Research Article

An Efficient Modified AZPRP Conjugate Gradient Method for Large-Scale Unconstrained Optimization Problem

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To find a solution of unconstrained optimization problems, we normally use a conjugate gradient (CG) method since it does not cost memory or storage of second derivative like Newton's method or Broyden-Fletcher-Goldfarb-Shanno (BFGS) method. Recently, a new modification of Polak and Ribiere method was proposed with new restart condition to give a so-call AZPRP method. In this paper, we propose a new modification of AZPRP CG method to solve large-scale unconstrained optimization problems based on a modification of restart condition. The new parameter satisfies the descent property and the global convergence analysis with the strong Wolfe-Powell line search. The numerical results prove that the new CG method is strongly aggressive compared with CG_Descent method. The comparisons are made under a set of more than 140 standard functions from the CUTEst library. The comparison includes number of iterations and CPU time.

1. Introduction

The conjugate gradient (CG) method aims to find a solution of optimization problems without constraint. Suppose that the following optimization problem is considered:

$$\min f(x), \quad x \in \mathbb{R}^n, \quad (1)$$

where $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is continuous, the function is differentiable, and the gradient $\nabla f(x)$ is available. The iterative method is given by the following sequence:

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 1, 2, \dots, \quad (2)$$

where x_k is the starting point and $\alpha_k > 0$ is a step length. The search direction d_k of the CG method is defined as follows:

$$d_k = \begin{cases} -\nabla f(x), & \text{if } k = 1, \\ -\nabla f(x) + \beta_k d_{k-1}, & \text{if } k \geq 2, \end{cases} \quad (3)$$

where $\nabla f(x) = g(x_k)$ and β_k is a parameter.

To obtain the step length, we normally use the inexact line search, since the exact line search which is defined as follows,

$$f(x_k + \alpha_k d_k) = \min f(x_k + \alpha d_k), \quad \alpha > 0, \quad (4)$$

requires many iterations to obtain the step length. Normally, we use the strong version of Wolfe-Powell (SWP) [1, 2] line search which is given by

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k, \quad (5)$$

$$|\nabla f(x_k + \alpha_k d_k)^T d_k| \leq \sigma |g_k^T d_k|, \quad (6)$$

where $0 < \delta < \sigma < 1$.

The weak Wolfe-Powell (WWP) line search is defined by (5) and

$$\nabla f(x_k + \alpha_k d_k)^T d_k \geq \sigma g_k^T d_k, \quad (7)$$

where $\nabla f = g_k = g(x_k)$. The famous parameters of β_k are the Hestenes-Stiefel (HS) [3], Fletcher-Reeves (FR) [4], and Polak-Ribière-Polyak (PRP) [5] formulas, which are given by

$$\begin{aligned} \beta_k^{\text{HS}} &= \frac{g_k^T y_k}{d_{k-1}^T y_k}, \\ \beta_k^{\text{FR}} &= \frac{g_k^T g_k}{\|g_{k-1}\|^2}, \\ \beta_k^{\text{PRP}} &= \frac{g_k^T y_k}{\|g_{k-1}\|^2}, \end{aligned} \quad (8)$$

where $y_k = g_k - g_{k-1}$.

Powell [6] shows that there exists a nonconvex function such that the PRP method does not globally converge. Gilbert and Nocedal [7] show that if $\beta_k^{\text{PRP}+} = \max\{0, \beta_k^{\text{PRP}}\}$ with the WWP and the descent property is satisfied, then it is globally convergent.

Al-Baali [8] proved that the CG method with FR coefficient is convergent with SWP line search when $\sigma \leq 1/2$. Hager and Zhang [9, 10] presented a new CG parameter with descent property, i.e., $g_k^T d_k \leq -(7/8)\|g_k\|^2$. This formula is given as follows:

$$\beta_k^{\text{HZ}} = \max\{\beta_k^N, \eta_k\}, \quad (9)$$

where $\beta_k^N = (1/d_k^T y_k)(y_k - 2d_k(\|y_k\|^2/d_k^T y_k))^T g_k$; $\eta_k = -(1/\|d_k\| \min\{\eta, \|g_k\|\})$; and $\eta > 0$ is a constant. In the numerical experiments, they set $\eta = 0.01$ in (9). Al-Baali et al. [11] compared β_k^{HZ} with a new three-term CG method (G3TCG).

Regarding the speed, memory requirements, number of iterations, function evaluations, gradient evaluations, and robustness to solve unconstrained optimization problems which have prompted the development of the CG method, the readers are advised to refer references [10–15] for more information on these new formulas.

2. The New Formula and the Algorithm

Alhawarat et al. [15] presented the following simple formula:

$$\beta_k^{\text{AZPRP}} = \begin{cases} \frac{\|g_k\|^2 - \mu_k |g_k^T g_{k-1}|}{\|g_{k-1}\|^2}, & \text{if } \|g_k\|^2 > \mu_k |g_k^T g_{k-1}|, \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

Dai and Laio [12] presented the following formula:

$$\beta_k^{\text{DL}+} = \max\{\beta_k^{\text{HS}}, 0\} - t \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, \quad (11)$$

where $s_{k-1} = x_k - x_{k-1}$ and $t \geq 0$.

The new formula is a modification of β_k^{AZPRP} and $\beta_k^{\text{DL}+}$ is defined as follows:

$$\beta_k^A = \begin{cases} \frac{\|g_k\|^2 - \mu_k |g_k^T s_{k-1}|}{\|g_{k-1}\|^2}, & \text{if } \|g_k\|^2 > \mu_k |g_k^T s_{k-1}|, \\ -t \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, & \text{otherwise,} \end{cases} \quad (12)$$

where $\mu_k = (\|s_{k-1}\|/\|y_{k-1}\|)$ and $t > 0$.

We obtain the following relations (Algorithm 1):

$$\begin{aligned} \beta_k^A &\geq 0, \\ \beta_k^A &\leq \frac{\|g_k\|^2 - \mu_k |g_k^T s_{k-1}|}{\|g_{k-1}\|^2} \leq \frac{\|g_k\|^2}{\|g_{k-1}\|^2} = \beta_k^{\text{FR}}. \end{aligned} \quad (13)$$

3. Convergence Analysis of Coefficient β_k^A with CG Method

Assumption 1

(A) The level set $\Psi = \{x | f(x) \leq f(x_1)\}$ is bounded, that is, a positive constant T exists such that

$$\|x\| \leq T, \quad \forall x \in \Psi. \quad (14)$$

(B) In some neighbourhoods N of Ψ , f is continuous and the gradient is available and its gradient is Lipschitz continuous; that is, for all $x, y \in N$, there exists a constant $L > 0$ such that

$$\|g(x) - g(y)\| \leq L\|x - y\|. \quad (15)$$

This assumption shows that there exists a positive constant B such that

$$\|g(u)\| \leq B, \quad \forall u \in N. \quad (16)$$

The descent condition

$$g_k^T d_k \leq -\|g_k\|^2, \quad \forall k \geq 1. \quad (17)$$

(17) plays an important role in the CG method. The sufficient descent condition proposed by Al-Baali [8] is a modification of (17) as follows:

$$g_k^T d_k \leq -c\|g_k\|^2, \quad \forall k \geq 1, \quad (18)$$

where $c \in (0, 1)$. Note that the general form of the sufficient descent condition is (18) with $c > 0$.

ALGORITHM 1: Steps of CG method with new modification to obtain the stationary point of functions.

3.1. Global Convergence for β_k^A with the SWP Line Search. The following theorem demonstrates that β_k^A ensures that the sufficient descent condition (21) is satisfied with the SWP line search.

The following theorem shows that β_k^A satisfies the descent condition. The proof is similar to that presented in [8].

Theorem 1. Let $\{g_k\}$ and $\{d_k\}$ be generated using (2), (3), and $\beta_k^A = (\|g_k\|^2 - \mu_k |g_k^T s_{k-1}| / \|g_{k-1}\|^2)$, where α_k is computed by the SWP line search (5) and (6). If $\sigma \in (0, 1/2]$, then the sufficient descent condition (18) holds.

Algorithm 1 shows the steps to obtain the solution of optimization problem using strong Wolfe-Powell line search.

Descent condition is (18) with $c > 0$.

Proof. By multiplying () by g_k^T , we obtain

$$g_k^T d_k = g_k^T (-g_k^T + \beta_k d_{k-1}) = -\|g_k\|^2 + \beta_k g_k^T d_{k-1}. \quad (19)$$

Divide (19) by $\|g_k\|^2$; using

$$|\nabla f(x_k + \alpha_k d_k)^T d_k| \leq \sigma |g_k^T d_k|, \quad (20)$$

and (12), we obtain

$$-1 + \sigma \frac{g_{k-1}^T d_{k-1}}{\|g_{k-1}\|^2} \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -1 - \sigma \frac{g_{k-1}^T d_{k-1}}{\|g_{k-1}\|^2}. \quad (21)$$

From (3), we obtain $g_1^T d_1 = -\|g_1\|^2$. Assume that it is true until $k-1$, i.e., $g_i^T d_i < 0$, for $i = 1, 2, \dots, k-1$. Repeating the process for (21), we obtain

$$-\sum_{j=0}^{k-1} \sigma^j \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -2 + \sum_{j=0}^{k-1} \sigma^j. \quad (22)$$

As

$$\sum_{j=0}^{k-1} (\sigma)^j < \frac{1 - (\sigma)^k}{1 - \sigma}, \quad (23)$$

hence,

$$-\frac{1 - (\sigma)^k}{1 - \sigma} \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -2 + \frac{1 - (\sigma)^k}{1 - \sigma}, \quad (24)$$

and when $\sigma \leq (1/2)$, we obtain $(1 - (\sigma)^k / (1 - \sigma)) < 2$. Let $c = 2 - (1 - (\sigma)^k / (1 - \sigma))$, then

$$c - 2 \leq \frac{g_k^T d_k}{\|g_k\|^2} \leq -c. \quad (25)$$

The proof is complete. $\square$

Theorem 2. Let $\{g_k\}$ and $\{d_k\}$ be obtained by using (2), (3), and $\beta_k^A = -t (g_k^T s_{k-1} / d_{k-1}^T y_{k-1})$ where α_k is computed by SWP line search (5) and (6), then the descent condition holds.

Proof

$$\beta_k^{\text{DL-HS}} = -\mu_k \frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}. \quad (26)$$

By multiplying (3) by g_k^T , and substituting β_k^A , we obtain

$$\begin{aligned}
g_k^T d_k &= -\|g_k\|^2 - \mu_k \frac{\alpha_{k-1} g_k^T d_{k-1}}{d_{k-1}^T y_{k-1}} g_k^T d_{k-1} \\
&= -\|g_k\|^2 - \mu_k \frac{\alpha_{k-1} \|g_k^T d_{k-1}\|^2}{d_{k-1}^T y_{k-1}} < 0,
\end{aligned} \tag{27}$$

which completes the proof.

Zoutendijk [16] presented a useful lemma for global convergence property of the CG method. The condition is given as follows. $\square$

Lemma 1. *Let Assumption 1 hold and consider any method in the form of (2) and (3), where α_k is obtained by the WWP line search (6) and (7), in which the search direction is descent. Then, the following condition holds:*

$$\sum_{k=0}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty. \tag{28}$$

Theorem 3. *Suppose Assumption 1 holds. Consider any form of equations (2) and (3), with the new formula (12), in which α_k is obtained from the SWP line search (5) and (6) with $\sigma \leq 1/2$. Then,*

$$\liminf_{k \rightarrow \infty} \|g_k\| = 0. \tag{29}$$

The proof is similar to that presented in [8].

Proof. We will prove the theorem by contradiction. Assume that the conclusion is not true, then a constant $\varepsilon > 0$ exists such that

$$\|g_k\| \geq \varepsilon, \quad \forall k \geq 1. \tag{30}$$

Squaring both sides of equation (3), we obtain

$$\|d_k\|^2 = \|g_k\|^2 - 2\beta_k g_k^T d_{k-1} + \beta_k^2 \|d_{k-1}\|^2. \tag{31}$$

Divide (31) by $\|g_k\|^4$, we get

$$\frac{\|d_k\|^2}{\|g_k\|^4} = \frac{1}{\|g_k\|^2} - \frac{2\beta_k g_k^T d_{k-1}}{\|g_k\|^4} + \frac{\beta_k^2 \|d_{k-1}\|^2}{\|g_k\|^4}. \tag{32}$$

Using (6), (12), and (32), we obtain

$$\frac{\|d_k\|^2}{\|g_k\|^4} \leq \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4} + \frac{1}{\|g_k\|^2} + \frac{2\sigma |g_{k-1}^T d_{k-1}|}{\|g_{k-1}\|^2 \|g_k\|^2} \leq \frac{\|d_{k-1}\|^2}{\|g_{k-1}\|^4} + \frac{1 + 2\sigma(2-c)}{\|g_k\|^2}. \tag{33}$$

Repeating the process for (33) and using the relationship $(1/\|g_1\|) = (1/\|d_1\|)$ yields

$$\frac{\|d_k\|^2}{\|g_k\|^4} \leq (1 + 2\sigma(2-c)) \sum_{i=1}^k \frac{1}{\|g_i\|^2}. \tag{34}$$

From (33), we obtain

$$\frac{\|g_k\|^4}{\|d_k\|^2} \geq \frac{\varepsilon^2}{k(1 + 2\sigma(2-c))}. \tag{35}$$

Therefore,

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} = \infty. \tag{36}$$

This result contradicts (32), thus $\liminf_{k \rightarrow \infty} \|g_k\| = 0$. The proof is complete. $\square$

4. Numerical Results

To investigate the effectiveness of the new parameter, several test problems in Table 1 from CUTEst [17] are chosen. We performed a comparison with the CG_Descent 5.3 based on the CPU time and the number of iterations. We employed the SWP line search with the line as mentioned in [1, 2] with $\delta = 0.01$ and $\sigma = 0.1$. The modified CG_Descent 6.8 where the memory (mem) equals zero is employed to obtain all results. The code can be downloaded from Hager web page <http://users.clas.ufl.edu/hager/papers/Software/>.

The CG_Descent 5.3 results are obtained by run CG_Descent 6.8 with memory which equals zero. The host computer is an AMD A4-7210 with RAM 4 GB. The results are shown in Figures 1 and 2 in which a performance measure introduced by Dolan and Moré [18] was employed. As shown in Figure 1, formula A strongly outperforms over CG_Descent in number of iterations. In Figure 2, we notice that the new CG formula A is strongly competitive with CG_Descent.

4.1. Multimodal Function with Its Graph. In this section, we present six-hump camel back function, which is a multimodal function to test the efficiency of the optimization algorithm. The function is defined as follows:

$$\begin{aligned}
\text{functions} \cdot f(x) &= \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right)x_1^2 + x_1x_2 \\
&\quad + (-4 + 4x_2^2)x_2^2.
\end{aligned} \tag{37}$$

The number of variables (n) equals 2. This function has six local minima, with two of them being global. Thus, this function is a multimodal function usually used to test global minima. Global minima are $x_1^* = (-0.0898, 0.7126)$ and $x_2^* = (0.0898, -0.7126)$. The function value is $f(x^*) = -1.0316$. As its name describes, this function looks like the back of an upside down camel with six humps (see Figure 3 for a three-dimensional graph); for more information about two-dimensional functions, the reader can refer to [19].

Finally, note that CG method can be applied in image restoration problems and neural network and others. For more information, the reader can refer to [20, 21].

TABLE 1: The test functions.

Function	Dimension	CG_Descent 5.3		β_k^A	
		Number of iterations	CPU time	Number of iterations	CPU time
AKIVA	2	10	0.02	8	0.02
ALLINITU	4	12	0.02	9	0.02
ARGLINA	200	1	0.02	1	0.02
ARGLINB	200	5	0.02	6	0.11
ARWHEAD	5000	7	0.02	6	0.03
BARD	3	16	0.02	12	0.02
BDQRTIC	5000	136	0.58	161	0.75
BEALE	2	15	0.02	11	0.02
BIGGS6	6	27	0.02	24	0.02
BOX3	3	11	0.02	10	0.02
BOX	1000	8	0.08	7	0.08
BRKMCC	2	5	0.02	5	0.02
BROWNAL	200	9	0.02	9	0.02
BROWNBS	2	13	0.02	10	0.02
BROWNDEN	4	16	0.02	16	0.02
BROYDN7D	5000	1411	5.47	64	0.37
BRYBND	5000	85	0.38	39	0.22
CHAINWOO	4000	318	0.866	379	1.08
CHNROSNB	50	287	0.02	340	0.02
CLIFF	2	18	0.02	10	0.02
COSINE	10000	11	0.19	14	0.26
CRAGGLVY	5000	103	0.45	104	0.48
CUBE	2	32	0.02	17	0.02
CURLY10	10000	47808	173.7	42454	145.16
CURLY20	10000	66587	383.94	67279	366.03
CURLY30	10000	79030	639.63	74375	509.59
DECONVU	63	400	2.00E-02	227	0.02
DENSCHNA	2	9	0.02	6	0.02
DENSCHNB	2	7	0.02	6	0.02
DENSCHNC	2	12	0.02	11	0.02
DENSCHND	3	47	0.02	14	0.02
DENSCHNE	3	18	0.02	12	0.02
DENSCHNF	2	8	0.02	9	0.02
DIXMAANA	3000	7	0.02	6	0.02
DIXMAANB	3000	6	0.02	6	0.02
DIXMAANC	3000	6	0.02	6	0.02
DIXMAAND	3000	7	0.02	8	0.02
DIXMAANE	3000	222	0.33	218	0.33
DIXMAANF	3000	161	0.13	116	0.09
DIXMAANG	3000	157	0.12	173	0.14
DIXMAANH	3000	173	0.22	190	0.2
DIXMAANI	3000	3856	4.25	3160	3.34
DIXMAANJ	3000	327	0.36	360	0.39
DIXMAANK	3000	283	0.28	416	0.36
DIXMAANL	3000	237	0.2	399	0.36
DIXON3DQ	10000	10000	19.12	10000	19.12
DJTL	2	82	0.02	75	0.02
DQDRTIC	5000	5	0.02	5	0.02
DQRTIC	5000	17	0.03	15	0.03
EDENSCH	2000	26	0.03	32	0.05
EG2	1000	5	0.02	3	0.02
EIGENALS	2550	10083	178.36	7247	133.4
EIGENBLS	2550	15301	237	18846	290.3
EIGENCLS	2652	10136	174.19	11152	186.86
ENGVAL1	5000	27	0.06	23	0.12
ENGVAL2	3	26	0.02	26	0.02
ERRINROS	50	380	0.02	95504	2.36

TABLE 1: Continued.

Function	Dimension	CG_Descent 5.3		β_k^A	
		Number of iterations	CPU time	Number of iterations	CPU time
EXPFIT	2	13	0.02	9	0.02
EXTROSNB	1000	3808	1.25	2370	0.87
FLETGBV2	5000	1	0.02	1	0.02
FLETCHCR	1000	152	0.05	84	0.05
FMINSRF2	5625	346	1.09E + 00	485	1.4
FMINSURF	5625	473	1.51	542	1.64
FREUROTH	5000	25	0.11	29	0.19
GENROSE	500	1078	0.17	2098	0.45
GROWTHLS	3	156	0.02	109	0.02
GULF	3	37	0.02	33	0.02
HAIRY	2	36	0.02	17	0.02
HATFLDD	3	20	0.02	17	0.02
HATFLDE	3	30	0.02	13	0.02
HATFLDFL	3	39	0.02	21	0.02
HEART6LS	6	684	0.02	375	0.02
HEART8LS	8	249	0.02	253	0.02
HELIX	3	23	0.02	23	0.02
HIELOW	3	14	0.02	13	0.05
HILBERTA	2	2	0.02	2	0.02
HILBERTB	10	4	0.02	4	0.02
HIMMELBB	2	10	0.02	4	0.02
HIMMELBF	4	26	0.02	23	0.02
HIMMELBG	2	8	0.02	7	0.02
HIMMELBH	2	7	0.02	5	0.02
HUMPS	2	52	0.02	45	0.02
JENSMP	2	15	0.02	12	0.02
JIMACK	35449	8314	1182.25	7297	1030.3
KOWOSB	4	17	0.02	16	0
LIARWHD	5000	21	0.03	15	0.05
LOGHAIRY	2	27	0.02	26	0.02
MANCINO	100	11	0.08	11	0.08
MARATOSB	2	1145	0.02	589	0.02
MEXHAT	2	20	0.02	14	0.02
MOREBV	5000	161	0.41	161	0.38
MSQRTALS	1024	2905	8.64	2788	9.08
MSQRTBLS	1024	2280	6.91	2181	6.84
NCB20B	500	2035	46.36	4181	70.16
NCB20	5010	879	11.83	959	13
NONCVXU2	5000	6610	15.89	6379	15.92
NONDIA	5000	7	0.03	7	0.03
NONDQUAR	5000	1942	2.45	3058	3.88
OSBORNEA	5	94	0.02	82	0.02
OSBORNEB	11	62	0.02	57	0.02
PALMER1C	8	11	0.02	12	0.02
PALMER1D	7	11	0.02	10	0.02
PALMER2C	8	11	0.02	11	0.02
PALMER3C	8	11	0.02	11	0.02
PALMER4C	8	11	0.02	11	0.02
PALMER5C	6	6	0.02	6	0.02
PALMER6C	8	11	0.02	11	0.02
PALMER7C	8	11	0.02	11	0.02
PALMER8C	8	11	0.02	11	0.02
PARKCH	15	672	29.45	823	39.39
PENALTY1	1000	28	0.02	41	0.02
PENALTY2	200	191	0.05	200	0.03
PENALTY3	200	99	1.78	88	1.98

TABLE 1: Continued.

Function	Dimension	CG_Descent 5.3		β_k^A	
		Number of iterations	CPU time	Number of iterations	CPU time
POWELLSG	5000	26	0.02	27	0.05
POWER	10000	372	0.76	543	1.2
QUARTC	5000	17	0.03	15	0.02
ROSENBR	2	34	0.02	28	0.02
S308	2	8	0.02	7	0.02
SCHMVETT	5000	43	0.23	40	0.27
SENSORS	100	21	0.25	50	0.8
SINEVAL	2	64	0.02	46	0.02
SINQUAD	5000	14	0.09	15	0.08
SISSER	2	6	0.02	5	0.02
SNAIL	2	100	0.02	61	0.02
SPARSINE	5000	18358	73	21328	83
SPARSQUR	10000	28	0.31	35	0.98
SPMSRTL	4999	203	0.59	219	0.61
SROSENBR	5000	11	0.02	9	0.03
STRATEC	10	462	19.98	170	6.23
TESTQUAD	5000	1577	1.52E + 00	1573	1.42
TOINTGOR	50	135	0.02	120	0.02
TOINTGSS	5000	4	0.02	5	0.02
TOINTPSP	50	143	0.02	157	0.02
TOINTQOR	50	29	0.02	29	0.02
TQUARTIC	5000	14	0.03	11	0.03
TRIDIA	5000	782	0.84	783	1.11
VAREIGVL		23	0.02	24	0.02
VIBRBEAM	50	138	0.02	98	0.02
WATSON	8	49	0.02	61	0.02
WOODS	12	22	0.06	22	0.03
YFITU	4000	84	0.02	68	0.02
ZANGWIL2	3	1	0.02	1	0.02

FIGURE 1: Performance measure based on the number of iterations.

FIGURE 2: Performance measure based on the CPU time.

FIGURE 3: Six-hump camel back function in 3D.

5. Conclusions

In this study, a modified version of the CG algorithm (A) is suggested and its performance is investigated. The modified formula is restarted based on the value of the Lipchitz constant. The global convergence is established by using SWP line search. Our numerical results show that the new coefficient produces efficient and competitive results compared with other methods, such as CG_Descent 5.3. In the future, an application of the new version of CG method will be combined with feed-forward neural network (back-propagation (BP) algorithm) to improve the training process and produce fast training multilayer algorithm. This will help in reducing time needed to train neural network when the training samples are massive.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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