Research Article

Fuzzy Adaptive Variable Structure Control of Second-Order Robotic Manipulators with Dead Zones

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In this study, with respect to certain second-order robotic systems with dead zones, a fuzzy adaptive variable structure controller (VSC) is implemented. Some suitable adaptive fuzzy systems are used to estimate uncertain functions. Based on Lyapunov stability theorems, parameter adaptive laws are designed, and it is proven that all signals involved will remain bounded and the stability of the controlled system is also guaranteed. Our controller is effective for the system with or without sector nonlinearity. Finally, a simulation example is presented to illustrate the correctness of the theoretical derivation.

1. Introduction

In the real world, almost all systems are multivariate in essence, and the multivariate control theory system can be immediately applied to various problems (aerospace technology, electric motors, and robot technology) [1–4]. Because of the coupling between control inputs and outputs, how to design an effective controller for multivariable systems is a challenging work. When multivariable systems are subject to system uncertainties, the control problems become more complicated [5, 6]. In this field, theoretical results and constructive procedures for designing satisfactory controllers are constantly evolving, and only a few works have been reported up to now.

In recent years, fuzzy logic system (FLS) has been shown to be one of the powerful tools in functional approximation. Fuzzy control mainly utilizes the basic ideas of fuzzy mathematics and theoretical control methods. With respect to a traditional control method, the accuracy of the dynamic mode of the control system is the most important factor affecting the control performance. The more detailed the system dynamic information is, the more precise the control can be achieved [7–13]. But, with respect to a complicated system, because there are a lot of system variables, it is very hard to accurately model the system, so scholars utilize kinds

of methods to describe dynamics to achieve certain purposes. That is, the traditional control theory has a strong control ability for known systems, but it is powerless for systems that are too complicated or difficult to accurately describe. So scholars try to deal with these control problems via FLS. In [9], an adaptive fuzzy control (AFC) method was proposed for a chaotic system with unknown control direction. In [11], an AFC backstepping method was proposed, where it is shown that the FLS has a very powerful approximation ability. An adaptive fuzzy observer was proposed in [13]. For more related works about the FLC, one can refer to [4, 14–19] and the references therein.

Since the 1980s, the control of robots has become a very meaningful work with a wide range of practical applications and has been increasingly studied by experts and scholars in various fields. The robot model is highly coupled and complex. In the early research on robot control, researchers often need to know the specific model of the robot. For some robots that are easy to model, many control methods have been proposed, such as active control, inversion based control, and backstepping control [20–23]. However, in real life, robot systems often contain many uncertainties, such as modeling estimation errors, system external disturbances, and parameter measurement errors. If the unknowns of these systems cannot be handled properly, it will affect the stability of the system and

even make the robot system uncontrollable. Consequently, FLSs are used to tackle the uncertainties in the robotic systems. For example, in [8, 24, 25], FLSs were used to model the robotic manipulators, and the model errors are satisfactory. More results can be referred to [24, 26–29]. However, the stability of the complicated robotic systems is hard to be theoretically discussed in the above literature.

There are two main reasons that derive the investigation of this work. First, although many control strategies have been proposed for robotic manipulators, the dead-zone input has been rarely considered. Secondly, in the traditional VSC method, the chattering phenomenon does exist, and how to solve this problem is a challenging work. Inspired by the above discussion, in this paper, an adaptive fuzzy VSC method is proposed for an n -link robotic manipulator with dead zone. The main contributions of this work are listed as follows: (1) to handle the dead zone in the robotic manipulator, an adaptive fuzzy VSC method is proposed, and the VSC can estimate the dead-zone model, and (2) the stability of the controlled robotic manipulator is proven strictly based on Lyapunov stability criteria.

2. Preliminaries and Problem Description

Consider the following n -link robotic manipulator:

$$M(\xi)\ddot{\xi} + C(\xi, \dot{\xi})\dot{\xi} + G(\xi) = \Phi(u) + a(t). \quad (1)$$

From system (1), we can get the following formula:

$$\begin{aligned} \ddot{\xi} = & -M^{-1}(\xi)C(\xi, \dot{\xi})\dot{\xi} - M^{-1}(\xi)G(\xi) + M^{-1}(\xi)\Phi(u) \\ & + M^{-1}(\xi)a(t). \end{aligned} \quad (2)$$

So, the robot system (2) is rewritten as

$$\ddot{y} = F(x) + G(x)\Phi(u) + D(t), \quad (3)$$

where

$$\begin{cases} F(x) = -M^{-1}(\xi)C(q, \dot{\xi})\dot{\xi} - M^{-1}(\xi)G(\xi), \\ G(x) = M^{-1}(\xi), \\ D(t) = M^{-1}(\xi)a(t). \end{cases} \quad (4)$$

$F(\cdot), \Phi(\cdot), D(\cdot) \in R^n, G(\cdot) \in R^{n \times n}, y = [y_1, y_2, \dots, y_n]^T = [\xi_1, \xi_2, \dots, \xi_n]^T \in R^n$ is the output, $x = [y_1, \dot{y}_1, \dots, y_n, \dot{y}_n]^T = [\xi_1, \xi_1, \dots, \xi_n, \xi_n]^T \in R^{2n}$ is the measurable state, $u = [u_1, u_2, \dots, u_n]^T \in R^n$ denotes the controller to be designed, $f_i(x)$ is unknown, $g_{ij}, i, j = 1, 2, \dots, n$ is an unknown constant control gain, $\Phi(u) = [\Phi_1(u_1), \Phi_2(u_2), \dots, \Phi_n(u_n)]^T$ means dead zone, and $d_i(t)$ is an external disturbance.

Let us denote

$$\begin{aligned} \ddot{y} &= [\ddot{y}_1, \ddot{y}_2, \dots, \ddot{y}_n]^T, \\ F(x) &= [f_1(x), f_2(x), \dots, f_n(x)]^T, \\ D(t) &= [d_1(t), \dots, d_n(t)]^T. \end{aligned} \quad (5)$$

Now, we need the following assumptions.

Assumption 1. We assume that the desired trajectory vector $x_d = [y_{d1}, \dot{y}_{d1}, \dots, y_{dn}, \dot{y}_{dn}, \ddot{y}_{d1}, \dots, \ddot{y}_{dn}]^T$ is continuous and bounded and can be obtained from the measurement. So $x_d \in \Omega_{x_d} \subset R^{2n+n}$, where Ω_{x_d} is an open compact set with a finite radius.

Assumption 2

- (i) Without loss of generality, one assumes that $G(x)$ is symmetric and positive definite. Therefore, there is an unknown positive constant δ_{g0} , such that $G(x) \geq \delta_{g0}E_n$, where E_n is the identity matrix.
- (ii) We assume that this internal disturbance $d_i(t)$ is bounded and satisfies the following property: $|d_i(t)| \leq \bar{d}_i, \forall i = 1, 2, \dots, n$, where $\bar{d}_i$ is an unknown positive constant.

Remark 1. It should be noted that, for a robotic system, Assumption 2 is reasonable, and this assumption guarantees the controllability of system (3).

The main purpose is to find a way to implement u so that the output vector y can better follow the desired tracking vector $y_d = [y_{d1}, \dots, y_{dn}]^T \in R^n$, and we assume that the signals involved are bounded.

Define

$$\begin{aligned} e &= y - y_d, \\ S &= \gamma e + \dot{e}, \end{aligned} \quad (6)$$

where $\gamma > 0$. Here, S is a sliding surface variable. Then, we have

$$\dot{S} = \gamma \dot{e} + \ddot{e} = \gamma \dot{e} + \ddot{y} - \ddot{y}_d. \quad (7)$$

Thereafter, (7) will be used in the controller design.

2.1. Description of the Fuzzy Logic System. The basic mechanism of a fuzzy logic system includes a fuzzy inference engine and a fuzzifier, some fuzzy IF-THEN rules, and a defuzzifier.

The i -th fuzzy rule is in the following form: $R^{(i)}$: if x_1 is B_1^i and $\dots$ and x_n is B_n^i , then $\hat{f}$ is f^i , where $B_1^i, B_2^i, \dots, B_n^i$ are fuzzy sets and f^i denotes the singleton for the output in the i -th rule. The FLS can be rewritten as follows:

$$\hat{f}(\bar{x}) = \frac{\sum_{i=1}^l f^i \left(\prod_{j=1}^n \nu_{B_j^i}(x_j) \right)}{\sum_{i=1}^l \left(\prod_{j=1}^n \nu_{B_j^i}(x_j) \right)} = \theta^T \varrho(\bar{x}), \quad (8)$$

where $\nu_{B_j^i}(x_j)$ is the degree of membership, $\theta^T = [f^1, \dots, f^l]$ can be adjusted online, and $\varrho^T = [\varrho^1, \dots, \varrho^l]$, with

$$\varrho^i(\bar{x}) = \frac{\prod_{j=1}^n \nu_{B_j^i}(x_j)}{\sum_{i=1}^l \left(\prod_{j=1}^n \nu_{B_j^i}(x_j) \right)}. \quad (9)$$

Throughout the paper, we always assume that $\sum_{i=1}^l \left(\prod_{j=1}^n \nu_{B_j^i}(x_j) \right) > 0$.

2.2. Input Nonlinearity. Suppose that $\Phi_i(u_i)$ satisfies

$$\Phi_i(u_i) = \begin{cases} \phi_{i+}(u_i)(u_i - u_{i+}), & u_i > u_{i+}, \\ 0, & -u_{i-} \leq u_i \leq u_{i+}, \\ \phi_{i-}(u_i)(u_i + u_{i-}), & u_i < -u_{i-}, \end{cases} \quad (10)$$

where $\phi_{i+}(u_i) > 0$ and $\phi_{i-}(u_i) > 0$ are functions with respect to u_i and $u_{i+} > 0, u_{i-} > 0$.

Suppose that $\Phi_i(u_i)$ satisfies

$$\begin{cases} (u_i - u_{i+})\Phi_i(u_i) \geq l_{i+}^*(u_i - u_{i+})^2, & u_i > u_{i+}, \\ (u_i + u_{i-})\Phi_i(u_i) \geq l_{i-}^*(u_i + u_{i-})^2, & u_i < -u_{i-}, \end{cases} \quad (11)$$

where l_{i+}^* and l_{i-}^* are constants called “gain reduction tolerances”. We can write $\zeta_i = \min\{l_{i+}^*, l_{i-}^*\}$.

In order to facilitate the study of the characteristics of this input nonlinear function in the control problems, we give the following assumption.

Assumption 3. Suppose that l_{i+}^* and l_{i-}^* are unknown, that is, ζ_i is also unknown. The explicit mathematical expression of the nonlinear function $\Phi_i(u_i)$ is uncertain, but this property (11) and constants u_{i+} and u_{i-} are known.

Remark 2. One knows from (10) and (11) that, for $u_{i+} = u_{i-} = 0$, the input $\Phi_i(u_i)$ is reduced to the special sector nonlinear function. We can make the following restriction: the gain reduction tolerances l_{i+}^* and l_{i-}^* and their minimum $\zeta_i = \min\{l_{i+}^*, l_{i-}^*\}$ are known.

3. Main Results

In this article, a fuzzy adaptive VSC scheme is proposed for a class of two-link robotic manipulator systems described in (3).

By bringing (3) into (7), the dynamic equation of S can become

$$\dot{S} = \gamma \dot{e} + F(x) + G(x)\Phi(u) + D(t) - \ddot{y}_d. \quad (12)$$

Posing $G_1(x) = G^{-1}(x)$ and $D_h(t) = G^{-1}(x)D(t)$, we have

$$\begin{aligned} G^{-1}(x)\dot{S} &= \gamma G^{-1}(x)\dot{e} + G^{-1}(x)F(x) + \Phi(u) \\ &\quad + G^{-1}(x)D(t) - G^{-1}(x)\ddot{y}_d, \end{aligned} \quad (13)$$

Then, S can be expressed by

$$\begin{aligned} G_1(x)\dot{S} &= G_1(x)[\gamma \dot{e} + F(x) - \ddot{y}_d] + \Phi(u) + D_h(t) \\ &= \beta(x, \omega) + \Phi(u) + D_h(t), \end{aligned} \quad (14)$$

where $\beta(x, \omega) = [\beta_1(x, \omega), \dots, \beta_n(x, \omega)]^T = G_1(x)[\omega + F(x)]$ and $\omega = \gamma \dot{e} - \ddot{y}_d$.

Assumption 4. One can find a function $\bar{\beta}_i(x)$ such that

$$|\beta_i(x, \omega)| \leq \zeta \bar{\beta}_i(x), \quad \forall x \in \Omega_x \subset \mathbb{R}^{2n}, \quad (15)$$

where $\zeta = \min_i\{\zeta_i\}$.

Remark 3. Assumption 4 is reasonable because

- (i) $\zeta = \min_i\{\zeta_i\} a \zeta \bar{\beta}_i(x)$ is unknown, where ζ is a positive scalar
- (ii) Nothing that ω is a function of (x, x_d) , $x_d \in L_\infty$, and $\beta_i(x, \omega)$ is a continuous function; therefore, such a function $\bar{\beta}_i(x)$ always exists

Multiplying (14) by S^T and using Assumption 4, we have

$$\begin{aligned} \frac{1}{\zeta} S^T G_1(x) \dot{S} &= \frac{1}{\zeta} S^T \beta(x, \omega) + \frac{1}{\zeta} S^T \Phi(u) + \frac{1}{\zeta} S^T D_h(t) \\ &= \frac{1}{\zeta} \sum_{i=1}^n S_i \beta_i(x, \omega) + \frac{1}{\zeta} S^T \Phi(u) + \frac{1}{\zeta} S^T D_h(t) \\ &\leq \frac{1}{\zeta} \sum_{i=1}^n |S_i| |\beta_i(x, \omega)| + \frac{1}{\zeta} S^T \Phi(u) + \frac{1}{\zeta} S^T D_h(t) \\ &\leq \sum_{i=1}^n |S_i| \bar{\beta}_i(x) + \frac{1}{\zeta} S^T \Phi(u) + \frac{1}{\zeta} S^T D_h(t). \end{aligned} \quad (16)$$

On a compact set Ω_x , the unknown continuous nonlinear function $\bar{\beta}_i(x)$ can be approximated by the fuzzy system (8) as follows:

$$\widehat{\bar{\beta}}_i(x, \theta_i) = \theta_i^T \mathbf{q}_i(x). \quad (17)$$

Next, we can define the optimal parameter as

$$\theta_i^* = \arg \min_{\theta_i} \left[\sup_{x \in \Omega_x} |\bar{\beta}_i(x) - \widehat{\bar{\beta}}_i(x, \theta_i)| \right]. \quad (18)$$

Define

$$\tilde{\theta}_i = \theta_i - \theta_i^*, \quad (19)$$

as the parameter estimation error, with

$$\varepsilon_i(x) = \bar{\beta}_i(x) - \widehat{\bar{\beta}}_i(x, \theta_i^*), \quad (20)$$

being the approximation error, with $\widehat{\bar{\beta}}_i(x, \theta_i^*) = \theta_i^{*T} \mathbf{q}_i(x)$.

As in the literature, the fuzzy systems used on the compact set Ω_x do not violate this universal approximation property. So, we can assume that

$$|\varepsilon_i(x)| \leq \bar{\varepsilon}_i, \quad \forall x \in \Omega_x, \quad (21)$$

with $\bar{\varepsilon}_i$ being unknown.

Then, one has

$$\begin{aligned} \widehat{\bar{\beta}}_i(x, \theta_i) - \bar{\beta}_i(x) &= \widehat{\bar{\beta}}_i(x, \theta_i) - \widehat{\bar{\beta}}_i(x, \theta_i^*) + \widehat{\bar{\beta}}_i(x, \theta_i^*) - \bar{\beta}_i(x) \\ &= \widehat{\bar{\beta}}_i(x, \theta_i) - \widehat{\bar{\beta}}_i(x, \theta_i^*) - \varepsilon_i(x) \\ &= \theta_i^T \mathbf{q}_i(x) - \theta_i^{*T} \mathbf{q}_i(x) - \varepsilon_i(x) \\ &= \tilde{\theta}_i^T \mathbf{q}_i(x) - \varepsilon_i(x). \end{aligned} \quad (22)$$

From (16) and (22), we can get

$$\begin{aligned}
\frac{1}{\zeta} S^T G_1(x) \dot{S} &\leq \sum_{i=1}^n |S_i| \bar{\beta}_i(x) + \frac{1}{\zeta} S^T \Phi(u) + \frac{1}{\zeta} S^T D_h(t) \\
&= \sum_{i=1}^n |S_i| \left(\theta_i^T \varrho_i(x) - \tilde{\theta}_i^T \varrho_i(x) + \varepsilon_i(x) \right) + \frac{1}{\zeta} S^T \Phi(u) \\
&\quad + \frac{1}{\zeta} S^T D_h(t) \\
&\leq \sum_{i=1}^n |S_i| \theta_i^T \varrho_i(x) - \sum_{i=1}^n |S_i| \tilde{\theta}_i^T \varrho_i(x) + \sum_{i=1}^n |S_i| |\varepsilon_i(x)| \\
&\quad + \frac{1}{\zeta} S^T \Phi(u) + \left| \frac{1}{\zeta} S^T D_h(t) \right| \\
&\leq \sum_{i=1}^n |S_i| \theta_i^T \varrho_i(x) - \sum_{i=1}^n |S_i| \tilde{\theta}_i^T \varrho_i(x) + \sum_{i=1}^n |S_i| \bar{\varepsilon}_i \\
&\quad + \frac{1}{\zeta} S^T \Phi(u) + \sum_{i=1}^n |S_i| \bar{d}_{hi} \\
&\leq \sum_{i=1}^n |S_i| \theta_i^T \varrho_i(x) - \sum_{i=1}^n |S_i| \tilde{\theta}_i^T \varrho_i(x) + \frac{1}{\zeta} S^T \Phi(u) \\
&\quad - \sum_{i=1}^n |S_i| \widetilde{\kappa_{0i}} + \sum_{i=1}^n |S_i| \kappa_{0i},
\end{aligned} \tag{23}$$

where $\tilde{\theta}_i = \theta_i - \theta_i^*$ and $\widetilde{\kappa_{0i}} = \kappa_{0i} - \kappa_{0i}^* = \kappa_{0i} - \bar{\varepsilon}_i - \bar{d}_{hi}$. Note that $\bar{d}_{hi}$ are unknown positive constants which satisfy $\sum_{i=1}^n \bar{d}_{hi} |S_i| \geq |S^T D_h(t)/\zeta|$.

In order to meet the control goal, we propose a suitable fuzzy adaptive VSC as follows:

$$u_i = \begin{cases} -\bar{\omega}_i(t) \text{sign}(S_i) - u_{i-}, & S_i > 0, \\ 0, & S_i = 0, \\ -\bar{\omega}_i(t) \text{sign}(S_i) + u_{i+}, & S_i < 0, \end{cases} \tag{24}$$

with

$$\bar{\omega}_i(t) = \kappa_{0i} + \kappa_{1i} |S_i| + \theta_i^T \varrho_i(x), \quad \forall i = 1, 2, \dots, n, \tag{25}$$

$$\dot{\kappa}_{0i} = -\chi_{0i} \delta_{0i} \kappa_{0i} + \chi_{0i} |S_i|, \quad \kappa_{0i}(0) \geq 0, \tag{26}$$

$$\dot{\theta}_i = -\chi_{1i} \delta_{1i} \theta_i + \chi_{1i} |S_i| \varrho_i(x), \quad \theta_{ij}(0) \geq 0, \tag{27}$$

where $\chi_{0i}, \chi_{1i}, \delta_{0i}, \delta_{1i}, \kappa_{1i} > 0$, and κ_{0i} and θ_i are the online estimates of the uncertain terms $\kappa_{0i}^* = \bar{\varepsilon}_i + \bar{d}_{hi}$ and θ_i^* , respectively.

From (11) and (24), we can easily get that $u_i < -u_{i-}$ for $S_i > 0$,

$$\begin{aligned}
(u_i + u_{i-}) \Phi_i(u_i) &= -\bar{\omega}_i(t) \text{sign}(S_i) \Phi_i(u_i) \geq l_{i-}^* (u_i + u_{i-})^2 \\
&= l_{i-}^* \bar{\omega}_i^2(t) \geq \zeta \bar{\omega}_i^2(t),
\end{aligned} \tag{28}$$

and $u_i > u_{i+}$ for $S_i < 0$,

$$\begin{aligned}
(u_i - u_{i+}) \Phi_i(u_i) &= -\bar{\omega}_i(t) \text{sign}(S_i) \Phi_i(u_i) \geq l_{i+}^* (u_i - u_{i+})^2 \\
&= l_{i+}^* \bar{\omega}_i^2(t) \geq \zeta \bar{\omega}_i^2(t).
\end{aligned} \tag{29}$$

Then, for $S_i > 0$ and for $S_i < 0$, we have

$$-\bar{\omega}_i(t) \text{sign}(S_i) \Phi_i(u_i) \geq \zeta \bar{\omega}_i^2(t). \tag{30}$$

Next, we can use $S_i^2 \geq 0$ and $S_i \text{sign}(S_i) = |S_i|$, and we get

$$-\bar{\omega}_i(t) S_i^2 \text{sign}(S_i) \Phi_i(u_i) \geq \zeta \bar{\omega}_i^2(t) S_i^2 = \zeta \bar{\omega}_i^2(t) |S_i|^2. \tag{31}$$

Finally, for all S_i (i.e., for $S_i > 0$, $S_i = 0$, and $S_i < 0$), we get

$$S_i \Phi_i(u_i) \leq -\zeta \bar{\omega}_i(t) |S_i|. \tag{32}$$

Theorem 1. For the robotic manipulator (3) with Assumptions 1–4, the proposed adaptive fuzzy VSC (24)–(27) guarantee that the tracking error converges to an adjustable region.

Proof. Let

$$V = \frac{1}{2\zeta} S^T G_1(x) S + \frac{1}{2} \sum_{i=1}^n \frac{1}{\chi_{0i}} \widetilde{\kappa_{0i}}^2 + \frac{1}{2} \sum_{i=1}^n \frac{1}{\chi_{1i}} \tilde{\theta}_i^T \tilde{\theta}_i. \tag{33}$$

Its time derivative is given by

$$\dot{V} = \frac{1}{2\zeta} S^T G_1(x) \dot{S} + \sum_{i=1}^n \frac{1}{\chi_{0i}} \widetilde{\kappa_{0i}} \dot{\kappa}_{0i} + \sum_{i=1}^n \frac{1}{\chi_{1i}} \tilde{\theta}_i^T \dot{\theta}_i, \tag{34}$$

with $\dot{G}_1(x) = 0$.

Using expressions (16), (26), (27), and (32), (34) can become

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^n |S_i| \kappa_{0i} + \sum_{i=1}^n |S_i| \theta_i^T \varrho_i(x) + \frac{1}{\zeta} \sum_{i=1}^n S_i \Phi_i(u_i) \\
&\quad - \sum_{i=1}^n \delta_{0i} \widetilde{\kappa_{0i}} \kappa_{0i} - \sum_{i=1}^n \delta_{1i} \tilde{\theta}_i^T \theta_i \\
&\leq \sum_{i=1}^n |S_i| \kappa_{0i} + \sum_{i=1}^n |S_i| \theta_i^T \varrho_i(x) + \sum_{i=1}^n -\bar{\omega}_i(t) |S_i| \\
&\quad - \sum_{i=1}^n \delta_{0i} \widetilde{\kappa_{0i}} \kappa_{0i} - \sum_{i=1}^n \delta_{1i} \tilde{\theta}_i^T \theta_i \\
&= -\sum_{i=1}^n \kappa_{1i} S_i^2 - \sum_{i=1}^n \delta_{0i} \widetilde{\kappa_{0i}} \kappa_{0i} - \sum_{i=1}^n \delta_{1i} \tilde{\theta}_i^T \theta_i.
\end{aligned} \tag{35}$$

We can easily verify that

$$-\delta_{0i} \widetilde{\kappa_{0i}} \kappa_{0i} \leq -\frac{\delta_{0i}}{2} \widetilde{\kappa_{0i}^2} + \frac{\delta_{0i}}{2} \kappa_{0i}^{*2}, \quad (36)$$

$$-\delta_{1i} \widetilde{\theta_i} \theta_i \leq -\frac{\delta_{1i}}{2} \|\widetilde{\theta_i}\|^2 + \frac{\delta_{1i}}{2} \|\theta_i^*\|^2.$$

Using the previous inequalities, (35) becomes

$$\dot{V} \leq -\sum_{i=1}^n \kappa_{1i} \varsigma_i^2 - \sum_{i=1}^n \frac{\delta_{0i}}{2} \widetilde{\kappa_{0i}^2} + \sum_{i=1}^n \frac{\delta_{0i}}{2} \kappa_{0i}^{*2} - \sum_{i=1}^n \frac{\delta_{1i}}{2} \|\widetilde{\theta_i}\|^2 + \sum_{i=1}^n \frac{\delta_{1i}}{2} \|\theta_i^*\|^2. \quad (37)$$

Since $G(x) \geq \delta_{g0} E_n$ (recall that the matrix $G(x)$ is positive definite and symmetric), then

$$S^T G^{-1}(x) S = S^T G_1(x) S \leq \frac{1}{\delta_{g0}} \|S\|^2. \quad (38)$$

From (37) and (38), we have

$$\dot{V} \leq -\omega V + \tau, \quad (39)$$

where

$$\tau = \sum_{i=1}^n \frac{\delta_{0i}}{2} \kappa_{0i}^{*2} + \sum_{i=1}^n \frac{\delta_{1i}}{2} \|\theta_i^*\|^2, \\ \omega = \min\{\min_i\{2\zeta\delta_{g0}\kappa_{1i}\}, \min_i\{\chi_{0i}\delta_{0i}\}, \min_i\{\chi_{1i}\delta_{1i}\}\}. \quad (40)$$

Multiplying (39) by $e^{\omega t}$ yields

$$\frac{d}{dt}(V e^{\omega t}) \leq \tau e^{\omega t}. \quad (41)$$

Integrating (41) over $[0, t]$, we have

$$0 \leq V(t) \leq \frac{\pi}{\omega} + \left(V(0) - \frac{\tau}{\omega}\right) e^{-\omega t}. \quad (42)$$

So, all signals κ_{0i} , θ_i , S_i , and x are bounded.

From (33), $V(0)$ can be written as follows:

$$V(0) = \frac{1}{2\zeta} S(0)^T G_1(x) S(0) + \frac{1}{2} \sum_{i=1}^n \frac{1}{\chi_{0i}} (\kappa_{0i}(0) - \kappa_{0i}^*)^2 \\ + \frac{1}{2} \sum_{i=1}^n \frac{1}{\chi_{1i}} (\theta_i(0) - \theta_i^*)^T (\theta_i(0) - \theta_i^*). \quad (43)$$

Since $G_1(x)$ is symmetric and positive definite (i.e., it exists an unknown positive constant δ_{g1} , such that $G_1(x) \geq \delta_{g1} E_n$), from (33) and (42), we have

$$|S_i| \leq \left(\frac{2\zeta}{\delta_{g1}} \left(\frac{\tau}{\omega} + \left(V(0) - \frac{\tau}{\omega} \right) e^{-\omega t} \right) \right)^{1/2}. \quad (44)$$

Finally, the solution of S_i exponentially converges to a bounded region $\Omega_{S_i} = \{S_i | |S_i| \leq ((2\zeta/\delta_{g1})(\tau/\omega))^{1/2}\}$. $\square$

Remark 4. If $\Phi_i(u_i) = u_i$ (or when $u_{i+} = u_{i-} = 0$, and $\phi_{i+}(u_i) = \phi_{i-}(u_i) = 1$), i.e., there are no dead-zone nonlinear function terms in the input function, we can clearly show

that the proposed controller is still suitable for such two-link robotic manipulator systems.

Remark 5. In the case where $u_{i+} = u_{i-} = u_{i0}$, (24) is reduced to the following expression:

$$u_i = -(\omega_{it} + u_{i0}) \text{sign}(S_i), \quad (45)$$

where $\omega_i(t) = \kappa_{0i} + \kappa_{1i}|S_i| + \theta_i^T \varrho_i(x)$.

4. Simulation Results

In this part, a simulation example is given to show the effectiveness of the proposed adaptive fuzzy controller. System (3) is given by

$$\ddot{\xi} = M^{-1}(\xi)[-C(\xi, \dot{\xi})\dot{\xi} - G(\xi) + \Phi(u)] + D(t). \quad (46)$$

And the details of system model (46) are given as follows:

$$M(\xi) = \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix}, \\ C(\xi, \dot{\xi})\dot{\xi} = \begin{bmatrix} -\frac{1}{2}m_2 l_1 l_2 (2\dot{\xi}_1 \dot{\xi}_2 + \dot{\xi}_2^2) \sin q_2 \\ \frac{1}{2}m_2 l_1 l_2 \dot{\xi}_1^2 \sin \xi_2 \end{bmatrix}, \\ G(\xi) = \begin{bmatrix} \left(\frac{1}{2}m_1 + m_2\right) g l_1 \cos \xi_2 + \frac{1}{2}m_2 l_2 g \cos(\xi_1 + \xi_2) \\ \frac{1}{2}m_2 g l_2 \cos(\xi_1 + \xi_2) \end{bmatrix}, \\ D(t) = \begin{bmatrix} \sin t \\ \sin t \end{bmatrix}. \quad (47)$$

The input nonlinearities $\Phi_1(u_1)$ are described by

$$\Phi_1(u_1) = \begin{cases} (u_1 - 2)(1 - 0.3 \sin(u_1)), & u_1 > 2, \\ 0, & -2 \leq u_1 \leq 2, \\ (u_1 + 2)(0.8 - 0.3 \cos(u_1)), & u_1 < -2, \end{cases} \quad (48)$$

and the input nonlinearity $\Phi_2(u_2)$ is supposed to be

$$\Phi_2(u_2) = \begin{cases} (u_2 - 5)(1 - 0.3 \sin(u_2)), & u_2 > 5, \\ 0, & -5 \leq u_2 \leq 5, \\ (u_2 + 5)(0.8 - 0.3 \cos(u_2)), & u_2 < -5. \end{cases} \quad (49)$$

Then, we have $M_{11} = ((1/4)m_1 + m_2)l_1^2 + (1/4)m_2 l_2^2 + m_2 l_1 l_2 \cos q_2$, $M_{12} = (1/4)m_2 l_2^2 + (1/2)m_2 l_1 l_2 \cos q_2$, and $M_{22} = (1/4)m_2 l_2^2$, m_1 and m_2 are the mass, and l_1 and l_2 denote the length. In the simulation, $m_1 = 4$, $m_2 = 2$, $l_1 = 2$, $l_2 = 1$, and $g = 9.8$.

FIGURE 1: Simulation results for (a) $y_1(t)$ and (b) $y_2(t)$.

FIGURE 2: Tracking errors for (a) e_1 and (b) e_2 .

FIGURE 3: Control input signal for u_1 and u_2 . FIGURE 4: Control input function for $\Phi_1(u_1)$ and $\Phi_2(u_2)$.

Define $y = [y_1, y_2]^T = [\xi_1, \xi_2]^T$, $u = [u_1, u_2]^T$, $\Phi(u) = [\Phi_1(u_1), \Phi_2(u_2)]^T$, $D(t) = [d_1(t), d_2(t)]^T$, and $x = [x_1^T, x_2^T]^T \in \mathcal{R}^4$, where $x_1(t) = [\xi_1, \xi_2]^T$ and $x_2(t) = [\dot{\xi}_1, \dot{\xi}_2]^T$. The initial conditions of the system are taken as $x(0) = [y_1(0), y_2(0), \dot{y}_1(0), \dot{y}_2(0)]^T = [0, 0, 0, 0]^T$. The desired signal is $y_d = [\sin 2t, \sin 2t]^T$, and the external disturbances are selected as follows: $d_1(t) = d_2(t) = \sin t$.

The adaptive fuzzy systems $\theta_i^T \phi_i(x)$, with $i = 1, 2$, have the vector $x = [y_1, y_2, \dot{y}_1, \dot{y}_2]^T$ as input, and we define three triangular membership functions uniformly distributed on the intervals $[-5, 5]$, so we will use $3 \times 3 \times 3 \times 3 = 81$ fuzzy rules in the simulation. The design parameters are chosen as follows: $\chi_{01} = \chi_{02} = 10$, $\chi_{11} = \chi_{12} = 5$, $\delta_{01} = \delta_{02} = 0.001$, $\delta_{11} = \delta_{12} = 0.0005$, $\gamma_1 = \gamma_2 = 2$, and $\kappa_{11} = \kappa_{12} = 2$. The

FIGURE 5: Estimation of input function for $f_1(x)$ and $f_2(x)$. FIGURE 6: Fuzzy parameter for θ_1 .

initial condition is $\kappa_{01}(0) = \kappa_{02}(0) = 0$, $\theta_{1j} = \theta_{2j} = 0$, $j = 1, 2, \dots, 81$.

The simulation results are shown in Figures 1–7. Figure 1 clearly shows the simulation results of the output vector y tracking the expected vector y_d , which shows that the tracking performance is very good. Figures 2(a) and 2(b) show the tracking errors and their derivatives of the two-link robotic manipulator system, i.e., e_i and $\dot{e}_i$, for $i = 1, 2$. Figure 3 presents the control input vector u_1 and u_2 , and the boundary of the control input signal is bounded. It is worth noting that, in all these simulation results, the assumptions of the two-link robotic manipulator system model are completely unknown, but for some functions, i.e., $\Phi_i(u_i)$, $i = 1, 2$ and $G(x)$, their basic properties are already known. Figure 4 shows the control input function of the system. Figure 5 shows the estimated value of the control

FIGURE 7: Fuzzy parameter for θ_2 .

input function. The adaptive fuzzy parameters of the fuzzy system are shown in Figures 6 and 7. Through these simulation results, we can see the boundedness of these control signals in closed-loop systems, and through the proposed fuzzy adaptive control scheme, internal disturbances in two-link robotic manipulator system and uncertain nonlinear input functions can be effectively resolved. Finally, we can get that, through the control signal, these input vectors can quickly converge to the tracking vector at time $t = 0$.

5. Conclusions

In this study, a fuzzy adaptive VSC for uncertain robotic manipulators system with dead zones has been proposed. It is proven that the proposed VSC can well handle the dead zone. The FLSs are used to model the robotic manipulators, and it is also shown that the model error is small enough by using the proposed adaptation law. Meanwhile, the VSC can maintain the good tracking performance even in the presence of dead zones. How to extend the proposed method to tackle the input saturation is one of our research directions.

Data Availability

All data used to support the findings of this are included in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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