Research Article

Interpolative Kannan Contractions in T_0 -Quasi-Metric Spaces

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In this paper, we update the well-known fixed point theorem of Kannan using the interpolation notion in the realm of quasi-metric spaces. We consider some asymmetric versions. We also present some illustrative examples in support of the obtained results.

1. Introduction and Preliminaries

In 2018, Karapinar [1] published a new type of contractions obtained from the definition of the Kannan contraction by interpolation as follows.

Theorem 1 (see [11, Theorem 2.2]). *Let (X, d) be a complete metric space and $T: X \rightarrow X$ be an interpolative Kannan type contraction, i.e., a self-map such that there exist $\lambda \in [0, 1)$, $\alpha \in (0, 1)$ so that*

$$d(Tx, Ty) \leq \lambda d(x, Tx)^\alpha d(y, Ty)^{1-\alpha}, \quad (1)$$

for all $x, y \in X$ with $x \neq Tx$. Then, T has a unique fixed point in X .

This theorem has been generalized in 2019 by Gaba et al. [2] where they introduced the concept of (λ, α, β) -interpolative Kannan contractions as follows.

Definition 1. Let (X, d) be a metric space and $T: X \rightarrow X$ be a self-map. We shall call T a relaxed (λ, α, β) -interpolative Kannan contraction, if there exist $0 \leq \lambda < 1$, $0 < \alpha, \beta \leq 1$ with $\alpha + \beta < 1$ such that

$$d(Tx, Ty) \leq \lambda d(x, Tx)^\alpha d(y, Ty)^\beta. \quad (2)$$

The interpolative method has been successfully applied to a diverse range of contractions (see [3–8]).

In 1982, Reilly et al. [9] obtained a quasi-metric version of the celebrated Banach contraction principle. Since then, and especially in the last decade, several authors have contributed to the development of the fixed point theory in the framework of quasi-metric spaces (see [10, 11]). It is in the same spirit that we provide here a quasi-metric version of the interpolative Kannan fixed point theorem.

Our basic reference for quasi-metric spaces is [12].

Definition 2 (see [13]). Let $q: X \times X \rightarrow [0, \infty)$ be a function where X is a nonempty set. This function is called a **quasi-pseudometric** (respectively, **T_0 -quasi-metric**) on X if (q_1) and (q_2) (respectively, $(q_1)^*$ and (q_2)) hold, where

$$(q_1) \quad q(\xi, \xi) = 0, \text{ for all } \xi \in X$$

$$(q_1)^* \quad q(\xi, \eta) = 0 = q(\eta, \xi) \text{ implies } \xi = \eta$$

$$(q_2) \quad q(\xi, \zeta) \leq q(\xi, \eta) + q(\eta, \zeta) \text{ for all } \xi, \eta, \zeta \in X$$

The condition $(q_1)^*$ is known as the T_0 -condition. Furthermore, given a quasi-pseudometric q on X , there is a natural function $q^{-1}: X \times X \rightarrow [0, \infty)$, defined by $q^{-1}(\xi, \eta) = q(\eta, \xi)$ for all $\xi, \eta \in X$. It is named as the

conjugate of q . For a T_0 -quasi-metric q on X , the distance function $q^s: X \times X \longrightarrow [0, \infty)$ defined by $q^s(\xi, \eta) = \max\{q(\xi, \eta), q(\eta, \xi)\}$ for all $(\xi, \eta) \in X \times X$ is a metric on X .

The classical example of a T_0 -quasi-metric is the **truncated difference**.

Example 1 (truncated difference). Set $\mathbb{R}_0^+ = [0, \infty)$. Given $\delta: \mathbb{R}_0^+ \times \mathbb{R}_0^+ \longrightarrow \mathbb{R}_0^+$ as

$$\delta(\xi, \eta) = \max\{0, \xi - \eta\} \quad \xi, \eta \in X. \quad (3)$$

Under these conditions, δ forms a T_0 -quasi-metric. Further, the pair $(\mathbb{R}_0^+, \delta)$ becomes a T_0 -quasi-metric space.

Each quasi-metric d on X induces a T_0 topology τ_d on X which has as a base the family of open balls $\{B_d(\xi, \eta): x \in X, \varepsilon > 0\}$ where $B_d(\xi, \eta) = \{\xi \in X: d(\xi, \eta) < \varepsilon\}$.

If τ_d satisfies the separation axiom T_1 (resp. T_2) on X , we say that (X, d) is a T_1 (resp. a Hausdorff) quasi-metric space. Note that a T_0 -quasi-metric space (X, d) is T_1 if and only if for each $x, y \in X$, the condition $d(x, y) = 0$ implies $x = y$.

A quasi-metric space (X, d) is called bicomplete if the metric space (X, d^s) is complete.

Definition 3 (convergence in quasi-pseudometric spaces, see [12]).

Let (X, q) be a quasi-pseudometric space. We say that the sequence $\{\xi_n\}$ **q -converges** to ξ , or **left-converges** to ξ , if

$$q(\xi_n, \xi) \longrightarrow 0. \quad (4)$$

We denote this by $\xi_n \longrightarrow^d \xi$. More precisely, $\{\xi_n\}$ converges to ξ with respect to $\tau(q)$.

In a similar manner, a sequence $\{\xi_n\}$ **q^{-1} -converges** to ξ or **right-converges** to ξ with respect to $\tau(q^{-1})$, if

$$q(\xi, \xi_n) \longrightarrow 0. \quad (5)$$

We denote this by $\xi_n \longrightarrow^{q^{-1}} \xi$. A sequence $\{\xi_n\}$ in the setting of a quasi-pseudometric space (X, q) **q^s -converges** to ξ in the case that the sequence converges to ξ from left and right, that is,

$$\xi_n \xrightarrow{q} \xi, \xi_n \xrightarrow{q^{-1}} \xi. \quad (6)$$

Moreover, it is denoted as $\xi_n \xrightarrow{q^s} \xi$ (or, $\xi_n \longrightarrow \xi$, if there is no confusion).

Remark 1. From the definition of q^s -convergence, we have

$$q^s\text{-convergence implies } q\text{-convergence.} \quad (7)$$

As demonstrated in [7, Example 1.7.], the reverse implication does not hold in general.

Definition 4 (compare [13]).

(1) A sequence $\{\xi_n\}$ in a quasi-pseudometric (X, q) is called **left K -Cauchy** if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\text{for all } n, k: \quad n_0 \leq k \leq n, q(\xi_k, \xi_n) < \varepsilon. \quad (8)$$

(2) Similarly, we define **right K -Cauchyness**. A sequence $\{\xi_n\}$ in a quasi-metric space (X, q) is called right K -Cauchy if it is a left K -Cauchy sequence in the quasi-metric space (X, q^{-1}) .

(3) The quasi-metric space (X, d) is called left (right) K -sequentially complete if every left (right) K -Cauchy sequence converges with respect to the topology τ_d .

(4) (X, q) is called q -sequentially complete if every Cauchy sequence in the metric space (X, q^s) converges with respect to the topology τ_d .

Remark 2. One can easily be convinced that both bicompleteness and left (right) K -sequential completeness imply q -sequential completeness. However, the rest of implications does not hold in general. For more details, the reader can consult [12].

2. Revisiting the Interpolative Kannan Mappings

Let us recall that an interpolative Kannan contraction on a metric space (X, m) is a self-mapping $T: X \longrightarrow X$ such that there exist $\lambda \in [0, 1), \alpha \in (0, 1)$ for which

$$m(Tx, Ty) \leq \lambda m(x, Tx)^\alpha m(y, Ty)^{1-\alpha}, \quad (9)$$

for all $x, y \in X$ with $x \neq Tx$.

Before proceeding to the main results of this paper, we would like to correct an inaccuracy that appears in the proof of Theorem 2.2 in the paper by Karapinar [1].

Theorem 2. (see [11, Theorem 2.2]).

Let (X, d) be a complete metric space and $T: X \longrightarrow X$ be an interpolative Kannan type contraction, i.e., a self-map such that there exist $\lambda \in [0, 1), \alpha \in (0, 1)$ so that

$$d(Tx, Ty) \leq \lambda d(x, Tx)^\alpha d(y, Ty)^{1-\alpha}, \quad (10)$$

for all $x, y \in X$ with $x \neq Tx$. Then, T has a unique fixed point in X .

Proof. Following the proof presented by Karapinar, let $x_0 \in X$, and construct the sequence $\{x_n\}$ by $x_{n+1} = T^n(x_0)$ for all positive integers n . Taking $x = x_n$ and $y = x_{n-1}$ in (10), we derive that

$$\begin{aligned} d(x_{n+1}, x_n) &= d(Tx_n, Tx_{n-1}) \leq \lambda [d(x_n, Tx_n)]^\alpha [d(x_{n-1}, Tx_{n-1})]^{1-\alpha} \\ &= \lambda [d(x_n, x_{n+1})]^\alpha [d(x_{n-1}, x_n)]^{1-\alpha}, \end{aligned} \quad (11)$$

which yields

$$[d(x_{n+1}, x_n)]^{1-\alpha} \leq \lambda [d(x_{n-1}, x_n)]^{1-\alpha}. \quad (12)$$

It is not difficult to see that inequality (10) is asymmetric, and to fill in this gap, we also consider the case of $x = x_{n-1}$ and $y = x_n$ in (10). So taking $x = x_{n-1}$ and $y = x_n$ in (10), we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Tx_{n-1}, Tx_n) \leq \lambda [d(x_{n-1}, Tx_{n-1})]^\alpha \\ &\quad [d(x_n, Tx_n)]^{1-\alpha} \\ &= \lambda [d(x_{n-1}, x_n)]^\alpha [d(x_n, x_{n+1})]^{1-\alpha}, \end{aligned} \quad (13)$$

which yields

$$[d(x_n, x_{n+1})]^\alpha \leq \lambda [d(x_{n-1}, x_n)]^\alpha. \quad (14)$$

Since $0 < \alpha < 1$, it is obvious that $0 < 1 - \alpha < 1$; hence, it is routine to check that (x_n) is a Cauchy sequence which converges to the unique fixed point x^* of T . $\square$

This gap was revealed in Example 2.3 of [1]. Indeed, according to [11, Example 2.3], let $X = \{x, y, z, w\}$ be a set endowed with a metric m such that $m(x, x) = m(y, y) = m(z, z) = m(w, w) = 0$, $m(x, y) = m(y, x) = 3$, $m(x, z) = m(z, x) = 4$, $m(y, z) = m(z, y) = 3/2$, $m(w, x) = m(x, w) = 5/2$, $m(w, y) = m(y, w) = 2$, $m(w, z) = m(z, w) = 3/2$. Define the self-mapping T on X by $Tx = x$, $Ty = w$, $Tz = x$ and $Tw = y$. The author claimed that for $\alpha = 1/8$ and $\lambda = 9/10$, the self-mapping T forms an interpolative Kannan type contraction. However, considering the pair (z, y) , note that $Tz \neq z$. We have

$$\begin{aligned} 2.5 &= m(Tz, Ty) = m(x, w) \\ &= \frac{5}{2} > \frac{9}{10} [m(z, Tz)]^{1/8} [m(y, Ty)]^{7/8} \\ &= (4)^{1/8} (2)^{7/8} \approx 1.962913. \end{aligned} \quad (15)$$

That is, inequality (10) fails for $m(Tz, Ty)$.

However, considering the pair (y, z) , note that $Ty \neq y$. We have

$$\begin{aligned} 2.5 &= m(Ty, Tz) = m(w, x) \\ &= \frac{5}{2} \leq \frac{9}{10} [m(y, Ty)]^{1/8} [m(z, Tz)]^{7/8} \\ &= \frac{9}{10} (2)^{1/8} (4)^{7/8} \approx 3.301214. \end{aligned} \quad (16)$$

This suggests a modification of the so-called **interpolative Kannan contraction** in the following way.

Definition 5. Let (X, m) be a metric space. A self-mapping $T: X \rightarrow X$ is called a generalized interpolative Kannan contraction if there exist $\lambda \in [0, 1)$, $\alpha \in (0, 1)$ for which

$$\begin{aligned} m(Tx, Ty) &\leq \lambda \max\{m(x, Tx)^\alpha m(y, Ty)^{1-\alpha}, \\ &\quad m(x, Tx)^{1-\alpha} m(y, Ty)^\alpha\}, \end{aligned} \quad (17)$$

for all $x, y \in X$ with $x \neq Tx$ and $y \neq Ty$.

Remark 3. Observe then that for $\alpha = 1/2$ in (17), a generalized interpolative Kannan contraction is actually just an interpolative Kannan contraction in the sense and spirit of Karapinar.

Using this definition, we get the following.

Theorem 3 (compare [11, Theorem 2.2]). *Let (X, d) be a complete metric space and $T: X \rightarrow X$ be a generalized interpolative Kannan contraction. Then, T has a unique fixed point in X .*

Proof. The proof is merely a copy of that of [11, Theorem 2.2.] and needs not be repeated. One just has to observe that the right hand side of inequality (17) is symmetric in $x, y \in X$. This has actually been captured in the proof of Theorem 2. Moreover, [11, Example 2.3.] satisfies the hypothesis of Theorem 3. $\square$

3. Interpolative Kannan Mappings and Fixed Points in Quasi-Metric Spaces

The discussion we made in the previous section suggests, in a natural way, the following notions.

Definition 6. Let (X, d) be a quasi-metric space.

A d -interpolative Kannan contraction on (X, d) is a mapping $T: X \rightarrow X$ such that there exist $\lambda \in [0, 1)$, $\alpha \in (0, 1)$ for which

$$d(Tx, Ty) \leq \lambda d(Tx, x)^\alpha d(y, Ty)^{1-\alpha}, \quad (18)$$

for all $x, y \in X$ with $x \neq Tx$ and $y \neq Ty$.

A d^{-1} -interpolative Kannan contraction on (X, d) is a mapping $T: X \rightarrow X$ such that there exist $\lambda \in [0, 1)$, $\alpha \in (0, 1)$ for which

$$d(Tx, Ty) \leq \lambda d(x, Tx)^\alpha d(Ty, y)^{1-\alpha}, \quad (19)$$

for all $x, y \in X$ with $x \neq Tx$, $y \neq Ty$.

Then, we have the following easy, but useful, consequence of the interpolative Kannan contraction for metric spaces.

Proposition 1. *Let (X, d) be a T_0 -quasi-metric space. If T is a d -interpolative Kannan contraction or a d^{-1} -interpolative Kannan contraction on (X, d) , then the following conditions hold:*

- (1) T is a generalized interpolative Kannan contraction on the metric space (X, d^s) .
- (2) For any $x_0 \in X$, $(T_n x_0)_{n \in \mathbb{N}}$ is a Cauchy sequence in the metric space (X, d^s) .

Proof. For (1), suppose that T is a d -interpolative Kannan contraction on (X, d) . So there exist $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$ for which

$$\begin{aligned} d(Tx, Ty) &\leq \lambda d(Tx, x)^\alpha d(y, Ty)^{1-\alpha} \\ &\leq \lambda [d^s(y, Ty)]^\alpha [d^s(Tx, x)]^{1-\alpha}, \end{aligned} \quad (20)$$

for all $x, y \in X$ with $x \neq Tx$ and $y \neq Ty$. Similarly, given $x, y \in X$, we also have

$$\begin{aligned} d(Ty, Tx) &\leq \lambda d(Ty, y)^\alpha d(x, Tx)^{1-\alpha} \\ &\leq \lambda [d^s(x, Tx)]^\alpha [d^s(Ty, y)]^{1-\alpha}, \end{aligned} \quad (21)$$

for all $x, y \in X$ with $x \neq Tx$ and $y \neq Ty$.

Thus, given $x, y \in X$,

$$\begin{aligned} d^s(Tx, Ty) &\leq \lambda \max\{[d^s(y, Ty)]^\alpha [d^s(Tx, x)]^{1-\alpha}, \\ &\quad [d^s(y, Ty)]^\alpha [d^s(Tx, x)]^{1-\alpha}\}. \end{aligned} \quad (22)$$

Then, T is a generalized interpolative Kannan contraction on the metric space (X, d^s) .

(2) Since, by (1), T is a generalized interpolative Kannan contraction on the metric space (X, d^s) , it follows from the modified proof of classical interpolative Kannan contraction principle that $(T^n x_0)_{n \in \mathbb{N}}$ is a Cauchy sequence in the metric space (X, d^s) . $\square$

By using the preceding proposition, three quasi-metric versions of the generalized interpolative Kannan principle are easily deduced.

Theorem 4. *Every d -(resp. every d^{-1} -) interpolative Kannan contraction on a bicomplete T_0 -quasi-metric space on (X, d) has a unique fixed point.*

Proof. Let T be a d or a d^{-1} interpolative Kannan contraction. Since (X, d^s) is a complete metric space and, by Proposition 1 (1), T is a generalized interpolative Kannan on (X, d^s) , we deduce a modified proof of classical interpolative Kannan contraction principle that T has a unique fixed point. $\square$

Theorem 5. *Every d -interpolative Kannan contraction on a Hausdorff d -sequentially complete T_0 -quasi-metric space (X, d) has a unique fixed point.*

Proof. Let T be a d -interpolative Kannan contraction on the Hausdorff d -sequentially complete T_0 -quasi-metric space (X, d) . Fix an $x_0 \in X$. By Proposition 1 (2), the sequence $(T^n x_0)_{n \in \mathbb{N}}$ is a Cauchy sequence in the metric space (X, d^s) . Hence, there is $y \in X$ such that $(T^n x_0)_{n \in \mathbb{N}}$ converges to y with respect to τ_d , i.e., $d(y, T^n x_0) \rightarrow 0$ as $n \rightarrow \infty$. Since T is a d -interpolative Kannan contraction, there exist $\lambda \in [0, 1), \alpha \in (0, 1)$ for which

$$d(T^{n+1} x_0, Ty) \leq \lambda d(T^{n+1} x_0, T^n x_0)^\alpha d(y, Ty)^{1-\alpha}. \quad (23)$$

Consequently, $d(T^{n+1} x_0, Ty) \rightarrow 0$ as $n \rightarrow \infty$. From Hausdorffness of (X, d) , we deduce that $y = Ty$. Finally, suppose that $z \in X$ is a fixed point of T . From (10), we have

$$\begin{aligned} d(y, z) &= d(Ty, Tz) \leq \lambda [d(Ty, y)]^\alpha [d(z, Tz)]^{1-\alpha} \Rightarrow d(y, z) = 0, \\ d(z, y) &= d(Tz, Ty) \leq \lambda [d(Tz, z)]^\alpha [d(y, Ty)]^{1-\alpha} \Rightarrow d(z, y) = 0, \end{aligned} \quad (24)$$

and thus $y = z$ since the space (X, d) is T_0 . This concludes the proof. $\square$

Corollary 1. *Every d -interpolative Kannan contraction on a T_0 -quasi-metric space (X, d) such that (X, d^{-1}) is Hausdorff and d^{-1} -sequentially complete has a unique fixed point.*

Proof. Let T be a d -interpolative Kannan contraction on (X, d) . Put $q = d^{-1}$; then, T is a q -interpolative Kannan contraction on a Hausdorff q -sequentially complete T_0 -quasi-metric space (X, q) . From Theorem 5, we deduce that T has a unique fixed point. $\square$

Theorem 6. *Every d^{-1} -interpolative Kannan contraction on a T_1 -sequentially complete T_0 -quasi-metric space (X, d) has a unique fixed point.*

Proof. Let T be a d^{-1} -interpolative Kannan contraction on the T_1 -sequentially complete T_0 -quasi-metric space (X, d) . Fix $x_0 \in X$. As in the proof of Theorem 5 (see Proposition 1), $(T^n x_0)_{n \in \mathbb{N}}$ is a Cauchy sequence in the metric space (X, d^s) . Hence, there is $y \in X$ such that $(T^n x_0)_{n \in \mathbb{N}}$ converges to y with respect to τ_d , i.e., $d(y, T^n x_0) \rightarrow 0$ as $n \rightarrow \infty$. Since T is a d^{-1} -interpolative Kannan contraction, there exist $\lambda \in [0, 1), \alpha \in (0, 1)$ for which

$$d(T^{n+1} x_0, Ty) \leq \lambda d(T^n x_0, T^{n+1} x_0)^\alpha d(Ty, y)^{1-\alpha}, \quad (25)$$

for all $n \in \mathbb{N}$. Consequently, $d(T^{n+1} x_0, Ty) \rightarrow 0$ as $n \rightarrow \infty$. From the triangle inequality,

$$d(y, Ty) \leq d(y, T^n x_0) + d(T^n x_0, Ty). \quad (26)$$

We deduce $d(y, Ty) = 0$. Therefore, $y = Ty$ because (X, d) is a T_0 -quasi-metric space, which is T_1 . Finally, suppose that $z \in X$ is a fixed point of T . Then,

$$d(y, z) = d(Ty, Tz) \leq \lambda d(y, Ty)^\alpha d(Tz, z)^{1-\alpha}, \quad (27)$$

and thus $y = z$. This concludes the proof. $\square$

Corollary 2. *Every d -contraction on a T_0 -quasi-metric space (X, d) which is T_1 such that (X, d^{-1}) is d^{-1} -sequentially complete has a unique fixed point.*

Proof. Let T be a d -interpolative Kannan contraction on (X, d) . Put $q = d^{-1}$. Then, T is a q^{-1} -interpolative Kannan contraction on the T_1 -sequentially complete T_0 -quasi-metric space (X, q) . From Theorem 6, we deduce that T has a unique fixed point. $\square$

Remark 4. Note that Theorems 5 and 6 remain valid if “ d -sequentially complete” is replaced with “left K -sequentially complete” or “right K -sequentially complete.”

We conclude the paper with an example which shows that Theorem 5 cannot be generalized to T_1d -sequentially complete T_0 -quasi-metric spaces.

Example 2. Let $X = \mathbb{N}$ and let d be the T_1T_0 -quasi-metric on X given by $d(n, n) = 0$ for all $n \in X$, and $d(n, m) = 1/m$ otherwise. Clearly, (X, d) is both left and right K -sequentially complete, so it is d -sequentially complete. Now, define $T: X \rightarrow X$ as $Tn = 2n$ for all $n \in X$. Of course, T has no fixed point. However, it is a d -interpolative Kannan contraction with $\lambda = 3/4$ and $\alpha = 1/2$ since for each $n, m \in X$ with $n \neq m$, one has

$$\begin{aligned} d(Tn, Tm) &= d(2n, 2m) = \frac{1}{2m} \leq \frac{3}{4} \sqrt{n} \sqrt{2m} \\ &= \frac{3}{4} d(Tn, n)^{1/2} d(m, Tm)^{1/2}. \end{aligned} \quad (28)$$

In light of the above example, a question is in order.

Open Question 1. Can Theorem 6 be generalized to d -sequentially complete T_0 -quasi-metric spaces? It is our belief that the answer is “No”; however, so far, we have failed to provide a counter-example.

4. Conclusion

As mentioned in the introduction, the interesting concept of (λ, α, β) -interpolative Kannan contractions was introduced in [2]. So, the authors plan, in a different manuscript, to discuss asymmetric versions of such a concept by first outlining the asymmetric nature of this notion, even for a classical interpolative Kannan contraction in a metric space.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

Our proofs are inspired by the recent work of Dağ et al. [14]. The presentation of the manuscript is as proceedings.

Conflicts of Interest

The authors declare that they have no conflicts of interest concerning the publication of this article.

Authors' Contributions

All authors contributed equally and significantly in writing this article.

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