aligned} \quad (20)$$

Defining $\widetilde{\mathbf{Q}}_{s_i} = \mathbf{Q}_{s_i} - \widehat{\mathbf{Q}}_{s_i}$ and $\widetilde{\mathbf{J}}_{s_i} = \mathbf{J}_{s_i} - \widehat{\mathbf{J}}_{s_i}$, system (20) is reorganized into

$$\begin{aligned} \mathbf{D}^{\nu} \mathbf{x}(t) &= \sum_{i=1}^r \vartheta_1(\boldsymbol{\kappa}(t)) \{ (\mathbf{Q}_i - \mathbf{E}_i) \mathbf{x}(t) \\ &\quad + (\mathbf{J}_i - \mathbf{F}_i) \mathbf{x}(t-\iota) + \widetilde{\mathbf{Q}}_{s_i} \mathbf{x}(t) + \widetilde{\mathbf{J}}_{s_i} \boldsymbol{\eta}(t) \}. \end{aligned} \quad (21)$$

3.2. Stability Analysis. To verify the stability of system (21), a useful theorem is proposed.

Theorem 2. Consider the uncertain FOTDS (15). If model (15) is controlled by control laws (17)–(19), then the system will converge to origin asymptotically.

Proof. Consider the following Lyapunov function candidate

$$V(\mathbf{x}, \mathbf{Q}_{s_i}, \mathbf{J}_{s_i}) = \mathbf{x}^T(t) \mathbf{B} \mathbf{x}(t) + \sum_{i=1}^r \text{tr} \left(\frac{\widetilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \widetilde{\mathbf{Q}}_{s_i}}{\varrho_{s_i}} \right) + \sum_{i=1}^r \text{tr} \left(\frac{\widetilde{\mathbf{J}}_{s_i}^T \mathbf{B} \widetilde{\mathbf{J}}_{s_i}}{\varrho_{t_i}} \right), \quad (22)$$

where $\text{tr}(\mathbf{C})$ represents the trace of a matrix and \mathbf{B} is a symmetric positive definite matrix.

Using Lemmas 1 and 2, one obtains

$$\mathbf{D}^{\nu} V \leq 2 \mathbf{x}^T(t) \mathbf{B} \mathbf{D}^{\nu} \mathbf{x}(t) + 2 \sum_{i=1}^r \text{tr} \left(\frac{\widetilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^{\nu} \widetilde{\mathbf{Q}}_{s_i}}{\varrho_{s_i}} \right) + 2 \sum_{i=1}^r \text{tr} \left(\frac{\widetilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^{\nu} \widetilde{\mathbf{J}}_{s_i}}{\varrho_{t_i}} \right). \quad (23)$$

By utilizing equation (21) and $\mathbf{D}^{\nu} \widetilde{\mathbf{Q}}_{s_i} = -\mathbf{D}^{\nu} \widehat{\mathbf{Q}}_{s_i}$ and $\mathbf{D}^{\nu} \widetilde{\mathbf{J}}_{s_i} = -\mathbf{D}^{\nu} \widehat{\mathbf{J}}_{s_i}$, one has

$$\begin{aligned} \mathbf{D}^{\nu} V &\leq 2 \mathbf{x}^T(t) \mathbf{B} \cdot \sum_{i=1}^r \vartheta_1(\boldsymbol{\kappa}(t)) \{ (\mathbf{Q}_i - \mathbf{E}_i) \mathbf{x}(t) + (\mathbf{J}_i - \mathbf{F}_i) \mathbf{x}(t-\iota) + \widetilde{\mathbf{Q}}_{s_i} \mathbf{x}(t) + \widetilde{\mathbf{J}}_{s_i} \boldsymbol{\eta}(t) \} \\ &\quad + 2 \sum_{i=1}^r \text{tr} \left(\frac{\widetilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot (-\mathbf{D}^{\nu} \widehat{\mathbf{Q}}_{s_i})}{\varrho_{s_i}} \right) + 2 \sum_{i=1}^r \text{tr} \left(\frac{\widetilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot (-\mathbf{D}^{\nu} \widehat{\mathbf{J}}_{s_i})}{\varrho_{t_i}} \right). \end{aligned} \quad (24)$$

According to the equations

$$\begin{cases} \mathbf{x}^T(t) \mathbf{B} \mathbf{C} \mathbf{x}(t) = (\mathbf{x}^T(t) \mathbf{B} \mathbf{C} \mathbf{x}(t))^T = \mathbf{x}^T(t) \mathbf{C}^T \mathbf{B} \mathbf{x}(t), \\ 2 \mathbf{x}^T(t) \mathbf{B} \mathbf{C} \mathbf{x}(t) = \mathbf{x}^T(t) (\mathbf{C}^T \mathbf{B} + \mathbf{B} \mathbf{C}) \mathbf{x}(t), \end{cases} \quad (25)$$

one has

$$\begin{aligned}
\mathbf{D}^y V &\leq \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) (2\mathbf{x}^T(t) \mathbf{B}(\mathbf{Q}_i - \mathbf{E}_i) \mathbf{x}(t)) + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) (2\mathbf{x}^T(t-i) \mathbf{B}(\mathbf{J}_i - \mathbf{F}_i) \mathbf{x}(t-i)) \\
&\quad + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) (2\mathbf{x}^T(t) \mathbf{B} \tilde{\mathbf{Q}}_{s_i} \mathbf{x}(t)) + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) (2\mathbf{x}^T(t) \mathbf{B} \tilde{\mathbf{J}}_{s_i} \boldsymbol{\eta}(t)) - 2 \sum_{i=1}^r \text{tr} \left(\frac{\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{Q}}_{s_i}}{\varrho_{s_i}} \right) - 2 \sum_{i=1}^r \text{tr} \left(\frac{\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{J}}_{s_i}}{\varrho_{t_i}} \right) \\
&= \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t) ((\mathbf{Q}_i - \mathbf{E}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{Q}_i - \mathbf{E}_i)) \mathbf{x}(t) \} + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t-i) ((\mathbf{J}_i - \mathbf{F}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{J}_i - \mathbf{F}_i)) \mathbf{x}(t-i) \} \\
&\quad + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) (2\mathbf{x}^T(t) \mathbf{B} \tilde{\mathbf{Q}}_{s_i} \mathbf{x}(t)) + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) (2\mathbf{x}^T(t) \mathbf{B} \tilde{\mathbf{J}}_{s_i} \boldsymbol{\eta}(t)) - 2 \sum_{i=1}^r \text{tr} \left(\frac{\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{Q}}_{s_i}}{\varrho_{s_i}} \right) - 2 \sum_{i=1}^r \text{tr} \left(\frac{\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{J}}_{s_i}}{\varrho_{t_i}} \right).
\end{aligned} \tag{26}$$

For the vectors $\mathbf{x} \in \mathbf{R}^{n \times 1}$ and $\mathbf{y} \in \mathbf{R}^{n \times 1}$, $\mathbf{x}^T \mathbf{x} = \text{trace}(\mathbf{x} \mathbf{x}^T)$ and $\mathbf{x}^T \mathbf{y} = \text{trace}(\mathbf{x} \mathbf{y}^T)$ are right, i.e., $\mathbf{x}^T(t) \mathbf{B} \tilde{\mathbf{Q}}_{s_i} \mathbf{x}(t) = \text{tr}(\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \mathbf{x}^T(t))$ and $\mathbf{x}^T(t) \mathbf{B} \tilde{\mathbf{J}}_{s_i} \boldsymbol{\eta}(t) = \text{tr}(\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \boldsymbol{\eta}^T(t))$; inequality (26) is rewritten as

$$\begin{aligned}
\mathbf{D}^y V &\leq \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t) ((\mathbf{Q}_i - \mathbf{E}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{Q}_i - \mathbf{E}_i)) \mathbf{x}(t) \} + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t-i) ((\mathbf{J}_i - \mathbf{F}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{J}_i - \mathbf{F}_i)) \mathbf{x}(t-i) \} \\
&\quad + 2 \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \left(\text{tr}(\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \mathbf{x}^T(t)) \right) + 2 \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \left(\text{tr}(\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \boldsymbol{\eta}^T(t)) \right) \\
&\quad - 2 \sum_{i=1}^r \text{tr} \left(\frac{\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{Q}}_{s_i}}{\varrho_{s_i}} \right) - 2 \sum_{i=1}^r \text{tr} \left(\frac{\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{J}}_{s_i}}{\varrho_{t_i}} \right) \\
&= \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t) ((\mathbf{Q}_i - \mathbf{E}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{Q}_i - \mathbf{E}_i)) \mathbf{x}(t) \} + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t-i) ((\mathbf{J}_i - \mathbf{F}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{J}_i - \mathbf{F}_i)) \mathbf{x}(t-i) \} \\
&\quad + 2 \text{tr} \left\{ \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \left(\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \mathbf{x}^T(t) \right) - \sum_{i=1}^r \frac{\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{Q}}_{s_i}}{\varrho_{s_i}} \right\} + 2 \text{tr} \left\{ \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \left(\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \boldsymbol{\eta}^T(t) \right) - \sum_{i=1}^r \frac{\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot \mathbf{D}^y \tilde{\mathbf{J}}_{s_i}}{\varrho_{t_i}} \right\}.
\end{aligned} \tag{27}$$

Substituting equations (18) and (19) into equation (27), one obtains

$$\begin{aligned}
\mathbf{D}^y V &\leq \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t) ((\mathbf{Q}_i - \mathbf{E}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{Q}_i - \mathbf{E}_i)) \mathbf{x}(t) \} + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t-i) ((\mathbf{J}_i - \mathbf{F}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{J}_i - \mathbf{F}_i)) \mathbf{x}(t-i) \} \\
&\quad + 2 \text{tr} \left\{ \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \left(\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \mathbf{x}^T(t) \right) - \sum_{i=1}^r \frac{\tilde{\mathbf{Q}}_{s_i}^T \mathbf{B} \cdot \varrho_{s_i} \vartheta_i(\boldsymbol{\kappa}(t)) \mathbf{x}(t) \mathbf{x}^T(t)}{\varrho_{s_i}} \right\} \\
&\quad + 2 \text{tr} \left\{ \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \left(\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \mathbf{x}(t) \boldsymbol{\eta}^T(t) \right) - \sum_{i=1}^r \frac{\tilde{\mathbf{J}}_{s_i}^T \mathbf{B} \cdot \varrho_{t_i} \vartheta_i(\boldsymbol{\kappa}(t)) \mathbf{x}(t) \boldsymbol{\eta}^T(t)}{\varrho_{t_i}} \right\} \\
&= \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t) ((\mathbf{Q}_i - \mathbf{E}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{Q}_i - \mathbf{E}_i)) \mathbf{x}(t) \} + \sum_{i=1}^r \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{x}^T(t-i) ((\mathbf{J}_i - \mathbf{F}_i)^T \mathbf{B} + \mathbf{B}(\mathbf{J}_i - \mathbf{F}_i)) \mathbf{x}(t-i) \}.
\end{aligned} \tag{28}$$

So, $D^{\nu}V \leq 0$ is equivalent to

$$\begin{cases} (\mathbf{Q}_1 - \mathbf{E}_1)^T \mathbf{B} + \mathbf{B}(\mathbf{Q}_1 - \mathbf{E}_1) < 0, \\ (\mathbf{J}_1 - \mathbf{F}_1)^T \mathbf{B} + \mathbf{B}(\mathbf{J}_1 - \mathbf{F}_1) < 0. \end{cases} \quad (29)$$

To sum up, if there exists \mathbf{E}_1 and \mathbf{F}_1 and a symmetric positive matrix \mathbf{B} such that inequality (29) holds, which shows that $D^{\nu}V \leq 0$ and the uncertain system (15) with time delay converges to zero asymptotically. The proof is over. \square

Remark 1. In a word, the fuzzy T-S system with Gauss membership function or other membership function can approach any a continuous function. It is worth emphasizing that compared with other literatures, such as [7, 27, 28, 52], in comparison with other techniques, this method has the advantage of the designed T-S fuzzy controller has a simple linear structure. In the simulation of this paper, a special membership function is adopted to control the FOTDS, it should be noted that due to the limitations of existing modeling methods, only this type of membership function is utilized for research.

Remark 2. Based on controller (17), the proportional relationship between $\mathbf{u}(t)$ and the parameters \mathbf{E}_1 and \mathbf{F}_1 can be obtained. In other words, the smaller the values of \mathbf{E}_1 and \mathbf{F}_1 , the smaller the control effect caused, and the reverse is also correct. In the adaptive laws (18) and (19), the sizes of ϱ_{s1} and ϱ_{t1} will also affect the update parameters \mathbf{E}_1 and \mathbf{F}_1 accordingly, and furthermore, it will also affect this control input $\mathbf{u}(t)$. Therefore, the greater the values of ϱ_{s1} and ϱ_{t1} , the greater the control impact caused. On the contrary, one can choose the appropriate ϱ_{s1} and ϱ_{t1} to make the control effect better. And according to inequality (29), \mathbf{E}_1 and \mathbf{F}_1 should be considered to achieve the asymptotic convergence of system (15).

4. Simulation Results

In the part, an example is provided to demonstrate the availability of the designed controller.

Consider the following uncertain T-S fuzzy FOS with time delay:

$$D^{\nu} \mathbf{x}(t) = \sum_{i=1}^2 \vartheta_i(\boldsymbol{\kappa}(t)) \{ \mathbf{Q}_i \mathbf{x}(t) + \mathbf{J}_i \mathbf{x}(t - \iota) + \mathbf{Q}_{s1} \mathbf{x}(t) \} + \mathbf{u}(t). \quad (30)$$

The fuzzy rules is as follows.

Rule 1: if $x_1(t)$ is $\mathfrak{R}_1(\mathbf{x}(t))$, then $D^{\nu} \mathbf{x}(t) = \mathbf{Q}_1 \mathbf{x}(t) + \mathbf{J}_1 \mathbf{x}(t - \iota) + \mathbf{Q}_{s1} \mathbf{x}(t) + \mathbf{u}(t)$

Rule 2: if $x_2(t)$ is $\mathfrak{R}_2(\mathbf{x}(t))$, then $D^{\nu} \mathbf{x}(t) = \mathbf{Q}_2 \mathbf{x}(t) + \mathbf{J}_2 \mathbf{x}(t - \iota) + \mathbf{Q}_{s2} \mathbf{x}(t) + \mathbf{u}(t)$

From the above, we can get that $\gamma = 0.9$, $\mathbf{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$ and $\mathbf{u}(t) = [u_1(t), u_2(t), u_3(t)]^T$.

The fuzzy membership functions are

$$\begin{aligned} \mathfrak{R}_1(\mathbf{x}(t)) &= \frac{1}{2} \left(1 + \frac{x_1(t)}{\delta} \right), \\ \mathfrak{R}_2(\mathbf{x}(t)) &= \frac{1}{2} \left(1 - \frac{x_1(t)}{\delta} \right). \end{aligned} \quad (31)$$

So, $\vartheta_i(\boldsymbol{\kappa}(t))$ as follows:

$$\begin{aligned} \vartheta_1(\boldsymbol{\kappa}(t)) &= \mathfrak{R}_1(\boldsymbol{\kappa}(t)) \times \mathbf{W}_1(\boldsymbol{\kappa}(t)), \\ \vartheta_2(\boldsymbol{\kappa}(t)) &= \mathfrak{R}_2(\boldsymbol{\kappa}(t)) \times \mathbf{W}_2(\boldsymbol{\kappa}(t)), \end{aligned} \quad (32)$$

where $\mathbf{W}_1(\boldsymbol{\kappa}(t)) = \mathbf{W}_2(\boldsymbol{\kappa}(t)) = 1$.

Let

$$\begin{aligned} \mathbf{Q}_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \delta & -\omega_2 & -\omega_3 \end{bmatrix}, \\ \mathbf{Q}_2 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\delta & -\omega_2 & -\omega_3 \end{bmatrix}, \\ \mathbf{Q}_{s1} = \mathbf{Q}_{s2} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \omega_2 & \omega_3 \end{bmatrix}, \\ \mathbf{J}_1 &= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ \delta & -\omega_2 & -\omega_3 \end{bmatrix}, \\ \mathbf{J}_2 &= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ -\delta & -\omega_2 & -\omega_3 \end{bmatrix}, \end{aligned} \quad (33)$$

where $\omega_1 = 5$, $\omega_2 = 3.85$, $\omega_3 = 2$, $\omega_4 = 2$, and $\delta = 21$.

Now, based on adaptive control laws (17)–(19), one obtains

$$\mathbf{u}(t) = - \sum_{i=1}^2 \vartheta_i(\boldsymbol{\kappa}(t)) (\mathbf{E}_i \mathbf{x}(t) + \mathbf{F}_i \mathbf{x}(t - \iota) + \widehat{\mathbf{Q}}_{si} \mathbf{x}(t)), \quad (34)$$

$$D^{\nu} \widehat{\mathbf{Q}}_{si} = \varrho_{si} \vartheta_i(\boldsymbol{\kappa}(t)) \mathbf{x}(t) \mathbf{x}^T(t), \quad (35)$$

where $\mathbf{E}_1 = \mathbf{E}_2 = \text{diag}(2, 2, 2)$, $\mathbf{F}_1 = \mathbf{F}_2 = \text{diag}(3, 3, 3)$, $\widehat{\mathbf{Q}}_{s1}$ is an estimation of \mathbf{Q}_{s1} , $\varrho_{s1} = 2$, $\iota = 1, 2$, and $\widehat{\mathbf{Q}}_{s1}(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Suppose $x_1(0) = -3$, $x_2(0) = 7$, and $x_3(0) = 12$. The simulation results are displayed in Figures 1–4. The time

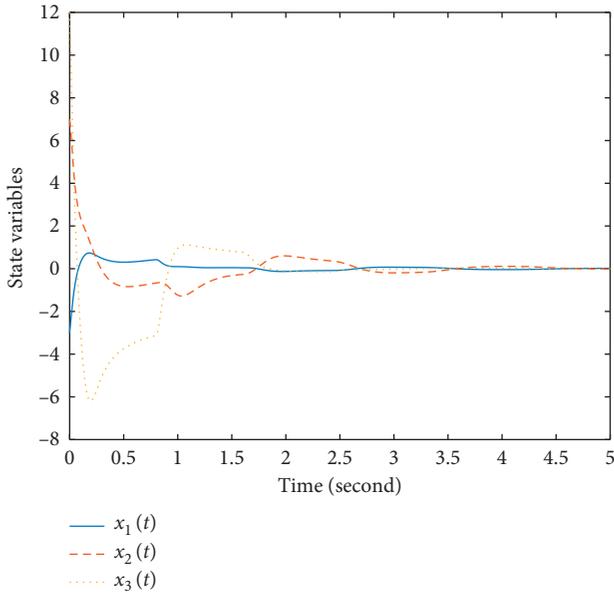


FIGURE 1: State trajectory of the closed-loop system.

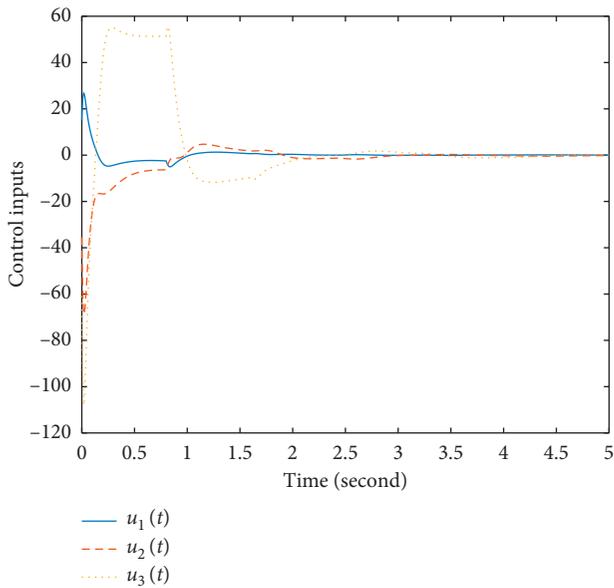


FIGURE 2: The control inputs of the close-loop system.

responses of $\mathbf{x}(t)$ are plotted in Figure 1. Obviously, the trajectories of $\mathbf{x}(t)$ converge to zero, which shows that the good control performance. Figure 2 describes the time responses of control input (34), which reveals that the controller can be implemented in dynamic applications. The time histories of the updated parameters $\hat{\mathbf{Q}}_{s1}$ and $\hat{\mathbf{Q}}_{s2}$ are described in Figures 3 and 4. Obviously, all adaption parameters reach a certain constant, which ensures that the internal stability of the model and they are bounded.

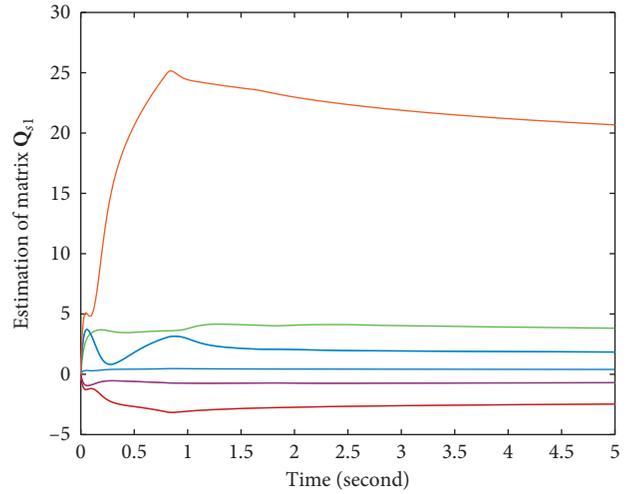


FIGURE 3: Estimated parameters for matrix \mathbf{Q}_{s1} .

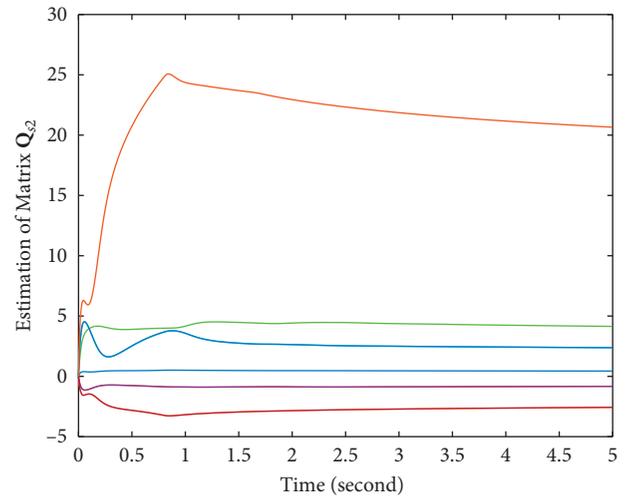


FIGURE 4: Estimated parameters for matrix \mathbf{Q}_{s2} .

5. Conclusions

This article studies the stability and control of uncertain FOTDSs by utilizing the T-S fuzzy structure. At the beginning, a model with if-then rules is developed to depict the differential equations of the system. Next, the fuzzy adaptive method is adopted to update the uncertainty and unknown parameters. Then, according to the Lyapunov function and T-S fuzzy control method, some control rules are adopted such that the system states tend to zero. Finally, the designed controller is used to stabilize FOTDSs. Simulation examples are proposed to verify the rationality of the devised adaptive control method. Our experimental results can not only enrich the control theory of FOSs but also can be implemented to other FOSs. Future research work will be devoted to adaptive control of incommensurate nonlinear FOSs.

Data Availability

All datasets generated for this study are included in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] S. Cheng, Y. Wei, D. Sheng, Y. Chen, and Y. Wang, "Identification for hammerstein nonlinear armax systems based on multi-innovation fractional order stochastic gradient," *Signal Processing*, vol. 142, pp. 1–10, 2018.
- [2] I. R. Birs, C. I. Muresan, I. Nascu, and C. M. Ionescu, "A survey of recent advances in fractional order control for time delay systems," *IEEE Access*, vol. 7, pp. 30951–30965, 2019.
- [3] C. Flores and V. Milanés, "Fractional-order-based acc/cacc algorithm for improving string stability," *Transportation Research Part C: Emerging Technologies*, vol. 95, pp. 381–393, 2018.
- [4] M.-R. Chen, G.-Q. Zeng, Y.-X. Dai, K.-D. Lu, and D.-Q. Bi, "Fractional-order model predictive frequency control of an islanded microgrid," *Energies*, vol. 12, no. 1, p. 84, 2018.
- [5] Z. Yu, Y. Zhang, Z. Liu, Y. Qu, and C.-Y. Su, "Distributed adaptive fractional-order fault-tolerant cooperative control of networked unmanned aerial vehicles via fuzzy neural networks," *IET Control Theory & Applications*, vol. 13, no. 17, pp. 2917–2929, 2019.
- [6] S. He, K. Sun, H. Wang, X. Mei, and Y. Sun, "Generalized synchronization of fractional-order hyperchaotic systems and its dsp implementation," *Nonlinear Dynamics*, vol. 92, no. 1, pp. 85–96, 2018.
- [7] M. Lupupa and S. Hadjiloucas, "Fractional-order system identification and equalization in massive mimo systems," *IEEE Access*, vol. 8, pp. 86481–86494, 2020.
- [8] A. Boukhouima, K. Hattaf, E. M. Lotfi, M. Mahrouf, D. F. M. Torres, and N. Yousfi, "Lyapunov functions for fractional-order systems in biology: methods and applications," *Chaos, Solitons & Fractals*, vol. 140, Article ID 110224, 2020.
- [9] R. Li and D. Huang, "Stability analysis and synchronization application for a 4d fractional-order system with infinite equilibria," *Physica Scripta*, vol. 95, no. 1, Article ID 015202, 2019.
- [10] A. Yousefpour, H. Jahanshahi, J. M. Munoz-Pacheco, S. Bekiros, and Z. Wei, "A fractional-order hyper-chaotic economic system with transient chaos," *Chaos, Solitons & Fractals*, vol. 130, Article ID 109400, 2020.
- [11] L.-F. Wang, H. Wu, D.-Y. Liu, D. Boutat, and Y.-M. Chen, "Lur'e Postnikov Lyapunov functional technique to global Mittag-Leffler stability of fractional-order neural networks with piecewise constant argument," *Neurocomputing*, vol. 302, pp. 23–32, 2018.
- [12] A. Cima, A. Gasull, and F. Maosas, "A note on the lyapunov and period constants," *Qualitative Theory of Dynamical Systems*, vol. 19, no. 1, pp. 1–13, 2020.
- [13] Y.-C. Chang, N. Roohi, and S. Gao, "Neural lyapunov control," in *Advances in Neural Information Processing Systems*, pp. 3245–3254, 2019.
- [14] H. Isozaki and E. L. Korotyaev, *Inverse Spectral Theory*, Academic Press, Orlando, FL, USA, 2019.
- [15] G. Horváth, I. Horváth, S. A.-D. Almousa, and M. Telek, "Numerical inverse laplace transformation using concentrated matrix exponential distributions," *Performance Evaluation*, vol. 137, Article ID 102067, 2020.
- [16] J. S.-H. Tsai, J.-S. Fang, J.-J. Yan, M.-C. Dai, S.-M. Guo, and L.-S. Shieh, "Hybrid robust discrete sliding mode control for generalized continuous chaotic systems subject to external disturbances," *Nonlinear Analysis: Hybrid Systems*, vol. 29, pp. 74–84, 2018.
- [17] A. K. Behera, B. Bandyopadhyay, and X. Yu, "Periodic event-triggered sliding mode control," *Automatica*, vol. 96, pp. 61–72, 2018.
- [18] A. K. Behera and B. Bandyopadhyay, "Discrete event-triggered sliding mode control," *Advances in Variable Structure Systems and Sliding Mode Control—Theory and Applications*, pp. 289–304, 2018.
- [19] P. Delellis, F. Garofalo, and F. Lo Iudice, "The partial pinning control strategy for large complex networks," *Automatica*, vol. 89, pp. 111–116, 2018.
- [20] C. Huang, X. Lu, J. Zhou, H. Qian, and B. Qin, "Equilibrium-pinning control for complex networks with inter-node coupling strength saturation improvement," *International Journal of Modern Physics C*, vol. 31, no. 1, 2019.
- [21] L. Tan and J. Jiang, "Adaptive volterra filters for active control of nonlinear noise processes," *IEEE Trans Signal Process*, vol. 49, no. 8, pp. 1667–1676, 2019.
- [22] L. Fang, S. Ding, J. H. Park, and L. Ma, "Adaptive fuzzy control for nontriangular stochastic high-order nonlinear systems subject to asymmetric output constraints," *IEEE Transactions on Cybernetics*, 2020.
- [23] D. Son and H. Choi, "Iterative feedback tuning of the proportional-integral-differential control of flow over a circular cylinder," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 4, pp. 1385–1396, 2019.
- [24] G. Bejarano, J. A. Alfaya, D. Rodríguez, F. Morilla, and M. G. Ortega, "Benchmark for PID control of refrigeration systems based on vapour compression," *IFAC-PapersOnLine*, vol. 51, no. 4, pp. 497–502, 2018.
- [25] H. Liu, Y. Pan, J. Cao, H. Wang, and Y. Zhou, "Adaptive neural network backstepping control of fractional-order nonlinear systems with actuator faults," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 12, pp. 5166–5177, 2020.
- [26] X. Bu, G. He, and K. Wang, "Tracking control of air-breathing hypersonic vehicles with non-affine dynamics via improved neural back-stepping design," *ISA Transactions*, vol. 75, 2018.
- [27] H. Delavari and M. Mohadeszadeh, "Robust finite-time synchronization of non-identical fractional-order hyperchaotic systems and its application in secure communication," *IEEE/CAA Journal of Automatica Sinica*, vol. 6, pp. 228–235, 2019.
- [28] K. Khettab, S. Ladaci, and Y. Bensafia, "Fuzzy adaptive control of a fractional order chaotic system with unknown control gain sign using a fractional order Nussbaum gain," *IEEE/CAA Journal of Automatica Sinica*, vol. 6, no. 3, pp. 816–823, 2019.

- [29] A. Selvam, D. Baleanu, J. Alzabut, D. Vignesh, and S. Abbas, "On Hyers–Ulam Mittag-Leffler stability of discrete fractional Duffing equation with application on inverted pendulum," *Advances in Difference Equations*, vol. 2020, no. 1, pp. 1–15, 2020.
- [30] J. Zhou, Y. Zheng, Y. Xu, H. Liu, and D. Chen, "A heuristic t-s fuzzy model for the pumped-storage generator-motor using variable-length tree-seed algorithm-based competitive agglomeration," *Energies*, vol. 11, no. 4, p. 944, 2018.
- [31] L. Qiao and Y. Yang, "Fault-tolerant control for t-s fuzzy systems with sensor faults: application to a ship propulsion system," *Journal of the Franklin Institute*, vol. 355, no. 12, pp. 4854–4872, 2018.
- [32] Y. L. Wang, Q. L. Han, M. R. Fei, and C. Peng, "Network-based tcs fuzzy dynamic positioning controller design for unmanned marine vehicles," *IEEE Transactions on Cybernetics*, vol. PP, no. 9, pp. 1–14, 2018.
- [33] S. A. Karthick, R. Sakthivel, Y. K. Ma, S. Mohanpriya, and A. Leelamani, "Disturbance rejection of fractional-order t-s fuzzy neural networks based on quantized dynamic output feedback controller," *Applied Mathematics and Computation*, vol. 361, pp. 846–857, 2019.
- [34] Z. Xia, "Design of static output feedback controller for fractional-order t-s fuzzy system," *Mathematical Problems in Engineering*, vol. 2020, Article ID 7898109, 9 pages, 2020.
- [35] X. Zhang and Z. Zhao, "Normalization and stabilization for rectangular singular fractional order t-s fuzzy systems," *Fuzzy Sets and Systems*, vol. 381, pp. 140–153, 2020.
- [36] K. Arvo, K. Ullé, and T. Maris, "Realization of nonlinear time-delay inputoutput equations," *IEEE Control Systems Letters*, vol. 2, no. 3, pp. 369–374, 2018.
- [37] S. Gumussoy, "Coprime inner/outer factorization of SISO time-delay systems and fir structure of their optimal h-infinity controllers," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 9, 2020.
- [38] W. H. Kwon and P. G. Park, *Stability of Time-Delay Systems*, Springer Science & Business Media, Berlin, Germany, 2019.
- [39] J. Favier, A. Pinelli, and U. Piomelli, "Control of the separated flow around an airfoil using a wavy leading edge inspired by humpback whale flippers," *Comptes Rendus Mecanique*, vol. 340, no. 1C2, pp. 107–114, 2020.
- [40] D. Wang, D. Wang, and W. Wang, "Necessary and sufficient conditions for containment control of multi-agent systems with time delay," *Automatica*, vol. 103, pp. 418–423, 2019.
- [41] M. Li, H. Fan, Y. Xiang, Y. Li, and Y. Zhang, "Cryptanalysis and improvement of a chaotic image encryption by first-order time-delay system," *IEEE Multimedia*, 2018.
- [42] X. Li, X. Yang, and S. Song, "Lyapunov conditions for finite-time stability of time-varying time-delay systems," *Automatica*, vol. 103, pp. 135–140, 2019.
- [43] E. A. Newman, "Disturbance ecology in the anthropocene," *Frontiers in Ecology and Evolution*, vol. 7, p. 147, 2019.
- [44] H. Haes Alhelou, M. Hamedani Golshan, T. Njenda, and P. Siano, "Wams-based online disturbance estimation in interconnected power systems using disturbance observer," *Applied Sciences*, vol. 9, no. 5, p. 990, 2019.
- [45] Z. Zhu, J. Zhang, Z. Yang et al., "Continuous monitoring of land disturbance based on landsat time series," *Remote Sensing of Environment*, vol. 238, p. 111116, 2020.
- [46] O. Mofid, S. Mobayen, and M.-H. Khooban, "Sliding mode disturbance observer control based on adaptive synchronization in a class of fractional-order chaotic systems," *International Journal of Adaptive Control and Signal Processing*, vol. 33, no. 3, pp. 462–474, 2019.
- [47] S. Mobayen, "Optimal lmi-based state feedback stabilizer for uncertain nonlinear systems with time-varying uncertainties and disturbances," *Complexity*, vol. 21, no. 6, pp. 356–362, 2016.
- [48] M. Jafari and S. Mobayen, "Second-order sliding set design for a class of uncertain nonlinear systems with disturbances: an lmi approach," *Mathematics and Computers in Simulation*, vol. 156, pp. 110–125, 2019.
- [49] A. Atangana and J. F. Gomez-Aguilar, "Decolonisation of fractional calculus rules: breaking commutativity and associativity to capture more natural phenomena," *European Physical Journal Plus*, vol. 133, no. 4, p. 166, 2018.
- [50] T. Hu and Z. Lin, "Composite quadratic Lyapunov functions for constrained control systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 3, 2003.
- [51] A. Fernandez and D. Baleanu, "The mean value theorem and Taylor's theorem for fractional derivatives with Mittag-Leffler kernel," *Advances in Difference Equations*, vol. 2018, no. 1, p. 86, 2018.
- [52] Z. Li, L. Gao, W. Chen, and Y. Xu, "Distributed adaptive cooperative tracking of uncertain nonlinear fractional-order multi-agent systems," *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 1, pp. 292–300, 2020.