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Retraction

Retracted: On Three Types of Soft Rough Covering-Based Fuzzy Sets

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Atef, S. Nada, A. Gumaei, and A. S. Nawar, "On Three Types of Soft Rough Covering-Based Fuzzy Sets," *Journal of Mathematics*, vol. 2021, Article ID 6677298, 9 pages, 2021.

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Research Article

On Three Types of Soft Rough Covering-Based Fuzzy Sets

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Recently, the concept of a soft rough fuzzy covering (briefly, SRFC) by means of soft neighborhoods was defined and their properties were studied by Zhan's model. As a generalization of Zhan's method and in order to increase the lower approximation and decrease the upper approximation, the present work aims to define the complementary soft neighborhood and hence three types of soft rough fuzzy covering models (briefly, 1-SRFC, 2-SRFC, and 3-SRFC) are proposed. We discuss their axiomatic properties. According to these results, we investigate three types of fuzzy soft measure degrees (briefly, 1-SMD, 2-SMD, and 3-SMD). Also, three kinds of ψ-soft rough fuzzy coverings (briefly, 1-ψ-SRFC, 2-ψ-SRFC, and 3-ψ-SRFC) and three kinds of $\mathscr{D}$-soft rough fuzzy coverings (briefly, 1-$\mathscr{D}$-SRFC, 2-$\mathscr{D}$-SRFC, and 3-$\mathscr{D}$-SRFC) are discussed and some of their properties are studied. Finally, the relationships among these three models and Zhan's model are presented.

1. Introduction

Pawlak [1, 2] developed the rough set theory for addressing the vagueness and granularity of information systems and data analysis. His theory and its generalizations since then have produced applications in different areas [3–15]. As mentioned above, a large variety of generalized rough set models have been investigated. These extensions include variable precision rough sets, covering-based rough sets (CRSs), fuzzy rough sets and rough fuzzy sets, covering-based multigranulation fuzzy rough sets, decision-theoretic rough sets, soft fuzzy rough sets, and probabilistic rough sets [16–19].

Covering-based rough sets are arguably one of the most studied generalizations of rough sets. Pomykala [20, 21] produced two pairs of operators with dual approximation. The definitions of neighborhood and granularity gave further insights of these approximation operators I (cf., Yao [22, 23]). Under the assumption of incomplete knowledge, Couso and Dubois [24] studied both pairs as well. Bonikowski et al. [25] proposed a model of CRS that depends on the concept of minimal description. There are other CRS models and relationships between them in [26–29]. Some CRS models were proposed by Tsang et al. [30] and Xu and Zhang [31]. Liu and Sai [32] compared CRS models defined by Zhu [26] and Xu and Zhang [31]. Ma [33] developed some neighborhood-related forms of covering rough sets using the neighborhood and complementary neighborhood concepts in 2012.

The fuzzy covering from a fuzzy relation is introduced by Deng et al. [34] in 2007. In 2016, Ma [35] introduced the concept of a fuzzy β-neighborhood to generate two types of fuzzy rough coverings. In 2017, Yang and Hu [36] defined the fuzzy β-complementary neighborhood to establish some types of the fuzzy covering-based rough sets. Also, Yang and Hu [37] in 2019 introduced the concept of fuzzy β-minimal description and fuzzy β-maximal description to propose four types of fuzzy neighborhood operators and studied their properties. D'eer et al. [38] discussed the fuzzy neighborhoods according to fuzzy coverings.

Dubois and Prade [39] presented the concepts of rough fuzzy set and fuzzy rough set in 1990. Lately, some scholars worked on covering-based rough fuzzy sets and fuzzy rough sets, for more information see [40–45].

Molodtsov [46] conceived the soft set theory as another valuable mathematical method for tackling the uncertainty problem. The soft set theory has a unique benefit compared to conventional mathematical methods, namely, parameterization by attributes. Maji et al. [47] introduced the concept of fuzzy soft sets (briefly, FSSs) in 2002. Recently, many researchers have studied the soft set theory, see [48–62]. Recently, the notion of a soft rough fuzzy covering by using soft neighborhoods was defined and their properties were studied by Zhan and Sun [63].

The aim of the paper is to increase the lower approximation and decrease the upper approximation of Zhan's model; this paper's contribution is to introduce three new kinds of soft rough fuzzy covering based on soft neighborhoods and complementary soft neighborhoods. Also, some of the related properties are studied. Further, the relationships among these models are discussed. The outline of this paper is as follows. Section 2 gives technical preliminaries. Sections 3 and 4 describe the three types of SRFC by using the notions of soft neighborhoods and complementary soft neighborhoods. In Section 5, we establish relationships among our model and Zhan's model. We conclude in Section 6.

2. Preliminaries

In this section, we review some concepts and results related to RST, CRS, SST, and SRFC.

Definition 1 (see [64]). Let Ω be a nonempty finite universe. A fuzzy subset on the universe Ω is defined by the mapping $\mathscr{A}(\bullet)\colon \Omega \longrightarrow [0,1]$, where the $\mathscr{A}(x)$ denotes the membership grade of the element $x\,(x \in \Omega)$ in the fuzzy set $\mathscr{A}.\mathscr{F}(\Omega)$ for the set of all fuzzy subsets of the Ω.

Definition 2 (see [65]). Let Ω be a universe of discourse, $\mathscr{A}, \mathscr{B} \in \mathscr{F}(\Omega)$. Then, we have the following statements:

(1) $\mathscr{A} \subseteq \mathscr{B} \Leftrightarrow \mathscr{A}(x) \leq \mathscr{B}(x)$,

(2) $\mathscr{A} = \mathscr{B} \Leftrightarrow \mathscr{A} \subseteq \mathscr{B}$ and $\mathscr{B} \subseteq \mathscr{A}$,

(3) $(\mathscr{A} \cap \mathscr{B})(x) = \mathscr{A}(x) \wedge \mathscr{B}(x)$ and $(\mathscr{A} \cup \mathscr{B})(x) = \mathscr{A}(x) \vee \mathscr{B}(x)$,

(4) $\mathscr{A}^c(x) = 1 - \mathscr{A}(x)$.

Definition 3 (see [26]). Let Ω be a universe and $\mathbf{C}$ be a family of subsets of Ω. If the empty set does not belong to $\mathbf{C}$ and $\Omega = \cup_{C \in \mathbf{C}} C$, then $\mathbf{C}$ is called a covering of Ω, and the ordered pair $(\Omega, \mathbf{C})$ is called a covering approximation space.

Definition 4 (see [26]). Let $(\Omega, \mathbf{C})$ be a covering approximation space. Then, for each $x \in \Omega$, define the neighborhood of x as follows:

$$N_{\mathbf{C}}(x) = \cap \{C \in \mathbf{C}\colon x \in C\}. \tag{1}$$

As already mentioned, the notion of soft sets was introduced in [46]. The beauty of soft sets lies in their quality of hybridization with other theories such as fuzzy sets and rough sets.

Definition 5 (see [46]). Let Ω be a universe of discourse, and let $\mathscr{E}$ be a finite set of relevant parameters regarding Ω. The pair $\mathscr{S} = (\tilde{\mathscr{F}}, tA)$ is a soft set over Ω, when $\mathscr{A} \subseteq \mathscr{E}$ and $\tilde{\mathscr{F}}\colon \mathscr{A} \longrightarrow \mathscr{P}(\Omega)$ (i.e., $\tilde{\mathscr{F}}$ is a set-valued mapping from the subset of attributes $\mathscr{A}$ to Ω and $\mathscr{P}(\Omega)$ denotes the set of all subsets of Ω).

Definition 6 (see [52, 54]). The soft set $\mathscr{S} = (\tilde{\mathscr{F}}, tA)$ is called a full soft set if $\cup_{a \in \mathscr{A}} \tilde{\mathscr{F}}(a) = \Omega$ and a full soft set $\mathscr{S} = (\tilde{\mathscr{F}}, tA)$ is called a soft covering (briefly, SC) over Ω if for each $a \in \mathscr{A}$, then $\tilde{\mathscr{F}}(a) \neq \varnothing$. In addition, $(\Omega, \tilde{\mathscr{F}}, \mathscr{A})$ is called a soft covering approximation space (briefly, SCAS).

Zhan et al. [63] introduced the concept of soft rough fuzzy covering (briefly, SRFC). So, in the following, some basic concepts related to SRFC are given.

Definition 7 (see [63]). Let $(\Omega, \tilde{\mathscr{F}}, \mathscr{A})$ be an SCAS. For each $x \in \Omega$, then we define a soft neighborhood of x as follows:

$$N_S(x) = \cap \left\{\tilde{\mathscr{F}}(a)t\colon naq \in hA_{\text{¸}}xx7 \in C\tilde{\mathscr{F}};(a)\right\}. \tag{2}$$

Definition 8 (see [63]). Let $(\Omega, \tilde{\mathscr{F}}, \mathscr{A})$ be a SCAS of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathscr{S}^{-0}(\mathscr{A})$ (resp. $\mathscr{S}^{+0}(\mathscr{A})$) is called the soft covering lower approximation (resp. the soft covering upper approximation), briefly 0-SCLA (resp. 0-SCUA), where

$$\begin{aligned}
\mathscr{S}^{-0}(\mathscr{A})(x) &= \wedge\{\mathscr{A}(y)\colon y \in N_S(x)\},\\
\mathscr{S}^{+0}(\mathscr{A})(x) &= \vee\{\mathscr{A}(y)\colon y \in N_S(x)\}, \quad \forall x \in \Omega.
\end{aligned} \tag{3}$$

If $\mathscr{S}^{-0}(\mathscr{A}) \neq \mathscr{S}^{+0}(\mathscr{A})$, then $\mathscr{A}$ is called a soft rough covering-based fuzzy set (briefly, 0-SRFC); otherwise, it is definable.

3. The First Kind of Soft Rough Covering-Based Fuzzy Sets

This section deals with the 1-SRFC, 1-SMD, 1-ψ-SRFC, and 1-$\mathscr{D}$-SRFC as complementary soft neighborhoods and studies some of their properties.

Definition 9 Let $(\Omega, \tilde{\mathscr{F}}, \mathscr{A})$ be an SCAS. Then, for each $x \in \Omega$, define the complementary soft neighborhood of x as follows:

$$M_S(x) = \{y \in \Omega, x \in N_S(y)\}. \tag{4}$$

Example 1. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS and $(\widetilde{\mathscr{F}}, tA)$ be a soft set given as Table 1.

Compute the soft neighborhoods and complementary soft neighborhoods as the following:

$$\begin{aligned}
N_S(x_1) &= \{x_1, x_2\},\\
N_S(x_2) &= \{x_1, x_2\},\\
N_S(x_3) &= \{x_3\},\\
N_S(x_4) &= \{x_4, x_5\},\\
N_S(x_5) &= \{x_5\},\\
N_S(x_6) &= \{x_3, x_5, x_6\},\\
M_S(x_1) &= \{x_1, x_2\},\\
M_S(x_2) &= \{x_1, x_2\},\\
M_S(x_3) &= \{x_3, x_6\},\\
M_S(x_4) &= \{x_4\},\\
M_S(x_5) &= \{x_4, x_5, x_6\},\\
M_S(x_6) &= \{x_6\}.
\end{aligned} \tag{5}$$

Definition 10. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathcal{S}^{-1}(\mathscr{A})$ (resp. $\mathcal{S}^{+1}(\mathscr{A})$) is called the first type of a soft covering lower approximation (resp. the first type of a soft covering upper approximation), briefly 1-SCLA (resp. 1-SCUA), where

$$\begin{aligned}
\mathcal{S}^{-1}(\mathscr{A})(x) &= \wedge\{\mathscr{A}(y)\colon y \in M_S(x)\},\\
\mathcal{S}^{+1}(\mathscr{A})(x) &= \vee\{\mathscr{A}(y)\colon y \in M_S(x)\}, \quad \forall x \in \Omega.
\end{aligned} \tag{6}$$

If $\mathcal{S}^{-1}(\mathscr{A}) \neq \mathcal{S}^{+1}(\mathscr{A})$, then $\mathscr{A}$ is called a soft rough covering-based fuzzy set (briefly, 1-SRFC); otherwise, it is definable.

Example 2 (continued from Example 1). If we take fuzzy set $\mathscr{A} = (0.1/x_1) + (0.3/x_2) + (0.8/x_3) + (0.2/x_4) + (0.5/x_5) + (0.7/x_6)$, then we have the following results:

$$\begin{aligned}
\mathcal{S}^{-1}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.7}{x_3} + \frac{0.2}{x_4} + \frac{0.2}{x_5} + \frac{0.7}{x_6},\\
\mathcal{S}^{+1}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.7}{x_5} + \frac{0.7}{x_6}.
\end{aligned} \tag{7}$$

Therefore, $\mathscr{A}$ is a 1-SRFC. In addition, we can obtain

$$\begin{aligned}
\mathcal{S}^{-0}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.5}{x_6},\\
\mathcal{S}^{+0}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.5}{x_4} + \frac{0.5}{x_5} + \frac{0.8}{x_6}.
\end{aligned} \tag{8}$$

Thus, $\mathscr{A}$ is a 0-SRFC.

Remark 1. From Example 2, we can see that

Table 1: Table for $(\widetilde{\mathscr{F}}, tA)$.

Ω	ν_1	ν_2	ν_3	ν_4	ν_5
x_1	1	1	1	0	0
x_2	1	1	1	0	0
x_3	0	1	0	1	1
x_4	0	0	1	1	0
x_5	0	0	1	1	1
x_6	0	0	0	1	1

(1) $\mathcal{S}^{-1}(\mathscr{A}) \nsubseteq \mathcal{S}^{-0}(\mathscr{A})$ and $\mathcal{S}^{-0}(\mathscr{A}) \nsubseteq \mathcal{S}^{-1}(\mathscr{A})$,

(2) $\mathcal{S}^{+1}(\mathscr{A}) \nsubseteq \mathcal{S}^{+0}(\mathscr{A})$ and $\mathcal{S}^{+0}(\mathscr{A}) \nsubseteq \mathcal{S}^{+1}(\mathscr{A})$.

Therefore, it is clear that 0-SRFC model and 1-SRFC model cannot contain each other.

Theorem 1. *Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{A}, \mathscr{B} \in \mathscr{F}(\Omega)$. Then, we have the following properties:*

(1) (L1) $\mathcal{S}^{-1}(\mathscr{A}^c) = (\mathcal{S}^{+1}(\mathscr{A}))^c$.

(H1) $\mathcal{S}^{+1}(\mathscr{A}^c) = (\mathcal{S}^{-1}(\mathscr{A}))^c$.

(2) If $\mathscr{A} \subseteq \mathscr{B}$, then

(L2) $\mathcal{S}^{-1}(\mathscr{A}) \subseteq \mathcal{S}^{-1}(\mathscr{B})$.

(H2) $\mathcal{S}^{+1}(\mathscr{A}) \subseteq \mathcal{S}^{+1}(\mathscr{B})$.

(3) (L3) $\mathcal{S}^{-1}(\mathscr{A} \cap \mathscr{B}) = \mathcal{S}^{-1}(\mathscr{A}) \cap \mathcal{S}^{-1}(\mathscr{B})$.

(H3) $\mathcal{S}^{+1}(\mathscr{A} \cap \mathscr{B}) \subseteq \mathcal{S}^{+1}(\mathscr{A}) \cap \mathcal{S}^{+1}(\mathscr{B})$.

(4) (L4) $\mathcal{S}^{-1}(\mathscr{A} \cup \mathscr{B}) \supseteq \mathcal{S}^{-1}(\mathscr{A}) \cup \mathcal{S}^{-1}(\mathscr{B})$.

(H4) $\mathcal{S}^{+1}(\mathscr{A} \cup \mathscr{B}) = \mathcal{S}^{+1}(\mathscr{A}) \cup \mathcal{S}^{+1}(\mathscr{B})$.

(5) (L5) $\mathcal{S}^{-1}(\mathscr{A}) = \mathcal{S}^{-1}(\mathcal{S}^{-1}(\mathscr{A}))$.

(H5) $\mathcal{S}^{+1}(\mathscr{A}) = \mathcal{S}^{+1}(\mathcal{S}^{+1}(\mathscr{A}))$.

(6) (LH) $\mathcal{S}^{-1}(\mathscr{A}) \subseteq \mathscr{A} \subseteq \mathcal{S}^{+1}(\mathscr{A})$.

Proof. We shall only prove (L1), (L2), (L3), (L5), and (LH), since (L1) (resp. (L2), (L4), and (L5)) is equivalent to (H1) (resp. (H2), (H4), and (H5)) and (L3), (L4), (H3), and (H4) are all equivalent to each other.

(1) (L1):

$$\begin{aligned}
\mathcal{S}^{-1}(\mathscr{A}^c) &= \wedge\{\mathscr{A}^c(y)\colon y \in M_S(x)\}\\
&= \wedge\{1 - \mathscr{A}(y)\colon y \in M_S(x)\}\\
&= 1 - \vee\{\mathscr{A}(y)\colon y \in M_S(x)\} = \left(\mathcal{S}^{+1}(\mathscr{A})\right)^c.
\end{aligned} \tag{9}$$

(2) (L2): let $\mathscr{A}, \mathscr{B} \in \mathscr{F}(\Omega)$ such that $\mathscr{A} \subseteq \mathscr{B}$ and $x \in \Omega$. Then, we get the following result:

$$\begin{aligned}
\mathcal{S}^{-1}(\mathscr{A})(x) &= \wedge\{\mathscr{A}(y)\colon y \in M_S(x)\}\\
&\leq \wedge\{\mathscr{B}(y)\colon y \in M_S(x)\} = \mathcal{S}^{-1}(\mathscr{B})(x).
\end{aligned} \tag{10}$$

(3) (L3): if $x \in \Omega$, then we have

$$\begin{aligned}\mathcal{S}^{-1}(\mathcal{A}\cap\mathcal{B})(x) &= \wedge\{(\mathcal{A}\cap\mathcal{B})(y)\colon\ y\in M_S(x)\}\\ &= \wedge\{\mathcal{A}(y)\colon\ y\in M_S(x)\}\cap\wedge\{\mathcal{B}(y)\colon\ y\in M_S(x)\} = \mathcal{S}^{-1}(\mathcal{A})(x)\cap\mathcal{S}^{-1}(\mathcal{B})(x).\end{aligned}\tag{11}$$

(4) (L5):

$$\begin{aligned}\mathcal{S}^{-1}\left(\mathcal{S}^{-1}(\mathcal{A})\right)(x) &= \wedge\left\{\mathcal{S}^{-1}(\mathcal{A})(y)\colon\ y\in M_S(x)\right\} = \wedge\{\wedge\{\mathcal{A}(w)\colon\ w\in M_S(y)\}\colon\ y\in M_S(x)\}\\ &= \wedge\{\mathcal{A}(w)\colon\ w\in M_S(y)\wedge y\in M_S(x)\}\\ &= \wedge\{\mathcal{A}(w)\colon\ w\in M_S(y)\subseteq M_S(x)\} = \wedge\{\mathcal{A}(w)\colon\ w\in M_S(x)\} = \mathcal{S}^{-1}(\mathcal{A})(x).\end{aligned}\tag{12}$$

(5) (LH): it is clear from Definition 10. □

Let us define the first type of a soft measure degree (briefly, 1-SMD) as follows.

Definition 11. Let $(\Omega,\widetilde{\mathcal{F}},\mathcal{A})$ be an SCAS of Ω and $x, y\in\Omega$. The first kind of a soft measure degree between x and y (briefly, 1-SMD), denoted by $\mathcal{D}_S^1(x,y)$, is defined as follows:

$$\mathcal{D}_S^1(x,y) = \frac{|M_S(x)\cap M_S(y)|}{|M_S(x)\cup M_S(y)|}.\tag{13}$$

Obviously, $\mathcal{D}_S^1(x,x)=1$ and $\mathcal{D}_S^1(x,y)=\mathcal{D}_S^1(y,x)$. Also, $0\le\mathcal{D}_S^1(x,y)\le 1$.

Example 3 (continued from Example 1). We have the following results as shown in Table 2.

From the concept of 1-SMD, we define a new kind called a first type of a soft rough covering-based ψ-fuzzy set (briefly, 1-ψ-SRFC) as follows.

Definition 12. Let $(\Omega,\widetilde{\mathcal{F}},\mathcal{A})$ be an SCAS of Ω and $\mathcal{D}_S^1(x,y)$ be a 1-SMD of Ω. For each $\mathcal{A}\in\mathcal{F}(\Omega)$, the set $\mathcal{S}_\psi^{-1}(\mathcal{A})$ (resp. $\mathcal{S}_\psi^{+1}(\mathcal{A})$) is called the first type of a soft covering ψ-lower approximation (resp. the first type of a soft covering ψ-upper approximation), briefly 1-ψ-SCLA (resp. 1-ψ-SCUA), where

$$\begin{aligned}&\mathcal{S}_\psi^{-1}(\mathcal{A})(x) = \wedge\left\{\mathcal{A}(y)\colon\ \mathcal{D}_S^1(x,y)>\psi\right\},\\ &\mathcal{S}_\psi^{+1}(\mathcal{A})(x) = \vee\left\{\mathcal{A}(y)\colon\ \mathcal{D}_S^1(x,y)>\psi\right\},\quad \forall x\in\Omega.\end{aligned}\tag{14}$$

If $\mathcal{S}_\psi^{-1}(\mathcal{A})\neq\mathcal{S}_\psi^{+1}(\mathcal{A})$, then $\mathcal{A}$ is called 1-ψ-SRFC; otherwise, it is definable.

Example 4 (continued from Example 3). If $\psi=0.2$ and $\mathcal{A}=(0.1/x_1)+(0.3/x_2)+(0.8/x_3)+(0.2/x_4)+(0.5/x_5)+(0.7/x_6)$, then we have the following results:

$$\begin{aligned}&\mathcal{S}_\psi^{-1}(\mathcal{A}) = \frac{0.1}{x_1}+\frac{0.1}{x_2}+\frac{0.5}{x_3}+\frac{0.2}{x_4}+\frac{0.2}{x_5}+\frac{0.5}{x_6},\\ &\mathcal{S}_\psi^{+1}(\mathcal{A}) = \frac{0.3}{x_1}+\frac{0.3}{x_2}+\frac{0.8}{x_3}+\frac{0.5}{x_4}+\frac{0.8}{x_5}+\frac{0.8}{x_6}.\end{aligned}\tag{15}$$

The proof of the following theorem is similar to Theorem 1, so we omit it.

Theorem 2. *Let $(\Omega,\widetilde{\mathcal{F}},\mathcal{A})$ be an SCAS of Ω and $\mathcal{A},\mathcal{B}\in\mathcal{F}(\Omega)$. Then, we have the following properties:*

(1) (L1) $\mathcal{S}_\psi^{-1}(\mathcal{A}^c)=(\mathcal{S}_\psi^{+1}(\mathcal{A}))^c$.

(H1) $\mathcal{S}_\psi^{+1}(\mathcal{A}^c)=(\mathcal{S}_\psi^{-1}(\mathcal{A}))^c$.

(2) If $\mathcal{A}\subseteq\mathcal{B}$, then

(L2) $\mathcal{S}_\psi^{-1}(\mathcal{A})\subseteq\mathcal{S}_\psi^{-1}(\mathcal{B})$.

(H2) $\mathcal{S}_\psi^{+1}(\mathcal{A})\subseteq\mathcal{S}_\psi^{+1}(\mathcal{B})$.

(3) (L3) $\mathcal{S}_\psi^{-1}(\mathcal{A}\cap\mathcal{B})=\mathcal{S}_\psi^{-1}(\mathcal{A})\cap\mathcal{S}_\psi^{-1}(\mathcal{B})$.

(H3) $\mathcal{S}_\psi^{+1}(\mathcal{A}\cap\mathcal{B})\subseteq\mathcal{S}_\psi^{+1}(\mathcal{A})\cap\mathcal{S}_\psi^{+1}(\mathcal{B})$.

(4) (L4) $\mathcal{S}_\psi^{-1}(\mathcal{A}\cup\mathcal{B})\supseteq\mathcal{S}_\psi^{-1}(\mathcal{A})\cup\mathcal{S}_\psi^{-1}(\mathcal{B})$.

(H4) $\mathcal{S}_\psi^{+1}(\mathcal{A}\cup\mathcal{B})=\mathcal{S}_\psi^{+1}(\mathcal{A})\cup\mathcal{S}_\psi^{+1}(\mathcal{B})$.

(5) If $\alpha\le\beta$, then

(L5) $\mathcal{S}_\alpha^{-1}(\mathcal{A})\subseteq\mathcal{S}_\beta^{-1}(\mathcal{A})$.

(H5) $\mathcal{S}_\alpha^{+1}(\mathcal{A})\subseteq\mathcal{S}_\beta^{+1}(\mathcal{A})$.

(6) (LH) $\mathcal{S}_\psi^{-1}(\mathcal{A})\subseteq\mathcal{A}\subseteq\mathcal{S}_\psi^{+1}(\mathcal{A})$.

Next, we define other SRFC models induced by 1-SMD as follows.

Definition 13. Let $(\Omega,\widetilde{\mathcal{F}},\mathcal{A})$ be an SCAS of Ω and $\mathcal{D}_S^1(x,y)$ be a 1-SMD of Ω. For each $\mathcal{A}\in\mathcal{F}(\Omega)$, the set $\mathcal{S}_\mathcal{D}^{-1}(\mathcal{A})$ (resp. $\mathcal{S}_\mathcal{D}^{+1}(\mathcal{A})$) is called the first type of soft covering $\mathcal{D}$-lower approximation (resp. the first type of soft covering $\mathcal{D}$-upper approximation), briefly 1-$\mathcal{D}$-SCLA (resp. 1-$\mathcal{D}$-SCUA), where

$$\begin{aligned}&\mathcal{S}_\mathcal{D}^{-1}(\mathcal{A})(x) = \bigwedge_{y\in\Omega}\left\{\left(1-\mathcal{D}_S^1\right)(x,y)\vee\mathcal{A}(y)\right\},\\ &\mathcal{S}_\mathcal{D}^{+1}(\mathcal{A})(x) = \bigvee_{y\in\Omega}\left\{\mathcal{D}_S^1(x,y)\wedge\mathcal{A}(y)\right\},\quad \forall x\in\Omega.\end{aligned}\tag{16}$$

If $\mathcal{S}_\mathcal{D}^{-1}(\mathcal{A})\neq\mathcal{S}_\mathcal{D}^{+1}(\mathcal{A})$, then $\mathcal{A}$ is called 1-$\mathcal{D}$-SRFC; otherwise, it is definable.

Example 5 (continued from Example 3). If we take the fuzzy set $\mathcal{A}=(0.1/x_1)+(0.3/x_2)+(0.8/x_3)+(0.2/x_4)+(0.5/x_5)+(0.7/x_6)$, then we have the following results:

Table 2: Table for $\mathscr{D}_S^1(x_i, x_j)\,\forall i,\ j \in \{1, 2, \ldots, 6\}$.

Ω	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1	1	0	0	0	0
x_2	1	1	0	0	0	0
x_3	0	0	1	0	(1/4)	(1/2)
x_4	0	0	0	1	(1/3)	0
x_5	0	0	(1/4)	(1/3)	1	(1/3)
x_6	0	0	(1/2)	0	(1/3)	1

$$\begin{aligned}\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.7}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6},\\ \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.3}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6}.\end{aligned} \tag{17}$$

Theorem 3. *Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{A}, \mathscr{B} \in \mathscr{F}(\Omega)$. Then, we have the following properties:*

(1) (L1) $\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}^c) = (\mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A}))^c$.
(H1) $\mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A}^c) = (\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}))^c$.
(2) If $\mathscr{A} \subseteq \mathscr{B}$, then
(L2) $\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}) \subseteq \mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{B})$.
(H2) $\mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A}) \subseteq \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{B})$.
(3) (L3) $\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A} \cap \mathscr{B}) = \mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}) \cap \mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{B})$.
(H3) $\mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A} \cap \mathscr{B}) \subseteq \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A}) \cap \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{B})$.
(4) (L4) $\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A} \cup \mathscr{B}) \supseteq \mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}) \cup \mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{B})$.
(H4) $\mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A} \cup \mathscr{B}) = \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A}) \cup \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{B})$.
(5) (LH) $\mathcal{S}_{\mathscr{D}}^{-1}(\mathscr{A}) \subseteq \mathscr{A} \subseteq \mathcal{S}_{\mathscr{D}}^{+1}(\mathscr{A})$.

Proof. It is similar to Theorem 1. □

4. The Other Two SRFC Models

The implementation of the other two types of SRFC models (i.e., 2-SRFC and 3-SRFC) will be the subject of this section by merging soft neighborhoods and complementary soft neighborhoods. We list only the baseline concepts and omit the properties.

4.1. Type 2-SRFC

Definition 14. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathcal{S}^{-2}(\mathscr{A})$ (resp. $\mathcal{S}^{+2}(\mathscr{A})$) is called the second type of a soft covering lower approximation (resp. the second type of a soft covering upper approximation), briefly 2-SCLA (resp. 2-SCUA), where

$$\begin{aligned}\mathcal{S}^{-2}(\mathscr{A})(x) &= \wedge\{\mathscr{A}(y)\colon\ y \in (N_S \cap M_S)(x)\},\\ \mathcal{S}^{+2}(\mathscr{A})(x) &= \vee\{\mathscr{A}(y)\colon\ y \in (N_S \cap M_S)(x)\}, \quad \forall x \in \Omega.\end{aligned} \tag{18}$$

If $\mathcal{S}^{-2}(\mathscr{A}) \neq \mathcal{S}^{+2}(\mathscr{A})$, then $\mathscr{A}$ is called a soft rough covering-based fuzzy set (briefly, 2-SRFC); otherwise, it is definable.

Example 6. Let us consider Examples 1 and 2. Then, for all $x \in \Omega$, we have

$$\begin{aligned}(N_S \cap M_S)(x_1) &= \{x_1, x_2\},\\ (N_S \cap M_S)(x_2) &= \{x_1, x_2\},\\ (N_S \cap M_S)(x_3) &= \{x_3\},\\ (N_S \cap M_S)(x_4) &= \{x_4\},\\ (N_S \cap M_S)(x_5) &= \{x_5\},\\ (N_S \cap M_S)(x_6) &= \{x_6\}.\end{aligned} \tag{19}$$

Also, we get $\mathcal{S}^{-2}(\mathscr{A})$ and $\mathcal{S}^{+2}(\mathscr{A})$ as the following:

$$\begin{aligned}\mathcal{S}^{-2}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6},\\ \mathcal{S}^{+2}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6}.\end{aligned} \tag{20}$$

We define the second type of a soft measure degree (briefly, 2-SMD) as follows.

Definition 15. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $x, y \in \Omega$. The second type of a soft measure degree between x and y (briefly, 2-SMD), denoted by $\mathscr{D}_S^2(x, y)$, is defined as follows:

$$\mathscr{D}_S^2(x, y) = \frac{\left|(N_S \cap M_S)(x) \cap (N_S \cap M_S)(y)\right|}{\left|(N_S \cap M_S)(x) \cup (N_S \cap M_S)(y)\right|}. \tag{21}$$

Obviously, $\mathscr{D}_S^2(x, x) = 1$ and $\mathscr{D}_S^2(x, y) = \mathscr{D}_S^2(y, x)$. Also, $0 \leq \mathscr{D}_S^2(x, y) \leq 1$.

Example 7 (continued from Example 6). We have the following results as set in Table 3.

From the concept of 2-SMD, we define a second type of a soft rough covering-based ψ-fuzzy set (briefly, 2-ψ-SRFC) as follows.

Definition 16. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{D}_S^2(x, y)$ be a 2-SMD of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathcal{S}_{\psi}^{-2}(\mathscr{A})$ (resp. $\mathcal{S}_{\psi}^{+2}(\mathscr{A})$) is called the second type of a soft covering ψ-lower approximation (resp. the second type of a soft covering ψ-upper approximation), briefly 2-ψ-SCLA (resp. 2-ψ-SCUA), where

$$\begin{aligned}\mathcal{S}_{\psi}^{-2}(\mathscr{A})(x) &= \wedge\left\{\mathscr{A}(y)\colon\ \mathscr{D}_S^2(x, y) > \psi\right\},\\ \mathcal{S}_{\psi}^{+2}(\mathscr{A})(x) &= \vee\left\{\mathscr{A}(y)\colon\ \mathscr{D}_S^2(x, y) > \psi\right\}, \quad \forall x \in \Omega.\end{aligned} \tag{22}$$

If $\mathcal{S}_{\psi}^{-2}(\mathscr{A}) \neq \mathcal{S}_{\psi}^{+2}(\mathscr{A})$, then $\mathscr{A}$ is called 2-ψ-SRFC; otherwise, it is definable.

Example 8. Let us consider Example 7. If we take $\psi = 0.2$ and $\mathscr{A} = (0.1/x_1) + (0.3/x_2) + (0.8/x_3) + (0.2/x_4) + (0.5/x_5) + (0.7/x_6)$, then 2-$\psi$-SCLA and 2-$\psi$-SCUA are obtained as follows:

Table 3: Table for $\mathscr{D}_S^2(x_i, x_j)\, \forall i,\ j \in \{1, 2, \ldots, 6\}$.

Ω	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1	1	0	0	0	0
x_2	1	1	0	0	0	0
x_3	0	0	1	0	0	0
x_4	0	0	0	1	0	0
x_5	0	0	0	0	1	0
x_6	0	0	0	0	0	1

$$\begin{aligned}\mathscr{S}_\psi^{-2}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6},\\ \mathscr{S}_\psi^{+2}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6}.\end{aligned} \tag{23}$$

Now, we define other SRFC models induced by 2-SMD as follows.

Definition 17. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{D}_S^2(x, y)$ be a 2-SMD of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathscr{S}_{\mathscr{D}}^{-2}(\mathscr{A})$ (resp. $\mathscr{S}_{\mathscr{D}}^{+2}(\mathscr{A})$) is called the second type of soft covering $\mathscr{D}$-lower approximation (resp. the second type of soft covering $\mathscr{D}$-upper approximation), briefly 2-$\mathscr{D}$-SCLA (resp. 2-$\mathscr{D}$-SCUA), if

$$\begin{aligned}\mathscr{S}_{\mathscr{D}}^{-2}(\mathscr{A})(x) &= \underset{y\in\Omega}{\wedge}\left\{\left(1 - \mathscr{D}_S^2\right)(x, y) \vee \mathscr{A}(y)\right\},\\ \mathscr{S}_{\mathscr{D}}^{+2}(\mathscr{A})(x) &= \underset{y\in\Omega}{\vee}\left\{\mathscr{D}_S^2(x, y) \wedge \mathscr{A}(y)\right\}, \quad \forall x \in \Omega.\end{aligned} \tag{24}$$

If $\mathscr{S}_{\mathscr{D}}^{-2}(\mathscr{A}) \neq \mathscr{S}_{\mathscr{D}}^{+2}(\mathscr{A})$, then $\mathscr{A}$ is called 2-$\mathscr{D}$-SRFC; otherwise, it is definable.

Example 9. Let us consider Example 7 and fuzzy set $\mathscr{A} = (0.1/x_1) + (0.3/x_2) + (0.8/x_3) + (0.2/x_4) + (0.5/x_5) + (0.7/x_6)$ obtains 2-$\mathscr{D}$-SCLA and 2-$\mathscr{D}$-SCUA as follows:

$$\begin{aligned}\mathscr{S}_{\mathscr{D}}^{-2}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6},\\ \mathscr{S}_{\mathscr{D}}^{+2}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6}.\end{aligned} \tag{25}$$

4.2. Type 3-SRFC

Definition 18. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathscr{S}^{-3}(\mathscr{A})$ (resp. $\mathscr{S}^{+3}(\mathscr{A})$) is called the third type of a soft covering lower approximation (resp. the third type of a soft covering upper approximation), briefly 3-SCLA (resp. 3-SCUA), if

$$\begin{aligned}\mathscr{S}^{-3}(\mathscr{A})(x) &= \wedge\{\mathscr{A}(y) \colon y \in (N_S \cup M_S)(x)\},\\ \mathscr{S}^{+3}(\mathscr{A})(x) &= \vee\{\mathscr{A}(y) \colon y \in (N_S \cup M_S)(x)\}, \quad \forall x \in \Omega.\end{aligned} \tag{26}$$

If $\mathscr{S}^{-3}(\mathscr{A}) \neq \mathscr{S}^{+3}(\mathscr{A})$, then $\mathscr{A}$ is called a soft rough covering-based fuzzy set (briefly, 3-SRFC); otherwise, it is definable.

Example 10. Let us consider Examples 1 and 2. Then, for all $x \in \Omega$, we have

$$\begin{aligned}(N_S \cup M_S)(x_1) &= \{x_1, x_2\},\\ (N_S \cup M_S)(x_2) &= \{x_1, x_2\},\\ (N_S \cup M_S)(x_3) &= \{x_3, x_6\},\\ (N_S \cup M_S)(x_4) &= \{x_4, x_5\},\\ (N_S \cup M_S)(x_5) &= \{x_4, x_5, x_6\},\\ (N_S \cup M_S)(x_6) &= \{x_3, x_5, x_6\}.\end{aligned} \tag{27}$$

Also, $\mathscr{S}^{-3}(\mathscr{A})$ and $\mathscr{S}^{+3}(\mathscr{A})$ are obtained as follows:

$$\begin{aligned}\mathscr{S}^{-3}(\mathscr{A}) &= \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.7}{x_3} + \frac{0.2}{x_4} + \frac{0.2}{x_5} + \frac{0.5}{x_6},\\ \mathscr{S}^{+3}(\mathscr{A}) &= \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.5}{x_4} + \frac{0.7}{x_5} + \frac{0.8}{x_6}.\end{aligned} \tag{28}$$

In the following definition, third type of a soft measure degree (briefly, 3-SMD) is given.

Definition 19. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $x, y \in \Omega$. The third kind of a soft measure degree (briefly, 3-SMD) between x and y, denoted by $\mathscr{D}_S^3(x, y)$, is defined by

$$\mathscr{D}_S^3(x, y) = \frac{\left|(N_S \cup M_S)(x) \cap (N_S \cup M_S)(y)\right|}{\left|(N_S \cup M_S)(x) \cup (N_S \cup M_S)(y)\right|}. \tag{29}$$

Obviously, $\mathscr{D}_S^3(x, x) = 1$ and $\mathscr{D}_S^3(x, y) = \mathscr{D}_S^3(y, x)$. Also, $0 \le \mathscr{D}_S^3(x, y) \le 1$.

Example 11 (continued from Example 10). We have the following results as summarized in Table 4.

From the concept of 3-SMD, we define a third type of a soft rough covering-based ψ-fuzzy set (briefly, 3-ψ-SRFC) as follows.

Definition 20. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{D}_S^3(x, y)$ be a 3-SMD of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathscr{S}_\psi^{-3}(\mathscr{A})$ (resp. $\mathscr{S}_\psi^{+3}(\mathscr{A})$) is called the third type of a soft covering ψ-lower approximation (resp. the third type of a soft covering ψ-upper approximation), briefly 3-ψ-SCLA (resp. 3-ψ-SCUA), if

$$\begin{aligned}\mathscr{S}_\psi^{-3}(\mathscr{A})(x) &= \wedge\left\{\mathscr{A}(y) \colon \mathscr{D}_S^3(x, y) > \psi\right\},\\ \mathscr{S}_\psi^{+3}(\mathscr{A})(x) &= \vee\left\{\mathscr{A}(y) \colon \mathscr{D}_S^3(x, y) > \psi\right\}, \quad \forall x \in \Omega.\end{aligned} \tag{30}$$

If $\mathscr{S}_\psi^{-3}(\mathscr{A}) \neq \mathscr{S}_\psi^{+3}(\mathscr{A})$, then $\mathscr{A}$ is called 3-ψ-SRFC; otherwise, it is definable.

TABLE 4: Table for $\mathscr{D}_S^3(x_i, x_j)\,\forall i,\ j \in \{1, 2, \ldots, 6\}$.

Ω	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1	1	0	0	0	0
x_2	1	1	0	0	0	0
x_3	0	0	1	0	0	0
x_4	0	0	0	1	(2/3)	(1/4)
x_5	0	0	0	(2/3)	1	(1/2)
x_6	0	0	0	(1/4)	(1/2)	1

Example 12. Let us consider Example 11. If we take $\psi = 0.2$ and $\mathscr{A} = (0.1/x_1) + (0.3/x_2) + (0.8/x_3) + (0.2/x_4) + (0.5/x_5) + (0.7/x_6)$, then 3-$\psi$-SCLA and 3-$\psi$-SCUA of fuzzy sets $\mathscr{A}$ are obtained as follows:

$$\mathcal{S}_\psi^{-3}(\mathscr{A}) = \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.2}{x_5} + \frac{0.2}{x_6},$$
$$\mathcal{S}_\psi^{+3}(\mathscr{A}) = \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.7}{x_4} + \frac{0.7}{x_5} + \frac{0.7}{x_6}. \tag{31}$$

We define other SRFC models induced by 3-SMD as follows.

Definition 21. Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{D}_S^3(x, y)$ be a 3-SMD of Ω. For each $\mathscr{A} \in \mathscr{F}(\Omega)$, the set $\mathcal{S}_\mathscr{D}^{-3}(\mathscr{A})$ (resp. $\mathcal{S}_\mathscr{D}^{+3}(\mathscr{A})$) is called the third type of soft covering $\mathscr{D}$-lower approximation (resp. the third type of soft covering $\mathscr{D}$-upper approximation), briefly 3-$\mathscr{D}$-SCLA (resp. 3-$\mathscr{D}$-SCUA), where

$$\mathcal{S}_\mathscr{D}^{-3}(\mathscr{A})(x) = \bigwedge_{y\in\Omega}\left\{\left(1 - \mathscr{D}_S^3\right)(x, y) \vee \mathscr{A}(y)\right\},$$
$$\mathcal{S}_\mathscr{D}^{+3}(\mathscr{A})(x) = \bigvee_{y\in\Omega}\left\{\mathscr{D}_S^3(x, y) \wedge \mathscr{A}(y)\right\}, \quad \forall x \in \Omega. \tag{32}$$

If $\mathcal{S}_\mathscr{D}^{-3}(\mathscr{A}) \neq \mathcal{S}_\mathscr{D}^{+3}(\mathscr{A})$, then $\mathscr{A}$ is called 3-$\mathscr{D}$-SRFC; otherwise, it is definable.

Example 13. Consider Example 11 and fuzzy set $\mathscr{A} = (0.1/x_1) + (0.3/x_2) + (0.8/x_3) + (0.2/x_4) + (0.5/x_5) + (0.7/x_6)$, then 3-$\mathscr{D}$-SCLA and 3-$\mathscr{D}$-SCUA of fuzzy set $\mathscr{A}$ are obtained as follows:

$$\mathcal{S}_\mathscr{D}^{-3}(\mathscr{A}) = \frac{0.1}{x_1} + \frac{0.1}{x_2} + \frac{0.8}{x_3} + \frac{0.2}{x_4} + \frac{0.3}{x_5} + \frac{0.5}{x_6},$$
$$\mathcal{S}_\mathscr{D}^{+3}(\mathscr{A}) = \frac{0.3}{x_1} + \frac{0.3}{x_2} + \frac{0.8}{x_3} + \frac{0.5}{x_4} + \frac{0.5}{x_5} + \frac{0.7}{x_6}. \tag{33}$$

5. The Relationships between Zhan's Model and Our's

Now, we proceed to explain some relationships among the models presented in previous sections.

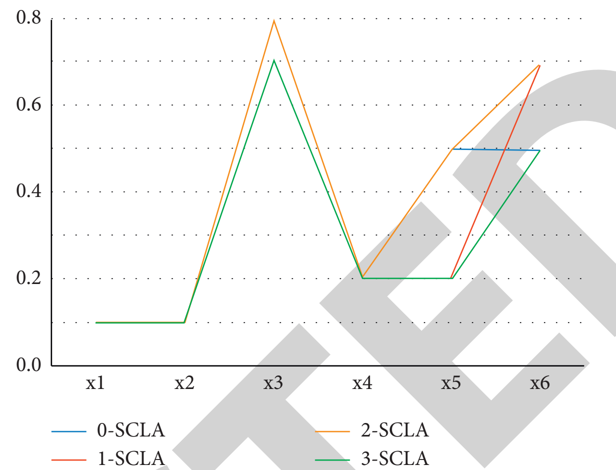

FIGURE 1: The representations of the four types of SCLA models.

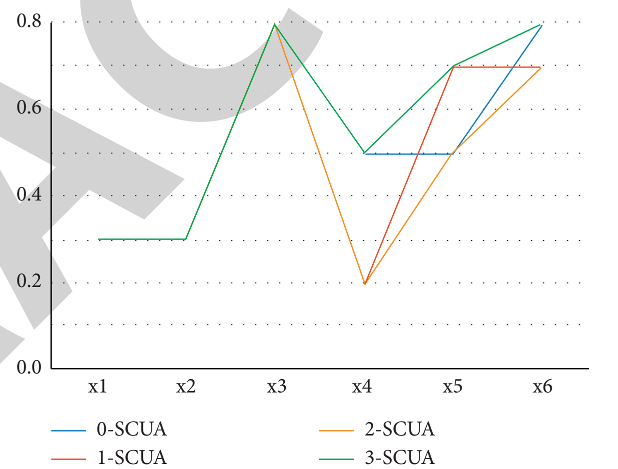

FIGURE 2: The representations of the four types of SCUA models.

Proposition 1. *Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{A} \in \mathscr{F}(\Omega)$. Then, we have the following properties.*

(1) $\mathcal{S}^{-3}(\mathscr{A}) \subseteq \mathcal{S}^{-1}(\mathscr{A}) \subseteq \mathcal{S}^{-2}(\mathscr{A})$.
(2) $\mathcal{S}^{-3}(\mathscr{A}) \subseteq \mathcal{S}^{-0}(\mathscr{A}) \subseteq \mathcal{S}^{-2}(\mathscr{A})$.
(3) $\mathcal{S}^{+2}(\mathscr{A}) \subseteq \mathcal{S}^{+1}(\mathscr{A}) \subseteq \mathcal{S}^{+3}(\mathscr{A})$.
(4) $\mathcal{S}^{+2}(\mathscr{A}) \subseteq \mathcal{S}^{+0}(\mathscr{A}) \subseteq \mathcal{S}^{+3}(\mathscr{A})$.

Proof. The proof is clear from Definitions 8, 10, 14, and 18. □

Proposition 2. *Let $(\Omega, \widetilde{\mathscr{F}}, \mathscr{A})$ be an SCAS of Ω and $\mathscr{A} \in \mathscr{F}(\Omega)$. Then, we have the following properties:*

(1) $\mathcal{S}^{-2}(\mathscr{A}) = \mathcal{S}^{-0}(\mathscr{A}) \cup \mathcal{S}^{-1}(\mathscr{A})$.
(2) $\mathcal{S}^{+2}(\mathscr{A}) = \mathcal{S}^{+0}(\mathscr{A}) \cap \mathcal{S}^{+1}(\mathscr{A})$.
(3) $\mathcal{S}^{-3}(\mathscr{A}) = \mathcal{S}^{-0}(\mathscr{A}) \cap \mathcal{S}^{-1}(\mathscr{A})$.
(4) $\mathcal{S}^{+3}(\mathscr{A}) = \mathcal{S}^{+0}(\mathscr{A}) \cup \mathcal{S}^{+1}(\mathscr{A})$.

Proof. Straightforward. □

The comparison of the results is given in Figures 1 and 2. Clearly, it is easy to see that the 2-SRFC model is better than 0-SRFC, 1-SRFC, and 2-SRFC model. Thus, this study indicates that our models are reasonable and effective.

6. Conclusion

In this paper, three new types of SRFC models are constructed as a generalization of definitions given in [63] by Zhan and Sun and their related properties are studied. The relationships between our model and Zhan's model are established. From Figures 1 and 2, it is obvious to see that the 2-SRFC is the best model (i.e., the increasing of the lower approximation and the decreasing of the upper approximation against Zhan's method) among the other models which are presented.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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