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Research Article

On a Flow-Shop Scheduling Problem with Fuzzy Pentagonal Processing Time

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Scheduling involves planning and arranging jobs across a coordinated set of events to satisfy the customer's demands. In this article, we present a simple approach for the flow-shop (FS) scheduling problem under fuzzy environment in which processing time of jobs are represented by pentagonal fuzzy numbers. This study is intended to reduce the rental cost of the machine in compliance with the rental policy. The fuzzy FS scheduling problem is solved without converting the processing time into its equivalent crisp numbers using a robust ranking technique and a fuzzy arithmetic pentagonal fuzzy numbers. A numerical illustration indicates that the approach is workable, accurate, and relevant.

1. Introduction

Scheduling requires a variety of activities to accomplish a particular purpose with time and budget. The job schedule and management of its flows through a planning step are the most fundamental facets of any industrial production procedure. The management of a certain number of machines or facilities for a certain number of tasks or employment is one difficulty. The control of staff in any manner poses a largely unresolved obstacle to accomplish a certain objective. In planning a production or plan, a decision maker has difficulties encouraging prompt implementation and reducing demands such as flow times or completion periods. The scheduler's goal is to set starting times to achieve maximum performance subject to energy and technological constraints. The problem with flow-shop preparation is that the simplistic version of which all staffs travel in the same sequence on all machinery is one of the most critical issues in production management. To minimize a machine's completion time, Argawal et al. [1] encountered three FS device scheduling problems. Vahedi-Nouri et al. [2] presented a more broad version of the FS strategy to minimize the average flow rate. In order to optimize the publication time, Ren et al. [3] explored the topic of FS programming. Laribi et al. [4] have introduced a mathematical model for two FS-limited machines that address FS time reduction problems where renewables are not constrained. Yazdani and Naderi [5] considered the scheduling problem with no-idle hybrid flow-shop and applied a mixed integer linear programming to formulate the problem. Qu et al. [6] designed a flower pollination algorithm based on the hormone modulation mechanism for no-wait flow-shop scheduling problem, where the method uses available neighborhood search strategy based on dynamic self-adaptive variable work piece in the local search.

This would be made possible by a blueprint for several research fields such as computer analysis and organizational science. Decision-making in a fuzzy environment, developed by Bellman and Zadeh [7], has an improvement and a great help in the management decision problems. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade [8] spread the expansion of algebraic operations into fuzzy numbers by a fuzzy hypothesis of real numbers. Panda and Pal developed the theory of pentagonal andantes fuzzy numbers (2016). Chakraborty et al. [9] extended the properties of the pentagonal fuzzy number to

interval-value fuzzy and pentagonal fuzzy numbers. Many scientists [10, 11] were involved in the problem of FS preparation. A method containing a variety of statistical and fuzzy flow-shop sets was introduced by Yao and Lin [12]. Hejari et al. [13] enhanced McCahon and Lee's [14] algorithm for work sequence with a fuzzy processing time (1990). For a smooth processing time, Ishibuchi and Lee established FS scheduling problem [15]. Temiz and Erol [16] also employed the fuzzy scheduling principle of the FS, updated the branch-and-bound of Ignall and Schrage [17], and reassigned the Ignall and Schrage algorithm to three FS frame problems. Gupta et al. [18] applied a heuristic algorithm for minimizing the rental cost of the machines for a specially structured three-stage flow-shop scheduling where triangular fuzzy numbers represent the processing times of the machine. Neira et al. [19] provided a general overview of the FS scheduling problem under uncertainties and its role in production logistics to draw up opportunities for further research. Komaki et al. [20] conducted a joint survey of their templates for assembly flow shops. Janaki and Ismail [21] explored the interval, modeling work using an efficient and heuristic make-up approach. Khalifa [2] analyzed the single-machine preparation issue in a fizzy dates setting. Khalifa et al. [23] proposed a fuzzy binding approach for solving constrained flow-shop scheduling problem without converting the problem into its crisp.

In this paper, we consider the problem of flow-shop scheduling in a fuzzy environment where processing times are represented as pentagonal fuzzy numbers. We investigate a method to find the rental cost of the machines without converting the pentagonal fuzzy numbers into its classical number.

This study aims to minimize the rental costs of the machine as described in the rental policy. The remainder of the article is organized as follows. Section 2 addresses some preliminaries. Section 3 includes some fundamental assumptions and concepts that are central to the problem. Section 4 contains the pentagonal fuzzy number processing times' assertion of the FS programming problem. Section 5 offers an approach to the problem of fuzzy flow-shop scheduling. Section 6 includes a numerical illustration example. Section 7 presents a comparison between the solutions from the proposed approach with the others. Finally, concluding remarks are included in Section 8.

2. Preliminaries

In order to easily discuss the problem, it recalls basic rules and findings related to fuzzy numbers, pentagonal fuzzy numbers, and arithmetic operations of pentagonal fuzzy numbers and its ranking.

Definition 1 (see [21–24]). A fuzzy set $\widetilde{A}$ a set of real numbers R is called fuzzy if its membership function $\mu_{\widetilde{A}}(x)$: $\mathbb{R} \longrightarrow [0,1]$ have the following properties:

(1) $\mu_{\widetilde{A}}(x)$ is an upper semicontinuous membership function

(2) $\widetilde{A}$ is convex fuzzy set, i.e., $\mu_{\widetilde{A}}(\delta x + (1-\delta) y) \geq \min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$ for all $x, y \in \mathbb{R}; 0 \leq \delta \leq 1$

(3) $\widetilde{A}$ is normal, i.e., $\exists x_0 \in \mathbb{R}$ for which $\mu_{\widetilde{A}}(x_0) = 1$

(4) Supp $(\widetilde{A}) = \{x \in \mathbb{R}: \mu_{\widetilde{A}}(x) > 0\}$ is the support of $\widetilde{A}$, and the closure $cl(\text{Supp}(\widetilde{A}))$ is compact set

Definition 2 (see [8, 22, 23 to 9, 25, 26]). A fuzzy number $\widetilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$, $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ on $\mathbb{R}$ is said to be a pentagonal fuzzy number if its membership function is

$$\mu_{\widetilde{A}_P} = \begin{cases} 0, & x < a_1, \\ w_1\left(\dfrac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \leq x \leq a_2, \\ 1-(1-w_1)\left(\dfrac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \leq x \leq a_3, \\ 1, & \text{for } x = a_3, \\ 1-(1-w_2)\left(\dfrac{a_4-x}{a_4-a_3}\right), & \text{for } a_3 \leq x \leq a_4, \\ w_2\left(\dfrac{a_5-x}{a_5-a_4}\right), & \text{for } a_4 \leq x \leq a_5, \\ 0, & \text{for } x > a_5. \end{cases} \quad (1)$$

Figure 1 shows the graphical representation of the pentagonal fuzzy number $\widetilde{A}_P$.

Definition 3. Let $\widetilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\widetilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be two pentagonal fuzzy numbers and $v \neq 0$. The arithmetic operations on $\widetilde{A}_P$ and $\widetilde{B}_P$ are

(i) $\widetilde{A}_P \oplus \widetilde{B}_P = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5)$

(ii) $\widetilde{A}_P \ominus \widetilde{B}_P = (a_1-b_5, a_2-b_4, a_3-b_3, a_4-b_2, a_5-b_1)$

(iii) $\widetilde{A}_P \otimes \widetilde{B}_P = 1/5\gamma_b (a_1, a_2, a_3, a_4, a_5), \gamma_b = (b_1 + b_2 + b_3 + b_4 + b_5)$

(iv) $\widetilde{A}_P \oslash \widetilde{B}_P = (5/\gamma_b)(a_1, a_2, a_3, a_4, a_5), 0 \neq \gamma_b = (b_1 + b_2 + b_3 + b_4 + b_5)$

(v) $k\widetilde{A}_P = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5)m, & k > 0, \\ (ka_5, ka_4, ka_3, ka_2, ka_1), & k < 0 \end{cases}$

Definition 4 (see [25]). The ranking function of $\widetilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ is defined as

$$R(\widetilde{A}_P) = \int_0^1 0.5\left[(a_2-a_1)\alpha + a_1, \ -(a_5-a_4)\alpha + a_5\right] d\alpha. \quad (2)$$

Definition 5. The order of relation between $\widetilde{A}_P$ and $\widetilde{B}_P$ based on the ranking function is defined as

(i) $\widetilde{A}_P \pm \widetilde{B}_P$ if and only if $R(\widetilde{A}_P) \geq R(\widetilde{B}_P)$

(ii) $\widetilde{A}_P \circ \widetilde{B}_P$ if and only if $R(\widetilde{A}_P) \leq R(\widetilde{B}_P)$

(iii) $\widetilde{A}_P \cong \widetilde{B}_P$ if and only if $R(\widetilde{A}_P) = R(\widetilde{B}_P)$

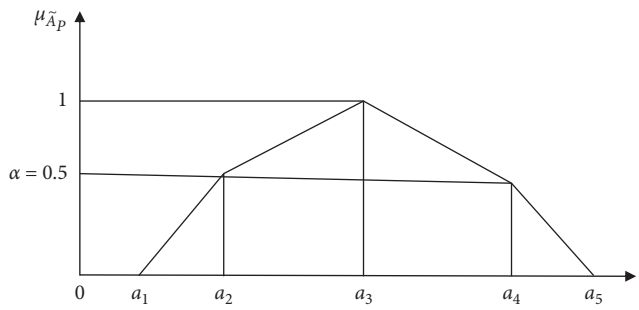

Figure 1: Graphical representation of pentagonal fuzzy number [4].

3. Assumptions, Notations, and Rental Policy

In this flow-shop scheduling problem, the following assumptions are made.

3.1. Assumptions

(i) No job pre-emption is allowed

(ii) Just one function can be performed by the machine

(iii) All jobs are available at the beginning of schedule time

(iv) The initialization times of the machines are negligible

(v) All workers are processed throughout the deterministic phase

(vii) Due dates are pentagonal fuzzy numbers

(viii) The machine may be idle

(viii) The production period is separate from the schedule

(ix) The first machine must be completed to provide a second device with feeding

(x) For each mission, M operations are needed

(xi) Any job must be done as it has begun

3.2. Notations. The following notations are allowed in the flow-shop scheduling problem.

S_k: Sequence resulted by applying Johnson's procedure, $k(k = 1, 2, \ldots, m)$

M_j: Machine j $(j = 1, 2, \ldots, m)$

$\tilde{M}_P$: Machine makes span

$(\tilde{A}_P)_{ij}$: Pentagonal fuzzy processing time of ith job on machine M_j, $i = 1, 2, \ldots, n$ and $j = (1, 2, \ldots, m)$

$(\tilde{T}_P)_{ij}(S_k)$: Completion time of ith job of sequence S_k on machine M_j $(j = 1, 2, \ldots, m)$

$(\tilde{I}_P)_{ij}(S_k)$: Idle time of machine M_j $(j = 1, 2, \ldots, m)$, for job i $(i = 1, 2, \ldots, n)$ in the sequence S_k

$(\tilde{U}_P)_j(S_k)$: Utilization time of which machine M_j $(j = 1, 2, \ldots, m)$ is required

$\tilde{R}_P(S_k)$: Total rental cost for the sequence S_k for all machines M_j $(j = 1, 2, \ldots, m)$

$(\tilde{C}_P)_i$: Rental cost of ith machine

$\tilde{C}_P(S_i)$: Total completion time of the jobs for sequence i $(i = 1, 2, \ldots, n)$

3.3. Rental Policy. The first machine will be taken on rent in the beginning of the processing of the jobs; the second machine will be taken on rent when the first job is completed on the first machine and transported to the second machine. The third machine will be taken on rent when the first job is completed on the second machine and transported to the third machine and so on.

4. Problem Formulation

Assume that job $i(i = 1, 2, \ldots, n)$ is to be processed on machine $j(j = 1, 2, \ldots, m)$ in the existence of specified rental policy. Let $(\tilde{A}_P)_{ij}(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ be the processing time of ith job on jth machine characterized by pentagonal fuzzy numbers. The problem can be formulated as

$$\min \tilde{R}_P(S_k) = \sum_{i=1}^{n} \left[\left(\tilde{A}_P\right)_{i1} \times \left(\tilde{C}_P\right)_1 + (\tilde{U}_P)_2(S_k) \times \left(\tilde{C}_P\right)_2 + (\tilde{U}_P)_3(S_k) \times \left(\tilde{C}_P\right)_3 \right], \tag{3}$$

which subjects to rental policy P.

5. Solution Method

In this section, the method to minimize the utilization time and then the rental cost of flow-shop scheduling problem under fuzzy environment is provided in the following steps.

Step 1 : convert the problem into a problem of two machines if one of the following conditions is reached:

(i) $\min_i (\tilde{A}_P)_{i1} \pm \max_i (\tilde{A}_P)_{ij}, j = 2, 3, \ldots, m-1$

(ii) $\min_i (\tilde{A}_P)_{im} \pm \max_i (\tilde{A}_P)_{ij}, j = 2, 3, \ldots, m-1$

Step 2 : convert this problem to a problem of two machines and add two fictional H 1 and H 2 machines, $(\tilde{H}_P)_1^i = \sum_{j=1}^{m-1} (\tilde{A}_P)_{ij}, i = 1, 2, \ldots, n$, and $(\tilde{H}_P)_2^i = \sum_{j=2}^{m} (\tilde{A}_P)_{ij}$, $i = 1, 2, \ldots, n$, where $(\tilde{H}_P)_1^i$ and

Table 1: Pentagonal fuzzy processing times $\widetilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$.

Job	Machine M_1	Machine M_2	Machine M_3
11	(2, 6, 8, 10, 14)	(2, 5, 7, 8, 9)	(1, 3, 4, 5, 6)
22	(11, 12, 13, 14, 16)	(4, 5, 6, 7, 8)	(3, 4, 5, 6 8)
33	(6, 8, 10, 12, 13)	(3, 4, 5, 6 8)	(2, 5, 7, 8, 9)
4	(8, 9, 11, 12, 13)	(4, 5, 6, 7, 8)	(9, 11, 12, 13, 15)
5	(7, 9, 10, 11, 13)	(4, 5, 6, 8, 10)	(6, 8, 9, 11, 12)

Table 2: Pentagonal fuzzy processing times for fictions machines H_1 and H_2.

Job	H_1	H_2
1	(4, 11, 15, 18, 23)	(3, 8, 11, 13, 15)
2	(15, 17, 19, 21, 24)	(7, 9, 11, 13, 16)
3	(9, 12, 15, 18, 21)	(5, 9, 12, 14, 17)
4	(12, 14, 17, 19, 21)	(13, 16, 18, 20, 23)
5	(11, 14, 16, 19, 23)	(10, 13, 15, 19, 22)

Table 3: Time in and time out for machines.

Job	Machine 1		Machine 2		Machine 3	
Time in	Time out	Time in	Time out	Time in	Time out	
1	(0, 0, 0, 0, 0)	(8, 9, 11, 12, 13)	(8, 9, 11, 12, 13)	(12, 14, 17, 19, 21)	(12, 14, 17, 19, 21)	(21, 25, 29, 32, 37)
2	(8, 9, 11, 12, 13)	(15, 18, 21, 23, 26)	(15, 18, 21, 23, 26)	(19, 23, 27, 30, 34)	(21, 25, 29, 32, 37)	(27, 33, 38, 43, 49)
3	(15, 18, 21, 23, 26)	(24, 30, 36, 41, 47)	(24, 30, 36, 41, 47)	(25, 30, 35, 39, 44)	(27, 33, 38, 43, 49)	(29, 38, 45, 51, 58)
4	(24, 30, 36, 41, 47)	(39, 47, 55, 62, 71)	(39, 47, 55, 62, 71)	(36, 44, 51, 58, 62)	(36, 44, 51, 58, 62)	(39, 48, 56, 64, 70)
5	(39, 47, 55, 62, 71)	(41, 53, 63, 72, 85)	(41, 53, 63, 72, 85)	(40, 55, 66, 76, 85)	(40, 55, 66, 76, 85)	(41, 58, 70, 81, 91)

$(\widetilde{H}_P)_2^i$ are the fuzzy pentagonal processing time for job i on machines H_1 and H_2, respectively.

Step 3 : apply the algorithm introduced by [27] on machines H_1 and H_2, to obtain the sequence S_k.

6. Numerical Example

Consider 5 jobs and 3 machines' flow-shop scheduling problem having fuzzy processing times represented by pentagonal fuzzy numbers as in Table 1. The rental cost per unit time for machines M_1, M_2, and M_3 are 5, 3, and 4 units, respectively. Our aim is to minimize the total rental cost [28].

Table 1 illustrates the three machines scheduling problem with processing times characterized by pentagonal fuzzy numbers.

(1) For M_1: processing time min is $(2, 6, 8, 10, 14)$

(2) For M_2: processing time max is $(2, 5, 7, 8, 9)$

(3) For M_3: processing time min is $(1, 3, 4, 5, 6)$

Here, all the decision parameters are represented by pentagonal fuzzy numbers.

It is clear that Min time of M_1 $(2, 6, 8, 10, 14) \geqslant$ Max time of M_2 $(2, 5, 7, 8, 9)$. Let H_1 and H_2 be two fictitious machine such that

$$(\widetilde{H}_P)_1^i = \sum_{j=1}^{2} (\widetilde{A}_P)_{ij},$$
$$(\widetilde{H}_P)_2^i = \sum_{j=2}^{3} (\widetilde{A}_P)_{ij}. \tag{4}$$

Then, Table 2 shows the fictitious machine H_1 and H_2.

Based on the algorithm introduced by [27], the order of sequencing is $S_k = 4 \longrightarrow 5 \longrightarrow 3 \longrightarrow 2 \longrightarrow 1$.

Hence, Table 3 provides the time in and out of the machines.

In addition, the times idle and rental costs are calculated in Table 4. The total elapsed time = $(41, 58, 70, 81, 91)$ unit of time.

Thus, the total rental cost is $(205, 265, 315, 360, 425) \oplus (-194, -96, 63, 186, 381) \oplus (-152, 40, 216, 260, 572) = (-141, 209, 594, 806, 378)$ units of cost.

7. Comparative Study

From Table 5, it is clear that the objective function value resulted from the Sathish and Ganesan algorithm is the same with the proposed approach, and the overall rental costs for the machines resulting from this strategy are clearly lower than those found by Gupta et al. [18].

Table 4: Idle times and rental costs on machines.

Machine	Idle time	Rental cost
1	$(-44, -14, 7, 28, 50)$ units of time	$5 \times (41, 53, 63, 72, 85) = (205, 265, 315, 360, 425)$units of cost
2	$(-6, -1, 4, 9, 14) \oplus (-10, 0, 9, 18, 28) \oplus (-5, 8, 20, 32, 46) \oplus (-21, -5, 12, 28, 49) = (-42, 14, 45, 87, 137)$ units of time	$3 \times (-97, -32, 21, 62, 127) = (-194, -96, 63, 186, 381)$units of cost
3	$(-22, -7, 6, 20, 33) \oplus (-30, -9, 10, 28, 46) = (-52, -16, 16, 48, 79)$ units of time	$4 \times (-38, 10, 54, 65, 143) = (-152, 40, 216, 260, 572)$units of cost

Table 5: Comparison of the example results.

Proposed approach	Algorithm by Sathish and Ganesan [28]	Method by Gupta et al. [18]
Total rental cost equal to $(-141, 209, 452, 806, 1378)$ units of cost	Total rental cost equal to $(452, 8, 8)$ units of cost	Total rental cost equal to $(377, 613, 662)$ units of cost

8. Concluding Remarks

In this paper, we provided a clear approach for the preparation of the flow-shop problem under fuzzy environment in which processing time of jobs are represented by pentagonal fuzzy numbers. The fuzzy FS scheduling problem was solved without converting the processing time into its equivalent crisp numbers using a robust ranking technique and a fuzzy arithmetic pentagonal fuzzy numbers. The strategy aims to reduce the rental costs of the machine in accordance with the leasing system. The advantage of this approach is more flexible where it allows the decision maker to choose the targets he/she is willing and useful for potential study in widespread fluctuations.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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