Hindawi
Journal of Mathematics
Volume 2021, Article ID 6648818, 9 pages
https://doi.org/10.1155/2021/6648818

Research Article

New Exact Traveling Wave Solutions of Fractional Time Coupled Nerve Fibers via Two New Approaches

Saud Owyed

Department of Mathematics, College of Sciences, University of Bisha, P.O. Box 344, Bisha 61922, Saudi Arabia

Correspondence should be addressed to Saud Owyed; saudowyed@gmail.com

Received 2 November 2020; Revised 23 December 2020; Accepted 16 January 2021; Published 27 January 2021

Academic Editor: Kottakkaran Sooppy Nisar

In this paper, we obtain new soliton solutions of one of the most important equations in biology (fractional time coupled nerve fibers) using two algorithm schemes, namely, $\exp(-\psi(\xi))$ expansion function method and $(\theta'(\xi)/\theta^2(\xi))$ expansion methods. The equation and the solution methods have free parameters which help to make the obtained solutions are dynamics and more readable for dealing with fractional parameter and the initial and boundary value problem. As a result, various new exact soliton solutions for the considered model are derived which include the hyperbolic, rational, and trigonometric functions, and other solutions are obtained. In addition, the obtained results proved that the used methods give better performance compared with existing methods in the literature.

1. Introduction

Differential equations attained great importance with several applications in nature and live environment [1–19]. Differential equations were widely used in the modeling and simulation of the biological system long time ago, for example, population dynamics and spreading and transmission of viruses. Nerve conduction is one of the most important phenomena in biological systems. Several studies have attempted to provide an interpretation to the nature of the nerve conduction. These studies started in the early last century in 1925 and seemed to indicate that local circuit currents were involved in the longitudinal propagation of activity [20–23]. This study presents a model of action potential propagation in bundles of myelinated nerve fibers. The nature of the conduction process on an isolated nerve axon is studied numerically and compared with the theoretical models [24]. Reutskiy et al. introduced a new model that combines the single-cable formulation of Goldman and Albus (1967) with a basic representation of the ephaptic interaction among the fibers. The conduction velocity (CV) behavior is investigated in the presence of various conductance parameters and temperatures [25].

Recently, the fractional calculus plays a major role in various fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematics, and chemical physics, and geochemistry [26–33] is the specific one which attracts our attention. Many effective methods for obtaining exact solutions of FNLEEs have been presented [34–42]. In the present work, we propose a system which is governed by a fractional order derivative. The fractional order derivative is a concept that has been known since the early 17th century [43–45]. The model studied here is the ephaptically coupled myelinated nerve fibers. The myelinated nerve fibers might be responsible for diagnostic dilemmas in cases of visual loss [46–48]. They allow an increase in the speed of a nerve impulse while decreasing the diameter of the nerve fiber. The first work concerning myelinated nerve fibers was developed by Rushton [49].

The primary content of the article is organized as follows. In section 2, the governing equations of the model are introduced. The properties of conformable fractional derivatives are given in Section 3. Two algorithms of the proposed analytical method, namely, $\exp(-\psi(\xi))$ expansion function method and $(\theta'(\xi)/\theta^2(\xi))$ expansion method for solving the reduced equation for fractional time coupled nerve fibers are introduced in Section 4. Finally, we briefly make a conclusion in Section 5.

2. Governing Model and Mathematical Analysis

To consider the equations for the coupled nerve fibers, consider their electrical analogy as in [50, 51]:

$$\begin{aligned}
&M_n^1 - M_{n+1}^1 = (R_i + R_0)N_n^1 + R_0\left[AN_n^2 + (1-A)N_{n-1}^2\right],\\
&M_n^2 - M_{n+1}^2 = (R_i + R_0)N_n^2 + R_0\left[AN_n^1 + (1-A)N_{n-1}^1\right],\\
&N_{n-1}^j - N_n^j = N_{C,n}^j + N_{ion,n}^j,\ j = 1,2.,
\end{aligned} \tag{1}$$

where n represents successive active nodes, M_n represents the voltage across the membrane, N_n represents the current flowing longitudinally through the fiber from node n to node $n+1$, R_i and R_0 are the inside and outside resistances, $N_{C,n}$ is the current supplying the capacity of the active node n, while $N_{\text{ion},n}$ is the ionic current, comprising both sodium and potassium components. According to [51–53], the complex impedance of a capacitor reads.

$$Z_C = \frac{1}{C(jw)^{\alpha}}. \tag{2}$$

Then, the ionic current becomes

$$N_{C,n}^j = C\frac{d^{\alpha}M_n^j}{dt^{\alpha}}. \tag{3}$$

The ionic current is given by the following relation:

$$N_{\text{ion},n}^j = \frac{G}{M_b(M_b - M_a)}M_n^j\left(M_n^j - M_b\right)\left(M_n^j - M_b\right), \tag{4}$$

where M_b and M_a is the threshold voltage at sodium current and potential at the current returns to zero. By setting $v_n^j = (M_n^j/M_b)$, $i_n^j = R\ f\,(N_n^j/M_b)$, $a = (M_b/M_a)$, $r = R_i + R_j$, $\eta = (R_0/R)$, $D = (R_f/R)$, $\beta = R_f G(\ 1-a)$, equation (1) can be rewritten as

$$\begin{aligned}
&D\left[(1-\eta)(v_{n+1} - 2v_n + v_{n+1}) - \eta(w_{n+1} - 2w_n + w_{n+1})\right] - R_f C\frac{d^{\alpha}v_n}{dt^{\alpha}} - \beta v_n(v_n - a)(v_n - 1),\\
&D\left[(1-\eta)(w_{n+1} - 2w_n + w_{n+1}) - \eta(v_{n+1} - 2v_n + v_{n+1})\right] - R_f C\frac{d^{\alpha}w_n}{dt^{\alpha}} - \beta w_n(w_n - a)(w_n - 1),
\end{aligned} \tag{5}$$

where $v_n^1 = v_n$ and $v_n^2 = w_n$.

As long as δ_n tends to x, equation (5) admits to

$$\begin{aligned}
&D\left[(1-\eta)\delta^2\frac{d^2v}{dx^2} - \eta\delta^2\frac{d^2w}{dx^2}\right] - R_f C\frac{d^{\alpha}v}{dt^{\alpha}} - \beta v(v-a)(v-1),\\
&D\left[(1-\eta)\delta^2\frac{d^2w}{dx^2} - \eta\delta^2\frac{d^2v}{dx^2}\right] - R_f C\frac{d^{\alpha}w}{dt^{\alpha}} - \beta w(w-a)(w-1).
\end{aligned} \tag{6}$$

3. Conformable Fractional Derivatives and Its Properties

For a given χ: $(0,\infty) \longrightarrow R$, some definition and useful properties about conformable fractional derivatives can be written as follows [22, 23]:

$$\begin{aligned}
&w_{\alpha}(\chi)(t) = \lim_{\varepsilon\longrightarrow 0}\frac{\chi\left(t + \varepsilon t^{1-\alpha}\right) - \chi(t)}{\varepsilon}, \quad t > 0, 0 < \alpha < 1,\\
&w_{\alpha}(b\chi + ch) = Bw_{\alpha}(\chi) + Cw_{\alpha}(h), \quad B, C \in R,\\
&w_{\alpha}t^{\lambda} = \lambda t^{\lambda-\alpha}, \quad \lambda \in R,\\
&w_{\alpha}(\chi h) = \chi w_{\alpha}(h) + hw_{\alpha}(\chi),\\
&w_{\alpha}\left(\frac{\chi}{h}\right) = \frac{hw_{\alpha}(\chi) - \chi w_{\alpha}(h)}{h^2},
\end{aligned} \tag{7}$$

and $w_{\alpha}(\chi)(t) = t^{1-\alpha}(d\chi/dh)$.

In the limiting case χ: $(0,\infty) \longrightarrow R$, then

$$w_{\alpha}(\chi * h)(t) = t^{1-\alpha}h'(t)\chi\prime(h(t)). \quad (8)$$

4. Revisitation of the Two Analytical Techniques

Let us consider the general form of nonlinear evolution equation of fractal order as

$$\phi\left(v, D_t^{\alpha}v, D_x^{\beta}v, D_y^{\gamma}v, D_z^{\delta}, D_x^{\beta}D_t^{\alpha}v, D_x^{\beta}D_x^{\beta}v, \ldots\ldots\ldots\ldots\right) = 0, \quad 0 < \alpha, \beta, \delta < 1. \quad (9)$$

By using the traveling wave variable $v(x,y,t) = V(\xi)$, $\xi = k(x + y - (vt^{\alpha}/\alpha))$, equation (9) reduces to

$$\chi\left(V, V\prime, V\prime, V'', \ldots,\right) = 0, \quad (10)$$

where the prime denotes the differentiation with respect to ξ.

4.1. The $\exp(-\psi(\xi))$ *Expansion Function Method.* This method proposes that the solution of equation (9) can be written as

$$V(\xi) = \sum_{i=0}^{N} \alpha_i \exp(-\psi(\xi))^i, \quad (11)$$

and the constant α_i will be evaluated later and $\psi = \psi(\xi)$ verifies

$$\psi'(\xi) = \exp(-\psi(\xi)) + \mu\exp(\psi(\xi)) + \lambda, \quad (12)$$

where λ and μ are constants and will be computed with the flow of the paper. The integer N is determined by balancing between the highest order derivative. The solutions of equation (12) read.

Family 1. When $\mu \neq 0$ and $\Delta = \lambda^2 - 4\mu > 0$, then the hyperbolic function solution is

$$\psi_1(\xi) = \ln\left[\frac{-\Delta}{2\mu}\tanh\left(\frac{\Delta}{2}(\xi + c_1)\right) - \frac{\lambda}{2\mu}\right]. \quad (13)$$

Family 2. When $\mu \neq 0$ and $\Delta = \lambda^2 - 4\mu < 0$, then the trigonometric function solution is

$$\psi_2(\xi) = \ln\left[\frac{\Delta_1}{2\mu}\tan\left(\frac{\Delta_1}{2}(\xi + c_1)\right) - \frac{\lambda}{2\mu}\right]. \quad (14)$$

where $\Delta_1 = -\lambda^2 + 4\mu$.

Family 3. When $\mu = 0, \lambda \neq 0$, and $\Delta = \lambda^2 - 4\mu > 0$, then the hyperbolic function solution is

$$\psi_3(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + c_1) - 1)}\right). \quad (15)$$

Family 4. (rational function solutions). When $\mu \neq 0, \lambda \neq 0$, and $\Delta = \lambda^2 - 4\mu = 0$,

$$\psi_4(\xi) = \ln\left(-\frac{2\lambda(\xi + c_1) + 2}{\lambda^2(\xi + c_1)}\right). \quad (16)$$

Family 5. When $\mu = 0, \lambda = 0$, and $\Delta = \lambda^2 - 4\mu = 0$, then the rational function solution is

$$\psi_5(\xi) = \ln(\xi + c_1), \quad (17)$$

where c_1 is constant.

The parameters a_i, μ, λ, c, k, and δ can be found by inserting equations (11) and (12) in equation (9) and taking some approximations for the parameters of $\exp(-\psi(\xi))$ and making them equal to zero. This will produce a group of equations which can be solved to find equation parameters. The exact solution of equation (9) can be found by inserting the parameter values in equation (11).

4.2. The Extended $(\theta'(\xi)/\theta^2(\xi))$ *Expansion Method.* In view of the $(\theta'(\xi)/\theta^2(\xi))$ expansion method, the quick gain of this method is the solution of

$$V(\xi) = \sum_{i=1}^{N} \alpha_i \left(\frac{\theta'(\xi)}{\theta^2(\xi)}\right)^i, \quad (18)$$

$$\left(\frac{\theta'(\xi)}{\theta^2(\xi)}\right) = \mu + \lambda\left(\frac{\theta'(\xi)}{\theta^2(\xi)}\right)^2. \quad (19)$$

where λ and μ are constants to be determined later. It is to be noted that the solution of equation (19) is given as follows.

Family 6. As long as $\lambda\mu > 0$,

$$\frac{\theta'(\xi)}{\theta^2(\xi)} = \sqrt{\frac{\mu}{\lambda}}\frac{\left[k_1\cos\sqrt{\mu\lambda}\,\xi + k_2\sin\sqrt{\mu\lambda}\,\xi\right]}{\left[k_2\cos\sqrt{\mu\lambda}\,\xi - k_1\sin\sqrt{\mu\lambda}\,\xi\right]}. \quad (20)$$

Family 7. If $\lambda\mu < 0$, then

$$\frac{\theta'(\xi)}{\theta^2(\xi)} = -\frac{\sqrt{|\mu\lambda|}}{\lambda}\frac{\left[k_1\sinh\sqrt{|\mu\lambda|}\,\xi + k_2\cosh\sqrt{|\mu\lambda|}\,\xi\right]}{\left[k_1\sinh\sqrt{|\mu\lambda|}\,\xi + k_1\cosh\sqrt{|\mu\lambda|}\,\xi\right]}. \quad (21)$$

Family 8. In the limiting case $\lambda \neq 0$ and $\mu = 0$, we have

$$\frac{\theta'(\xi)}{\theta^2(\xi)} = -\frac{\xi_1}{\lambda(\xi_1 + \xi_2\xi)}, \quad (22)$$

where ξ_1 and ξ_2 are constants to be determined later.

Here, the term N in equation (19) can be computed taking into account the balancing between the highest order derivative and the nonlinear term in equation (10). Making use of equation (18) with equation (19) into equation (10), combining all the terms of the same power of $(w\prime(\xi)/w^2(\xi))$, and performing some steps, we can get the values of α_i, β_i, w, and k. By inserting these values in equation (18) along with general solutions of equation (19), the solutions of equation (9) can be obtained directly.

4.3. New Exact Solutions of the Reduced Equation via $\exp(-\psi(\xi))$ *Expansion Function Method.* To solve the reduced equation (6), via $\exp(-\psi(\xi))$ expansion, by using the wave transformation $\xi = k(x + y - (\nu t^{\alpha}/\alpha))$, equation (6) yields

$$\begin{gathered} -\nu k R_f C\frac{\mathrm{d}v}{\mathrm{d}\xi} - k^2\delta^2 D\left[(1-\eta)\frac{\mathrm{d}^2 v}{\mathrm{d}^2\xi}\right] + \beta\left[v^3 - (1+a)v^2 + av\right], \\ -\nu k R_f C\frac{\mathrm{d}w}{\mathrm{d}\xi} - k^2\delta^2 D\left[(1-\eta)\frac{\mathrm{d}^2 w}{\mathrm{d}^2\xi}\right] + \beta\left[w^3 - (1+a)w^2 + aw\right]. \end{gathered} \tag{23}$$

Considering the balance principle to equation (23), we obtain $N=1$. Then, the solution of equation (23) admits to

$$V(\xi) = a_0 + a_1\exp(-\psi(\xi)), \tag{24}$$

$$W(\xi) = b_0 + b_1\exp(-\psi(\xi)). \tag{25}$$

Inserting equations (25) and (24) into equation (23) with equation (12), collecting all power of $\exp(-\psi(\xi))$, and using symbolic computation program, it yields as follows.

Set 1

$$\begin{aligned} k &= k, \delta = \delta, \nu = \pm\left[\frac{-\left(-1/\left(-\lambda^2+4\mu\right)\right)^{1/2}(2\eta-1)D\delta^2 k\left(4\mu+4\mu a-\lambda^2-a\lambda^2\right)}{R_f C(a-1)}\right], b_0 = a_0, \\ a_0 &= \pm\left[\frac{a}{2} + \frac{\left(-1/\left(-\lambda^2+4\mu\right)\right)^{1/2}\lambda a}{2} - \frac{\left(-1/\left(-\lambda^2+4\mu\right)\right)^{1/2}\lambda}{2} + \frac{1}{2}\right], \\ a_1 &= b_1 = \pm\left[\left(-1/\left(-\lambda^2+4\mu\right)\right)^{1/2}(a-1)\right]. \end{aligned} \tag{26}$$

Set 2

$$\begin{aligned} k &= k, \nu = \frac{\sqrt{-1-\lambda^2+4\mu}(2\eta-1)D\delta^2 k\left(4\mu+4\mu a-\lambda^2-a\lambda^2\right)}{R_f C(a-1)}, \\ a_0 &= \pm\frac{a}{2} + \frac{\sqrt{\left(-1/-\lambda^2+4\mu\right)}\lambda a}{2} - \frac{\sqrt{\left(-1/-\lambda^2+4\mu\right)}\lambda}{2} + \frac{1}{2}, \\ \delta &= \delta, b_0 = a_0, b_1 = a_1 = \pm\left[\sqrt{\frac{-1}{-\lambda^2+4\mu}}(a-1)\right]. \end{aligned} \tag{27}$$

Substituting Set 1 into equations (23) and (24), it gains the new solutions as follows:

$$V_i(\xi) = \pm\left[\frac{a}{2} + \frac{\left(-1/\left(-\lambda^2+4\mu\right)\right)^{(1/2)}\lambda a}{2} - \frac{\left(-1/\left(-\lambda^2+4\mu\right)\right)^{(1/2)}\lambda}{2} + \frac{1}{2}\right]$$
$$\pm\left[\left(-1/\left(-\lambda^2+4\mu\right)\right)^{(1/2)}(a-1)\right]\exp(-\psi_i\xi), \quad i = 1,2,3,4,5,$$
$$W_i(\xi) = \pm\left[\frac{a}{2} + \frac{\left(-1/\left(-\lambda^2+4\mu\right)\right)^{(1/2)}\lambda a}{2} - \frac{\left(-1/\left(-\lambda^2+4\mu\right)\right)^{(1/2)}\lambda}{2} + \frac{1}{2}\right]$$
$$\pm\left[\left(-1/\left(-\lambda^2+4\mu\right)\right)^{(1/2)}(a-1)\right]\exp(-\psi_i\xi). \tag{28}$$

Substituting Set 2 into equations (25) and (24), it gains the new exact solution of equation (6) as follows:

$$V_i(\xi) = \pm\left[\frac{a}{2} + \frac{\sqrt{\left(-1/-\lambda^2+4\mu\right)}\lambda a}{2} - \frac{\sqrt{\left(-1/-\lambda^2+4\mu\right)}\lambda}{2} + \frac{1}{2}\right]$$
$$\pm\left[\sqrt{\frac{-1}{-\lambda^2+4\mu}}(a-1)\right]\exp(-\psi_i\xi), \quad i = 1,2,3,4,5, \tag{29}$$

$$W_i(\xi) = \pm\left[\frac{a}{2} + \frac{\sqrt{\left(-1/-\lambda^2+4\mu\right)}\lambda a}{2} - \frac{\sqrt{\left(-1/-\lambda^2+4\mu\right)}\lambda}{2} + \frac{1}{2}\right] \pm\left[\sqrt{\frac{-1}{-\lambda^2+4\mu}}(a-1)\right]\exp(-\psi_i\xi), \tag{30}$$

where $\psi_i(\xi), i = 1,\ldots,5$ is given in equations (13)–(17) and $\xi = k(x+y-(\nu t^\alpha/\alpha))$.

4.4. New Solutions for Reduced Equation (6) via Fractional $(\theta'(\xi)/\theta^2(\xi))$ Expansion Method. This section is devoted for obtaining the soliton solutions of equation (6) via the $(\theta'(\xi)/\theta^2(\xi))$. Considering the balance between w'' and v^3, we get $N=1$. Therefore, the solutions of equation (23) become

$$V(\xi) = a_0 + a_1\left[\frac{\theta'(\xi)}{\theta^2(\xi)}\right], \tag{31}$$

$$W(\xi) = c_0 + c_1\left[\frac{\theta'(\xi)}{\theta^2(\xi)}\right]. \tag{32}$$

Inserting equations (31) and (32) with equation (19) into equation (23) and collecting same power of $w\prime(\xi)/w^2(\xi)$, equating to zero, we have a set of algebraic equations. By solving it, we attain

$$k = \pm\frac{(\beta/(-8D\lambda\mu+16D\eta\lambda\mu))^{(1/2)}(-1+a)}{\delta},$$
$$\nu = \frac{1}{2}(1+a)(-1/4\lambda/\mu)^{(1/2)}\beta/R_f/C/(\beta/(-8D\lambda\mu+16D\eta\lambda\mu))^{(1/2)}\delta/\lambda,$$
$$c_0 = a_0 = \frac{1}{2}+\frac{a}{2},$$
$$a_1 = c_1 = \pm\sqrt{\frac{-\lambda}{4\mu}}(-1+a). \tag{33}$$

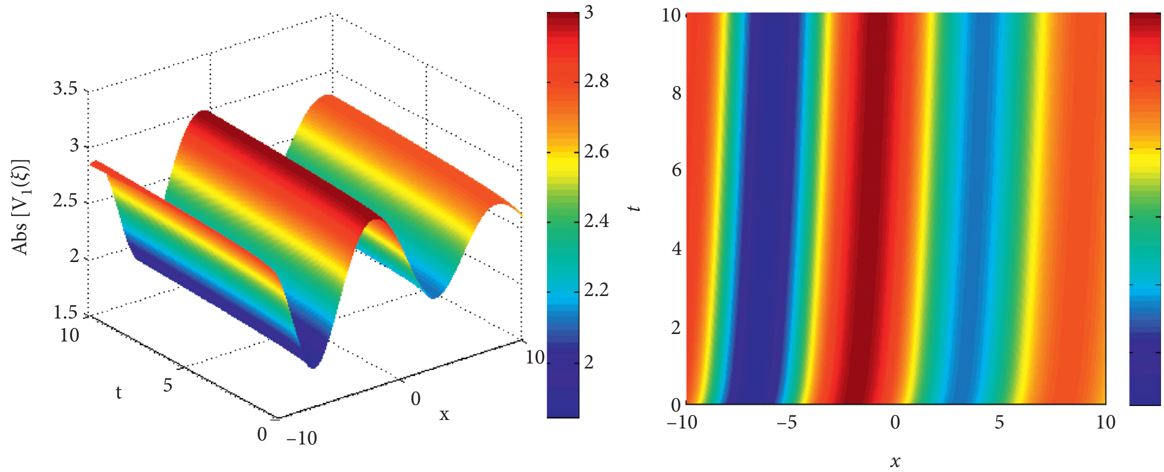

FIGURE 1: Soliton solutions of equation (29) in 3D and contour plots, where $y = 2$; $k = 0.2$; $v = 0.2$; $\alpha = 0.5$; $u = 3$; $\lambda = 4$; $a = 2$; and $c_1 = 2$.

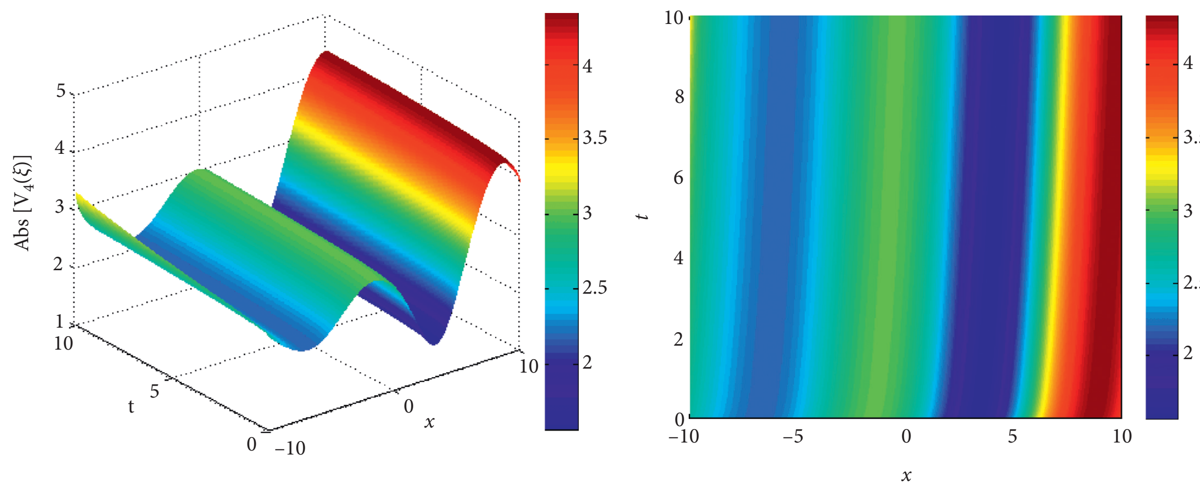

FIGURE 2: Soliton solutions of equation (29) in 3D and contour plots, where $y = 2$; $k = 0.2$; $v = 0.2$; $\alpha = 0.5$; $u = 3$; $\lambda = 4$; $a = 2$; $D = (\lambda^2) - (4u)$; and $c_1 = 2$.

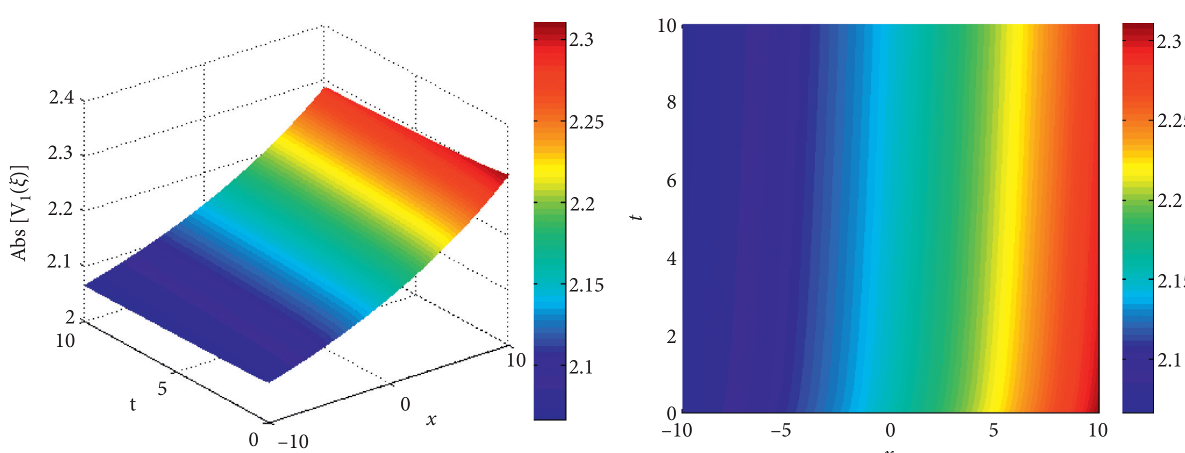

FIGURE 3: Soliton solutions of equation (34) in 3D and contour plots, where $y = 2$; $v = 0.2$; $a = 3$; $u = 0.2$; $k1 = 3$; $k2 = 4$; $k = 0.02$; $\lambda = 4$; and $\alpha = 0.5$.

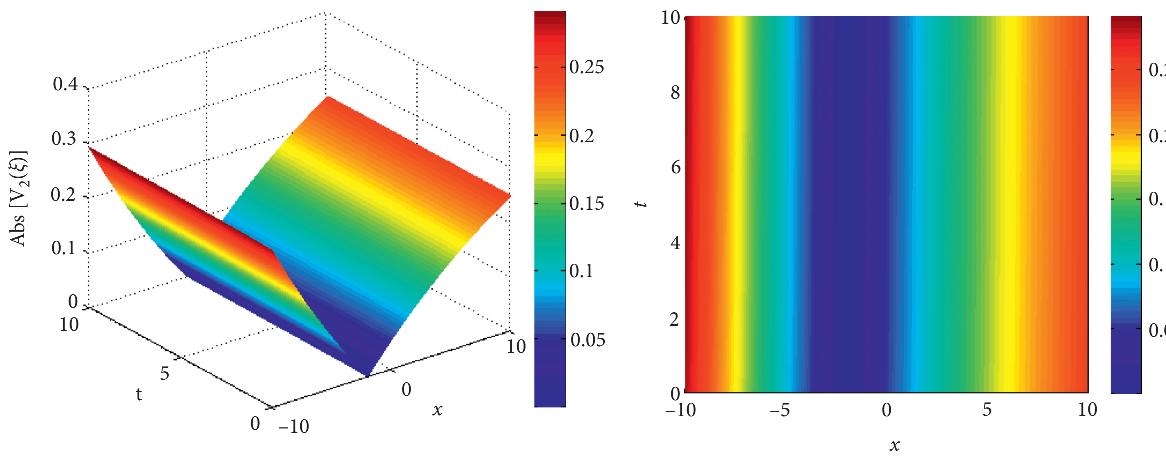

FIGURE 4: Soliton solutions of equation (34) in 3D and contour plots, where $y = 2$; $\nu = 0.2$; $a = 2$; $u = -0.5$; $k1 = 2$; $k2 = 6$; $k = 0.02$; $\lambda = 4$; and $\alpha = 0.5$.

Substituting equation (33) into equations (31) and (32), we get the new exact solutions of equation (6) as follows:

$$V_i(\xi) = \frac{1}{2} + \frac{a}{2} \pm \sqrt{\frac{-\lambda}{4\mu}}(-1 + a)\left[\frac{\theta'(\xi)}{\theta^2(\xi)}\right], \tag{34}$$

$$W_i(\xi) = \frac{1}{2} + \frac{a}{2} \pm \sqrt{\frac{-\lambda}{4\mu}}(-1 + a)\left[\frac{\theta'(\xi)}{\theta^2(\xi)}\right], \tag{35}$$

where $(\theta'(\xi)/\theta^2(\xi))$ is given by equations (20)–(22) and $\xi = k(x + y - (\nu t^\alpha/\alpha))$. It is worth noting that, in the limiting case $r = \mu$, $q = \lambda$, and $p = 0$, the results obtained here are the same obtained in [28].

5. Conclusion

In this paper, we introduced new solutions to one of the most important differential equations in biology. In this study, we have proposed a new model of myelinated nerve fibers described by a time fractional nonlinear evolution equation. Two algorithm schemes, namely, $\exp(-\psi(\xi))$ expansion function method and $(\theta'(\xi)/\theta^2(\xi))$ expansion method are used for constructing the new soliton solutions and other solutions such as hyperbolic function, trigonometric function, and rational function. To our knowledge, these new solutions have not been reported in former literature, and hence, they may be of significant importance for the explanation of some special physical phenomena. The results as in equations (29) and (34) are plotted in Figures 1–4. The obtained solutions here can be useful for applications in mathematical physics, engineering, and nonlinear optics. The achieved results here show the effectiveness and reliability of the proposed technique.

Data Availability

The data used to support the findings of this study are available within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] C. Ravichandran, K. Logeswari, S. K. Panda, and K. S. Nisar, "On new approach of fractional derivative by Mittag-Leffler kernel to neutral integro-differential systems with impulsive conditions," *Chaos, Solitons & Fractals*, vol. 139, Article ID 110012, 2020.

[2] M. M. A. Khater, R. A. M. Attia, A.-H. Abdel-Aty, W. Alharbi, and D. Lu, "Abundant analytical and numerical solutions of the fractional microbiological densities model in bacteria cell as a result of diffusion mechanisms, Chaos," *Solitons & Fractals*, vol. 136, Article ID 10982, 2020.

[3] A. Ahmed, L. Y. Cheong, N. Zakaria, N. Metwally, and H. Eleuch, "Statistical properties of a Raman three-level atom interacting with a cavity field," *AIP Conference Proceedings*, vol. 1482, pp. 373–375, 2012.

[4] M. M. A. Khater, R. A. M. Attia, A.-H. Abdel-Aty, M. A. Abdou, H. Eleuch, and D. Lu, "Analytical and semi-analytical ample solutions of the higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term," *Results in Physics*, vol. 16, Article ID 103000, 2020.

[5] C. Park, M. M. A. Khater, A.-H. Abdel-Aty, R. A. M. Attia, and D. Lu, "On new computational and numerical solutions of the modified Zakharov-Kuznetsov equation arising in electrical engineering," *Alexandria Engineering Journal*, vol. 59, no. 3, pp. 1099–1105, 2020.

[6] S. K. Panda, T. Abdeljawad, and C. Ravichandran, "Novel fixed point approach to Atangana-Baleanu fractional and Lp-Fredholm integral equations," *Alexandria Engineering Journal*, vol. 59, no. 4, pp. 1959–1970, 2020.

[7] D. Kumar, J. Singh, A. Prakash, and R. Swroop, "Numerical simulation for system of time-fractional linear and nonlinear differential equations," *Progress in Fractional Differentiation and Applications*, vol. 5, no. 1, pp. 65–77, 2019.

[8] M. Abdelkawy and I. G. Ameen, "A spectral collocation method for coupled system of two-dimensional abel integral equations of the second kind," *Information Sciences Letters*, vol. 8, pp. 89–93, 2019.

[9] G. Akram, Z. Elahi, and S. S. Siddiqi, "Laguerre approximations for system of linear pantograph differential equations," *Mathematical Sciences Letters*, vol. 7, pp. 125–131, 2018.

[10] D. Kumar, J. Singh, A. Prakash, R. Swroop, and R. Swroop, "Numerical simulation for system of time-fractional linear and nonlinear differential equations," *Progress in Fractional Differentiation and Applications*, vol. 5, no. 1, pp. 65–77, 2019.

[11] R. Cherniha, K. Gozak, and J. Waniewski, "Exact and numerical solutions of a spatially-distributed mathematical model for fluid and solute transport in peritoneal dialysis," *Symmetry*, vol. 8, no. 6, p. 50, 2016.

[12] A. K. Golmankhaneh, C. Tunc, S. M. Nia, and A. K. Golmankhaneh, "A review on local and non-local fractal calculus," *International Journal for Numerical Methods in Engineering*, vol. 1, pp. 19–31, 2019.

[13] T. Taiwo, "Four-parameter potential function with negative energy bound states," *Information Sciences Letters*, vol. 8, pp. 25–31, 2019.

[14] E. G. Tsega, "A finite volume solution of unsteady incompressible Navier-Stokes equations using MATLAB," *International Journal for Numerical Methods in Engineering*, vol. 1, p. 117, 2019.

[15] A. S. J. Al-Saif and M. S. Abdul-Wahab, "Application of new simulation scheme for the nonlinear biological population model," *International Journal for Numerical Methods in Engineering*, vol. 1, p. 89, 2019.

[16] O. Tasbozan, Y. Cenesiz, A. Kurt, and O. S. Iyiola, "New analytical solutions and approximate solution of the space-time conformable sharma-tasso-olver equation," *Progress in Fractional Differentiation and Applications*, vol. 4, pp. 519–531, 2018.

[17] M. M. A. Khater, R. A. M. Attia, and A.-H. Abdel-Aty, "Computational analysis of a nonlinear fractional emerging telecommunication model with higher–order dispersive cubic–quintic," *Information Sciences Letters*, vol. 9, pp. 83–93, 2020.

[18] A.-H. Abdel-Aty, A. N. Khedr, Y. B. Youssef, and Y. B. Saddeek, "Thermal entanglement in quantum annealing processor," *International Journal of Quantum Information*, vol. 16, no. 01, Article ID 1850006, 2018.

[19] S. Owyed, A. Abdel-Aty, M. Mabrok, and N. Zakaria: Mathematical modeling and simulation of 3-qubits quantum annealing processor. In Proceedings of the 2019 2nd International Conference on Mathematics and Statistics (ICoMS'19). Association for Computing Machinery, New York, NY, USA, (2019).

[20] R. Subashini, K. Jothimani, K. S. Nisar, and C. Ravichandran, "New results on nonlocal functional integro-differential equations via Hilfer fractional derivative," *Alexandria Engineering Journal*, vol. 59, no. 5, pp. 2891–2899, 2020.

[21] K. Jothimani, K. Kaliraj, Z. Hammouch, and C. Ravichandran, "New results on controllability in the framework of fractional integrodifferential equations with nondense domain," *The European Physical Journal Plus*, vol. 134, p. 441, 2019.

[22] K. Jothimani, K. Kaliraj, K. Kaliraj, S. Kumari Panda, K. Sooppy Nisar, and C. Ravichandran, "Results on controllability of non-densely characterized neutral fractional delay differential system," *Evolution Equations & Control Theory*, vol. 134, no. 9, 2019.

[23] S. Binczak et al., "Ephaptic coupling of myelinated nerve fibers," *Physica D: Nonlinear Phenomena*, vol. 148, no. 1-2, pp. 159–174, 2001.

[24] S. Reutskiy, E. Rossoni, and B. Tirozzi, "Conduction in bundles of demyelinated nerve fibers: computer simulation," *Biological Cybernetics*, vol. 89, no. 6, pp. 439–448, 2003.

[25] J. Bell, "Modeling parallel, unmyelinated axons: pulse trapping and ephaptic transmission," *SIAM Journal on Applied Mathematics*, vol. 41, no. 1, pp. 168–180.

[26] R. Hirota and K. Suzuki, "Theoretical and experimental studies of lattice solitons in nonlinear lumped networks," *Proceedings of the IEEE*, vol. 61, no. 10, pp. 1483–1491, 1973.

[27] H. Nagashima and Y. Amagishi, "Experiment on the toda lattice using nonlinear transmission lines," *Journal of the Physical Society of Japan*, vol. 45, no. 2, pp. 680–688, 1978.

[28] S. I. Mostafa, "Analytical study for the ability of nonlinear transmission lines to generate solitons," *Chaos, Solitons & Fractals*, vol. 39, no. 5, pp. 2125–2132, 2009.

[29] A. Saidou, M. Alidou, D. Ousmanou, and Y. D. Serge, "Exact Solutions of the nonlinear differential difference equations associated with the nonlinear electrical transmission line through a variable-coefficient discrete (G/G)- expansion method," *Chinese Physics B*, vol. 23, pp. 1205-1206, 2014.

[30] Y. D. Serge, "Propagation of dark solitary waves in the Korteveg–Devries– Burgers equation describing the nonlinear RLC transmission," *International Journal of Modern Physics*, vol. 5, pp. 394–401, 2014.

[31] A. Ehsan and H. Ali, "Nonlinear transmission lines for pulse shaping in silicon," *IEEE Journal of Solid State Circuits*, vol. 40, pp. 744–752, 2005.

[32] B. P. Franois, C. K. Timeleon, F. Nikolas, and R. Michel, "Wave modulations in the nonlinear biinductance transmission line," *Journal of the Physical Society of Japan*, vol. 70, pp. 2568–2577, 2001.

[33] Y. David and K. Fabien, "Compact envelope dark solitary wave in a discrete nonlinear electrical transmission line," *Physics Letters*, vol. 373, pp. 3801–3809, 2008.

[34] F. Kazuhiro, W. Miki, and N. Yoshimasa, "Envelope soliton in a new nonlinear transmission line," *Journal of the Physical Society of Japan*, vol. 49, pp. 1593–1597, 1980.

[35] M. Patrick, J. M. Bilbault, and M. Remoissenet, "Observation of nonlinear localized modes in an electrical lattice," *Physical Review E*, vol. 5, pp. 6127–6133, 1995.

[36] K. Fabien, B. N. Guy, Y. David, and F. Anaclet, "Nonlinearsupratransmission in a discrete nonlinear electrical," *Chaos Solitons Fractals*, vol. 75, pp. 263–271, 2015.

[37] A. B. T. Motcheyo, C. Tchawoua, S. M. Siewe, and J. D. Tchinang, "Supratransmission phenomenon in a discrete electrcal latticewith nonlineardispersion," *Communications in Nonlinear Science and Numerical*, vol. 18, pp. 946–952, 2013.

[38] S. A. El-Wakil and M. A. Abdou, "New exact travelling wave solutions of two nonlinear physical models," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 68, no. 2, pp. 235–245, 2008.

[39] M. A. Abdou, "The extended F-expansion method and its application for a class of nonlinear evolution equations," *Chaos, Solitons & Fractals*, vol. 31, no. 1, pp. 95–104, 2007.

[40] M. A. Abdou, "Further improved F-expansion and new exact solutions for nonlinear evolution equations," *Nonlinear Dynamics*, vol. 52, no. 3, pp. 277–288, 2007.

[41] M. A. Abdou and S. Zhang, "New periodic wave solutions via extended mapping method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 1, pp. 2–11, 2009.

[42] S. .A. El-Wakil and M. A. Abdou, "New applications of the homotopy analysis method," *Zeitschrift Fur Naturforschung*, vol. 63A, pp. 1–8, 2008.

[43] M. A. Abdou, "On the variational iteration method," *Physics Letters A*, vol. 366, no. 1-2, pp. 61–68, 2007.

[44] M. A. Abdou and A. Elhanbaly, "Decomposition method for solving a system of coupled fractional-time nonlinear equations," *Physica Scripta*, vol. 73, no. 4, pp. 338–348, 2006.

[45] J.-H. He and M. A. Abdou, "New periodic solutions for nonlinear evolution equations using Exp-function method," *Chaos, Solitons & Fractals*, vol. 34, no. 5, pp. 1421–1429, 2007.

[46] M. A. Abdou, "Generalized solitary and periodic solutions for nonlinear partial differential equations by the Exp-function method," *Nonlinear Dynamics*, vol. 52, no. 1-2, pp. 1–9, 2008.

[47] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh, "A new definition of fractional derivative," *Journal of Computational and Applied Mathematics*, vol. 264, pp. 65–70, 2014.

[48] M. Eslami and H. Rezazadeh, "The first integral method for Wu-Zhang system with conformable time-fractional derivative," *Calcolo*, vol. 53, no. 3, pp. 475–485, 2016.

[49] A. Grzybowski and I. Winiarczyk, "Myelinated retinal nerve fibers (MRNF) - dilemmas related to their influence on visual function," *Saudi Journal of Ophthalmology*, vol. 29, no. 1, pp. 85–88, 2015.

[50] W. A. H. Rushton, "A theory of the effects of fibre size in medullated nerve," *The Journal of Physiology*, vol. 115, no. 1, pp. 101–122, 1951.

[51] E. Tala-Tebue, H. Rezazadeh, M. Eslami, and A. Bekir, "New Approach to Model Coupled Nerve Fibers and Exact Solutions of the System," *Chinese Journal of Physics*, vol. 62, pp. 179–186.

[52] A. K. Jonscher, *Dielectric Relaxation in Solids*, Chelsea Dielectric Press, London, UK, 1993.

[53] I. Maina, Discrete Impulses in Ephaptically Coupled Nereve Fibers, Chaos 25, 2015.