Research Article

The Calculations of Topological Indices on Certain Networks

Jia-Bao Liu ¹, Ting Zhang ¹, and Sakander Hayat ²

¹School of Mathematics and Physics, Anhui Jianzhu University, Hefei 230601, China

²Faculty of Engineering Sciences, GIK Institute of Engineering Sciences and Technology, Topi Swabi, Khyber Pakhtunkhwa, Pakistan

Correspondence should be addressed to Jia-Bao Liu; liujiabaoad@163.com and Ting Zhang; zhangting1996@ahjzu.edu.cn

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It is one of the core problems in the study of chemical graph theory to study the topological index of molecular graph and the internal relationship between its structural properties and some invariants. In recent years, topological index has been gradually applied to the models of QSAR and QSPR. In this work, using the definition of the ABC index, AZI index, GA index, the multiplicative version of ordinary first Zagreb index, the second multiplicative Zagreb index, and Zagreb index, we calculate the degree-based topological indices of some networks. Then, the above indices' formulas are obtained.

1. Introduction

The topological index is a numerical parameter in the structure graph of molecular compounds, and it can be used to predict the chemical and physical properties of molecules or to predict the biological activity [1–5]. In this paper, we mainly calculate several topological indices, which are invariants that can describe some properties of graph. The topological indices consist of three parts, namely, degree-based indices, spectrum-based indices, and distance-based indices; meanwhile, many indices based on both degree and distance are followed in [6, 7]. The Kirchhoff index is based on the boiling point of kerosene, and other indices can predict the chemical and biological properties of some substances. We are dealing with some degree-based indices, such as, ABC index, AZI index, and GA index. We also calculate some indices for some chemical networks, examples include the first and the second Zagreb index and Zagreb index, by which we can predict the stability or others properties of some networks, such as n -dimensional silicate networks (S_n), chain silicate networks (C_n), hexagonal networks (H_n), n -dimensional honeycomb networks (HC_n), cellular networks (CP_n), and Sierpiński networks ($s(p, m)$). The graph of various networks are shown in Figures 1–6. In the rest of the paper, we made the following arrangements.

In Section 1, we introduce some indices and their backgrounds. In Section 2, we show the important results of this paper. In Section 3, we make a summary.

All graphs and networks are limited to simple undirected graphs. Let $G = (V(G), E(G))$ represent the vertex set and edge set of the networks, respectively. The degree of vertex i is the number of edges associated with i , expressed by d_i . The standard notation and topological descriptors are mainly followed in [8].

According to the chemical molecules, the molecular graph is made up of atoms and bonds. The atom-bond connectivity index $ABC(G)$ is written as

$$ABC(G) = \sum_{ij \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i \times d_j}}, \quad (1)$$

and posted by Estrada et al. [9]. The geometric-arithmetic index $GA(G)$ is the valency-based topological index denoted by

$$GA(G) = \sum_{ij \in E(G)} \frac{2\sqrt{d_i \times d_j}}{d_i + d_j}, \quad (2)$$

which is proposed by Furtula and Vukicević followed in [10]. Compared with other indices, geometric-arithmetic index is

FIGURE 1: The silicate networks S_n . FIGURE 2: The chain silicate networks C_n . FIGURE 3: The hexagonal networks H_n . FIGURE 4: The oxide networks O_n . FIGURE 5: The cellular networks CP_n .

better than other topological indices for predicting the physical and chemical properties of some substances, and more properties of $GA(G)$ index is not introduced and readers can read the literature [11]. In 2010, Furtula et al. proposed $AZI(G)$ index [12], named as the augmented Zagreb index, denoted by

$$AZI(G) = \sum_{ij \in E(G)} \left(\frac{d_i \times d_j}{d_i + d_j - 2} \right)^3. \quad (3)$$

The ordinary first Zagreb index of the multiplication version is represented as

$$\Pi_1^*(G) = \prod_{ij \in E(G)} (d_i + d_j). \quad (4)$$

FIGURE 6: The generalized Sierpiński networks. (a) $t = 1$. (b) $t = 2$. (c) $t = 3$.

For the chemical properties and applications of this index, readers can refer to [13–19]. The second multiplication Zagreb index [20] is written as

$$\Pi_2(G) = \prod_{ij \in E(G)} (d_i \times d_j). \quad (5)$$

On the basis of the Zagreb index, Azari et al. [21] put forward their general form and defined it as

$$Z_{a,b}(G) = \sum_{ij \in E(G)} (d_i^a d_j^b + d_i^b d_j^a). \quad (6)$$

In 1972, the F index was put forward, but there is little research on it. In 2015, B. Furtula and I. Gutman [22] redefined it as the forgotten topological index, or the F index for short, and defined it as

$$F(G) = \sum_{ij \in E(G)} (d_i^2 + d_j^2). \quad (7)$$

About its related research, the reader may refer to [23–26].

2. Main Results and Discussion

In this section, according to the definition of the ABC index, AZI index, GA index, multiplicative version of ordinary first Zagreb index, second multiplicative Zagreb index, and Zagreb index, we calculate the correlation index formula of several kinds of networks and get their concrete expressions.

Silicate is one of the most abundant minerals in the world. It is a mixture of metal compounds and sand [27]. S_n represents a silicate network, where n is the number of hexagons between the boundary and the center. Then, one has $|V(S_n)| = 15n^2 + 3n$, $|E(S_n)| = 36n^2$ and the following results.

According to the distribution of networks vertices, there are three sets of vertex division based on valencies, as A_1, A_2 , and A_3 . The set A_1 consists of $6n$ edges ij , where $d_i = 3$ and $d_j = 3$. The set A_2 consists of $18n^2 + 6n$ edges ij ,

where $d_i = 3$ and $d_j = 6$. The set A_3 consists of $18n^2 - 12n$ edges ij , where $d_i = 6$ and $d_j = 6$.

Theorem 1. Suppose G is a silicate network. Then,

$$GA(G) = (18 + 12\sqrt{2})n^2 + (4\sqrt{2} - 6)n,$$

$$ABC(G) = (3\sqrt{14} + 3\sqrt{10})n^2 + (4 + \sqrt{14} - 2\sqrt{10})n,$$

$$AZI(G) = \left(\frac{18^4}{7^3} + \frac{18^4}{5^3}\right)n^2 + \left(\frac{3^7}{2^5} + \frac{6 \times 18^3}{7^3} - \frac{12 \times 18^3}{5^3}\right)n. \quad (8)$$

$$\begin{aligned}
 GA(G) &= \sum_{ij \in E(G)} \frac{2\sqrt{d_i \times d_j}}{d_i + d_j} \\
 &= 6n \times \frac{2\sqrt{3 \times 3}}{3+3} + (18n^2 + 6n) \times \frac{2\sqrt{3 \times 6}}{3+6} \\
 &\quad + (18n^2 - 12n) \times \frac{2\sqrt{6 \times 6}}{6+6} \\
 &= 6n + (18n^2 + 6n) \times \frac{2\sqrt{2}}{3} + (18n^2 - 12n) \\
 &= (18 + 12\sqrt{2})n^2 + (4\sqrt{2} - 6)n, \\
 ABC(G) &= \sum_{ij \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i \times d_j}} \\
 &= 6n \times \sqrt{\frac{3+3-2}{3 \times 3}} + (18n^2 + 6n) \times \sqrt{\frac{3+6-2}{3 \times 6}} + (18n^2 - 12n) \times \sqrt{\frac{6+6-2}{6 \times 6}} \\
 &= 6n \times \frac{2}{3} + (18n^2 + 6n) \times \frac{\sqrt{14}}{6} + (18n^2 - 12n) \times \frac{\sqrt{10}}{6} \\
 &= (3\sqrt{14} + 3\sqrt{10})n^2 + (4 + \sqrt{14} - 2\sqrt{10})n, \\
 AZI(G) &= \sum_{ij \in E(G)} \left(\frac{d_i \times d_j}{d_i + d_j - 2}\right)^3 \\
 &= 6n \times \left(\frac{3 \times 3}{3+3-2}\right)^3 + (18n^2 + 6n) \times \left(\frac{3 \times 6}{3+6-2}\right)^3 + (18n^2 - 12n) \times \left(\frac{6 \times 6}{6+6-2}\right)^3 \\
 &= 6n \times \left(\frac{9}{4}\right)^3 + (18n^2 + 6n) \times \left(\frac{18}{7}\right)^3 + (18n^2 - 12n) \times \left(\frac{18}{5}\right)^3 \\
 &= \left(\frac{18^4}{7^3} + \frac{18^4}{5^3}\right)n^2 + \left(\frac{3^7}{2^5} + \frac{6 \times 18^3}{7^3} - \frac{12 \times 18^3}{5^3}\right)n.
 \end{aligned} \quad (9)$$

Hexagonal networks is written as H_n , which is composed of n hexagons. According to the relationship of degree series, we mainly calculate the following indices. One can refer to more research on hexagonal networks [28–31].

Similarly, according to the degree distribution of the Hexagonal networks vertices, there are five sets of vertex division

Proof. Let G be a silicate network. Then, one has

based on valencies, as A_1, A_2, A_3, A_4 , and A_5 . The set A_1 consists of 12 edges ij , where $d_i = 3$ and $d_j = 4$. The set A_2 consists of 6 edges ij , where $d_i = 3$ and $d_j = 6$. The set A_3 consists of $6n - 18$ edges ij , where $d_i = 4$ and $d_j = 4$. The set A_4 consists of $12n - 24$ edges ij , where $d_i = 4$ and $d_j = 6$. The set A_5 consists of $9n^2 - 33n + 30$ edges ij , where $d_i = 6$ and $d_j = 6$.

Theorem 2. Suppose G is a hexagonal network. Then,

$$\begin{aligned} \text{GA}(G) &= 9n^2 + \left(\frac{24\sqrt{6}}{5} - 27\right)n + 12 + 4\sqrt{2} + \frac{48}{35}(5\sqrt{3} - 7\sqrt{6}), \\ \text{ABC}(G) &= \frac{3\sqrt{10}}{2}n^2 + \left(\frac{3}{2}\sqrt{6} + 4\sqrt{3} - \frac{11\sqrt{10}}{2}\right)n + \left(2\sqrt{15} + \sqrt{14} - \frac{9\sqrt{6}}{2} - 8\sqrt{3} + 5\sqrt{10}\right), \\ \text{AZI}(G) &= \frac{9 \times 18^3}{5^3}n^2 + \left(\frac{2 \times 8^3}{9} + 324 - \frac{33 \times 18^3}{5^3}\right)n + \left(\frac{12^4}{5^3} + \frac{6 \times 18^3}{7^3} - \frac{2 \times 8^3}{3} + \frac{6 \times 18^3}{25} - 648\right). \end{aligned} \quad (10)$$

Proof. Only consider that H_n is an n -dimensional hexagonal networks. So, $|V(H_n)| = 3n^2 - 3n + 1$, $|E(H_n)| = 9n^2 - 15n + 6$. Thus,

$$\begin{aligned} \text{GA}(G) &= \sum_{ij \in E(G)} \frac{2\sqrt{d_i \times d_j}}{d_i + d_j} \\ &= 12 \times \frac{2 \times \sqrt{3 \times 4}}{3 + 4} + 6 \times \frac{2 \times \sqrt{3 \times 6}}{3 + 6} + (6n - 18) \times \frac{2 \times \sqrt{4 \times 4}}{4 + 4} \\ &\quad + (12n - 24) \times \frac{2 \times \sqrt{4 \times 6}}{4 + 6} + (9n^2 - 33n + 30) \times \frac{2 \times \sqrt{6 \times 6}}{6 + 6} \\ &= 12 \times \frac{4\sqrt{3}}{7} + 6 \times \frac{2 \times 3\sqrt{2}}{9} + (6n - 18) \times \frac{2 \times 4}{8} + (12n - 24) \\ &\quad \times \frac{4\sqrt{6}}{10} + (9n^2 - 33n + 30) \times \frac{2 \times \sqrt{6 \times 6}}{6 + 6} \\ &= 9n^2 + \left(\frac{24\sqrt{6}}{5} - 27\right)n + 12 + 4\sqrt{2} + \frac{48}{35}(5\sqrt{3} - 7\sqrt{6}), \\ \text{ABC}(G) &= \sum_{ij \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i \times d_j}} \\ &= 12 \times \sqrt{\frac{3 + 4 - 2}{3 \times 4}} + 6 \times \sqrt{\frac{3 + 6 - 2}{3 \times 6}} + (6n - 18) \times \sqrt{\frac{4 + 4 - 2}{4 \times 4}} + (12n - 24) \times \sqrt{\frac{4 + 6 - 2}{4 \times 6}} \\ &\quad + (9n^2 - 33n + 30) \times \sqrt{\frac{6 + 6 - 2}{6 \times 6}} \\ &= 12 \times \sqrt{\frac{5}{12}} + 6 \times \sqrt{\frac{7}{18}} + (6n - 18) \times \sqrt{\frac{6}{16}} + (12n - 24) \times \sqrt{\frac{8}{24}} + (9n^2 - 33n + 30) \times \sqrt{\frac{10}{36}} \\ &= \frac{3\sqrt{10}}{2}n^2 + \left(\frac{3}{2}\sqrt{6} + 4\sqrt{3} - \frac{11\sqrt{10}}{2}\right)n + \left(2\sqrt{15} + \sqrt{14} - \frac{9\sqrt{6}}{2} - 8\sqrt{3} + 5\sqrt{10}\right), \end{aligned}$$

$$\begin{aligned}
AZI(G) &= \sum_{ij \in E(G)} \left(\frac{d_i \times d_j}{d_i + d_j - 2} \right)^3 \\
&= 12 \times \left(\frac{3 \times 4}{3 + 3 - 2} \right)^3 + 6 \times \left(\frac{3 \times 6}{3 + 6 - 2} \right)^3 + (6n - 18) \times \left(\frac{4 \times 4}{4 + 4 - 2} \right)^3 \\
&\quad + (12n - 24) \times \left(\frac{4 \times 6}{4 + 6 - 2} \right)^3 + (9n^2 - 33n + 30) \times \left(\frac{6 \times 6}{6 + 6 - 2} \right)^3 \\
&= 12 \times \frac{12^3}{5^3} + 6 \times \frac{18^3}{7^3} + (6n - 18) \times \frac{16^3}{6^3} + (12n - 24) \times \frac{24^3}{8^3} + (9n^2 - 33n + 30) \times \frac{36^3}{10^3} \\
&= \frac{9 \times 18^3}{5^3} n^2 + \left(\frac{2 \times 8^3}{9} + 324 - \frac{33 \times 18^3}{5^3} \right) n + \left(\frac{12^4}{5^3} + \frac{6 \times 18^3}{7^3} - \frac{2 \times 8^3}{3} + \frac{6 \times 18^3}{25} - 648 \right). \tag{11}
\end{aligned}$$

At present, we discuss about another member of the silicate networks and the chain silicate networks, which is a linear combination of n tetrahedrons, referred to as C_n . In the same way, the edges of the silicate networks can be divided into three sets of vertex division based on valencies, as A_1 , A_2 , and A_3 . For $n = 1$, the set A_1 consists of 6 edges ij , where $d_i = 3$ and $d_j = 3$, the set A_2 consists of 0 edges ij , where $d_i = 3$ and $d_j = 6$, and the set A_3 consists of 0 edges ij ,

where $d_i = 6$ and $d_j = 6$. For $n \geq 2$, the set A_1 consists of $n + 4$ edges ij , where $d_i = 3$ and $d_j = 3$, the set A_2 consists of $4n - 2$ edges ij , where $d_i = 3$ and $d_j = 6$, and the set A_3 consists of $n - 2$ edges ij , where $d_i = 6$ and $d_j = 6$. $\square$

Theorem 3. Suppose G is a chain silicate network with $6n$ edges and $3n + 1$ vertices. Then,

$$\begin{aligned}
GA(G) &= \begin{cases} 6, & n = 1, \\ \left(2 + \frac{8\sqrt{2}}{3} \right) n + \left(2 - \frac{4\sqrt{2}}{3} \right), & n \geq 2, \end{cases} \\
ABC(G) &= \begin{cases} 4, & n = 1, \\ \left(\frac{4 + 4\sqrt{14} + \sqrt{10}}{6} \right) n + \frac{8 - \sqrt{14} - \sqrt{10}}{3}, & n \geq 2, \end{cases} \tag{12} \\
AZI(G) &= \begin{cases} \frac{3^7}{2^5}, & n = 1, \\ \left(\frac{9^3}{4^3} + \frac{4 \times 18^3}{7^3} + \frac{18^3}{5^3} \right) n + \left(\frac{9^3}{16} - \frac{2 \times 18^3}{7^3} - \frac{2 \times 18^3}{5^3} \right), & n \geq 2. \end{cases}
\end{aligned}$$

Proof. Let G be a chain silicate network. Then,

$$GA(G) = \sum_{ij \in E(G)} \frac{2\sqrt{d_i \times d_j}}{d_i + d_j}, \quad (13)$$

where

$$\begin{aligned} n = 1, \quad GA(G) &= 6 \times \frac{2\sqrt{3 \times 3}}{3 + 3} = 6, \\ n \geq 2, \quad GA(G) &= (n + 4) \times \frac{2\sqrt{3 \times 3}}{3 + 3} \\ &\quad + (4n - 2) \times \frac{2\sqrt{3 \times 6}}{3 + 6} + (n - 2) \times \frac{2\sqrt{6 \times 6}}{6 + 6} \\ &= (n + 4) + (4n - 2) \times \frac{2\sqrt{2}}{3} + (n - 2) \\ &= \left(2 + \frac{8\sqrt{2}}{3}\right)n + \left(2 - \frac{4\sqrt{2}}{3}\right), \\ ABC(G) &= \sum_{ij \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i \times d_j}}, \end{aligned} \quad (14)$$

where

$$\begin{aligned} n = 1, \quad ABC(G) &= 6 \times \sqrt{\frac{3 + 3 - 2}{3 \times 3}} = 4, \\ n \geq 2, \quad ABC(G) &= (n + 4) \times \sqrt{\frac{3 + 3 - 2}{3 \times 3}} \\ &\quad + (4n - 2) \times \sqrt{\frac{3 + 6 - 2}{3 \times 6}} + (n - 2) \\ &\quad \times \sqrt{\frac{6 + 6 - 2}{6 \times 6}} \\ &= \frac{2}{3}(n + 4) + (4n - 2) \times \frac{\sqrt{14}}{6} \\ &\quad + (n - 2) \times \frac{\sqrt{10}}{6} \\ &= \left(\frac{4 + 4\sqrt{14} + \sqrt{10}}{6}\right)n + \frac{8 - \sqrt{14} - \sqrt{10}}{3}, \\ AZI(G) &= \sum_{ij \in E(G)} \left(\frac{d_i \times d_j}{d_i + d_j - 2}\right)^3, \end{aligned} \quad (15)$$

where

$$\begin{aligned} n = 1, \quad AZI(G) &= 6 \times \left(\frac{3 \times 3}{3 + 3 - 2}\right)^3 = \frac{3^7}{2^5}, \\ n \geq 2, \quad AZI(G) &= (n + 4) \times \left(\frac{3 \times 3}{3 + 3 - 2}\right)^3 + (4n - 2) \\ &\quad \times \left(\frac{3 \times 6}{3 + 6 - 2}\right)^3 + (n - 2) \times \left(\frac{6 \times 6}{6 + 6 - 2}\right)^3 \\ &= (n + 4) \frac{9^3}{4^3} + (4n - 2) \times \frac{18^3}{7^3} \\ &\quad + (n - 2) \times \frac{36^3}{10^3} \\ &= \left(\frac{9^3}{4^3} + \frac{4 \times 18^3}{7^3} + \frac{18^3}{5^3}\right)n \\ &\quad + \left(\frac{9^3}{16} - \frac{2 \times 18^3}{7^3} - \frac{2 \times 18^3}{5^3}\right). \end{aligned} \quad (16)$$

Oxide networks play an important role in silicate networks. When the silicon atoms in the silicate networks are removed, the oxide networks are obtained. The n -dimensional oxide networks are defined as O_n . By observing the edge division of oxide networks, there are two sets of vertex division based on valencies, as A_1 and A_2 . The set A_1 consists of $12n$ edges ij , where $d_i = 2$ and $d_j = 4$. The set A_2 consists of $18n^2 - 12n$ edges ij , where $d_i = 4$ and $d_j = 4$.

Theorem 4. Suppose G is an oxide network with $18n^2$ edges and $9n^2 + 3n$ vertices. Then,

$$\begin{aligned} GA(G) &= 18n^2 + (8\sqrt{2} - 12)n, \\ ABC(G) &= \frac{9\sqrt{6}}{2}n^2 + (6\sqrt{2} - 3\sqrt{6})n, \\ AZI(G) &= \frac{1024}{3}n^2 - \frac{1184}{9}n. \end{aligned} \quad (17)$$

Proof. Let G be an oxide network. Then, one has

$$\begin{aligned}
GA(G) &= \sum_{ij \in E(G)} \frac{2\sqrt{d_i \times d_j}}{d_i + d_j} \\
&= 12n \times \frac{2\sqrt{2 \times 4}}{2 + 4} + (18n^2 - 12n) \times \frac{2\sqrt{4 \times 4}}{4 + 4} \\
&= 18n^2 + (8\sqrt{2} - 12)n, \\
ABC(G) &= \sum_{ij \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i \times d_j}} \\
&= 12n \times \sqrt{\frac{2 + 4 - 2}{2 \times 4}} + (18n^2 - 12n) \times \sqrt{\frac{4 + 4 - 2}{4 \times 4}} \\
&= 12n \times \frac{\sqrt{2}}{2} + (18n^2 - 12n) \times \frac{\sqrt{6}}{4} \\
&= \frac{9\sqrt{6}}{2}n^2 + (6\sqrt{2} - 3\sqrt{6})n, \\
AZI(G) &= \sum_{ij \in E(G)} \left(\frac{d_i \times d_j}{d_i + d_j - 2} \right)^3 \\
&= 12n \times \left(\frac{2 \times 4}{2 + 4 - 2} \right)^3 + (18n^2 - 12n) \times \left(\frac{4 \times 4}{4 + 4 - 2} \right)^3 \\
&= 12n \times 2^3 + (18n^2 - 12n) \times \frac{8^3}{27} \\
&= \frac{1024}{3}n^2 - \frac{1184}{9}n.
\end{aligned} \tag{18}$$

Cellular networks is mainly composed of three parts: mobile station, network subsystem, and base station subsystem, denoted by CP_n . It plays an important role in computer graphics and in chemistry. Meanwhile, it also can be characterized as benzene hydrocarbons. In the same way, the edges of cellular networks can be divided into three sets based on valencies, as A_1 , A_2 , and A_3 . The set A_1 consists of 6 edges ij , where $d_i = 2$ and $d_j = 2$. The set A_2 consists of $12n - 12$ edges ij , where $d_i = 2$ and $d_j = 3$. The set A_3 consists of $9n^2 - 15n + 6$ edges ij , where $d_i = 3$ and $d_j = 3$. $\square$

Theorem 5. Suppose G is a cellular network with $6n^2$ edges and $9n^2 - 3n$ vertices. Then,

$$\begin{aligned}
GA(G) &= 9n^2 + \left(\frac{24\sqrt{6}}{5} - 15 \right)n + 12 - \frac{24}{5}\sqrt{6}, \\
ABC(G) &= 6n^2 + (6\sqrt{2} - 10)n + 4 - 3\sqrt{2}, \\
AZI(G) &= \frac{729}{64}(9n^2 - 15n + 6) + 96n - 48.
\end{aligned} \tag{19}$$

Proof. Let G be a cellular network. Then, one has

$$\begin{aligned}
GA(G) &= \sum_{ij \in E(G)} \frac{2\sqrt{d_i \times d_j}}{d_i + d_j} \\
&= 6 \times \frac{2\sqrt{2 \times 2}}{2 + 2} + (12n - 12) \times \frac{2\sqrt{2 \times 3}}{2 + 3} \\
&\quad + (9n^2 - 15n + 6) \times \frac{2\sqrt{3 \times 3}}{3 + 3} \\
&= 6 \times \frac{2 \times 2}{4} + (12n - 12) \times \frac{2\sqrt{6}}{5} \\
&\quad + (9n^2 - 15n + 6) \times \frac{2 \times 3}{6} \\
&= 9n^2 + \left(\frac{24\sqrt{6}}{5} - 15 \right)n + 12 - \frac{24}{5}\sqrt{6}, \\
ABC(G) &= \sum_{ij \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i \times d_j}} \\
&= 6 \times \sqrt{\frac{2 + 2 - 2}{2 \times 2}} + (12n - 12) \times \sqrt{\frac{2 + 3 - 2}{2 \times 3}} \\
&\quad + (9n^2 - 15n + 6) \times \sqrt{\frac{3 + 3 - 2}{3 \times 3}} \\
&= 3\sqrt{2} + (6n - 6)\sqrt{2} + (6n^2 - 10n + 4) \times 2 \\
&= 6n^2 + (6\sqrt{2} - 10)n + 4 - 3\sqrt{2}, \\
AZI(G) &= \sum_{ij \in E(G)} \left(\frac{d_i \times d_j}{d_i + d_j - 2} \right)^3 \\
&= 6 \times \left(\frac{2 \times 2}{2 + 2 - 2} \right)^3 + (12n - 12) \times \left(\frac{2 \times 3}{2 + 3 - 2} \right)^3 \\
&\quad + (9n^2 - 15n + 6) \times \left(\frac{3 \times 3}{3 + 3 - 2} \right)^3 \\
&= 6 \times 2^3 + (12n - 12) \times 2^3 + (9n^2 - 15n + 6) \times \left(\frac{9}{4} \right)^3 \\
&= \frac{729}{64}(9n^2 - 15n + 6) + 96n - 48.
\end{aligned} \tag{20}$$

Next, our step is to study the generalized Sierpiński networks $s(K_p, m)$ when its subgraph is a complete graph. By consulting [32], the edges of the $s(K_p, m)$ can be divided into two sets of vertex division based on valencies, as A_1 and A_2 . The set A_1 consists of $m(m - 1)$ edges ij , where $d_i = m$ and $d_j = m + 1$. The set A_2 consists of $(m^{t+1} - 2m^2 + m)/2$ edges ij , where $d_i = m + 1$ and $d_j = m + 1$. $\square$

Theorem 6. Suppose G is a Sierpiński networks and its subgraph is a complete graph. Then,

$$\begin{aligned}\Pi_2(G) &= m^{m(m-1)} \cdot (m+1)^{m^{t+1}-m^2}, \\ \Pi_1^*(G) &= (2m+1)^{m(m-1)} \cdot (2m+2)^{(m^{t+1}-2m^2+m)/2}, \\ F(G) &= m^{t+3} + 2m^{t+2} + m^{t+1} - 3m^3 - m^2, \\ Z_{a,b}(G) &= (m^2 - m) \cdot [m^a(m+1)^b + m^b(m+1)^a] \\ &\quad + (m+1)^{a+b} \cdot (m^{t+1} - 2m^2 + m).\end{aligned}\quad (21)$$

Proof. Let G be Sierpiński networks with the seed graph being a complete graph. Then, one has

$$\begin{aligned}\Pi_2(G) &= \prod_{ij \in E(G)} (d_i \times d_j) \\ &= [m \cdot (m+1)]^{m(m-1)} \cdot [(m+1)^2]^{(m^{t+1}-2m^2+m)/2} \\ &= [m \cdot (m+1)]^{m(m-1)} \cdot (m+1)^{m^{t+1}-2m^2+m} \\ &= m^{m(m-1)} \cdot (m+1)^{m^{t+1}-m^2}, \\ \Pi_1^*(G) &= \prod_{ij \in E(G)} (d_i + d_j) \\ &= [m + (m+1)]^{m(m-1)} + [(m+1) + (m+1)]^{(m^{t+1}-2m^2+m)/2} \\ &= (2m+1)^{m(m-1)} \cdot (2m+2)^{(m^{t+1}-2m^2+m)/2}, \\ F(G) &= \sum_{ij \in E(G)} (d_i^2 + d_j^2) \\ &= [m^2 + (m+1)^2] \cdot m(m-1) + [(m+1)^2 + (m+1)^2] \frac{m^{t+1} - 2m^2 + m}{2} \\ &= (2m^2 + 2m + 1) \cdot (m^2 - m) + (m^2 + 2m + 1) \cdot (m^{t+1} - 2m^2 + m) \\ &= m^{t+3} + 2m^{t+2} + m^{t+1} - 3m^3 - m^2, \\ Z_{a,b}(G) &= \sum_{ij \in E(G)} (d_i^a d_j^b + d_i^b d_j^a) \\ &= [m^a(m+1)^b + m^b(m+1)^a] \cdot m(m-1) + \frac{m^{t+1} - 2m^2 + m}{2} \cdot [2(m+1)^a(m+1)^b] \\ &= (m^2 - m) \cdot [m^a(m+1)^b + m^b(m+1)^a] + (m+1)^{a+b} \cdot (m^{t+1} - 2m^2 + m).\end{aligned}\quad (22)$$

Finally, we discuss the Sierpiński networks $s(p, m)$ when the seed graph is a m -regular graph without triangles. Similarly, the edges of the $s(p, m)$ can be divided into three sets of vertex division based on valencies, as A_1 , A_2 , and A_3 . The set A_1 consists of $((p^{t-1}m)/2)(p-2m)$ edges ij , where $d_i = m$ and $d_j = m$. The set A_2 consists of $(p^{t-1} + ((p^{t-1} - p)/(1-p)))m^2$ edges ij , where $d_i = m$ and $d_j = m+1$. The

set A_3 consists of $(pm/2)((1-p^{t-1})/(1-p)) + pm^2((1-p^{t-2})/(1-p))$ edges ij , where $d_i = m+1$ and $d_j = m+1$. $\square$

Theorem 7. Suppose G is a Sierpiński networks $s(p, m)$ and its subgraph is a m -regular graph without triangles. Then,

$$\begin{aligned}
\Pi_2(G) &= m^{p^{t-1}m(p-m)+((p^{t-1}-p)/(1-p))m^2} \cdot (m+1)^{p^{t-1}m^2+((m^2(p-p^{t-1})+pm(1-p^{t-1}))/((1-p))), \\
\Pi_1^*(G) &= 2^{((p^{t-1}m)/2)(p-2m)+((pm(1-p^{t-1}+2m-2mp^{t-2}))/((2(1-p))))} \cdot m^{((p^{t-1}m)/2)(p-2m)} \\
&\quad \cdot (m+1)^{(pm/2)((1-p^{t-1})/(1-p))+pm^2((1-p^{t-2})/(1-p))} \cdot (2m+1)^{(p^{t-1}+((p^{t-1}-p)/(1-p)))m^2}, \\
F(G) &= \left(p^{t-1} + \frac{p^{t-1}-p}{1-p}\right)m^4 + p^{t-1}(p-2m)m^3 + \left[p^{t-1}m^2 + \frac{m^2(p-p^{t-1})+pm(1-p^{t-1})}{1-p}\right](m+1)^2, \quad (23) \\
Z_{a,b}(G) &= p^{t-1}(p-2m) \cdot m^{a+b+1} + \left(p^{t-1} + \frac{p^{t-1}-p}{1-p}\right)m^2[m^a(m+1)^b + m^b(m+1)^a] \\
&\quad + \frac{pm(1-p^{t-1}+2m-2mp^{t-2})}{1-p}(m+1)^{a+b}.
\end{aligned}$$

Proof. Let G be Sierpiński networks and its subgraph be m -regular graph without triangles. Then, one has

$$\begin{aligned}
\Pi_2(G) &= \prod_{ij \in E(G)} (d_i \times d_j) \\
&= (m \cdot m)^{((p^{t-1}m)/2)(p-2m)} \cdot [m \cdot (m+1)]^{(p^{t-1}+((p^{t-1}-p)/(1-p)))m^2} \\
&\quad \cdot [(m+1) \cdot (m+1)]^{(pm/2)((1-p^{t-1})/(1-p))+pm^2((1-p^{t-2})/(1-p))} \\
&= m^{p^{t-1}m(p-2m)} \cdot m^{(p^{t-1}+((p^{t-1}-n)/1-))m^2} \cdot (m+1)^{(p^{t-1}+((p^{t-1}-p)/(1-p)))m^2+pm((1-p^{t-1})/(1-p))+2pm^2((1-p^{t-2})/(1-p))} \\
&= m^{p^{t-1}m(p-m)+((p^{t-1}-p)/(1-p))m^2} \cdot (m+1)^{p^{t-1}m^2+((m^2(p-p^{t-1})+pm(1-p^{t-1}))/((1-p))), \\
\Pi_1^*(G) &= \prod_{ij \in E(G)} (d_i + d_j) \\
&= (m+m)^{((p^{t-1}m)/2)(p-2m)} \cdot [m+(m+1)]^{(p^{t-1}+((p^{t-1}-p)/(1-p)))m^2} \\
&\quad \cdot [(m+1)+(m+1)]^{(pm/2)((1-p^{t-1})/(1-p))+pm^2((1-p^{t-2})/(1-p))} \\
&= 2^{((p^{t-1}m)/2)(p-2m)+((pm(1-p^{t-1}+2m-2mp^{t-2}))/((2(1-p))))} \cdot m^{((p^{t-1}m)/2)(p-2m)} \cdot (2m+1)^{(p^{t-1}+((p^{t-1}-p)/(1-p)))m^2} \\
&\quad \cdot (m+1)^{(pm/2)((1-p^{t-1})/(1-p))+pm^2((1-p^{t-2})/(1-p))} \\
&= 2^{(p^{t-1}m/2)(p-2m)+((pm(1-p^{t-1}+2m-2mp^{t-2}))/((2(1-p))))} \cdot m^{(p^{t-1}m/2)(p-2m)} \cdot (m+1)^{(pm/2)((1-p^{t-1})/(1-p))+pm^2((1-p^{t-2})/(1-p))} \\
&\quad \cdot (2m+1)^{(p^{t-1}+((p^{t-1}-p)/(1-p)))m^2},
\end{aligned}$$

$$\begin{aligned}
F(G) &= \sum_{ij \in E(G)} (d_i^2 + d_j^2) \\
&= [m^2 + m^2] \cdot \frac{p^{t-1}m}{2} (p-2m) + \left(\left(p^{t-1} + \frac{p^{t-1}-p}{1-p} \right) m^2 \right) [m^2 + (m+1)^2] \\
&\quad + \left[\frac{pm}{2} \left(\frac{1-p^{t-1}}{1-p} \right) + pm^2 \left(\frac{1-p^{t-2}}{1-p} \right) \right] \cdot [(m+1)^2 + (m+1)^2] \\
&= p^{t-1}m^3(p-2m) + \left(p^{t-1} + \frac{p^{t-1}-p}{1-p} \right) m^4 + \left(p^{t-1} + \frac{p^{t-1}-p}{1-p} \right) m^2(m+1)^2 \\
&\quad + \left[pm \frac{1-p^{t-1}}{1-p} + 2pm^2 \frac{1-p^{t-2}}{1-p} \right] \cdot (m+1)^2 \\
&= \left(p^{t-1} + \frac{p^{t-1}-p}{1-p} \right) m^4 + p^{t-1}(p-2m)m^3 + \left[p^{t-1}m^2 + \frac{m^2(p-p^{t-1}) + pm(1-p^{t-1})}{1-p} \right] (m+1)^2, \\
Z_{a,b}(G) &= \sum_{ij \in E(G)} (d_i^a d_j^b + d_i^b d_j^a) \\
&= \frac{p^{t-1}m}{2} (p-2m) \cdot (m^a m^b + m^b m^a) + \left(p^{t-1} + \frac{p^{t-1}-p}{1-p} \right) m^2 \cdot [m^a(m+1)^b + m^b(m+1)^a] \\
&\quad + \frac{pm}{2} \left(\frac{1-p^{t-1}}{1-p} \right) + pm^2 \left(\frac{1-p^{t-2}}{1-p} \right) \cdot (2(m+1)^{a+b}) \\
&= p^{t-1}(p-2m) \cdot m^{a+b+1} + \left(p^{t-1} + \frac{p^{t-1}-p}{1-p} \right) m^2 [m^a(m+1)^b + m^b(m+1)^a] \\
&\quad + \frac{pm(1-p^{t-1} + 2m - 2mp^{t-2})}{1-p} (m+1)^{a+b}, \tag{24}
\end{aligned}$$

as desired. $\square$

3. Conclusion

In this paper, we studied the ABC index of some chemical networks and obtained AZI index, GA index, the multiplicative version of ordinary first Zagreb index, the second multiplicative Zagreb indices, and the Zagreb index. By calculating the correlation index of several specific chemical networks, we can get the above indices formulas. This also provides potential help for scholars to study networks characteristics better. For further work, if the corresponding networks are replaced by other networks, we can also calculate and get the corresponding formulas.

Data Availability

The figures, tables, and other data used to support this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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