Research Article

More Constructions of 3-Weight Linear Codes

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Linear codes with few weights have become an interesting research topic and important applications of cryptography and coding theory. In this paper, we apply some ternary near-bent and 2-plateaued functions or r -ary functions to construct more 3-weight linear codes, where r is a prime. Moreover, we determine the weight distributions of the resulted linear codes by means of some exponential sums.

1. Introduction

Let r be a prime and $q = r^m$. If $\mathcal{C}$ is a k -dimensional subspace of $\mathbb{F}_r^n$, then $\mathcal{C}$ is called an $[n, k]$ linear code over $\mathbb{F}_r$. A code $\mathcal{C}$ is called an $[n, k, d]$ code if d is the minimum nonzero weight in $\mathcal{C}$. Let B_j be the number of code words of weight j in $\mathcal{C}$. The weight enumerator of $\mathcal{C}$ is defined by

$$1 + B_1x + B_2x^2 + \cdots + B_nx^n. \quad (1)$$

The list B_j for $1 \leq j \leq n$ is called the weight distribution of $\mathcal{C}$. In general, it is difficult to determine the weight distribution of a given linear code $\mathcal{C}$. Information on the weight distribution of linear codes could be seen in [1–9]. If the number of nonzero B_j in $(1, B_1, B_2, \dots, B_n)$ is equal to t , then $\mathcal{C}$ is called a t -weight code. Linear codes with few weights have been studied due to their importance in communication and combinatorial theory [10–15]. If the parameters of $\mathcal{C}$ meet a bound on linear codes [16], then $\mathcal{C}$ is called optimal.

For a subset $S = \{s_1, s_2, \dots, s_n\}$ of $\mathbb{F}_q$, we define

$$\mathcal{C}_S = \{c_a = (\text{tr}(as_1), \text{tr}(as_2), \dots, \text{tr}(as_n)): a \in \mathbb{F}_q\}, \quad (2)$$

where $\text{tr}(y) = \sum_{i=0}^{s-1} y^{r^i}$ is the absolute trace function from $\mathbb{F}_{r^s}$ to $\mathbb{F}_r$. The set S is called the defining set of $\mathcal{C}_S$.

Many linear codes can be generated by choosing the suitable defining set S [14, 17–21]. In [22], a class of binary

linear codes with 3 weights was obtained by using the exponential sum $\sum_{y \in \mathbb{F}_{2^m}} (-1)^{\text{tr}(ay^{2^h+1}+by)}$. Recently, by using some r -ary bent functions, the authors obtained several classes of linear codes with 2 or 3 weights in [23–26].

The Walsh transform of $g(y)$ is defined by

$$\widehat{\chi}_g(\lambda) = \sum_{y \in \mathbb{F}_{r^m}} \zeta_r^{g(y) - \text{tr}(\lambda y)}, \quad \lambda \in \mathbb{F}_{r^m}, \quad (3)$$

where $\zeta_r = e^{2\pi\sqrt{-1}/r}$ is a primitive r -th root of unity.

Walsh transform is a basic tool to study the properties of cryptography functions. A function g from $\mathbb{F}_{r^m}$ to $\mathbb{F}_r$ is bent if $|\widehat{\chi}_g(\lambda)| = r^{m/2}$ for all $\lambda \in \mathbb{F}_{r^m}$. For an integer $0 \leq k \leq m$, g is called k -plateaued if $|\widehat{\chi}_g(\lambda)| \in \{0, r^{(m+k)/2}\}$ for all $\lambda \in \mathbb{F}_{r^m}$. Plateaued function can be used as an ideal candidate function for the design of the cryptographic system because it has a variety of ideal cryptographic characteristics [27].

It should be noted that binary k -plateaued functions exist only when m and k have the same parity. The case $k = 0$ corresponds to bent functions by definition. 1-plateaued functions are also called near-bent (see [28]), and binary 1-plateaued and 2-plateaued functions are called semibent [29].

Based on the aforementioned work, this paper makes further endeavors to construct 3-weight linear codes. We use a class of quadratic ternary near-bent and 2-plateaued functions to construct two classes of ternary 3-weight linear codes.

Moreover, the weight distributions of the resulted linear codes can be completely given. Numerical results indicate the proposed linear codes contain some linear codes with good parameters. By analyzing the values of some Weil sums, a class of r -ary linear codes with 3 weights is presented.

The organization of this paper is as follows. Section 2 gives two classes of ternary linear codes with 3 weights. In Section 3, a class of r -ary linear codes with 3 weights is provided by using some Weil sums. Conclusions are finally drawn in Section 4.

2. Ternary Linear Codes with 3 Weights from Some Ternary Functions

Throughout this section, let r be an odd prime and m a positive integer. Let $\mathbb{F}_{3^k}$ denote the finite field with 3^k elements and $\mathbb{F}_{3^k}^* = \mathbb{F}_{3^k} \setminus \{0\}$. In [24, 26], some r -ary linear codes with 2 or 3 weights have been obtained by exploiting r -ary bent functions. We give two constructions of ternary linear codes with 3 weights by utilizing some ternary functions in this section.

Let $g(x)$ be a function satisfying $g(-x) = g(x)$ and $g(0) = 0$. In this section, the defining set S_a of $\mathcal{C}_{S_a}$ is given by

$$S_a = \{x \in \mathbb{F}_{3^m}^* : g(x) = a\}, \quad (4)$$

where $a \in \mathbb{F}_3$. Denote $n_a = |S_a|$. Clearly, n_a is the length n of $\mathcal{C}_{S_a}$. We compute

$$\begin{aligned} n_a &= \frac{1}{3} \sum_{x \in \mathbb{F}_{3^m}^*} \sum_{y \in \mathbb{F}_3} z_3^{y(g(x)-a)} \\ &= \frac{1}{3} \sum_{x \in \mathbb{F}_{3^m}^*} \left(1 + \zeta_3^{-a} \zeta_3^{g(x)} + \zeta_3^a \zeta_3^{-g(x)} \right) \end{aligned} \quad (5)$$

$$= \begin{cases} 3^{m-1} - 1 + \frac{1}{3} (\widehat{\chi}_g(0) + \overline{\widehat{\chi}_g(0)}) & \text{if } a = 0, \\ 3^{m-1} + \frac{1}{3} (\zeta_3^{-a} \widehat{\chi}_g(0) + \zeta_3^a \overline{\widehat{\chi}_g(0)}) & \text{if } a \neq 0. \end{cases}$$

Note that $\text{wt}(\mathbf{c}_b)$ of $\mathbf{c}_b$ is $n_a - M_b$, where

$$M_b = |\{x \in \mathbb{F}_{3^m}^* : g(x) = a \text{ and } \text{tr}(bx) = 0\}|, \quad (6)$$

for each $b \in \mathbb{F}_{3^m}^*$. Note that $f(-x) = f(x)$. We have

$$\begin{aligned} M_b &= \frac{1}{3^2} \sum_{x \in \mathbb{F}_{3^m}^*} \left(\sum_{y \in \mathbb{F}_3} \zeta_3^{y(g(x)-a)} \right) \left(\sum_{z \in \mathbb{F}_3} \zeta_3^{z \text{tr}(bx)} \right) \\ &= \frac{1}{3^2} \sum_{x \in \mathbb{F}_{3^m}^*} \left(1 + \zeta_3^{-a} \zeta_3^{g(x)} + \zeta_3^a \zeta_3^{-g(x)} \right) \left(1 + \zeta_3^{\text{tr}(bx)} + \zeta_3^{-\text{tr}(bx)} \right) \\ &= \frac{1}{3^2} (3^m - 3 - 3(\zeta_3^{-a} + \zeta_3^a)) + \frac{1}{3^2} (\zeta_3^{-a} \widehat{\chi}_g(0) + \zeta_3^a \overline{\widehat{\chi}_g(0)} + 2(\zeta_3^{-a} \widehat{\chi}_g(b) + \zeta_3^a \overline{\widehat{\chi}_g(b)})) \\ &= \begin{cases} 3^{m-2} - 1 + \frac{1}{3^2} (\widehat{\chi}_g(0) + \overline{\widehat{\chi}_g(0)} + 2(\widehat{\chi}_g(b) + \overline{\widehat{\chi}_g(b)})) & \text{if } a = 0, \\ 3^{m-2} + \frac{1}{3^2} (\zeta_3^{-a} \widehat{\chi}_g(0) + \zeta_3^a \overline{\widehat{\chi}_g(0)} + 2(\zeta_3^{-a} \widehat{\chi}_g(b) + \zeta_3^a \overline{\widehat{\chi}_g(b)})) & \text{if } a \neq 0. \end{cases} \end{aligned} \quad (7)$$

Generally, it is difficult to give the Walsh coefficients of a ternary function. In the following, we shall select some proper ternary plateaued functions to obtain linear codes $\mathcal{C}_{S_0}$ with 3 weights over $\mathbb{F}_3$, where S_0 is defined as in (4).

Lemma 1 (see [30], Theorem 2). Assume that $k > 3$, $\lambda, u, v \in \mathbb{F}_{3^k}^*$, and η is the quadratic character of $\mathbb{F}_{3^k}$. Define

$$\widehat{\chi}_g(a) = \begin{cases} \eta(\lambda)(-1)^k i^{k+1} 3^{(k+1)/2} \zeta_3^{-\text{tr}(a^2/\lambda)}, & \text{if } \left(\text{tr}\left(\frac{au}{\lambda}\right), \text{tr}\left(\frac{av}{\lambda}\right) \right) = (0, 0), \\ \eta(\lambda)(-1)^k i^{k+1} 3^{(k+1)/2} \zeta_3^{-\text{tr}(a^2/\lambda)+1}, & \text{if } \left(\text{tr}\left(\frac{au}{\lambda}\right), \text{tr}\left(\frac{av}{\lambda}\right) \right) = (1, 1) \text{ or } (2, 2), \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

$$g(y) = \text{tr}(\lambda y^2) + \text{tr}(uy)\text{tr}(vy). \quad (8)$$

(i) If $\text{tr}(uv/\lambda) = 2$, $\text{tr}(u^2/\lambda) = 1$, and $\text{tr}(v^2/\lambda) = 1$, then $g(y)$ is near-bent. Moreover, for $a \in \mathbb{F}_{3^k}$,

(ii) If $\text{tr}(uv/\lambda) = 1$, $\text{tr}(u^2/\lambda) = 0$, and $\text{tr}(v^2/\lambda) = 0$, then $g(y)$ is a 2-plateaued function. Moreover, for $a \in \mathbb{F}_{3^m}$,

$$\hat{\chi}_g(a) = \begin{cases} \eta(\lambda)(-1)^{k-1} i^k 3^{(k/2)+1} \zeta_3^{-\text{tr}(a^2/\lambda)} & \text{if } \left(\text{tr}\left(\frac{au}{\lambda}\right), \text{tr}\left(\frac{av}{\lambda}\right) \right) = (0, 0), \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

The following theorem gives two classes of linear codes with 3 weights over $\mathbb{F}_3$.

Theorem 1. Let k be a positive integer with $k > 3$ and $\lambda \in \mathbb{F}_{3^k}^*$. Let $u, v \in \mathbb{F}_{3^k}^*$ such that $\text{tr}(\lambda^{-1}uv) = 2$, $\text{tr}(\lambda^{-1}u^2) = 1$, and $\text{tr}(\lambda^{-1}v^2) = 1$. Let g be defined as in (8) and S_0 be defined as in (4).

- (i) When k is even, $\mathcal{C}_{S_0}$ is a $[3^{k-1} - 1, k]$ code with 3 weights and has weight distribution given in Table 1
- (ii) When k is odd, $\mathcal{C}_{S_0}$ is a $[3^{k-1} + 2\delta\eta(\lambda)3^{(k-1)/2} - 1, k]$ code with 3 weights and has weight distribution given in Table 2, where $\delta = 1$ if $k \equiv 1 \pmod{4}$ and $\delta = -1$ if $k \equiv 3 \pmod{4}$

Proof.

Note that $\text{tr}(\lambda^{-1}uv) = 2$, $\text{tr}(\lambda^{-1}u^2) = 1$, and $\text{tr}(\lambda^{-1}v^2) = 1$. By Lemma 1 (i), we have $\hat{\chi}_g(0) + \overline{\hat{\chi}_g(0)} = 0$ as k is even. It follows from (5) that $n_0 = 3^{k-1} - 1$. Hence, the length n of $\mathcal{C}_{S_0}$ is $n_0 = 3^{k-1} - 1$.

For $a \in \mathbb{F}_{3^k}^*$, to calculate M_a , we need to calculate $\hat{\chi}_g(a) + \overline{\hat{\chi}_g(a)}$ from (7).

If $k \equiv 0 \pmod{4}$, then $i^{k+1} = i$. We evaluate $\hat{\chi}_g(a) + \overline{\hat{\chi}_g(a)}$ by distinguishing the following three cases.

When $(\text{tr}(au/\lambda), \text{tr}(av/\lambda)) = (0, 0)$, it follows from (9) that

$$\hat{\chi}_g(a) + \overline{\hat{\chi}_g(a)} = \begin{cases} 0 & \text{if } \text{tr}\left(\frac{a^2}{\lambda}\right) = 0, \\ \eta(\lambda)3^{(k+1)/2} & \text{if } \text{tr}\left(\frac{a^2}{\lambda}\right) = 1, \\ -\eta(\lambda)3^{(k+1)/2} & \text{if } \text{tr}\left(\frac{a^2}{\lambda}\right) = 2. \end{cases} \quad (11)$$

When $(\text{tr}(au/\lambda), \text{tr}(av/\lambda)) = (1, 1)$ or $(2, 2)$, it follows from (9) that

$$\hat{\chi}_g(a) + \overline{\hat{\chi}_g(a)} = \begin{cases} -\eta(\lambda)3^{(k+1)/2} & \text{if } \text{tr}\left(\frac{a^2}{\lambda}\right) = 0, \\ \eta(\lambda)3^{(k+1)/2} & \text{if } \text{tr}\left(\frac{a^2}{\lambda}\right) = 1, \\ 0 & \text{if } \text{tr}\left(\frac{a^2}{\lambda}\right) = 2. \end{cases} \quad (12)$$

When $(\text{tr}(au/\lambda), \text{tr}(av/\lambda)) \in \mathbb{F}_3^2 \setminus \{(0, 0), (1, 1), (2, 2)\}$,

$$\hat{\chi}_g(a) + \overline{\hat{\chi}_g(a)} = 0. \quad (13)$$

It then follows from (7) with $b = 0$ that

$$M_a \in \{3^{k-2} - 1, 3^{k-2} - 2\eta(\lambda)3^{(k-2)/2} - 1, 3^{k-2} + 2\eta(\lambda)3^{(k-2)/2} - 1\}. \quad (14)$$

Hence, the Hamming weight $\text{wt}(\mathbf{c}_a)$ of $\mathbf{c}_a$ satisfies

$$\text{wt}(\mathbf{c}_a) = n_0 - M_a \in \{2 \cdot 3^{k-2} - 2\eta(\lambda)3^{(k-2)/2}, 2 \cdot 3^{k-2}, 2 \cdot 3^{k-2} + 2\eta(\lambda)3^{(k-2)/2}\}. \quad (15)$$

This implies that the code $\mathcal{C}_{S_0}$ has 3 weights in the above set.

If $k \equiv 2 \pmod{4}$, then $i^{k+1} = -i$. Similar to the proof above, we obtain

$$\text{wt}(\mathbf{c}_a) = n_0 - M_a \in \{2 \cdot 3^{k-2} - 2\eta(\lambda)3^{(k-2)/2}, 2 \cdot 3^{k-2}, 2 \cdot 3^{k-2} + 2\eta(\lambda)3^{(k-2)/2}\}, \quad (16)$$

for each $a \in \mathbb{F}_{3^k}^*$. Since $g(-x) = 0$ if $g(x) = 0$ and $0 \notin S_0$, we have the minimum distance of $\mathcal{C}_{S_0}^\perp$ is 2. It is easy to see that the number $B_2^\perp$ of code words with weight 2 in $\mathcal{C}_{S_0}^\perp$ is n . The first three Pless power

TABLE 1: Weight distribution of $\mathcal{C}_{S_0}$ for Theorem 1 (i).

Weight	Frequency
0	1
$2 \cdot 3^{k-2} - 2\eta(\lambda)3^{(k/2)-1}$	$3^{k-2} + \eta(\lambda)3^{(k/2)-1}$
$2 \cdot 3^{k-2}$	$3^k - 1 - 2 \cdot 3^{k-2}$
$2 \cdot 3^{k-2} + 2\eta(\lambda)3^{(k/2)-1}$	$3^{k-2} - \eta(\lambda)3^{(k/2)-1}$

TABLE 2: Weight distribution of $\mathcal{C}_{S_0}$ for Theorem 1 (ii).

Weight	Frequency
0	1
$2 \cdot 3^{k-2} + 4\delta\eta(\lambda)3^{(k-3)/2}$	$2 \cdot 3^{k-1}$
$2 \cdot 3^{k-2}$	$3^{k-2} + 2\delta\eta(\lambda)3^{k-3/2} - 1$
$2 \cdot 3^{k-2} + 6\delta\eta(\lambda)3^{(k-3)/2}$	$2 \cdot 3^{k-2} - 2\delta\eta(\lambda)3^{(k-3)/2}$

moments give the desired weight distribution. Since $k > 3$, $\text{wt}(\mathbf{c}_a) > 0$ for any $a \in \mathbb{F}_{3^k}^*$. Hence, the dimension of $\mathcal{C}_{S_0}$ is k .

- (ii) Conclusion (ii) can be proved by the similar proof as (i). $\square$

Example 1. Let $k = 4$, $\lambda = 1$, $u = -1$, and $v = 1$. Define $g(y) = \text{tr}(y^2) - \text{tr}(y)^2$. Then, $\mathcal{C}_{S_0}$ in Theorem 1 (i) is a ternary $[26, 4, 12]$ code, and its weight enumerator is $1 + 12z^{12} + 62z^{18} + 6z^{24}$.

Theorem 2. Let k be a positive integer with $k > 4$ and $\lambda \in \mathbb{F}_{3^k}^*$. Let $u, v \in \mathbb{F}_{3^k}^*$ such that $\text{tr}(\lambda^{-1}uv) = 1$, $\text{tr}(\lambda^{-1}u^2) = 0$, and $\text{tr}(\lambda^{-1}v^2) = 0$. Let S_0 be defined as in (4) and f be defined as in (8).

- (i) When k is even, $\mathcal{C}_{S_0}$ is a $[3^{k-1} - 2\delta\eta(\lambda)3^{(k/2)} - 1, k]$ 3-weight code and has weight distribution listed in Table 3, where $\delta = 1$ if $k \equiv 0 \pmod{4}$ and $\delta = -1$ if $k \equiv 2 \pmod{4}$.
- (ii) When k is odd, $\mathcal{C}_{S_0}$ is a $[3^{k-1} - 1, k]$ 3-weight code and has weight distribution listed in Table 4

Proof. We just omit the proof here since it is similar to that of Theorem 1. $\square$

Remark 1. It should be noted that if λ is a nonsquare in $\mathbb{F}_{3^4}^*$, the first conclusion in Theorem 2 is also true when $k = 4$.

Example 2. Assume $k = 4$ and β is the generator of $\mathbb{F}_{3^4}^*$, where β is a root of $x^4 - x^3 - 1 = 0$. Let $\lambda = \beta$, $u = \beta^{16}$, and $v = \beta^8$. Define $g(x) = \text{tr}(\beta x^2) + \text{tr}(\beta^{16}x)\text{tr}(\beta^8x)$. Then, the code $\mathcal{C}_{S_0}$ in Theorem 2 (i) is $[44, 4, 18]$ code and has weight enumerator $1 + 4z^{18} + 72z^{30} + 4z^{36}$.

Since $g(x) = g(-x)$, the set S_0 defined by (4) can be expressed as

$$S_0 = -\bar{S} \cup \bar{S}, \quad (17)$$

where $d_i d_j^{-1} \neq \pm 1$ for $d_i, d_j \in \bar{S}$ and $d_i \neq d_j$. Selecting $\bar{D}$ as the defining set can give more ternary 3-weight codes.

TABLE 3: Weight distribution of $\mathcal{C}_{S_0}$ for Theorem 2 (i).

Weight	Frequency
0	1
$2 \cdot 3^{k-2} - 4\delta\eta(\lambda)3^{(k/2)-1}$	$3^k - 3^{k-2}$
$2 \cdot 3^{k-2}$	$3^{k-3} - 2\delta\eta(\lambda)3^{(k/2)-2} - 1$
$2 \cdot 3^{k-2} - 6\delta\eta(\lambda)3^{(k/2)-1}$	$2 \cdot 3^{k-3} + 2\delta\eta(\lambda)3^{(k/2)-2}$

TABLE 4: Weight distribution of $\mathcal{C}_{S_0}$ for Theorem 2 (ii).

Weight	Frequency
0	1
$2 \cdot 3^{k-2} - 2\eta(\lambda)3^{(k+1/2)-1}$	$3^{k-3} + \eta(\lambda)3^{(k-3/2)}$
$2 \cdot 3^{k-2}$	$3^k - 2 \cdot 3^{k-3} - 1$
$2 \cdot 3^{k-2} + 2\eta(\lambda)3^{(k+1/2)-1}$	$3^{k-3} - \eta(\lambda)3^{(k-3/2)}$

Corollary 1. Assume that $k > 3$ and $\lambda \in \mathbb{F}_{3^k}^*$. Let $u, v \in \mathbb{F}_{3^k}^*$ such that $\text{tr}(\lambda^{-1}uv) = 2$, $\text{tr}(\lambda^{-1}u^2) = 1$, and $\text{tr}(\lambda^{-1}v^2) = 1$. Let $\bar{D}$ be given as in (17) and g be given as in (8).

- (i) When k is even, $\mathcal{C}_{\bar{S}}$ is a $[(3^{k-1} - 1)/2, tk]$ 3-weight code and has weight distribution given in Table 5
- (ii) When k is odd, $\mathcal{C}_{\bar{S}}$ is a $[(3^{k-1} - 1)/2, k]$ 3-weight code and has weight distribution given in Table 6, where $\delta = 1$ if $k \equiv 1 \pmod{4}$ and $\delta = -1$ if $k \equiv 3 \pmod{4}$

Example 3. Let $k = 4$. Let $\lambda = 1$, $u = -1$, and $v = 1$. Define $g(y) = \text{tr}(y^2) - \text{tr}(y)^2$. The code $\mathcal{C}_{\bar{S}}$ in Corollary 1 (i) is a $[13, 4, 6]$ code and has weight enumerator $1 + 12z^6 + 62z^9 + 6z^{12}$.

Example 4. Let $k = 5$. Let $\lambda = -1$, $u = -1$, and $v = 1$. Define $g(y) = \text{tr}(-y^2) - \text{tr}(y)^2$. Then, $\mathcal{C}_{\bar{S}}$ in Corollary 1 (ii) is a $[31, 5, 18]$ code and has weight enumerator $1 + 60z^{18} + 162z^{21} + 20z^{27}$. By using the Griesmer bound, we know this code is optimal.

Corollary 2. Assume that $k > 4$ and $\lambda \in \mathbb{F}_{3^k}^*$. Let $u, v \in \mathbb{F}_{3^k}^*$ such that $\text{tr}(\lambda^{-1}uv) = 1$, $\text{tr}(\lambda^{-1}u^2) = 0$, and $\text{tr}(\lambda^{-1}v^2) = 0$. Let $\bar{S}$ be defined as in (17) and f be defined as in (8).

- (i) When k is even, $\mathcal{C}_{\bar{S}}$ is a $[(3^{k-1} - 1)/2, k]$ 3-weight code, and its weight distribution is listed in Table 7, where $\delta = 1$ if $k \equiv 0 \pmod{4}$ and $\delta = -1$ if $k \equiv 2 \pmod{4}$
- (ii) When k is odd, $\mathcal{C}_{\bar{S}}$ is a $[(3^{k-1} - 1)/2, k]$ 3-weight code and has weight distribution listed in Table 8

Example 5. Assume β is a generator of $\mathbb{F}_{3^4}^*$, where β is a root of $x^4 - x^3 - 1 = 0$. Let $\lambda = \beta$, $u = \beta^{16}$, and $v = \beta^8$. Define $g(y) = \text{tr}(\beta y^2) + \text{tr}(\beta^{16}y)\text{tr}(\beta^8y)$. Then, the code $\mathcal{C}_{\bar{S}}$ in Corollary 2 (i) is a $[22, 4, 9]$ code and has weight enumerator $1 + 4z^9 + 72z^{15} + 4z^{18}$.

TABLE 5: Weight distribution of $\mathcal{C}_{\bar{S}}$ for Corollary 1 (i).

Weight	Frequency
0	1
$3^{k-2} - \eta(\lambda)3^{(k/2)-1}$	$3^{k-2} + \eta(\lambda)3^{(k/2)-1}$
3^{k-2}	$3^k - 1 - 2 \cdot 3^{k-2}$
$3^{k-2} + \eta(\lambda)3^{(k/2)-1}$	$3^{k-2} - \eta(\lambda)3^{(k/2)-1}$

TABLE 6: Weight distribution of $\mathcal{C}_{\bar{S}}$ for Corollary 1 (ii).

Weight	Frequency
0	1
$3^{k-2} + 2\delta\eta(\lambda)3^{(k-3)/2}$	$2 \cdot 3^{k-1}$
3^{k-2}	$3^{k-2} + 2\delta\eta(\lambda)3^{(k-3)/2} - 1$
$3^{k-2} + 3\delta\eta(\lambda)3^{(k-3)/2}$	$2 \cdot 3^{k-2} - 2\delta\eta(\lambda)3^{(k-3)/2}$

TABLE 7: Weight distribution of $\mathcal{C}_{\bar{S}}$ for Corollary 2 (i).

Weight	Frequency
0	1
$3^{k-2} - 2\delta\eta(\lambda)3^{(k/2)-1}$	$3^k - 3^{k-2}$
3^{k-2}	$3^{k-3} - 2\delta\eta(\lambda)3^{(k/2)-2} - 1$
$3^{k-2} - 3\delta\eta(\lambda)3^{(k/2)-1}$	$2 \cdot 3^{k-3} + 2\delta\eta(\lambda)3^{(k/2)-2}$

TABLE 8: Weight distribution of $\mathcal{C}_{\bar{S}}$ for Corollary 2 (ii).

Weight	Frequency
0	1
$3^{k-2} - \eta(\lambda)3^{((k+1)/2)-1}$	$3^{k-3} + \eta(\lambda)3^{(k-3)/2}$
3^{k-2}	$3^m - 2 \cdot 3^{m-3} - 1$
$3^{k-2} + \eta(\lambda)3^{((k+1)/2)-1}$	$3^{k-3} - \eta(\lambda)3^{(k-3)/2}$

3. A Class of r -Ary Linear Codes with 3 Weights

Denote $d = \gcd(h, m)$, where m and h are positive integers. For $\lambda, a \in \mathbb{F}_{r^m}$, let $f(x) = \text{tr}(\lambda x^{r^h+1} + ax)$.

Based on the fact that a quadratic r -ary bent function has full rank, Zhou et al. ([26], Section III.B) obtained a class of linear codes with 2 or 3 weights by using Gold functions. Following the work of Zhou et al. [26], we shall employ Gold class of plateaued functions whose rank is less than m to construct r -ary linear codes with 3 weights.

For $\lambda, a \in \mathbb{F}_{r^m}$, we denote by $W_h(\lambda, a)$ the Weil sum given by

$$W_h(\lambda, a) = \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(\lambda x^{r^h+1} + ax)}. \quad (18)$$

Lemma 2 (see [31], Theorem 4.1). *For $m = 2k$, the equation $\lambda^{r^h} x^{r^{2h}} + \lambda x = 0$ has solutions in $\mathbb{F}_{r^m}^*$ if and only if (m/d) is even and $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$.*

In [31, 32], the following two lemmas studied explicit evaluations of the exponential sum (18).

Lemma 3 (see [31]). *Let (m/d) be even with $m = 2k$. Then, $W_h(\lambda, 0) = W_h(-\lambda, 0)$, and*

$$W_h(\lambda, 0) = \begin{cases} r^{k+d} & \text{if } \lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)} \text{ and } \frac{k}{d} \text{ is odd,} \\ -r^{k+d} & \text{if } \lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)} \text{ and } \frac{k}{d} \text{ is even.} \end{cases} \quad (19)$$

Lemma 4 (see [32]). *Let (m/d) be even with $m = 2k$. Assume that $g(x) = \lambda^{r^h} x^{r^{2h}} + \lambda x$ is not a permutation polynomial over $\mathbb{F}_{r^m}$. Then, for $a \neq 0$, we have $W_h(\lambda, a) = 0$ unless the equation $g(x) = -a^{r^h}$ is solvable. If this equation is solvable, then*

$$W_h(\lambda, a) = -(-1)^{(k/d)} r^{k+d} \zeta_r^{-\text{tr}(\lambda x_0^{r^h+1})}, \quad (20)$$

where x_0 is such that $g(x_0) = -a^{r^h}$.

Let (m/d) be even with $m = 2k$ and $\lambda \in \mathbb{F}_{r^m}^*$ such that $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$. It follows from Lemmas 3 and 4 and the definition of the plateaued function that $f(x) = \text{tr}(\lambda x^{r^h+1})$ is a $2d$ -plateaued function if $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$.

Lemma 5. *Let (m/d) be even with $m = 2k$ and $\lambda \in \mathbb{F}_{r^m}^*$ such that $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$. Then,*

$$W_h(\lambda, 0) = W_h(2\lambda, 0) = \dots = W_h((r-1)\lambda, 0)$$

$$= \begin{cases} r^{k+d} & \text{if } \lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)} \text{ and } \frac{k}{d} \text{ is odd,} \\ -r^{k+d} & \text{if } \lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)} \text{ and } \frac{k}{d} \text{ is even.} \end{cases} \quad (21)$$

Proof. Since (m/d) is even, $r-1 \mid ((r^m-1)/(r^d+1))$. Hence, $(c\lambda)^{(r^m-1)/(r^d+1)} = \lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$ for each $c \in \mathbb{F}_r^*$. The desired result follows from Lemma 3. $\square$

Lemma 6. *Let (m/d) be even with $m = 2k$ and $\lambda \in \mathbb{F}_{r^m}^*$ such that $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$. For $a \in \mathbb{F}_{r^m}^*$, we have*

$$\sum_{z \in \mathbb{F}_r^*} \sum_{y \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(y\lambda x^{r^h+1} + zax)} \in \{0, -(-1)^{(k/d)}(r-1)^2 r^{k+d}, (-1)^{(k/d)} r^{k+d}(r-1)\}. \quad (22)$$

Proof. Since (m/d) is even and $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$, the equation $\lambda^{r^h} x^{r^{2h}} + \lambda x = 0$ is solvable for $x \in \mathbb{F}_{r^m}^*$ by Lemma 2.

For $a \in \mathbb{F}_{r^m}^*$, we claim that the equation $\lambda^{r^h} x^{r^{2h}} + \lambda x + a^{r^h} = 0$ is solvable if and only if the equation $(y\lambda)^{r^h} x^{r^{2h}} + (y\lambda)x +$

$(za)^{r^h} = 0$ is solvable for each $(y, z) \in \mathbb{F}_r^* \times \mathbb{F}_r^*$. Suppose x_0 is a solution of $\lambda^{r^h} x^{r^{2h}} + \lambda x + a^{r^h} = 0$. Clearly, x_0 is also a root of $y^{-1} z \lambda^{r^h} x^{r^{2h}} + y^{-1} z \lambda x + y^{-1} z a^{r^h} = 0$. Then, $y^{-1} z \lambda^{r^h} x_0^{r^{2h}} + y^{-1} z \lambda x_0 + y^{-1} z a^{r^h} = 0$. We can rewrite this as $\lambda^{r^h} (y^{-1} z x_0)^{r^{2h}} + \lambda (y^{-1} z x_0) + y^{-1} z a^{r^h} = 0$ since $z, y \in \mathbb{F}_r^*$. It then follows that $(y\lambda)^{r^h} (y^{-1} z x_0)^{r^{2h}} + (y\lambda) (y^{-1} z x_0) + (za)^{r^h} = 0$. That is to say, the equation $(y\lambda)^{r^h} x^{r^{2h}} + (y\lambda)x + (za)^{r^h} = 0$ has a solution $y^{-1} z x_0$. Conversely, assume x_0 is a solution of $(y\lambda)^{r^h} x^{r^{2h}} + (y\lambda)x + (za)^{r^h} = 0$; then, $(y\lambda)^{r^h} x_0^{r^{2h}} + (y\lambda)x_0 + (za)^{r^h} = 0$. Note that $y, z \in \mathbb{F}_r^*$.

Hence, we have $\lambda^{r^h} (z^{-1} y x_0)^{r^{2h}} + \lambda (z^{-1} y x_0) + a^{r^h} = 0$. This implies that $\lambda^{r^h} x^{r^{2h}} + \lambda x + a^{r^h} = 0$ has a solution $z^{-1} y x_0$.

If $\lambda^{r^h} x^{r^{2h}} + \lambda x + a^{r^h} = 0$ has no solutions in $\mathbb{F}_{r^m}$, then by Lemma 4 and the discussions above, we have

$$\sum_{y \in \mathbb{F}_r^*} \sum_{z \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(y\lambda x^{r^h+1} + zax)} = 0. \quad (23)$$

If $\lambda^{r^h} x^{r^{2h}} + \lambda x + a^{r^h} = 0$ has a solution x_0 in $\mathbb{F}_{r^m}$, then we know that $(y\lambda)^{r^h} x^{r^{2h}} + (y\lambda)x + (za)^{r^h} = 0$ has a solution $y^{-1} z x_0$ from the discussions above. It follows from (20) that

$$\begin{aligned} \sum_{z \in \mathbb{F}_r^*} \sum_{y \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(y\lambda x^{r^h+1} + zax)} &= (-1)^{(k/d)+1} r^{k+d} \sum_{z \in \mathbb{F}_r^*} \sum_{y \in \mathbb{F}_r^*} \zeta_r^{-\text{tr}(y\lambda (y^{-1} z x_0)^{r^h+1})} \\ &= (-1)^{(k/d)+1} r^{k+d} \sum_{z \in \mathbb{F}_r^*} \sum_{y \in \mathbb{F}_r^*} \zeta_r^{-y^{-1} z^2 \text{tr}(\lambda x_0^{r^h+1})} \\ &= \begin{cases} (-1)^{(k/d)} r^{k+d} (r-1) & \text{if } \text{tr}(\lambda x_0^{r^h+1}) \neq 0, \\ (-1)^{(k/d)+1} r^{k+d} (r-1)^2 & \text{if } \text{tr}(\lambda x_0^{r^h+1}) = 0. \end{cases} \end{aligned} \quad (24)$$

Hence, for any $a \in \mathbb{F}_{r^m}^*$,

$$\sum_{z \in \mathbb{F}_r^*} \sum_{y \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(y\lambda x^{r^h+1} + zax)} \in \{0, (-1)^{(k/d)} r^{k+d} (r-1), (-1)^{(k/d)+1} r^{k+d} (r-1)^2\}. \quad (25)$$

Let h be a positive integer, $1 \leq h < k$, where $m = 2k > 4$ is an even integer. Define

$$S = \left\{ x \in \mathbb{F}_{r^m}^* : \text{tr}(\lambda x^{r^h+1}) = 0 \right\}, \quad (26)$$

where $\lambda \in \mathbb{F}_{r^m}^*$. $\square$

Theorem 3. Let h be a positive integer, $1 \leq h < k$, where $m = 2k > 4$ is an even integer. Let S be defined as in (26), where $\lambda \in \mathbb{F}_{r^m}^*$. Define $d = \gcd(h, m)$. Assume that (m/d) is even and $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$.

(i) If (k/d) is odd, then $\mathcal{C}_S$ is a $[(r-1)r^{k+d-1} + r^{m-1} - 1, m, r^{m-2}(r-1)]$ 3-weight code and has weight distribution given in Table 9

(ii) If (k/d) is even, then $\mathcal{C}_S$ is a $[(r-1)r^{k+d-1} - r^{m-1} - 1, m, r^{m-2}(r-1) - (r-1)r^{k+d-1}]$ 3-weight code with the weight distribution given in Table 10

Proof. We only present the proof of (i) since the others can be proven by a similar proof. Denote $n_S = |S \cup \{0\}|$. We have

$$\begin{aligned} n_S &= \frac{1}{r} \sum_{x \in \mathbb{F}_{r^m}} \sum_{y \in \mathbb{F}_r} \zeta_r^{y \text{tr}(\lambda x^{r^h+1})} \\ &= \frac{1}{r} \sum_{x \in \mathbb{F}_{r^m}} \left(1 + \zeta_r^{\text{tr}(\lambda x^{r^h+1})} + \zeta_r^{2 \text{tr}(\lambda x^{r^h+1})} + \dots + \zeta_r^{(r-1) \text{tr}(\lambda x^{r^h+1})} \right) \\ &= r^{m-1} + \frac{1}{r} (W_h(\lambda, 0) + W_h(2\lambda, 0) + \dots + W_h((r-1)\lambda, 0)). \end{aligned} \quad (27)$$

Note that (k/d) is odd and $\lambda^{(r^m-1)/(r^d+1)} = (-1)^{(k/d)}$. By Lemma 5, we have

$$n = n_S - 1 = r^{m-1} + (r-1)r^{k+d-1} - 1. \quad (28)$$

TABLE 9: Weight distribution of $\mathcal{C}_S$ for Theorem 3 (i).

Weight	Frequency
0	1
$r^{m-2}(r-1)$	$((r^{m-2} - r^{k+d-2})(r^{m-2} + r^{k+d-1}))/ (r^{m+2d-3})$
$r^{m-2}(r-1) + (r-1)^2 r^{k+d-2}$	$r^m - 1 - ((r^{m-2} - r^{k+d-2})(r^{m-1} + r^{k+d-1}))/ r^{m+2d-3}$
$r^{m-2}(r-1) + (r-1)r^{k+d-1}$	$((r^{m-2} - r^{k+d-2})(r-1)r^{m-2})/ (r^{m+2d-3})$

TABLE 10: Weight distribution of $\mathcal{C}_S$ for Theorem 3 (ii).

Weight	Frequency
0	1
$r^{m-2}(r-1) - (r-1)r^{k+d-1}$	$((r^{m-2} + r^{k+d-2})(r-1)r^{m-2})/ (r^{m+2d-3})$
$r^{m-2}(r-1) - (r-1)^2 r^{k+d-2}$	$r^m - 1 - (((r^{m-2} + r^{k+d-2})(r^{m-1} - r^{k+d-1}))/ (r^{m+2d-3}))$
$r^{m-2}(r-1)$	$((r^{m-2} + r^{k+d-2})(r^{m-2} - r^{k+d-1}))/ (r^{m+2d-3})$

For each $a \in \mathbb{F}_{r^m}^*$, let

$$M_a = \left| \left\{ x \in \mathbb{F}_{r^m} : \text{tr}(\lambda x^{r^{h+1}}) = 0 \text{ and } \text{tr}(ax) = 0 \right\} \right|. \quad (29)$$

Then, the Hamming weight $\text{wt}(\mathbf{c}_a)$ is $n_S - M_a$. We can compute

$$\begin{aligned}
M_a &= \frac{1}{r^2} \sum_{x \in \mathbb{F}_{r^m}} \left(\sum_{y \in \mathbb{F}_r} \zeta_r^{y \text{tr}(\lambda x^{r^{h+1}})} \right) \left(\sum_{z \in \mathbb{F}_r} \zeta_r^{z \text{tr}(ax)} \right) \\
&= r^{m-2} + \frac{1}{r^2} \left(\sum_{y \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{y \text{tr}(\lambda x^{r^{h+1}})} \right) + \frac{1}{r^2} \sum_{y \in \mathbb{F}_r^*} \sum_{z \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(y \lambda x^{r^{h+1}} + z a x)} \\
&= r^{m-2} + \frac{1}{r^2} \left(\sum_{y \in \mathbb{F}_r^*} W_h(y \lambda, 0) \right) + \frac{1}{r^2} \sum_{y \in \mathbb{F}_r^*} \sum_{z \in \mathbb{F}_r^*} \sum_{x \in \mathbb{F}_{r^m}} \zeta_r^{\text{tr}(y \lambda x^{r^{h+1}} + z a x)}.
\end{aligned} \quad (30)$$

It follows from Lemmas 5 and 6 that

$$M_a \in \{r^{m-2} + (r-1)^2 r^{k+d-2}, r^{m-2}, r^{m-2} + (r-1)r^{k+d-2}\}. \quad (31)$$

For each $a \in \mathbb{F}_{r^m}^*$, we have

$$\text{wt}(\mathbf{c}_a) = \{r^{m-2}(r-1), (r-1)r^{m-2} + (r-1)r^{k+d-1}, r^{m-2}(r-1) + (r-1)^2 r^{k+d-2}\}. \quad (32)$$

Then, $\mathcal{C}_S$ has three nonzero weights above. When $m > 4$, for each $a \in \mathbb{F}_{3^m}^*$, $\text{wt}(\mathbf{c}_a) > 0$, and thus, the dimension of the code is m .

Note that $0 \notin S$. The minimum distance of $\mathcal{C}_S^\perp$ cannot be one. Since $\text{tr}(\lambda(cx)^{r^{h+1}}) = 0$ for all $c \in \mathbb{F}_r^*$, if $\text{tr}(\lambda x^{r^{h+1}}) = 0$, the minimum Hamming weight of $\mathcal{C}_S^\perp$ is 2. Moreover, we can prove that $B_2^\perp$ is equal to $n \binom{r-1}{2}$. In fact, for each $r-1$

distinct element $v, 2v, \dots, (r-1)v \in S$, we can get $(r-1) \binom{r-1}{2}$ code words with weight 2 in $\mathcal{C}_S^\perp$. Define $w_1 = r^{m-2}(r-1)$, $w_2 = r^{m-2}(r-1) + r^{k+d-2}(r-1)^2$, and $w_3 = r^{m-2}(r-1) + r(r-1)r^{k+d-2}$.

Solving the following linear equations gives the desired weight distribution:

$$\begin{cases} B_{w_1} + B_{w_2} + B_{w_3} = r^m - 1, \\ w_1 B_{w_1} + w_2 B_{w_2} + w_3 B_{w_3} = (r-1)nr^{m-1}, \\ w_1^2 B_{w_1} + w_2^2 B_{w_2} + w_3^2 B_{w_3} = [(r-1)n((r-1)n+1) + 2\binom{r-1}{2}n]r^{m-2}. \end{cases} \quad (33)$$

□

Example 6. Let $m = 8$ and $h = 2$. Let $f(x) = \text{tr}(x^{10})$. Then, the code $\mathcal{C}_S$ is a $[1700, 8, 972]$ code and has weight enumerator $1 + 60z^{972} + 6480z^{1134} + 20z^{1458}$.

Example 7. Let $m = 6$ and $h = 1$. Let β be the generator of $\mathbb{F}_{5^6}^*$, where β is a root of $x^6 + x^4 - x^3 + x^2 + 2 = 0$. Let $\lambda = \beta^3$. Clearly, $\lambda^{(5^6-1)/6} = \beta^{7812} = -1$. Let $f(x) = \text{tr}(\beta^3 x^6)$. Then, the code $\mathcal{C}_S$ is a $[3624, 6, 2500]$ code and has weight enumerator $1 + 144z^{2500} + 15000z^{2900} + 480z^{3000}$.

4. Conclusion

We succeed in constructing some linear codes with 3 weights from some r -ary functions with few Walsh transform values in the odd characteristic. The weight distributions of linear codes in this paper are determined by analyzing Walsh coefficients of r -ary functions. Compared with other linear codes with 3 weights, the parameters of the linear codes with 3 weights of this paper are different from those obtained from bent functions in [19, 23, 24, 26]. As applications, some of the 3-weight codes can be used to obtain secret sharing schemes [15] and association schemes introduced in [11].

Data Availability

All datasets generated for this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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