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Research Article

Remarks on (α, β)-Admissible Mappings and Fixed Points under $\mathscr{Z}$-Contraction Mappings

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In this paper, we discuss about different types of (α, β)-admissible mappings and introduce some new (α, β)-contraction-type mappings under simulation function. Furthermore, we present the definition of S-metric-like space and its topological properties. Some fixed point theorems in this space are established, proved, and verified with examples.

1. Introduction and Preliminaries

Banach contraction principle is considered as one of the most important tools for examining the existence and nonexistence of fixed point. Due to its simplicity and applicability, it is generalised in different directions.

Samet et al. [1] introduced the concept of α-admissible and $\alpha - \psi$ contraction to generalise the Banach contraction principle. These concepts were further generalised by many researchers to $\alpha - \beta$ admissible [2], β-admissible [3, 4], α-admissible in S-metric space [5], (α, β)-admissible [6, 7], and γ-admissible [8]. Some results on different types on contractive mappings can be seen in [9–11].

Different from the (α, β)-admissible mapping introduced by Alizadeh et al. [6], Chandok [7] introduced a new type of (α, β)-admissible mapping to obtain some fixed point results.

On the contrary, Khojasteh et al. [12] introduced simulation function and proved several fixed point theorems. Argoubi et al. [13] made some modifications to the definition of Khojasteh et al. [12]. It can be noted that the definition of Khojasteh et al. [12] implies the definition of Argoubi et al. [13], but the converse is not true.

Various generalizations of metric space are found in the literature. The concept of S-metric space [14] is one of them. More results on S-metric space can be found in [15, 16]. Hitzler [17] also introduced the concept of dislocated metric by generalizing metric space. Amini-Harandi [18] rediscovered dislocated metric space under the new name "metric-like."

Mehravaran et al. [19] introduced the concept of dislocated S_b-metric space and dislocated S-metric space as a particular case of dislocated S_b-metric space when the parameter $b = 1$, but the topological properties of neither dislocated S_b-metric space nor dislocated S-metric space were given in [19].

In order to fill up the missing gap in [19], in our present study, first of all, we present the definition of dislocated S-metric space and its topological properties. For our convenience, it will be known as S-metric-like space in place of dislocated S-metric space.

Definition 1 (see [14]). In a nonempty set Ω, the function $S\colon \Omega^3 \longrightarrow [0, +\infty)$ is said to be S-metric if it satisfies the following:

(1) $S(\theta, \mu, \xi) \geq 0$,
(2) $S(\theta, \mu, \xi) = 0$ if and only if $\theta = \mu = \xi$,
(3) $S(\theta, \mu, \xi) \leq S(\theta, \sigma, \sigma) + S(\mu, \sigma, \sigma) + S(\xi, \sigma, \sigma)$,

for all $\theta, \mu, \xi, \sigma \in \Omega$. (Ω, S) is known as S-metric space.

Definition 2 (see [14]). In a S-metric space, we have $S(\theta,\theta,\mu) = S(\mu,\mu,\theta)$.

Definition 3 (see [14]). In a nonempty set Ω, the function $S\colon \Omega^3 \longrightarrow [0,+\infty)$ is said to be S-metric-like if it satisfies the following:

(1) $S(\theta,\mu,\xi) = 0$ implies $\theta = \mu = \xi$,

(2) $S(\theta,\mu,\xi) \le S(\theta,\sigma,\sigma) + S(\mu,\sigma,\sigma) + S(\xi,\sigma,\sigma)$,

for all $\theta,\mu,\xi,\sigma \in \Omega$. (Ω,S) is known as S-metric-like space.

Remark 1. It is true that every S-metric space is a S-metric-like space, but every S-metric-like space may not be a S-metric space.

Example 1. Let $\Omega = \{0,2\}$ and

$$S(\theta,\mu,\xi) = \begin{cases} 3, & \theta = \mu = \xi, \\ 1, & \text{otherwise.} \end{cases} \tag{1}$$

Here, (Ω,S) is S-metric-like space. But $S(0,0,0) = 3 \ne 0$ and hence not S-metric space.

We discuss some topological properties of S-metric-like space. Let τ_s be a topology generated on S-metric-like space (Ω,S) with base as the family of open S-balls:

$$B_S(\theta,\varepsilon) = \{\mu \in \Omega\colon\ |S(\theta,\theta,\mu) - S(\theta,\theta,\theta)| < \varepsilon\},\quad \text{for all } \theta \in \Omega \text{ and } \varepsilon > 0. \tag{2}$$

Mapping $R\colon \Omega \longrightarrow \Omega$ in a S-metric-like (Ω,S) is said to be continuous at $\theta \in \Omega$ if there exists $\delta > 0$ for all $\varepsilon > 0$ satisfying $R(B_S(\theta,\delta)) \subseteq B_S(R\theta,\varepsilon)$.

If $R\colon \Omega \longrightarrow \Omega$ is continuous, then $\lim_{n\longrightarrow+\infty}\theta_n = \theta$ implies $\lim_{n\longrightarrow+\infty}R\theta_n = R\theta$.

A sequence $\{\theta_n\}$ in Ω is said to be Cauchy if $\lim_{n,m\longrightarrow+\infty}S(\theta_n,\theta_n,\theta_m)$ exists and is finite.

The S-metric-like (Ω, S) is said to be complete if every Cauchy sequence $\{\theta_n\}$ in Ω satisfies

$$\lim_{n\longrightarrow+\infty} S(\theta_n,\theta_n,\theta) = S(\theta,\theta,\theta) = \lim_{n,m\longrightarrow+\infty} S(\theta_n,\theta_n,\theta_m), \tag{3}$$

for some $\theta \in \Omega$.

A subset $P \subseteq \Omega$ is said to be bounded if there is a point $\xi \in \Omega$ satisfying

$$S(\theta,\mu,\xi) \le L, \tag{4}$$

for all $\theta,\mu \in P$ and L is a positive constant.

Definition 4 (see [12]). A function $\zeta\colon [0,+\infty)\times[0,+\infty)\longrightarrow\mathbb{R}$ is said to be a simulation function if ζ satisfies the following:

(1) $\zeta(0,0) = 0$.

(2) $\zeta(\upsilon,\mu) < \mu - \upsilon$ for all $\upsilon,\mu > 0$.

(3) If $\{\upsilon_n\}$ and $\{\mu_n\}$ are sequences in $(0,+\infty)$ such that $\lim_{n\longrightarrow+\infty}\upsilon_n = \lim_{n\longrightarrow+\infty}\mu_n = l \in (0,+\infty)$, then

$$\lim_{n\longrightarrow+\infty} \sup \zeta(\upsilon_n,\mu_n) < 0. \tag{5}$$

Simulation function is modified by Argoubi et al. [13] as follows.

Definition 5 (see [13]). A mapping $\zeta\colon [0,+\infty)\times[0,+\infty) \longrightarrow \mathbb{R}$ is said to be a simulation function if satisfying the conditions (2) and (3) of Definition 4.

Now, we present different forms of (α,β)-admissible mappings.

Alizadeh et al. [6] defined a type of (α,β)-admissible mappings by extending definition given by Salimi et al. [20] which they called as cyclic (α,β)-admissible mapping.

Definition 6 (see [6]). Let $R\colon \Omega \longrightarrow \Omega$ be a mapping and $\alpha,\beta\colon \Omega \longrightarrow \mathbb{R}^+$ be two functions. R is said to be a cyclic (α,β)-admissible mapping if the following holds:

(i) For some $\theta \in \Omega$, $\alpha(\theta) \ge 1$ induces $\beta(R\theta) \ge 1$.

(ii) For some $\theta \in \Omega$, $\beta(\theta) \ge 1$ induces $\alpha(R\theta) \ge 1$.

The following definition was given by Chandok [7].

Definition 7 (see [7]). Let $R\colon \Omega \longrightarrow \Omega$ be a mapping in a nonempty set Ω and $\alpha,\beta\colon \Omega\times\Omega \longrightarrow \mathbb{R}^+$. R is said to be (α,β)-admissible if $\alpha(\theta,\mu) \ge 1$ and $\beta(\theta,\mu) \ge 1$ implies $\alpha(R\theta,R\mu) \ge 1$ and $\beta(R\theta,R\mu) \ge 1$ for all $\theta,\mu \in \Omega$.

Shatanawi [2] also introduced a type of (α,β)-admissible mapping which he named as (α,β)-admissibility and defined as follows.

Definition 8 (see [2]). Let $S,T\colon \Omega \longrightarrow \Omega$ be mappings in a nonempty set Ω and $\alpha,\beta\colon \Omega\times\Omega \longrightarrow \mathbb{R}^+ \cup \{0\}$ be functions. Then, (R,S) is said to be a pair of (α,β)-admissibility if $\theta,\mu \in \Omega$ and $\alpha(\theta,\mu) \ge \beta(\theta,\mu)$ implies $\alpha(R\theta,S\mu) \ge \beta(R\theta,S\mu)$ and $\alpha(S\theta,R\mu) \ge \beta(S\theta,R\mu)$.

For our study, we will consider the definition given by Chandok [7]. This definition will be extended in the framework of S-metric-like space to define some new contractive types under simulation function defined by Khojasteh et al. [12].

Following is the extension of the property of metric-like space to S-metric-like space.

2. Main Results

We start our result with the following definitions.

Definition 9. Let R be a self-mapping on a nonempty set Ω and $\alpha,\beta\colon \Omega\times\Omega\times\Omega \longrightarrow \mathbb{R}^+$. Then, R is said to be $(\alpha,\beta)_s$-admissible if $\alpha(\theta,\mu,\xi) \ge 1$ and $\beta(\theta,\mu,\xi) \ge 1$ imply that $\alpha(R\theta,R\mu,R\xi) \ge 1$ and $\beta(R\theta,R\mu,R\xi) \ge 1$ for all $\theta,\mu,\xi \in \Omega$.

Definition 10. Let R be a self-mapping on a nonempty set Ω and $\alpha,\beta\colon \Omega\times\Omega\times\Omega \longrightarrow \mathbb{R}^+$. Then, R is said to be triangular $(\alpha,\beta)_s$-admissible mapping if it is $(\alpha,\beta)_s$-admissible and $\alpha(\theta,\theta,t) \ge 1$, $\beta(\theta,\theta,t) \ge 1$, $\alpha(\mu,\mu,t) \ge 1$, $\beta(\mu,\mu,t) \ge 1$, and $\alpha(\xi,\xi,t) \ge 1$, $\beta(\xi,\xi,t) \ge 1$ imply $\alpha(\theta,\mu,\xi) \ge 1$, $\beta(\theta,\mu,\xi) \ge 1$.

Definition 11. Let R be a self-mapping on a S-metric-like space (Ω,S) and $\alpha,\beta\colon \Omega\times\Omega\times\Omega \longrightarrow \mathbb{R}^+$. R is said to be an

$(\alpha,\beta)_s$-admissible $\mathscr{Z}$-contraction of type I with respect to ζ if

$$\zeta(\alpha(\theta,\mu,\xi)\beta(\theta,\mu,\xi)S(R\theta,R\mu,R\xi),S(\theta,\mu,\xi))\geq 0, \tag{6}$$

for all $\theta,\mu,\xi\in\Omega$.

Definition 12. Let R be a self-mapping on a S-metric-like space (Ω,S) and $\alpha,\beta\colon \Omega\times\Omega\times\Omega\longrightarrow\mathbb{R}^+$. R is said to be an $(\alpha,\beta)_s$-admissible $\mathscr{Z}$-contraction of type II with respect to ζ if

$$\zeta(\alpha(\theta,\theta,\mu)\beta(\theta,\theta,\mu)S(R\theta,R\theta,R\mu),S(\theta,\theta,\mu))\geq 0, \tag{7}$$

for all $\theta,\mu\in\Omega$.

Lemma 1. *Let (Ω,S) be a S-metric-like space and $\{\theta_n\}$ be a sequence in Ω such that $\theta_n\longrightarrow\theta$ with $S(\theta,\theta,\theta)=0$. Then, for all $\mu\in\Omega$, we have $lim_{n\longrightarrow+\infty}S(\theta_n,\theta_n,\mu)=S(\theta,\theta,\mu)$.*

Proof. From (2) of Definition 3, we have

$$\left|S(\theta_n,\theta_n,\mu)-S(\theta,\theta,\mu)\right|\leq 2S(\theta_n,\theta_n,\theta). \tag{8}$$

Taking limit as $n\longrightarrow+\infty$, we have

$$\lim_{n\longrightarrow+\infty}S(\theta_n,\theta_n,\mu)=S(\theta,\theta,\mu). \tag{9}$$

Now, we prove the following theorem. □

Theorem 1. *Let $R\colon\Omega\longrightarrow\Omega$ be a mapping on a complete S-metric-like space (Ω,S) satisfying the following:*

(a) R is triangular $(\alpha,\beta)_s$-admissible.

(b) There exists $\theta_0\in\Omega$ such that $\alpha(\theta_0,\theta_0,R\theta_0)\geq 1$ and $\beta(\theta_0,\theta_0,R\theta_0)\geq 1$.

(c) R is $(\alpha,\beta)_s$-admissible $\mathscr{Z}$-contraction of type I on (Ω,S).

(d) R is S-continuous.

Ŝen, there is a unique fixed point $\xi\in\Omega$ of R with $S(\xi,\xi,\xi)=0$.

Proof. By (b), let $\theta_0\in\Omega$ with $\alpha(\theta_0,\theta_0,R\theta_0)\geq 1$ and $\beta(\theta_0,\theta_0,R\theta_0)\geq 1$. We construct the sequence $\{\theta_n\}$ by $\theta_{n+1}=R\theta_n$ for all $n\in\mathbb{N}\cup\{0\}$. For some n, if $\theta_n=\theta_{n+1}$, then we have $\theta_n=R\theta_n$. This gives that θ_n is a fixed point of R. In this case, proof is completed. Now, let $\theta_n\neq\theta_{n+1}$ for all $n\in\mathbb{N}\cup\{0\}$. By (a), we have

$$\begin{aligned}\alpha(\theta_0,\theta_0,R\theta_0)&=\alpha(\theta_0,\theta_0,\theta_1)\geq 1\Rightarrow\alpha(R\theta_0,R\theta_0,R\theta_1)\\&=\alpha(\theta_1,\theta_1,\theta_2)\geq 1.\end{aligned} \tag{10}$$

Deductively, we have

$$\alpha(\theta_n,\theta_n,\theta_{n+1})\geq 1,\quad \text{for all } n\geq 0, \tag{11}$$

$$\beta(\theta_n,\theta_n,\theta_{n+1})\geq 1,\quad \text{for all } n\geq 0. \tag{12}$$

By (6), (11), and (12), we have

$$\begin{aligned}0&\leq\zeta(\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(R\theta_n,R\theta_n,R\theta_{n+1}),S(\theta_n,\theta_n,\theta_{n+1}))\\&=\zeta(\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(\theta_{n+1},\theta_{n+1},\theta_{n+2}),S(\theta_n,\theta_n,\theta_{n+1}))\\&<S(\theta_n,\theta_n,\theta_{n+1})-\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(\theta_{n+1},\theta_{n+1},\theta_{n+2}).\end{aligned} \tag{13}$$

Hence,

$$\begin{aligned}S(\theta_{n+1},\theta_{n+1},\theta_{n+2})&\leq\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(\theta_{n+1},\theta_{n+1},\theta_{n+2})\\&<S(\theta_n,\theta_n,\theta_{n+1}),\end{aligned} \tag{14}$$

for all $n\geq 0$.

This shows that $\{S(\theta_n,\theta_n,\theta_{n+1})\}$ is a decreasing sequence, then we have

$$\lim_{n\longrightarrow+\infty}S(\theta_n,\theta_n,\theta_{n+1})=r,\quad r\geq 0. \tag{15}$$

We prove that

$$\lim_{n\longrightarrow+\infty}S(\theta_n,\theta_n,\theta_{n+1})=0. \tag{16}$$

Let $r>0$. From (14), we have

$$\lim_{n\longrightarrow+\infty}\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(\theta_{n+1},\theta_{n+1},\theta_{n+2})=r. \tag{17}$$

Taking $\upsilon_n=\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(\theta_{n+1},\theta_{n+1},\theta_{n+2})$ and $\mu_n=S(\theta_n,\theta_n,\theta_{n+1})$ and using (3) of Definition 4, we have

$$\begin{aligned}0\leq&\lim_{n\longrightarrow+\infty}\sup\zeta(\alpha(\theta_n,\theta_n,\theta_{n+1})\beta(\theta_n,\theta_n,\theta_{n+1})S(\theta_{n+1},\theta_{n+1},\theta_{n+2}),\\&S(\theta_n,\theta_n,\theta_{n+1}))<0,\end{aligned} \tag{18}$$

a contradiction. Hence, $r=0$.

Next, we have to prove that $\{\theta_n\}$ is Cauchy. If possible, let $\{\theta_n\}$ is not Cauchy. Then, there exists $\varepsilon>0$ for which $\{\theta_n\}$ has subsequences $\{\theta_{n_k}\}$ and $\{\theta_{m_k}\}$ with $n_k>m_k>k$ such that for every k,

$$\begin{aligned}S\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)&\geq\varepsilon,\\S\left(\theta_{n_k-1},\theta_{n_k-1},\theta_{m_k}\right)&<\varepsilon.\end{aligned} \tag{19}$$

We have

$$\begin{aligned}\varepsilon\leq S\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)&\leq 2S\left(\theta_{n_k},\theta_{n_k},\theta_{n_k-1}\right)+S\left(\theta_{n_k-1},\theta_{n_k-1},\theta_{m_k}\right)\\&<2S\left(\theta_{n_k},\theta_{n_k},\theta_{n_k-1}\right)+\varepsilon.\end{aligned} \tag{20}$$

Taking $k\longrightarrow+\infty$ and using (16),

$$\lim_{n\longrightarrow+\infty}S\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)=\varepsilon. \tag{21}$$

Also,

$$\left|S\left(\theta_{n_k+1},\theta_{n_k+1},\theta_{m_k}\right)-S\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)\right|\leq 2S\left(\theta_{n_k},\theta_{n_k},\theta_{n_k+1}\right). \tag{22}$$

Taking limit as $k\longrightarrow+\infty$ and by (16) and (21),

$$\lim_{k\longrightarrow+\infty}S\left(\theta_{n_k+1},\theta_{n_k+1},\theta_{m_k}\right)=\varepsilon. \tag{23}$$

Similarly,

$$\lim_{k\longrightarrow+\infty} S\left(\theta_{m_k+1},\theta_{m_k+1},\theta_{n_k}\right)=\varepsilon,\\ \lim_{k\longrightarrow+\infty} S\left(\theta_{n_k+1},\theta_{n_k+1},\theta_{m_k+1}\right)=\varepsilon. \tag{24}$$

By triangular $(\alpha,\beta)_s$-admissibility of R,

$$\alpha\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)\geq 1 \text{ and } \beta\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)\geq 1. \tag{25}$$

Since R is $(\alpha,\beta)_s$-admissible $\mathcal{Z}$-contraction of type I and using (21), (25), and (3) of Definition 4, we have

$$0\leq \lim_{k\longrightarrow+\infty}\sup\zeta\left(\alpha\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)\beta\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)S\left(\theta_{n_k+1},\theta_{n_k+1},\theta_{m_k+1}\right),S\left(\theta_{n_k},\theta_{n_k},\theta_{m_k}\right)\right)<0, \tag{26}$$

is a contradiction, and hence, $\{\theta_n\}$ is a Cauchy sequence. By completeness of S-metric-like space (Ω, S), we know that there exist some $\xi\in\Omega$ such that

$$\lim_{n\longrightarrow+\infty} S(\theta_n,\theta_n,\xi)=S(\xi,\xi,\xi)=\lim_{n,m\longrightarrow+\infty} S(\theta_n,\theta_n,\theta_m)=0, \tag{27}$$

and thus, $S(\xi,\xi,\xi)=0$. Since R is S-continuous,

$$\lim_{n\longrightarrow+\infty} S(\theta_{n+1},\theta_{n+1},R\xi)=\lim_{n\longrightarrow+\infty} S(R\theta_n,R\theta_n,R\xi)\\ =S(R\xi,R\xi,R\xi)=0. \tag{28}$$

Using Lemma 1 and (27), we have

$$\lim_{n\longrightarrow+\infty} S(\theta_{n+1},\theta_{n+1},R\xi)=S(\xi,\xi,R\xi). \tag{29}$$

Thus, $S(R\xi,R\xi,R\xi)=S(\xi,\xi,R\xi)$, that is, $R\xi=\xi$. For proving uniqueness of the fixed point, let $\eta\in\Omega$ such that $R\eta=\eta$ and $\eta\neq\xi$. Then,

$$0\leq\zeta(\alpha(\xi,\xi,\eta)\beta(\xi,\xi,\eta)S(R\xi,R\xi,R\eta),S(\xi,\xi,\eta))\\ <S(\xi,\xi,\eta)-\alpha(\xi,\xi,\eta)\beta(\xi,\xi,\eta)S(R\xi,R\xi,\eta)\leq 0, \tag{30}$$

is a contradiction, so $\xi=\eta$.

Next, we remove the continuity condition. □

Theorem 2. *Let R: $\Omega\longrightarrow\Omega$ be a mapping on a complete S-metric-like space (Ω,S) satisfying the following:*

(a) R is triangular $(\alpha,\beta)_s$-admissible.

(b) There exists $\theta_0\in\Omega$ such that $\alpha(\theta_0,\theta_0,R\theta_0)\geq 1$ and $\beta(\theta_0,\theta_0,R\theta_0)\geq 1$.

(c) R is $(\alpha,\beta)_s$-admissible $\mathcal{Z}$-contraction of type I on (Ω,S).

(d) If $\{\theta_n\}$ is a sequence in Ω such that $\alpha(\theta_n,\theta_n,\theta_{n+1})\geq 1$ and $\beta(\theta_n,\theta_n,\theta_{n+1})\geq 1$ for all n and $\theta_n\longrightarrow\xi\in\Omega$ as $n\longrightarrow+\infty$, then there exists a subsequence $\{\theta_{n_k}\}$ of $\{\theta_n\}$ such that $\alpha(\theta_{n_k},\theta_{n_k},\xi)\geq 1$ and $\beta(\theta_{n_k},\theta_{n_k},\xi)\geq 1$ for all $k\in\mathbb{N}$.

Șen, there is a unique fixed point $\xi\in\Omega$ of R with $S(\xi,\xi,\xi)=0$.

Proof. Proceeding as of Theorem 1, let $\{\theta_n\}$ be a sequence in Ω given by $\theta_{n+1}=R\theta_n$ converges to some $\xi\in\Omega$ with $\alpha(\theta_n,\theta_n,\theta_{n+1})\geq 1$ and $\beta(\theta_n,\theta_n,\theta_{n+1})\geq 1$ for all n and $S(\xi,\xi,\xi)=0$. From (d), there exists a subsequence $\{\theta_{n_k}\}$ of $\{\theta_n\}$ such that $\alpha(\theta_{n_k},\theta_{n_k},\xi)\geq 1$ and $\beta(\theta_{n_k},\theta_{n_k},\xi)\geq 1$ for all $k\in\mathbb{N}$. Thus, applying (6) for all k, we have

$$0\leq\zeta\left(\alpha\left(\theta_{n_k},\theta_{n_k},\xi\right)\beta\left(\theta_{n_k},\theta_{n_k},\xi\right)S\left(R\theta_{n_k},R\theta_{n_k},R\xi\right),S\left(\theta_{n_k},\theta_{n_k},\xi\right)\right)\\ =\zeta\left(\alpha\left(\theta_{n_k},\theta_{n_k},\xi\right)\beta\left(\theta_{n_k},\theta_{n_k},\xi\right)S\left(\theta_{n_k+1},\theta_{n_k+1},R\xi\right),S\left(\theta_{n_k},\theta_{n_k},\xi\right)\right)\\ <S\left(\theta_{n_k},\theta_{n_k},\xi\right)-\alpha\left(\theta_{n_k},\theta_{n_k},\xi\right)\beta\left(\theta_{n_k},\theta_{n_k},\xi\right)S\left(\theta_{n_k+1},\theta_{n_k+1},R\xi\right), \tag{31}$$

imply

$$S\left(\theta_{n_k+1},\theta_{n_k+1},R\xi\right)\leq\alpha\left(\theta_{n_k},\theta_{n_k},\xi\right)\beta\left(\theta_{n_k},\theta_{n_k},\xi\right)S\left(\theta_{n_k+1},\theta_{n_k+1},R\xi\right)\\ <S\left(\theta_{n_k},\theta_{n_k},\xi\right). \tag{32}$$

Taking $k\longrightarrow+\infty$, we have $S(\xi,\xi,R\xi)=0$, that is, $R\xi=\xi$. Proceeding as in Theorem 16, the uniqueness of fixed point of R can be proved. □ □

Definition 13. Let R: $\Omega\longrightarrow\Omega$ be a mapping on a S-metric-like space (Ω,S) and α,β: $\Omega\times\Omega\times\Omega\longrightarrow\mathbb{R}^+$. R is said to be a generalised $(\alpha,\beta)_s$-admissible $\mathcal{Z}$-contraction of type I with respect to ζ if

$$\zeta(\alpha(\theta,\mu,\xi)\beta(\theta,\mu,\xi)M(\theta,\mu,\xi),S(\theta,\mu,\xi))\geq 0, \tag{33}$$

where

$$M(\theta,\mu,\xi)=\max\{S(\theta,\mu,\xi),S(\theta,\theta,R\theta),S(\mu,\mu,R\mu),S(\xi,\xi,R\xi)\}, \tag{34}$$

for all $\theta,\mu,\xi\in\Omega$.

Definition 14. Let R: $\Omega\longrightarrow\Omega$ be a mapping on a S-metric-like space (Ω,S) and α,β: $\Omega\times\Omega\times\Omega\longrightarrow\mathbb{R}^+$. R is said to be a generalised $(\alpha,\beta)_s$-admissible $\mathcal{Z}$-contraction of type II with respect to ζ if

$$\zeta(\alpha(\theta,\theta,\mu)\beta(\theta,\theta,\mu)M(\theta,\theta,\mu),S(\theta,\theta,\mu))\geq 0, \tag{35}$$

where

$$M(\theta,\theta,\mu)=\{S(\theta,\theta,\mu),S(\theta,\theta,R\theta),S(\mu,\mu,R\mu)\}, \tag{36}$$

for all $\theta,\mu\in\Omega$.

Theorem 3. *Let R: $\Omega\longrightarrow\Omega$ be a mapping on a complete S-metric-like space (Ω,S) satisfying the following:*

(a) R is triangular $(\alpha,\beta)_s$-admissible.

(b) *There exists* $\theta_0 \in \Omega$ *such that* $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ *and* $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) *R is a generalised* $(\alpha, \beta)_s$*-admissible* $\mathscr{Z}$*-contraction of type I on* (Ω, S).

(d) *R is S-continuous.*

Then, there is a unique fixed point $\xi \in \Omega$ of R with $S(\xi, \xi, \xi) = 0$.

Proof. By (b), let $\theta_0 \in \Omega$ with $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ and $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$. We construct the sequence $\{\theta_n\}$ by $\theta_{n+1} = R\theta_n$ for all $n \in \mathbb{N} \cup \{0\}$. For some n, if $\theta_n = \theta_{n+1}$, then we have $\theta_n = R\theta_n$. This gives that θ_n is a fixed point of R. In this case, proof is completed. Now, let $\theta_n \neq \theta_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. By (a), we have

$$\begin{aligned}\alpha(\theta_0, \theta_0, R\theta_0) &= \alpha(\theta_0, \theta_0, \theta_1) \geq 1 \Rightarrow \alpha(R\theta_0, R\theta_0, R\theta_1) \\ &= \alpha(\theta_1, \theta_1, \theta_2) \geq 1.\end{aligned} \tag{37}$$

Deductively, we have

$$\alpha(\theta_n, \theta_n, \theta_{n+1}) \geq 1, \quad \text{for all } n \geq 0, \tag{38}$$

$$\beta(\theta_n, \theta_n, \theta_{n+1}) \geq 1, \quad \text{for all } n \geq 0. \tag{39}$$

By (33), (38), and (39),

$$0 \leq \zeta(\alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})M(\theta_n, \theta_n, \theta_{n+1}), S(\theta_n, \theta_n, \theta_{n+1})), \tag{40}$$

where

$$\begin{aligned}M(\theta_n, \theta_n, \theta_{n+1}) &= \max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_n, \theta_n, R\theta_n), \\ &\quad S(\theta_{n+1}, \theta_{n+1}, R\theta_{n+1})\} \\ &= \max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})\}.\end{aligned} \tag{41}$$

Thus,

$$\begin{aligned}0 &\leq \zeta(\alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})\max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})\}, S(\theta_n, \theta_n, \theta_{n-1})) \\ &< S(\theta_n, \theta_n, \theta_{n+1}) - \alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})\max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})\}.\end{aligned} \tag{42}$$

Consequently, we have

$$S(\theta_n, \theta_n, \theta_{n+1}) > \alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})\max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})\}. \tag{43}$$

If $\max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})\} = S(\theta_n, \theta_n, \theta_{n+1})$ for all $n \geq 0$, then

$$S(\theta_n, \theta_n, \theta_{n+1}) > \alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})S(\theta_n, \theta_n, \theta_{n+1}) \geq S(\theta_n, \theta_n, \theta_{n+1}), \tag{44}$$

is a contradiction. Thus, $\max\{S(\theta_n, \theta_n, \theta_{n+1}), S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})\} = S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2})$ for all $n \geq 0$. Hence,

$$S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2}) \leq \alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2}) < S(\theta_n, \theta_n, \theta_{n+1}), \tag{45}$$

for all $n \geq 0$.

This shows that $\{S(\theta_n, \theta_n, \theta_{n+1})\}$ is a decreasing sequence, and then, we have

$$\lim_{n \longrightarrow +\infty} S(\theta_n, \theta_n, \theta_{n+1}) = r, \quad r \geq 0. \tag{46}$$

We prove that

$$\lim_{n \longrightarrow +\infty} S(\theta_n, \theta_n, \theta_{n+1}) = 0. \tag{47}$$

Let $r > 0$. From (45), we have

$$\lim_{n \longrightarrow +\infty} \alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})S(\theta_{n+1}, \theta_{n+1}, \theta_{n+2}) = r. \tag{48}$$

Taking $\upsilon_n = \alpha(\theta_n, \theta_n, \theta_{n+1})\beta(\theta_n, \theta_n, \theta_{n+1})S(\theta_{n+1}, \theta_{n+1}, \theta_n)$ and $\mu_n = S(\theta_n, \theta_n, \theta_{n+1})$ and using (3) of Definition 4, we have

$$0 \leq \lim_{n \longrightarrow +\infty} \sup \zeta\left(\alpha\left(\theta_n, \theta_n, \theta_{n+1}\right)\beta\left(\theta_n, \theta_n, \theta_{n+1}\right)S\left(\theta_{n+1}, \theta_{n+1}, \theta_{n+2}\right), S\left(\theta_n, \theta_n, \theta_{n+1}\right)\right) < 0, \tag{49}$$

is a contradiction. Hence, $r = 0$.

Next, we have to prove that $\{\theta_n\}$ is Cauchy. If possible, let $\{\theta_n\}$ is not Cauchy. Then, there exists $\varepsilon > 0$ for which subsequences $\{\theta_{n_k}\}$ and $\{\theta_{m_k}\}$ of $\{\theta_n\}$ can be obtained with $n_k > m_k > k$ such that, for every k,

$$\begin{aligned} &S\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right) \geq \varepsilon,\\ &S\left(\theta_{n_k-1}, \theta_{n_k-1}, \theta_{m_k}\right) < \varepsilon. \end{aligned} \tag{50}$$

We have

$$\begin{aligned} \varepsilon \leq S\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right) &\leq 2S\left(\theta_{n_k}, \theta_{n_k}, \theta_{n_k-1}\right) + S\left(\theta_{n_k-1}, \theta_{n_k-1}, \theta_{m_k}\right)\\ &< 2S\left(\theta_{n_k}, \theta_{n_k}, \theta_{n_k-1}\right) + \varepsilon. \end{aligned} \tag{51}$$

Taking $k \longrightarrow +\infty$ and by (47),

$$\lim_{k \longrightarrow +\infty} S\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right) = \varepsilon. \tag{52}$$

Also,

$$\left|S\left(\theta_{n_k+1}, \theta_{n_k+1}, \theta_{m_k}\right) - S\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right)\right| \leq 2S\left(\theta_{n_k}, \theta_{n_k}, \theta_{n_k+1}\right). \tag{53}$$

Taking limit as $k \longrightarrow +\infty$ and by (47) and (52),

$$\lim_{k \longrightarrow +\infty} S\left(\theta_{n_k+1}, \theta_{n_k+1}, \theta_{m_k}\right) = \varepsilon. \tag{54}$$

Similarly,

$$\begin{aligned} &\lim_{k \longrightarrow +\infty} S\left(\theta_{m_k+1}, \theta_{m_k+1}, \theta_{n_k}\right) = \varepsilon,\\ &\lim_{k \longrightarrow +\infty} S\left(\theta_{n_k+1}, \theta_{n_k+1}, \theta_{m_k+1}\right) = \varepsilon. \end{aligned} \tag{55}$$

By triangular $(\alpha, \beta)_s$-admissibility of R,

$$\alpha\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right) \geq 1 \text{ and } \beta\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right) \geq 1. \tag{56}$$

Since R is generalised $(\alpha, \beta)_s$-admissible $\mathscr{Z}$-contraction of type I and using (52), (56), and (3) of Definition 4, we have

$$0 \leq \lim_{k \longrightarrow +\infty} \sup \zeta\left(\alpha\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right)\beta\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right)M\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right), S\left(\theta_{n_k}, \theta_{n_k}, \theta_{m_k}\right)\right) < 0, \tag{57}$$

is a contradiction, and hence, $\{\theta_n\}$ is Cauchy. By completeness of S-metric-like space (Ω, S), we know that there exist some $\xi \in \Omega$ such that

$$\lim_{n \longrightarrow +\infty} S\left(\theta_n, \theta_n, \xi\right) = S(\xi, \xi, \xi) = \lim_{n,m \longrightarrow +\infty} S\left(\theta_n, \theta_n, \theta_m\right) = 0, \tag{58}$$

which implies that $S(\xi, \xi, \xi) = 0$. The continuity of R implies that

$$\begin{aligned} \lim_{n \longrightarrow +\infty} S\left(\theta_{n+1}, \theta_{n+1}, R\xi\right) &= \lim_{n \longrightarrow +\infty} S\left(R\theta_n, R\theta_n, R\xi\right)\\ &= S(R\xi, R\xi, R\xi) = 0. \end{aligned} \tag{59}$$

Using Lemma 1 and (58), we have

$$\lim_{n \longrightarrow +\infty} S\left(\theta_{n+1}, \theta_{n+1}, R\xi\right) = S(\xi, \xi, R\xi). \tag{60}$$

Thus, $S(R\xi, R\xi, R\xi) = S(\xi, \xi, R\xi)$, that is, $R\xi = \xi$. To prove uniqueness of fixed point, let $\eta \in \Omega$ such that $R\eta = \eta$ and $\eta \neq \xi$. Then,

$$\begin{aligned} 0 &\leq \zeta(\alpha(\xi, \xi, \eta)\beta(\xi, \xi, \eta)S(R\xi, R\xi, R\eta), S(\xi, \xi, \eta))\\ &< S(\xi, \xi, \eta) - \alpha(\xi, \xi, \eta)\beta(\xi, \xi, \eta)S(R\xi, R\xi, \eta) \leq 0, \end{aligned} \tag{61}$$

is a contradiction, so $\xi = \eta$.

By removing the continuity condition, we have the following result. □

Theorem 4. *Let R: $\Omega \longrightarrow \Omega$ be a mapping on a complete S-metric-like space (Ω, S) satisfying the following:*

(a) R is triangular $(\alpha, \beta)_s$-admissible.

(b) There exists $\theta_0 \in \Omega$ such that $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ and $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) R is generalised $(\alpha, \beta)_s$-admissible $\mathscr{Z}$-contraction of type I on (Ω, S).

(d) If $\{\theta_n\}$ is a sequence in Ω such that $\alpha(\theta_n, \theta_n, \theta_{n+1}) \geq 1$ and $\beta(\theta_n, \theta_n, \theta_{n+1}) \geq 1$ for all n and $\theta_n \longrightarrow \xi \in \Omega$ as $n \longrightarrow +\infty$, then there exists a subsequence $\{\theta_{n_k}\}$ of $\{\theta_n\}$ such that $\alpha(\theta_{n_k}, \theta_{n_k}, \xi) \geq 1$ and $\beta(\theta_{n_k}, \theta_{n_k}, \xi) \geq 1$ for all $k \in \mathbb{N}$.

Then, there is a unique fixed point $\xi \in \Omega$ of R with $S(\xi, \xi, \xi) = 0$.

Proof. Proceeding as of Theorem 3, let $\{\theta_n\}$ be a sequence in Ω given by $\theta_{n+1} = R\theta_n$ converges to some $\xi \in \Omega$ with $\alpha(\theta_n, \theta_n, \theta_{n+1}) \geq 1$ and $\beta(\theta_n, \theta_n, \theta_{n+1}) \geq 1$ for all n and $S(\xi, \xi, \xi) = 0$. From (d), there exists a subsequence $\{\theta_{n_k}\}$ of $\{\theta_n\}$ such that $\alpha(\theta_{n_k}, \theta_{n_k}, \xi) \geq 1$ and $\beta(\theta_{n_k}, \theta_{n_k}, \xi) \geq 1$ for all $k \in \mathbb{N}$. Thus, applying (33) for all k, we have

$$\begin{aligned} 0 &\leq \zeta\left(\alpha\left(\theta_{n_k}, \theta_{n_k}, \xi\right)\beta\left(\theta_{n_k}, \theta_{n_k}, \xi\right)M\left(\theta_{n_k}, \theta_{n_k}, \xi\right), S\left(\theta_{n_k}, \theta_{n_k}, \xi\right)\right)\\ &< S\left(\theta_{n_k}, \theta_{n_k}, \xi\right)\\ &\quad - \alpha\left(\theta_{n_k}, \theta_{n_k}, \xi\right)\beta\left(\theta_{n_k}, \theta_{n_k}, \xi\right)M\left(\theta_{n_k}, \theta_{n_k}, \xi\right), \end{aligned} \tag{62}$$

where

$$\begin{aligned} M\left(\theta_{n_k}, \theta_{n_k}, \xi\right) &= \max\left\{S\left(\theta_{n_k}, \theta_{n_k}, \xi\right), S\left(\theta_{n_k}, \theta_{n_k}, R\theta_{n_k}\right), S(\xi, \xi, R\xi)\right\} \\ &= \max\left\{S\left(\theta_{n_k}, \theta_{n_k}, \xi\right), S\left(\theta_{n_k}, \theta_{n_k}, \theta_{n_k+1}\right), S(\xi, \xi, R\xi)\right\}. \end{aligned} \tag{63}$$

From (62), we get

$$\begin{aligned} M\left(\theta_{n_k}, \theta_{n_k}, \xi\right) &\leq \alpha\left(\theta_{n_k}, \theta_{n_k}, \xi\right)\beta\left(\theta_{n_k}, \theta_{n_k}, \xi\right)M\left(\theta_{n_k}, \theta_{n_k}, \xi\right) \\ &< S\left(\theta_{n_k}, \theta_{n_k}, \xi\right). \end{aligned} \tag{64}$$

Taking $k \longrightarrow +\infty$, we have $S(\xi, \xi, R\xi) = 0$, that is, $R\xi = \xi$. Similar to Theorem 3 uniqueness of fixed point of R can be proved. □

Next, we give the following definition. □

Definition 15. Let $R\colon \Omega \longrightarrow \Omega$ be a mapping on a S-metric-like space (Ω, S) and $\alpha, \beta\colon \Omega \times \Omega \times \Omega \longrightarrow \mathbb{R}^+$. R is said to be a generalised rational $(\alpha, \beta)_s$-admissible $\mathscr{Z}$-contraction of type I with respect to ζ if

$$\zeta\left(\alpha(\theta, \mu, \xi)\beta(\theta, \mu, \xi)M_r(\theta, \mu, \xi), S(\theta, \mu, \xi)\right) \geq 0, \tag{65}$$

where

$$M_r(\theta, \mu, \xi) = \max\left\{\begin{array}{c} S(\theta, \mu, \xi), S(R\theta, R\mu, R\xi), \dfrac{S(\theta, \theta, R\theta)S(\mu, \mu, R\mu)}{1 + S(\theta, \mu, \xi) + S(R\theta, R\mu, R\xi)}, \\ \dfrac{S(\mu, \mu, R\mu)S(\xi, \xi, R\xi)}{1 + S(\theta, \mu, \xi) + S(R\theta, R\mu, R\xi)}, \dfrac{S(\xi, \xi, R\xi)S(\theta, \theta, R\theta)}{1 + S(\theta, \mu, \xi) + S(R\theta, R\mu, R\xi)} \end{array}\right\}, \tag{66}$$

for all $\theta, \mu, \xi \in \Omega$.

Definition 16. Let $R\colon \Omega \longrightarrow \Omega$ be a mapping on a S-metric-like space (Ω, S) and $\alpha, \beta\colon \Omega \times \Omega \times \Omega \longrightarrow \mathbb{R}^+$. R is said to be a generalised rational $(\alpha, \beta)_s$-admissible $\mathscr{Z}$-contraction of type II with respect to ζ if

$$\zeta\left(\alpha(\theta, \theta, \mu)\beta(\theta, \theta, \mu)M_r(\theta, \theta, \mu), S(\theta, \theta, \mu)\right) \geq 0, \tag{67}$$

where

$$M_r(\theta, \theta, \mu) = \max\left\{S(\theta, \theta, \mu), S(R\theta, R\theta, R\mu), \frac{S(\theta, \theta, R\theta)S(\theta, \theta, R\theta)}{1 + S(\theta, \theta, \mu) + S(R\theta, R\theta, R\mu)}, \frac{S(\theta, \theta, R\theta)S(\mu, \mu, R\mu)}{1 + S(\theta, \theta, \mu) + S(R\theta, R\theta, R\mu)}\right\}, \tag{68}$$

for all $\theta, \mu, \xi \in \Omega$.

Theorem 5. *Let $R\colon \Omega \longrightarrow \Omega$ be a mapping on a complete S-metric-like space (Ω, S) satisfying the following:*

(a) R is triangular $(\alpha, \beta)_s$-admissible.

(b) There exists $\theta_0 \in \Omega$ such that $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ and $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) R is a generalised rational $(\alpha, \beta)_s$-admissible $\mathscr{Z}$-contraction of type I on (Ω, S).

(d) R is S-continuous.

Then, there is a unique fixed point $\xi \in \Omega$ of R with $S(\xi, \xi, \xi) = 0$.

Proof. Similar to Theorem 3. □

Theorem 6. *Let $R\colon \Omega \longrightarrow \Omega$ be a mapping on a complete S-metric-like space (Ω, S) satisfying the following:*

(a) R is triangular $(\alpha, \beta)_s$-admissible.

(b) There exists $\theta_0 \in \Omega$ such that $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ and $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) R is generalised rational $(\alpha, \beta)_s$-admissible $\mathscr{Z}$-contraction of type I on (Ω, S).

(d) If $\{\theta_n\}$ is a sequence in Ω such that $\alpha(\theta_n, \theta_n, \theta_{n+1}) \geq 1$ and $\beta(\theta_n, \theta_n, \theta_{n+1}) \geq 1$ for all n and $\theta_n \longrightarrow \xi \in \Omega$ as $n \longrightarrow +\infty$, then there exists a subsequence $\{\theta_{n_k}\}$ of $\{\theta_n\}$ such that $\alpha(\theta_{n_k}, \theta_{n_k}, \xi) \geq 1$ and $\beta(\theta_{n_k}, \theta_{n_k}, \xi) \geq 1$ for all $k \in \mathbb{N}$.

Then, there is a unique fixed point $\xi \in \Omega$ of R with $S(\xi, \xi, \xi) = 0$.

Proof. Similar to Theorem 4. □

3. Consequences

In this section, we give various results as consequences of the above results. First, we give a Banach type.

Corollary 1. *Let $R\colon \Omega \longrightarrow \Omega$ be a mapping on a complete S-metric-like space (Ω, S) satisfying the following:*

(a) R is triangular $(\alpha, \beta)_s$-admissible.

(b) There exists $\theta_0 \in \Omega$ such that $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ and $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) $\alpha(\theta, \mu, \xi)\beta(\theta, \mu, \xi)S(R\theta, R\mu, R\xi) \leq kS(\theta, \mu, \xi)$ *for all* $\theta, \mu, \xi \in \Omega$ *and* $k \in [0, 1)$.

(d) *R is S-continuous.*

Then, there is a unique fixed point $\xi \in \Omega$ *of R with* $S(\xi, \xi, \xi) = 0$.

Proof. Similar to Theorem 1, considering

$$\zeta(t, s) = ks - t. \tag{69}$$

□

Corollary 2. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a mapping on a complete S-metric-like space* (Ω, S) *satisfying the following:*

(a) *R is triangular* $(\alpha, \beta)_s$*-admissible.*

(b) *There exists* $\theta_0 \in \Omega$ *such that* $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ *and* $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) *There exists a lower semicontinuous function* $\phi\colon \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ *with* $\phi^{-1}\{0\} = \{0\}$ *such that*

$$\begin{aligned} &\alpha(\theta, \mu, \xi)\beta(\theta, \mu, \xi)S(R\theta, R\mu, R\xi) \\ &\quad \leq S(\theta, \mu, \xi) - \phi(S(\theta, \mu, \xi)), \end{aligned} \tag{70}$$

for all $\theta, \mu, \xi \in \Omega$.

(d) *R is S-continuous.*

Then, there is a unique fixed point $\xi \in \Omega$ *of R with* $S(\xi, \xi, \xi) = 0$.

Proof. Consider

$$\zeta(t, s) = s - \phi(s) - t. \tag{71}$$

Taking $\alpha(\theta, \mu, \xi) = \beta(\theta, \mu, \xi) = 1$ for all $\theta, \mu, \xi \in \Omega$ in Theorem 1, we have the following. □

Corollary 3. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a mapping on a complete S-metric-like space* (Ω, S)*. Assume that there exists a simulation function* ζ *such that*

$$\zeta(S(R\theta, R\mu, R\xi), S(\theta, \mu, \xi)) \geq 0, \tag{72}$$

for all $\theta, \mu, \xi \in \Omega$*. Then,* $\xi \in \Omega$ *is a unique fixed point of R with* $S(\xi, \xi, \xi) = 0$.

Corollary 4. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a mapping on a complete S-metric-like space* (Ω, S) *satisfying the following:*

(a) *R is triangular* $(\alpha, \beta)_s$*-admissible.*

(b) *There exists* $\theta_0 \in \Omega$ *such that* $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ *and* $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) $\alpha(\theta, \mu, \xi)\beta(\theta, \mu, \xi)M(R\theta, R\mu, R\xi) \leq kS(\theta, \mu, \xi)$ *for all* $\theta, \mu, \xi \in \Omega$ *and* $k \in [0, 1)$.

(d) *R is S-continuous.*

Then, there is a unique fixed point $\xi \in \Omega$ *of R with* $S(\xi, \xi, \xi) = 0$.

Proof. Similar to Theorem 3, considering,

$$\zeta(t, s) = ks - t. \tag{73}$$

□

Corollary 5. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a mapping on a complete S-metric-like space* (Ω, S) *satisfying the following:*

(a) *R is triangular* $(\alpha, \beta)_s$*-admissible.*

(b) *There exists* $\theta_0 \in \Omega$ *such that* $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ *and* $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) *There exists a lower semicontinuous function* $\phi\colon \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ *with* $\phi^{-1}\{0\} = \{0\}$ *such that*

$$\begin{aligned} &\alpha(\theta, \mu, \xi)\beta(\theta, \mu, \xi)M(R\theta, R\mu, R\xi) \\ &\quad \leq S(\theta, \mu, \xi) - \phi(S(\theta, \mu, \xi)), \end{aligned} \tag{74}$$

(i) *for all* $\theta, \mu, \xi \in \Omega$.

(d) *R is S-continuous.*

Then, there is a unique fixed point $\xi \in \Omega$ *of R with* $S(\xi, \xi, \xi) = 0$.

Proof. Consider

$$\zeta(t, s) = s - \phi(s) - t. \tag{75}$$

If we consider in Theorem 20, $\alpha(\theta, \mu, \xi) = \beta(\theta, \mu, \xi) = 1$ for all $\theta, \mu, \xi \in \Omega$, we have □

Corollary 6. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a S-continuous mapping on a complete S-metric-like space* (Ω, S)*. Suppose that there exists a simulation function* ζ *such that*

$$\zeta(M(R\theta, R\mu, R\xi), S(\theta, \mu, \xi)) \geq 0, \tag{76}$$

for all $\theta, \mu, \xi \in \Omega$*. Then, there is a unique fixed point* $\xi \in \Omega$ *of R with* $S(\xi, \xi, \xi) = 0$.

Corollary 7. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a mapping on a complete S-metric-like space* (Ω, S) *satisfying the following:*

(a) *R is triangular* $(\alpha, \beta)_s$*-admissible.*

(b) *There exists* $\theta_0 \in \Omega$ *such that* $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ *and* $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) $\alpha(\theta, \mu, \xi)\beta(\theta, \mu, \xi)M_r(R\theta, R\mu, R\xi) \leq kS(\theta, \mu, \xi)$ *for all* $\theta, \mu, \xi \in \Omega$ *and* $\theta \in [0, 1)$.

(d) *R is S-continuous.*

Then, there is a unique fixed point $\xi \in \Omega$ *of R with* $S(\xi, \xi, \xi) = 0$.

Proof. Similar to Theorem 5, considering,

$$\zeta(t, s) = ks - t. \tag{77}$$

□

Corollary 8. *Let* $R\colon \Omega \longrightarrow \Omega$ *be a mapping on a complete S-metric-like space* (Ω, S) *satisfying the following:*

(a) *R is triangular* $(\alpha, \beta)_s$*-admissible.*

(b) *There exists* $\theta_0 \in \Omega$ *such that* $\alpha(\theta_0, \theta_0, R\theta_0) \geq 1$ *and* $\beta(\theta_0, \theta_0, R\theta_0) \geq 1$.

(c) *There exists a lower semicontinuous function* $\phi\colon \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ *with* $\phi^{-1}\{0\} = \{0\}$ *such that*

$$\begin{aligned}&\alpha(\theta,\mu,\xi)\beta(\theta,\mu,\xi)M_r(R\theta,R\mu,R\xi)\\&\quad\le S(\theta,\mu,\xi)-\phi(S(\theta,\mu,\xi)),\end{aligned}\tag{78}$$

for all $\theta,\mu,\xi\in\Omega$.

(d) R is S-continuous.

Then, there is a unique fixed point $\xi\in\Omega$ of R with $S(\xi,\xi,\xi)=0$.

Proof. It suffices to take

$$\zeta(t,s)=s-\phi(s)-t.\tag{79}$$

If we consider in Theorem 5, $\alpha(\theta,\mu,\xi)=\beta(\theta,\mu,\xi)=1$ for all $\theta,\mu,\xi\in\Omega$, we have the following. □

Corollary 9. *Let R: $\Omega\longrightarrow\Omega$ be a S-continuous mapping on a complete S-metric-like space (Ω,S). Suppose that there exists a simulation function ζ such that*

$$\zeta(M_r(R\theta,R\mu,R\xi),S(\theta,\mu,\xi))\ge 0,\tag{80}$$

for all $\theta,\mu,\xi\in\Omega$. Then, there is a unique fixed point $\xi\in\Omega$ of R with $S(\xi,\xi,\xi)=0$.

We take the following example.

Example 2. Suppose $\Omega=[0,+\infty)$, $S(\theta,\mu,\xi)=(\theta+\mu)+(\mu+\xi)$ for all $\theta,\mu,\xi\in\Omega$ and R: $\Omega\longrightarrow\Omega$ as

$$R\theta=\begin{cases}\dfrac{\theta}{4}, & \text{if } 0\le\theta\le 1,\\ 4\theta, & \text{otherwise.}\end{cases}\tag{81}$$

Consider

$$\zeta(t,s)=cs-t,\quad\text{where } 0\le\frac{1}{4}<c<1.\tag{82}$$

Let α,β: $\Omega\times\Omega\times\Omega\longrightarrow\mathbb{R}^+$ be defined as

$$\begin{aligned}\alpha(\theta,\mu,\xi)&=\begin{cases}\dfrac{4}{3}, & \text{if } 0\le\theta,\mu,\xi\le 1,\\ 0, & \text{otherwise,}\end{cases}\\ \beta(\theta,\mu,\xi)&=\begin{cases}\dfrac{3}{2}, & \text{if } 0\le\theta,\mu,\xi\le 1,\\ 0, & \text{otherwise.}\end{cases}\end{aligned}\tag{83}$$

We will verify Corollary 1. Here, (Ω,S) is a complete S-metric-like space. Let $\theta,\mu,\xi\in\Omega$ such that $\alpha(\theta,\mu,\xi)\ge 1$ and $\beta(\theta,\mu,\xi)\ge 1$. Then, $\theta,\mu,\xi\in[0,1]$ and so $R\theta\in[0,1]$, $R\mu\in[0,1]$, $R\xi\in[0,1]$, and $\alpha(R\theta,R\mu,R\xi)\ge 1$ and $\beta(R\theta,R\mu,R\xi)\ge 1$. Hence, R is triangular $(\alpha,\beta)_s$-admissible. When $\theta_0=1$, condition (b) is true, $\theta_0=R^n\theta_1=(1/n)$ satisfies condition (d).

If $0\le\theta\le 1$, then $\alpha(\theta,\mu,\xi)=(4/3)$ and $\beta(\theta,\mu,\xi)=(3/2)$. We have

$$\begin{aligned}\zeta(\alpha(\theta,\mu,\xi)\beta(\theta,\mu,\xi)S(R\theta,f\mu,f\xi),S(\theta,\mu,\xi))&=cS(\theta,\mu,\xi)-\alpha(\theta,\mu,\xi)\beta(\theta,\mu,\xi)S(f\theta,f\mu,f\xi)\\&=\frac{3}{4}\{(\theta+\mu)+(\mu+\xi)\}-2.\frac{1}{4}\{(\theta+\mu)+(\mu+\xi)\}=\frac{1}{4}\{(\theta+\mu)+(\mu+\xi)\}.\end{aligned}\tag{84}$$

If $0\le\mu\le 1$ and $\xi>1$, then $\zeta(\alpha(\theta,\mu,\xi)\beta(\theta,\mu,\xi)S(f\theta,f\mu,f\xi),S(\theta,\mu,\xi))\ge 0$ since $\alpha(\theta,\mu,\xi)=\beta(\theta,\mu,\xi)=0$. Thus, $\xi=0$ is the unique fixed point of f.

We also notice that (72) is not satisfied. In fact, for $\theta=1$, $\mu=2$, $\xi=3$, we get

$$\begin{aligned}S(R1,R2,R3)&=S\left(\frac{1}{4},8,12\right)=\left(\frac{1}{4}+8\right)+(8+12)\\&=\frac{33}{4}+20=\frac{113}{4}>8=S(1,2,3).\end{aligned}\tag{85}$$

4. Conclusion

In this paper, we present S-metric-like space and some of its topological properties. Also, we present some new (α,β)-contraction-type mappings under simulation function by extending the definition of (α,β)-admissible mappings and prove some fixed point results in the setting of S-metric-like space. We also open the scope for extending various contractions in S-metric-like space.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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