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Research Article

Forecast of Freight Volume in Xi'an Based on Gray GM (1, 1) Model and Markov Forecasting Model

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Due to the continuous improvement of productivity, the transportation demand of freight volume is also increasing. It is difficult to organize freight transportation efficiently when the freight volume is quite large. Therefore, predicting the total amount of goods transported is essential in order to ensure efficient and orderly transportation. Aiming at optimizing the forecast of freight volume, this paper predicts the freight volume in Xi'an based on the Gray GM (1, 1) model and Markov forecasting model. Firstly, the Gray GM (1, 1) model is established based on related freight volume data of Xi'an from 2000 to 2008. Then, the corresponding time sequence and expression of restore value of Xi'an freight volume can be attained by determining parameters, so as to obtain the gray forecast values of Xi'an's freight volume from 2009 to 2013. In combination with the Markov chain process, the random sequence state is divided into three categories. By determining the state transition probability matrix, the probability value of the sequence in each state and the predicted median value corresponding to each state can be obtained. Finally, the revised predicted values of the freight volume based on the Gray–Markov forecasting model in Xi'an from 2009 to 2013 are calculated. It is proved in theory and practice that the Gray–Markov forecasting model has high accuracy and can provide relevant policy bases for the traffic management department of Xi'an.

1. Introduction

With the rapid development of the economy, China's transportation develops rapidly and the traffic volume increases quickly, which provides great convenience for the logistic system [1]. The logistic system is one of the most crucial aspects of the regional economy and the freight volume is the largest component of the logistic system [2]. Thus, freight volume can reflect the transportation development level to some extent. Since the policymakers cannot formulate appropriate strategies with limited information [3], then predicting freight volume is important for them to make some related strategies. The forecast of freight volume is the foundation of transportation facility planning and construction, and it can provide useful and appropriate policy-making information for the government department that conducts market supervision and management [4]. Scientific and accurate forecast of freight volume is the basis for formulating transportation development plans and rationally allocating resources, which is important to the development of national and local transportation [5]. Therefore, forecasting freight volume accurately plays a crucial role in the healthy development of transportation.

Due to the strong randomness, nonlinearity, and some other characteristics of freight volume change, the research of forecasting methods has always been the focus in this field.

Among these characteristics, randomness is a crucial factor that can cause an undesirable impact on the predicted results. The reason is that the actual probability of a result can be hard to determine by taking randomness into consideration [6]. Throughout the comprehensive transportation development research, there have been abundant research results in the prediction of traffic volume related to transportation. The traditional forecast of freight volume usually includes the combination of qualitative and quantitative methods or the combination of subjective and objective methods [7–9]. For instance, Tang et al. forecasted short-term passenger flow on the Shenzhen metro by using support vector regression (SVR) [10]. Li applied the support vector machine (SVM) to predict short-time traffic flow volume [11]. Li et al. established the gray model to forecast the annual passenger departure volume of railway stations [12]. Wang et al. employed the Gray–Markov model to improve the forecast accuracy of passenger flow entering and leaving metro stations [13]. The traditional volume prediction methods include the exponential smoothing model, gray forecasting model, and regression analysis [4]. Moreover, these models have been widely adopted in railway passenger volume forecast, road passenger volume forecast, bus passenger volume forecast, and other cases in different regions of China.

Compared with other forecasting methods, the gray forecasting model is a kind of forecasting method with a wide range of adaptation and more scientific theory. However, the accuracy of this model is easily affected by external factors in the practical application. The Gray–Markov forecasting model constructed by combining the Gray prediction model and the Markov chain can be more precise and efficient for prediction application [14]. This paper introduces the Markov chain into the Gray GM (1, 1) model and establishes the Gray–Markov forecasting model to predict the freight volume in Xi'an.

2. Modeling Methodologies

2.1. Gray GM (1, 1) Model. The basic principle of the Gray forecasting model is to dilute the randomness of the data sequence by accumulating the original data. It mainly studies the uncertain system with small sample and poor information and even allows as few as four modeling data [15]. The purpose is to improve the internal law of the data sequence and thus establish related dynamic differential equations. The Gray GM (1, 1) model adopts a first-order differential equation to characterize an unknown system [16]. The Gray forecasting model is built as follows.

Assume that the original time series of the random system is

$$x^{(0)}(t) = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right\}. \quad (1)$$

However, this is an unstable and irregular equidistant sequence. The Gray GM (1, 1) model is established by using the Gray theory as follows.

2.1.1. Step 1. Perform an accumulation generating operation:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \quad (2)$$

2.1.2. Step 2. A new series $x^{(1)}$ is generated as follows:

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\}. \quad (3)$$

2.1.3. Step 3. Construct a first-order differential equation:

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t} + ax^{(1)} = b, \quad (4)$$

where the parameters a is called the development coefficient and b is called gray input.

2.1.4. Step 4. After processing, the estimated values of parameters a and b can be obtained as follows:

$$\hat{a} = \begin{pmatrix} a \\ b \end{pmatrix} = \left(B^T B\right)^{-1} B^T Y, \quad (5)$$

$$B = \begin{bmatrix} -\frac{1}{2}\left(x^1(1) + x^2(2)\right) & -\frac{1}{2}\left(x^1(2) + x^2(3)\right) & -\frac{1}{2}\left(x^1(n-1) + x^2(n)\right) \\ 1 & 1 & 1 \end{bmatrix}^T, \quad (6)$$

$$Y = \left(x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right)^T. \quad (7)$$

2.1.5. Step 5. After substituting equation (5) into equation (4), the Gray GM (1, 1) model can be obtained as follows:

$$\widehat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}. \tag{8}$$

The original data is

$$x(k+1) = a^{(1)}x^{(1)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k) = \left(1 - e^{a}\left(x^{(0)}(1) - \frac{b}{a}\right)\right)e^{-ak}, \quad k = 1, 2, \ldots, n. \tag{9}$$

2.2. Markov Chain. The Gray forecasting model is easily biased by external interference when it is applied so that the forecasting precision might be low. In other words, the Gray forecasting model has some limitations which affect negatively the applicability and prediction accuracy of this model [17]. Thus, the Markov chain is combined on the basis of the Gray forecasting model to reduce forecast errors generated by long-term predicted data [13]. That is to say, adopting the Markov chain to forecast the random volatility range of time series could improve the prediction precision of the Gray forecasting model [18].

Markov chain is a mathematical model that describes a random system which changes states only depending on the transition rule of the current state [19]. In other words, the Markov chain can deal with some problems through dynamic program [20]. It is a method based on the concepts of state and state transition of the system. Therefore, state division must be carried out first.

2.2.1. State Division. Markov forecasting model divides the predicted target into a certain state, and the relation with this certain state is the probability that the system is in this state or will reach a certain state. The main principle of the Markov forecasting model is to obtain the probability that the system may reach some states in the future by using Markov chain theory according to the original state number of each state, which belongs to the probabilistic prediction model [21]. Its predicted results are only related to the current state and have the characteristics of no aftereffect. Therefore, the problem of data prediction with high random fluctuation can be solved by Markov chain.

An n-order Markov chain consists of n state sets $\{S_1, S_2, S_3, \ldots, S_n\}$ and a transition probability matrix of $n \times n$. It means that the state is S_i when the system is at time i; then, the system will transfer from this state to the next moment S_j with the probability of P_{ij}. The probability matrix of state transition is as follows:

$$P = \begin{bmatrix} P_{11} & \ldots & P_{1n} \\ \ldots & \ldots & \ldots \\ P_{n1} & \ldots & P_{nn} \end{bmatrix}. \tag{10}$$

2.2.2. Construct Markov Forecasting Model. After dividing the state, the transition probability matrix is constructed. For a random sequence with Markov characteristics, it can be divided into n states, and any state can be expressed as follows:

$$\begin{aligned} \otimes_i &= [\otimes_{1i}, \otimes_{2i}], \quad i = 1, 2, \ldots, n, \\ \otimes_{1i} &= y(k) + A_i, \quad i = 1, 2, \ldots, n, \\ \otimes_{2i} &= y(k) + B_i, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{11}$$

where A_i and B_i, respectively, represent the distances from the upper and lower boundary of state i to $y(k)$.

2.3. Gray–Markov Forecasting Model. The Gray GM (1, 1) model has lower prediction accuracy for data sequence with large random fluctuations [16]. However, the Markov forecasting model can be adopted to predict a randomly varying time series. Therefore, the Markov forecasting model can improve the Gray GM (1, 1) model especially when the dataset fluctuates greatly [22]. Therefore, these two models can be combined to form the Gray–Markov forecasting model which makes the two algorithms complementary [23]. The generally modeling steps of the Gray–Markov forecasting model is described below.

2.3.1. Step 1. The historical data is de-dimensioned to obtain the original sequence.

2.3.2. Step 2. The predicted values of the prediction sequence are obtained by using the Gray GM (1, 1) model.

2.3.3. Step 3. The Markov state is divided based on the relative variation between the predicted value and the actual value, which can generally be divided into 3 to 5 states.

2.3.4. Step 4. Calculate the one-step state transition matrix, and then obtain the two-step, three-step, etc. state matrix.

2.3.5. Step 5. Compare the fitting values of the Gray GM (1, 1) model and the Gray–Markov forecasting model with the real data at the same time to evaluate the prediction accuracy.

Table 1: Xi'an's freight volume from 2000 to 2008.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2009
Freight volume ($10^8 t$)	0.6999	0.7728	0.9482	0.9392	1.4845	1.2051	1.1832	1.5124	2.7560

3. Application

In order to test the prediction precision of the Gray–Markov forecasting model, this paper collects historical data of freight volume in Xi'an from 2000 to 2008 which is shown in Table 1 [24].

3.1. Gary GM (1, 1) Model. The Gray GM (1, 1) model is established based on the historical data of freight volume from 2000 to 2008 to test the prediction accuracy from 2009 to 2013.

According to the Gray GM (1, 1) model algorithm in the preceding section, the parameters are calculated by the Python algorithm. The values of a and b can be obtained as follows:

$$\begin{aligned} a &= -0.1704, \\ b &= 0.4933. \end{aligned} \tag{12}$$

Then, the following Gray GM (1, 1) model can be established based on related formulas:

$$x^{(1)} \cdot (k+1) = 3.5949 \cdot e^{0.1704 \cdot k} - 2.8950. \tag{13}$$

Therefore, the predicted values, original values, and their differences can be obtained based on the Gray GM (1, 1) model shown in Table 2.

3.2. Construct State Transition Matrix. According to the relative error and the actual situation of the sample data, the whole sequence is divided into 3 states (underestimated state, normal state, and overestimated state), as shown in Table 3.

The number of transitions from state E_i to state E_j is n_{ij}, and the total number of transitions from state E_i to the next state is N_i. Thus, the frequency of one-step transition from state E_i to state E_j is n_{ij}/N_i, which represents the one-step transition probability from state E_i to E_j.

The one-step state transition probability matrix between all states can be obtained according to the ratio of the original state sample number and the transferred sample number as follows:

$$P = \begin{bmatrix} \frac{2}{3} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{14}$$

Then, the k-step state transition probability matrix can be determined according to the one-step state transition probability matrix:

Table 2: Xi'an's predicted freight volume from 2009 to 2013 based on the Gray GM (1, 1) model.

Year	Freight volume ($10^8 t$)	Predicted value ($10^8 t$)	Relative error (%)
2009	3.0606	2.6050	−14.87
2010	3.4323	3.0894	9.99
2011	3.9239	3.6325	−7.43
2012	4.4924	4.3272	−3.68
2013	5.0119	5.1615	2.98

Table 3: State interval division.

State	Relative error interval (%)
E_1 (underestimated state)	[−15, −5]
E_2 (normal state)	[−5, 5]
E_3 (overestimated state)	[5, 15]

$$P_k = P^{(k-1)}. \tag{15}$$

3.3. Analysis of Freight Volume Forecast Results. The freight volume of each year is calculated by the Gray GM (1, 1) model. Then, the possible state of the next year can be predicted according to the state of the current year, and the weighted average correction can be conducted for the range of each state.

Let the state vector for the current year be m_0:

$$m_0 = (p_1, p_2, \ldots, p_i). \tag{16}$$

Then, the predicted state vector for the i^{th} year can be denoted by m_i:

$$m_i = m_0 \cdot P_{i-1}. \tag{17}$$

The purpose of obtaining the numerical interval of the actual value in each state is to reduce the influence of the random disturbance of the transition between states. The mean value of the upper and lower limits of the prediction interval is one column of predicted median values. The one-step state transition matrix can be obtained according to the formula so that the two-step, three-step, four-step, and five-step transition matrix can be also determined, so the probability of transition to each state in each year can be obtained. Finally, the predicted correction value based on the Markov chain is calculated. Table 4 shows the Gray–Markov-modified predicted results based on the Gray GM (1, 1) predicted results according to the corresponding state probability and state value.

The predicted values and corresponding errors of the Gray GM (1, 1) model and Gray–Markov model are compared, as shown in Table 5. It can be seen from Table 5 that when predicting the change in freight volume in Xi'an from

Table 4: Xi'an's predicted freight volume from 2009 to 2013 based on the Gray–Markov model.

Year	Gray predicted value ($10^8 t$)	Predicted interval ($10^8 t$)	Predicted median ($10^8 t$)	State probability	Gray–Markov-modified predicted value ($10^8 t$)
		[2.4189, 2.5467]	2.4828	0.67	
2009	2.6050	[2.5467, 2.6745]	2.6106	0.33	2.6106
		[2.6745, 2.8023]	2.7384	0	
		[2.9039, 3.0317]	2.9678	0.44	
2010	3.0894	[3.0317, 3.1595]	3.0956	0.56	3.0956
		[3.1595, 3.2873]	3.2234	0	
		[3.4791, 3.6069]	3.5430	0.30	
2011	3.6325	[3.6069, 3.7347]	3.6708	0.70	3.6708
		[3.7347, 3.8625]	3.7986	0	
		[4.1611, 4.2889]	4.2250	0.20	
2012	4.3272	[4.2889, 4.4167]	4.3528	0.80	4.3528
		[4.4167, 4.5445]	4.4806	0	
		[4.9698, 5.0976]	5.0337	0.13	
2013	5.1615	[5.0976, 5.2254]	5.1615	0.87	5.1449
		[5.2254, 5.3532]	5.2893	0	

Table 5: Predicted value comparison between the Gray GM (1, 1) model and Gray–Markov model.

Year	Gray predicted value ($10^8 t$)	Relative error (%)	Gray–Markov predicted value ($10^8 t$)	Relative error (%)
2009	2.6050	−14.89	2.6106	−14.70
2010	3.0894	−9.99	3.0956	−9.81
2011	3.6325	−7.43	3.6708	−6.45
2012	4.3272	−3.68	4.3528	−3.11
2013	5.1615	2.98	5.1449	2.65

2009 to 2013, the accuracy of the Gray–Markov forecasting model is higher than that of the Gray GM (1, 1) model. This result indicates that the correction with the Markov model has a quite significant effect on improving the Gray GM (1, 1) model and can predict data more precisely.

4. Conclusion

The Gray–Markov forecasting model is based on the short-term advantages of the Gray prediction model. By considering the randomness and volatility of the data, the state is divided into different degrees of the relative deviation of the predicted data, thereby establishing the state transition matrix. Then, the forecast data is revised according to the state transition matrix. This improved forecasting model can not only be beneficial to describe the development trend of predicted data in the time series but also can reflect the random fluctuation performance of the series. Therefore, the accuracy and scientific of the forecasting model can be improved significantly.

In this paper, the Gray–Markov forecasting model is used to predict the freight volume data of Xi'an, and the data obtained from the forecast is compared with the Gray GM (1, 1) model. The comparation results show that the prediction accuracy of the Gray–Markov forecasting model is improved, which reflects the superiority of the combined forecast. Using the Gray–Markov forecasting model can not only obtain the freight volume data of the in the future but also enable people to understand the trend of freight volume changes in different intervals, so as to more accurately grasp the overall development trend of Xi'an freight volume. Overall, the predicted results have the reference value for the construction and operation of the transportation system. More high-precision statistic data are needed for the further research to improve the accuracy of the freight volume forecast. Furthermore, it is suggested that the Gray GM (1, 1) model could be modified to improve the prediction accuracy, for example, combining variable weight construction background value and residual correction into Gray GM (1, 1).

Data Availability

All data generated or used during the study are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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