Research Article

Compactness on Soft Topological Ordered Spaces and Its Application on the Information System

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It is well known every soft topological space induced from soft information system is soft compact. In this study, we integrate between soft compactness and partially ordered set to introduce new types of soft compactness on the finite spaces and investigate their application on the information system. First, we initiate a notion of monotonic soft sets and establish its main properties. Second, we introduce the concepts of monotonic soft compact and ordered soft compact spaces and show the relationships between them with the help of examples. We give a complete description for each one of them by making use of the finite intersection property. Also, we study some properties associated with some soft ordered spaces and finite product spaces. Furthermore, we investigate the conditions under which these concepts are preserved between the soft topological ordered space and its parametric topological ordered spaces. In the end, we provide an algorithm for expecting the missing values of objects on the information system depending on the concept of ordered soft compact spaces.

1. Introduction

Compactness is a property that generalizes the notion of a closed and bounded subset of Euclidean space. It has been described by using the finite intersection property for closed sets. The important motivations beyond studying compactness have been given in [1]. Without doubt, the concept of compactness occupied a wide area of topologists' attention. Many relevant ideas to this concept have been introduced and studied. Generalizations of compactness have been formulated in many directions, one of them given by using generalized open sets; see, for example, [2].

In 1965, Nachbin [3] combined a partial order relation with a topological space to define a new mathematical structure, namely, a topological ordered space. Then, McCartan [4] formulated ordered separation axioms with respect to open sets and neighbourhoods which were described by increasing or decreasing operators. Shabir and Gupta [5] extended these ordered separation axioms in the cases of T_1-ordered and T_2-ordered spaces using semiopen sets. Recently, Al-Shami and Abo-Elhamayel [6] introduced new types of ordered separation axioms.

In 1999, Molotdsov [7] came up with the idea of soft sets for dealing with uncertainties and vagueness. Shabir and Naz [8], in 2011, exploited soft sets to introduce the concept of soft topological spaces and study soft separation axioms. Then, researchers started working to generalize topological notions on the soft topological frame. In this regard, compactness was one of the topics that received much attention. It was presented and explored firstly by Molodtsov Ayğunoğlu and Ayğun, and Zorlutuna et al. [9, 10]. Then, Hida [11] compared between two types of soft compactness. After that, Al-Shami et al. [12] defined almost soft compact and mildly soft compact spaces and investigated the main properties. Lack of consideration in the privacy of soft sets by some authors causes emerging some alleged results, especially those related to the properties of soft compactness. Therefore, the authors of [13–17] carried out some corrective studies in this regard.

In 2019, Al-Shami et al. [18] defined topological ordered spaces on soft setting. They studied monotonic soft sets and then utilized them to present p-soft T_i-ordered spaces. Also, they [19] presented soft $I(D, B)$-continuous mappings and established several generalizations of them using generalized

soft open sets. Moreover, Al-Shami and El-Shafei [20] initiated two types of ordered separation axioms, namely, soft T_i-ordered and strong soft T_i-ordered spaces. Recently, the concept of supra soft topological ordered spaces has been explored and discussed in [21].

This paper is organized as follows: Section 2 gives some main notions of monotonic soft sets and soft topological ordered spaces. In Section 3, we introduce the concept of monotonic soft sets and show some properties with the help of examples. Section 4 defines and explores a concept of monotonic soft compact spaces. By using a convenient technique, we show that many well-known results of soft compactness are valid on monotonic soft compact spaces. We divide Section 5 into two parts: the first one studies a concept of ordered soft compact spaces, and the second one makes use of it to present a practical application on the information system. Section 6 concludes the paper with summary and further works.

2. Preliminaries

To make this work self-contained, we mention some definitions and results that were introduced in soft set theory, soft topology, and ordered soft topology.

2.1. Soft Set

Definition 1 (see [7]). A soft set over $X \neq \varnothing$ is a mapping G from a parameter set E to the power set 2^X of X. It is denoted by (G, E).

Usually, we write a soft set (G, E) as a set of ordered pairs. In other words, $(G, E) = \{(e, G(e)): e \in E \text{ and } G(e) \in 2^X\}$.

Sometimes, we use some symbols such as A, B in place of E and F, H in place of G.

Definition 2 (G, E) over X is said to be

(i) a null soft set (resp. an absolute soft set) [22] if $G(e) = \varnothing$ (resp. $G(e) = X$) for each $e \in E$; and it is denoted by $\widetilde{\Phi}$ (resp. $\widetilde{X}$).

(ii) soft point [23, 24] if there are $e \in E$ and $x \in X$ such that $G(e) = \{x\}$ and $G(b) = \varnothing$ for each $b \in E \smallsetminus \{e\}$. It is denoted by P_e^x.

(iii) finite (resp. countable) soft set [23] if $G(e)$ is finite (resp. countable) for each $e \in E$. Otherwise, it is called infinite (resp. uncountable).

Definition 3 (see [25]). The relative complement of (G, E), denoted by $(G, E)^c$, is a mapping $G^c: E \longrightarrow 2^X$ defined by $G^c(e) = X \smallsetminus G(e)$ for each $e \in E$.

Definition 4 (see [26]). (G, A) is a subset of (F, B) if $A \subseteq B$ and $G(a) \subseteq F(a)$ for all $a \in A$.

Definition 5 (see [22]). The union of two soft sets (G, A) and (F, B) over X, denoted by $(G, A) \widetilde{\cup} (F, B)$, is the soft set

(V, D), where $D = A \cup B$, and a mapping $V: D \longrightarrow 2^X$ is defined as follows:

$$V(d) = \begin{cases} G(d): & d \in A - B, \\ F(d): & d \in B - A, \\ G(d) \cup F(d): & d \in A \cap B. \end{cases} \quad (1)$$

Definition 6 (see [25]). The intersection of soft sets (G, A) and (F, B) over X, denoted by $(G, A) \widetilde{\cap} (F, B)$, is the soft set V_D, where $D = A \cap B \neq \varnothing$, and a mapping $V: D \longrightarrow 2^X$ is defined by $V(d) = G(d) \cap F(d)$ for all $d \in D$.

Definition 7 (see [9, 10]). Let (G, A) and (F, B) be two soft sets over X and Y, respectively. Then, the Cartesian product of (G, A) and (F, B) is a soft set $(G \times F, A \times B)$ such that $(G \times F)(a, b) = G(a) \times F(b)$ for each $a \in A$ and $b \in B$.

Definition 8 (see [8, 27]). For a soft set (G, A) over X and $x \in X$, we say that

(i) $x \in (G, E)$ if $x \in G(e)$ for each $e \in E$, and $x \notin (G, E)$ if $x \notin G(e)$ for some $e \in E$

(ii) $x \widetilde{\in} (G, E)$ if $x \in G(e)$ for some $e \in E$, and $x \widetilde{\notin} (G, E)$ if $x \notin G(e)$ for each $e \in E$

Proposition 1 (see [10, 24]). *For a soft mapping $f_\phi: S(X_A) \longrightarrow S(Y_B)$, we have the following results:*

(i) $(G, A) \widetilde{\subseteq} f_\phi^{-1} f_\phi(G, A)$ *for each* $(G, A) \in S(X_A)$, *and* $f_\phi f_\phi^{-1}(F, B) \widetilde{\subseteq} (F, B)$ *for each* $(F, B) \in S(Y_B)$

(ii) *If* f_ϕ *is injective (resp. surjective), then* $(G, A) = f_\phi^{-1} f_\phi(G, A)$ *(resp.* $f_\phi f_\phi^{-1}(F, B) = (F, B))$

Definition 9 (see [28]). A binary relation $\prec$ is called a partial order relation if it is reflexive, antisymmetric, and transitive. The pair $(X, \prec)$ is called a partially ordered set.

The relation on a nonempty set X which is given by $\{(x, x): x \in X\}$ is called the equality relation and is denoted by $\triangle$.

Definition 10 (see [18]). $(X, E, \prec)$ is said to be a partially ordered soft set on $X \neq \varnothing$ if $(X, \prec)$ is a partially ordered set. For two soft points P_α^x and P_α^y in $(X, E, \prec)$, we say that $P_\alpha^x \prec P_\alpha^y$ if $x \prec y$.

Definition 11 (see [18]). Increasing operator i and decreasing operator d are two maps of $(S(X_E), \prec)$ into $(S(X_E), \prec)$ defined as follows: for each soft subset (G, E) of $S(X_E)$,

(i) $i(G, E) = (iG, E)$, where iG is a mapping of E into $P(X)$ given by $iG(e) = i(G(e)) = \{x \in X: b \prec x$ for some $b \in G(e)\}$

(ii) $d(G, E) = (dG, E)$, where dG is a mapping of E into $P(X)$ given by $dG(e) = d(G(e)) = \{x \in X: x \prec b$ for some $b \in G(e)\}$

Definition 12. Let $(X, E, \prec_1)$ and $(Y, E, \prec_2)$ be two partially ordered soft sets. The product relation $\prec$ of $\prec_1$ and $\prec_2$ on $X \times Y$ is defined as follows: $\prec = \{(a, b): a = (a_1, a_2), b = (b_1, b_2) \in X \times Y$ such that $(a_j, b_j) \in \prec_j$ for every $j = 1, 2\}$.

Definition 13 (see [18]). A soft subset (G, E) of $(X, E, \prec)$ is said to be increasing (resp. decreasing) provided that $(G, E) = i(G, E)$ (resp. $(G, E) = d(G, E)$).

Theorem 1 (see [18]). *ie finite product of increasing (resp. decreasing) soft sets is increasing (resp. decreasing).*

Definition 14 (see [18]). A soft map $f_\phi: (S(X_A), \prec_1) \longrightarrow (S(Y_B), \prec_2)$ is said to be a subjective ordered embedding provided that $P_\alpha^x \prec_1 P_\alpha^y$ if and only if $f_\phi(P_\alpha^x) \prec_2 f_\phi(P_\alpha^y)$.

Theorem 2 (see [18]). *Let $f_\phi: (S(X_A), \prec_1) \longrightarrow (S(Y_B), \prec_2)$ be a subjective ordered embedding soft mapping. Then, the image of each increasing (resp. decreasing) soft set is increasing (resp. decreasing).*

Proposition 2 (see [18])

(i) The union of increasing (resp. decreasing) soft sets is increasing (resp. decreasing)

(ii) The intersection of increasing (resp. decreasing) soft sets is increasing (resp. decreasing)

2.2. Soft Topological Space

Definition 15 (see [8]). The family τ of soft sets over X under a fixed parameter set E is said to be a soft topology on X if it contains $\widetilde{X}$ and $\widetilde{\Phi}$ and is closed under a finite intersection and an arbitrary union.

The triple (X, τ, E) is said to be a soft topological space. Every member of τ is called soft open, and its relative complement is called soft closed.

Definition 16 (see [8]). A soft subset (W, E) of (X, τ, E) is called soft neighborhood of $x \in X$ if there exists a soft open set (G, E) such that $x \in (G, E) \widetilde{\subseteq} (W, E)$.

Definition 17 (see [10, 24]). A soft mapping $f_\phi: (X, \tau, A) \longrightarrow (Y, \theta, B)$ is said to be

(i) soft continuous if the inverse image of each soft open set is soft open

(ii) soft open (resp. soft closed) if the image of each soft open (resp. soft closed) set is soft open (resp. soft closed)

(iii) soft bicontinuous if it is soft continuous and soft open

(iv) soft homeomorphism if it is bijective soft bicontinuous

Theorem 3 (see [9]). *Let (X, τ, A) and (Y, θ, B) be two soft topological spaces. Let $\Omega = \{(G, A) \times (F, B): (G, A) \in \tau$ and $(F, B) \in \theta\}$. Then, the family of all arbitrary union of elements of Ω is a soft topology on $X \times Y$.*

Definition 18 (see [29]). Let (X, τ, E) be a soft topological space. Then,

$$\tau^\star = \{(G, E): G(e) \in \tau_e, \quad \text{for each } e \in E\}, \tag{2}$$

is called an extended soft topology on X.

Many properties of extended soft topologies which help us to show the relationships between soft topology and its parametric topologies were studied in [30].

2.3. Soft Topological Ordered Space

Definition 19 (see [18]). A quadrable system $(X, \tau, E, \prec)$ is said to be a soft topological ordered space if (X, τ, E) is a soft topological space and $(X, \prec)$ is a partially ordered set.

Definition 20 (see [18]). A soft subset (W, E) of $(X, \tau, E, \prec)$ is said to be increasing (resp. decreasing) soft neighborhood of $x \in X$ if (W, E) is increasing (resp. decreasing) and a soft neighborhood of x.

Proposition 3 (see [18]). *In $(X, \tau, E, \prec)$, we find that, for each $e \in E$, the family $\tau_e = \{G(e): (G, E) \in \tau\}$ with a partial order relation $\prec$ forms an ordered topology on X.*

$(X, \tau_e, \prec)$ is said to be a parametric topological ordered space.

Definition 21 (see [18]). Let $Y \subseteq X$. Then, $(Y, \tau_Y, \prec_Y, E)$ is called a soft ordered subspace of $(X, \tau, \prec, E)$ if (Y, τ_Y, E) is a soft subspace of (X, τ, E) and $\prec_Y = \prec \cap Y \times Y$.

Lemma 1 (see [18]). *If (U, E) is an increasing (resp. a decreasing) soft subset of $(X, \tau, \prec, E)$, then $(U, E) \widetilde{\cap} \widetilde{Y}$ is an increasing (resp. a decreasing) soft subset of a soft ordered subspace $(Y, \tau_Y, \prec_Y, E)$.*

Definition 22 (see [18]). The product of a finite family of soft topological ordered spaces $\{(X_i, \tau_i, \prec_i, E_i): i \in \{1, 2, \ldots, n\}\}$ is a soft topological ordered space $(X, \tau, \prec, E)$, where $X = \prod_{i=1}^n X_i$, τ is the product soft topology on X, $E = \prod_{i=1}^n E_i$, and $\prec = \{(x, y): x, y \in X$ such that $(x_i, y_i) \in \prec_i$ for every $i\}$.

Definition 23 (see [18]). A soft ordered subspace $(Y, \tau_Y, \prec_Y, E)$ of $(X, \tau, E, \prec)$ is called soft compatibly ordered if for each increasing (resp. decreasing) soft closed subset (H, E) of $(Y, \tau_Y, \prec_Y, E)$, there exists an increasing (resp. a decreasing) soft closed subset $(H^\star, E)$ of $(X, \tau, E, \prec)$ such that $(H, E) = \widetilde{Y} \widetilde{\cap} (H^\star, E)$.

Definition 24 (see [18]). $(X, \tau, E, \prec)$ is said to be

(i) p-soft T_2-ordered if for every distinct points $x \nprec y$ in X, there exist disjoint soft neighborhoods (V, E) and (W, E) of x and y, respectively, such that (V, E) is increasing and (W, E) is decreasing

(ii) lower (upper) p-soft regularly ordered if for each decreasing (increasing) soft closed set (H, E) and $x \in X$ such that $x \widetilde{\notin} (H, E)$, there exist disjoint soft neighbourhoods (V, E) of (H, E) and (W, E) of x such that (V, E) is decreasing (increasing) and (W, E) is increasing (decreasing)

(iii) p-soft regularly ordered if it is both lower p-soft regularly ordered and upper p-soft regularly ordered

(iv) p-soft T_3-ordered if it is both lower p-soft T_3-ordered and upper p-soft T_3-ordered

If the phrase "soft neighborhoods" is replaced by "soft open sets," then the above soft axioms are called strong p-soft regularly ordered and strong p-soft T_i-ordered spaces, $i = 2, 3$.

3. Monotonically Soft Sets

In this section, we introduce a concept of monotonic soft sets and study the main properties with the help of illustrative examples.

Definition 25. A subset (G, E) of $(X, E, \prec)$ is called a monotonic soft set if (G, E) is increasing or decreasing. In other words, $(G, E) = i(G, E)$ or $(G, E) = d(G, E)$.

The following example shows that the union and intersection of monotonic soft sets are not always monotonic.

Example 1. Let $\prec = \triangle \cup \{(1, 3), (3, 5), (1, 5), (2, 4)\}$ be a partial order relation on $X = \{1, 2, 3, 4, 5\}$, and let $E = \{e_1, e_2\}$ be a set of parameters. We define the following three subsets (F, E), (G, E), and (H, E) of $(X, E, \prec)$ as follows:

$$(F, E) = \{(e_1, \{1, 3\}), (e_2, X)\},$$
$$(G, E) = \{(e_1, \{3, 5\}), (e_2, \varnothing)\}, \qquad (3)$$
$$(H, E) = \{(e_1, \{4\}), (e_2, \{4\})\}.$$

Since $d(F, E) = (F, E)$, $i(G, E) = (G, E)$, and $i(H, E) = (H, E)$, (F, E), (G, E), and (H, E) are monotonic soft sets. Now, $(F, E) \widetilde{\cap} (G, E) = \{(e_1, \{3\}), (e_2, \varnothing)\}$ and $(F, E) \widetilde{\cup} (H, E) = \{(e_1, \{1, 3, 4\}), (e_2, X)\}$. It is clear that $i((F, E) \widetilde{\cap} (G, E)) = \{(e_1, \{3, 5\}), (e_2, \varnothing)\} \neq d((F, E) \widetilde{\cap} (G, E)) = \{(e_1, \{1, 3\}), (e_2, \varnothing)\} \neq (F, E) \widetilde{\cap} (G, E)$ and $i((F, E) \widetilde{\cup} (H, E)) = \{(e_1, \{1, 3, 4, 5\}), (e_2, X)\} \neq d((F, E) \widetilde{\cup} (H, E)) = \{(e_1, \{1, 2, 3, 4\}), (e_2, X)\} \neq (F, E) \widetilde{\cup} (H, E)$. Hence, $(F, E) \widetilde{\cap} (G, E)$ and $(F, E) \widetilde{\cup} (H, E)$ are not monotonic soft sets.

The following result demonstrates under what conditions the union (intersection) of monotonic soft sets is monotonic.

Proposition 4. *Let (F, E) and (G, E) be two subsets of $(X, E, \prec_1)$ and $(Y, E, \prec_2)$, respectively. If (F, E) and (G, E) are both increasing (decreasing), then the union and intersection of (F, E) and (G, E) are monotonic soft sets.*

Proof. It follows immediately from Proposition 2.   □

Corollary 1. *Let (F, E) and (G, E) be two monotonic subsets of $(X, E, \prec_1)$ and $(Y, E, \prec_2)$, respectively. Then, we have the following two results:*

(i) $(F, E) \widetilde{\cup} (G, E)$ and $(F, E)^c \widetilde{\cup} (G, E)^c$ are monotonic or $(F, E)^c \widetilde{\cup} (G, E)$ and $(F, E) \widetilde{\cup} (G, E)^c$ are monotonic

(ii) $(F, E) \widetilde{\cap} (G, E)$ and $(F, E)^c \widetilde{\cap} (G, E)^c$ are monotonic or $(F, E)^c \widetilde{\cap} (G, E)$ and $(F, E) \widetilde{\cap} (G, E)^c$ are monotonic

Proposition 5. *Let (F, E) and (G, E) be two soft subsets of $(X, E, \prec_1)$ and $(Y, E, \prec_2)$, respectively. We have the following two results:*

(i) $i_\prec ((F, E) \times (G, E)) = i_{\prec_1}(F, E) \times i_{\prec_2}(G, E)$

(ii) $d_\prec ((F, E) \times (G, E)) = d_{\prec_1}(F, E) \times d_{\prec_2}(G, E)$

Proof. We only prove (i), and one can prove (ii) similarly.
$i_\prec ((F, E) \times (G, E)) = i_\prec (F \times G, E \times E) = \left\{ P^{(x,y)}_{(e,e')} \in (X \times Y, E \times E, \prec) : \text{there exists } P^{(a,b)}_{(e,e')} \in (F, E) \times (G, E) \text{ such that } (a, b) \prec (x, y) \right\}$.

(i) $= \{ P^x_e \in (X, E, \prec_1) \text{ and } P^y_{e'} \in (Y, E, \prec_2) : \text{there exist } P^a_e \in (F, E) \text{ and } P^b_{e'} \in (G, E) \text{ such that } a \prec_1 x \text{ and } b \prec_2 y \}$

(ii) $= \{ P^x_e \in (X, E, \prec_1) : \text{there exist } P^a_e \in (F, E) \text{ such that } a \prec_1 x \} \text{ and } \{ P^y_{e'} \in (Y, E, \prec_2) : \text{there exist } P^b_{e'} \in (G, E) \text{ such that } b \prec_2 y \}$

(iii) $= i_{\prec_1}(F, E) \times i_{\prec_2}(G, E)$ □

The next example illustrates that the product of two monotonic soft sets need not be a monotonic soft set.

Example 2. Let $\prec_1 = \triangle \cup \{(a, b)\}$ and $\prec_2 = \triangle \cup \{(1, 2)\}$ be the partial order relations on $X = \{a, b\}$ and $Y = \{1, 2\}$, respectively. From Definition 12, the partial order relation $\prec$ on $X \times Y = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$ is given as follows: $\prec = \triangle \cup \{((a, 1), (a, 2)), ((a, 1), (b, 1)), ((a, 1), (b, 2)), ((a, 2), (b, 2))\}$. Take $E = \{e_1, e_2\}$ as a set of parameters. Now, $(F, E) = \{(e_1, \{a\}), (e_2, \varnothing)\}$ is a monotonic subset of $(X, E, \prec_1)$ because $d_{\prec_1}(F, E) = \{(e_1, \{a\}), (e_2, \varnothing)\} = (F, E)$; and $(G, E) = \{(e_1, \{2\}), (e_2, Y)\}$ is a monotonic subset of $(Y, E, \prec_2)$ because $i_{\leq_2}(G, E) = \{(e_1, \{2\}), (e_2, Y)\} = (G, E)$. On the contrary, their product $(F \times G, E \times E) = \{((e_1, e_1), \{(a, 2)\}), ((e_1, e_2), \{(a, 1), (a, 2)\}), ((e_2, e_1), \varnothing), ((e_2, e_2), \varnothing)\}$ is not a monotonic subset of $(X \times Y, E \times E, \prec)$ because $i_\prec (F \times G, E \times E) = \{((e_1, e_1), \{(a, 2), (b, 2)\}), ((e_1, e_2), X \times Y), ((e_2, e_1), \varnothing), ((e_2, e_2), \varnothing)\} \neq (F \times G, E \times E)$ and $d_\prec (F \times G, E \times E) = \{((e_1, e_1), \{(a, 1), (a, 2)\}), ((e_1, e_2), \{(a, 1), (a, 2)\}), ((e_2, e_1), \varnothing), ((e_2, e_2), \varnothing)\} \neq (F \times G, E \times E)$.

The following result demonstrates under what conditions the product of two monotonic soft sets is a monotonic soft set.

Proposition 6. *Let (F, E) and (G, E) be two subsets of $(X, E, \prec_1)$ and $(Y, E, \prec_2)$, respectively, such that (F, E) and (G, E) are both increasing (decreasing). Then, (F, E) and*

(G, E) are monotonic soft sets iff $(F, E) \times (G, E)$ is a monotonic soft set.

Proof. It follows immediately from Proposition 5.

In view of Theorem 2, we obtain the following result. □

Theorem 4. *If* $g: (X, E, \prec_1) \longrightarrow (Y, E, \prec_2)$ *is an order embedding map, then*

  (i) *The image of each monotonic soft set is monotonic*

  (ii) *The inverse image of each monotonic soft set is monotonic*

4. Monotonically Soft Compact Spaces

In this section, we introduce a concept of monotonic soft compact spaces and study their main properties. Also, we characterize them and demonstrate their relationships with ST_i-ordered spaces ($i = 2, 3, 4$). Finally, we investigate some results that associated the concept of monotonic compact spaces with monotonic limit points and some types of monotonic maps.

Definition 26. The collection $\{(G_j, E): j \in J\}$ of soft open subsets of $(X, \tau, E, \prec)$ is called a monotonic soft open cover for $(F, E) \widetilde{\subseteq} \widetilde{X}$ provided that $(F, E) \widetilde{\subseteq} \widetilde{\cup}_{j \in J} (G_j, E)$ and all (G_j, E) are monotonic.

Definition 27. $(X, \tau, E, \prec)$ is said to be monotonic soft compact provided that every monotonic soft open cover of $\widetilde{X}$ has a finite subcover.

One can easily prove the following result.

Proposition 7. *Every soft compact space is monotonic soft compact.*

The next example elucidates that the converse of the above proposition is not always true.

Example 3. Let $\tau = \{\widetilde{\varnothing}, (G, E) \widetilde{\subseteq} \widetilde{\mathcal{N}}$ such that $1 \in (G, E)\}$ be the particular point soft topology on the set of natural numbers $\mathcal{N}$, $E = \{e_1, e_2\}$ be a set of parameters, and $\prec = \triangle \cup \{(x, y): x$ is smaller than $y\}$ be a partial order relation on $\mathcal{N}$. Obviously, $(\mathcal{N}, \tau, E, \prec)$ is not soft compact. On the contrary, the only increasing open subsets of $(\mathcal{N}, \tau, \prec)$ are $\widetilde{\varnothing}$ and $\widetilde{\mathcal{N}}$. The decreasing open subsets of $(\mathcal{N}, \tau, E, \prec)$ are given in the form $(G_n, E) = \{(e_i, \{1, 2, \ldots, n\})\}$ for each $i = 1, 2$. Hence, $(\mathcal{N}, \tau, E, \prec)$ is monotonic soft compact.

In the following, we establish the main properties of a monotonic soft compact space which are similar to their counterparts on a soft compact space.

Proposition 8. *Every monotonic soft closed subset* (F, E) *of a monotonic soft compact space* $(X, \tau, E, \prec)$ *is monotonic soft compact.*

Proof. Let $\{(G_j, E): j \in J\}$ be a monotonic soft open cover of a monotonic soft closed subset (F, E) of $(X, \tau, E, \prec)$. Since $(F, E)^c$ is monotonic soft open, $\cup_{j \in J} (G_j, E) \bigcup (F, E)^c$ is a monotonic soft open cover of $\widetilde{X}$. Therefore, $\widetilde{X} = \widetilde{\cup}_{j=1}^{n} (G_j, E) \widetilde{\cup} (F, E)^c$. Thus, $(F, E) \widetilde{\subseteq} \widetilde{\cup}_{j=1}^{n} (G_j, E)$. Hence, (F, E) is monotonic soft compact. □

Corollary 2. *The intersection of monotonic soft closed and monotonic soft compact sets is monotonic soft compact.*

Theorem 5. *Let* (F, E) *be a monotonic soft compact subset of a strong p-soft* T_2*-ordered space* $(X, \tau, E, \prec)$. *If* $x \notin (F, E)$, *then there exist disjoint monotonic soft open sets* (W, E) *and* (V, E) *containing* x *and* (F, E), *respectively.*

Proof. Let (F, E) be a monotonic soft closed set such that $x \notin (F, E)$ and $y \in (F, E)$. Without loss of generality, suppose that (F, E) is increasing. Then, $y \prec x$. Since $(X, \tau, E, \prec)$ is strong p-soft T_2-ordered, there are an increasing soft open set (W_i, E) containing y and a decreasing soft open set (V_i, E) containing x such that (W_i, E) and (V_i, E) are disjoint. Therefore, $\{(W_i, E)\}$ forms an increasing soft open cover of (F, E). Thus, $(F, E) \widetilde{\subseteq} \widetilde{\cup}_{i=1}^{i=n} W_{i_E}$. Obviously, the two disjoint soft open sets $\widetilde{\cup}_{i=1}^{i=n} (W_i, E)$ and $\cap_{i=1}^{i=n} (V_i, E)$ are increasing and decreasing, respectively. Hence, the proof is completed. □

Theorem 6. *Every monotonic soft compact and strong p-soft* T_2*-ordered space* $(X, \tau, E, \prec)$ *is strongly p-soft regular ordered.*

Proof. Let (F, E) be a monotonic soft closed subset of $(X, \tau, E, \prec)$ such that $x \notin (F, E)$. Since $(X, \tau, E, \prec)$ is monotonic soft compact, (F, E) is monotonic soft compact. Since $(X, \tau, E, \prec)$ is strong p-soft T_2-ordered, it follows from Theorem 5 that there are two disjoint monotonic soft open sets (U, E) and (V, E) containing x and (F, E), respectively. Hence, $(X, \tau, E, \prec)$ is strongly p-soft regular ordered. □

Corollary 3. *Every monotonic soft compact and strong p-soft* T_2*-ordered space* $(X, \tau, E, \prec)$ *is strong p-soft* T_3*-ordered.*

Definition 28 (see [28]). For a nonempty set X, a subcollection Λ of 2^X is said to have the finite intersection property (for short, FIP) if any finite subcollection of Λ has a nonempty intersection.

Theorem 7. $(X, \tau, E, \prec)$ *is a monotonic soft compact space iff every collection of monotonic soft closed subsets of X, satisfying the FIP, has a nonempty soft intersection.*

Proof. Necessity: let $\Omega = \{(F_j, E): j \in J\}$ be the collection of monotonic soft closed subsets of $\widetilde{X}$ which has the FIP. Suppose that $\widetilde{\cap}_{j \in J} (F_j, E) = \widetilde{\varnothing}$. Then, $\widetilde{X} = \widetilde{\cup}_{j \in J} (F_j, E)^c$. Since $\widetilde{X}$ is monotonic soft compact, $\widetilde{X} = \widetilde{\cup}_{j=1}^{n} (F_j, E)^c$. Therefore, $\widetilde{\cap}_{j=1}^{n} (F_j, E) = \widetilde{\varnothing}$, but this contradicts that Ω has the FIP. Hence, Ω has a nonempty soft intersection.

Sufficiency: let $\{(G_j, E): j \in J\}$ be a monotonic soft open cover of $\widetilde{X}$. Suppose that $\{(G_j, E): j \in J\}$ has no finitely monotonic subcover. Then, $\widetilde{X} \backslash \widetilde{\cup}_{j=1}^{n} (G_j, E) \neq \widetilde{\varnothing}$ for each $n \in \mathcal{N}$. Therefore, $\widetilde{\cap}_{j=1}^{n} (G_j, E)^c \neq \widetilde{\varnothing}$. This implies that $\{(G_j, E)^c: j \in J\}$ is the collection of monotonic soft closed

subsets of $\widetilde{X}$ which has the FIP. By hypothesis, $\widetilde{\cap}_{j \in J}(G_j, E)^c \neq \widetilde{\varnothing}$. Thus, $\widetilde{X} \neq \widetilde{\cup}_{j \in J}(G_j, E)$, but this contradicts that Ω is a monotonic soft open cover of $\widetilde{X}$. Hence, $\widetilde{X}$ is monotonic soft compact. $\square$

Theorem 8. *The subspace* $(Y, \tau_Y, E, \prec_Y)$ *of* $(X, \tau, E, \prec)$ *is monotonic soft compact iff* $\widetilde{Y}$ *is a monotonic soft compact set in* $(X, \tau, E, \prec)$.

Proof. Necessity: let $\Omega = \{(G_j, E) : j \in J\}$ be the collection of monotonic soft open subsets of $(X, \tau, E, \prec)$ which cover $\widetilde{Y}$. Then, $\Omega' = \{(G_j, E) \widetilde{\cap} \widetilde{Y} : j \in J\}$ is the collection of monotonic soft open subsets of $(Y, \tau_Y, E, \prec_Y)$ which cover $\widetilde{Y}$. By hypothesis, we have $\widetilde{Y} = \widetilde{\cup}_{j=1}^n ((G_j, E) \widetilde{\cap} \widetilde{Y})$. Thus, $\widetilde{Y} = \widetilde{\cup}_{j=1}^n (G_j, E)$. Hence, $\widetilde{Y}$ is a monotonic soft compact set.

Sufficiency: let $\{(H_j, E) : j \in J\}$ be the collection of monotonic soft open subsets of $(Y, \tau_Y, E, \prec_Y)$ which cover $\widetilde{Y}$. Since $(H_j, E) = \widetilde{Y} \widetilde{\cap} (G_j, E)$ for some $(G_j, E) \in \tau$, $\{(G_j, E) : j \in J\}$ is a monotonic soft open cover of $\widetilde{Y}$. By hypothesis, we have $\widetilde{Y} = \widetilde{\cup}_{j=1}^n (G_j, E)$. Thus, $\widetilde{Y} = \widetilde{\cup}_{j=1}^n ((G_j, E) \widetilde{\cap} \widetilde{Y})$. Hence, $(Y, \tau_Y, E, \prec_Y)$ is monotonic soft compact. $\square$

Definition 29. Let (H, E) be a soft subset of $(X, \tau, E, \prec)$ and $P_e^x \in \widetilde{X}$. A soft point P_e^x is said to be a monotonic soft limit point of (H, E) if $[(G, E) \setminus P_e^x] \widetilde{\cap} (H, E) \neq \widetilde{\varnothing}$ for every monotonic soft open set (G, E) containing P_e^x.

The soft set of all monotonic soft limit points of (H, E) is denoted by $(H, E)^{ml}$.

It is clear that the limit points of a set (H, E) are a subset of the monotonic limit point of (H, E). The next example shows that the converse need not be true in general.

Example 4. Consider $\tau = \{\varnothing, X, \{b\}, \{a, c\}\}$ is a topology on $X = \{a, b, c\}$, and let $\prec = \triangle \cup \{(a, b), (b, c), (a, c)\}$ be a partial order relation on X. Let $A = \{b, c\}$. Then, $A^l = \{a\}$ and $A^{ml} = X$. Hence, A^l is a proper subset of A^{ml}.

Theorem 9. *Let* (H, E) *be a subset of* $(X, \tau, E, \prec)$. *Then, the following results hold:*

(i) (H, E) *is monotonic soft closed iff* $(H, E)^{ml} \subseteq A$

(ii) $(H, E) \cup (H, E)^{ml}$ *is monotonic soft closed*

Proof. We only prove (i), and one can prove (ii) similarly.

Necessity: assume that (H, E) is a monotonic soft closed set and $P_e^x \notin (H, E)$. Then, $P_e^x \in (H, E)^c$. Since $(H, E)^c$ is monotonic soft open and $(H, E)^c \cap (H, E) = \widetilde{\varnothing}$, $P_e^x \notin (H, E)^{ml}$. Therefore, $(H, E)^{ml} \subseteq (H, E)$.

Sufficiency: let $P_e^x \in (H, E)^c$ and $(H, E)^{ml} \subseteq (H, E)$. Then, $P_e^x \notin (H, E)^{ml}$. Therefore, there is a monotonic soft open set (G_{xe}, E) such that $(G_{xe}, E) \setminus \{P_e^x\} \widetilde{\cap} (H, E) = \widetilde{\varnothing}$. Since $P_e^x \in (H, E)^c$, $(G_{xe}, E) \widetilde{\cap} (H, E) = \varnothing$. Now, $(G_{xe}, E) \subseteq (H, E)^c$. Therefore, $(H, E)^c = \widetilde{\cup} \{(G_{xe}, E) : P_e^x \in (H, E)^c\}$. Thus, $(H, E)^c$ is a monotonic soft open set. Hence, (H, E) is monotonic soft closed. $\square$

Corollary 4. *If* (H, E) *is a soft subset of a monotonic soft compact space* $(X, \tau, E, \prec)$, *then* $(H, E) \widetilde{\cup} (H, E)^{ml}$ *is monotonic soft compact.*

Theorem 10. *Every infinite soft subset of a monotonic soft compact space* $(X, \tau, E, \prec)$ *has a monotonic soft limit point.*

Proof. Suppose that (H, E) is an infinite soft subset of a monotonic soft compact space $(X, \tau, E, \prec)$. Suppose that (H, E) does not have a monotonic soft limit point. Then, for each $P_e^x \in \widetilde{X}$, we have a monotonic soft open set (G_{xe}, E) containing P_e^x such that $(G_{xe}, E) \widetilde{\cap} (H, E) \subseteq \{P_e^x\}$. Now, the collection $\Omega = \{(G_{xe}, E) : P_e^x \in \widetilde{X}\}$ forms a monotonic soft open cover of $\widetilde{X}$. Since $\widetilde{X}$ is monotonic soft compact, $\widetilde{X} = \widetilde{\cup}_{j=1}^n (G_{xe}, E)$. Therefore, $\widetilde{X}$ has at most n soft points of (H, E). This implies that (H, E) is finite, but this contradicts the infinity of (H, E). Thus, (H, E) has a monotonic soft limit point. $\square$

Definition 30. A soft map $g : (X, \tau, E, \prec_1) \longrightarrow (Y, \theta, E, \prec_2)$ is said to be

(i) monotonic soft continuous if the preimage of every monotonic soft open set is a monotonic soft open set

(ii) monotonic soft open (resp. monotonic soft closed) if the image of every monotonic soft open (resp. monotonic soft closed) set is a monotonic soft open (resp. monotonic soft closed) set

(iii) monotonic soft homeomorphism if it is bijective, monotonic soft continuous, and monotonic soft open

Proposition 9. *The property of being a monotonic soft compact set is preserved under a monotonic soft continuous map.*

Proof. Let $g : (X, \tau, E, \prec_1) \longrightarrow (Y, \theta, E, \prec_2)$ be a monotonic soft continuous map, and let (F, E) be a monotonic soft compact subset of $\widetilde{X}$. Suppose that $\{(H_j, E) : j \in J\}$ is a monotonic soft open cover of $g(F, E)$. Then, $(F, E) \subseteq \widetilde{\cup}_{j \in J} g^{-1}(H_j, E)$. Since g is monotonic soft continuous, $g^{-1}(H_j, E)$ is a monotonic soft open set for all $j \in J$. Since (F, E) is monotonic soft compact, $g(F, E) \subseteq \cup_{j=1}^n (H_j, E)$. Hence, $g(F, E)$ is monotonic soft compact.

The following is still an open problem. $\square$

Problem 1. Is the product of monotonic soft compact spaces a monotonic soft compact space?

Definition 31 (see [3]). A triple $(X, \tau, \prec)$ is said to be a topological ordered space if $(X, \prec)$ is a partially ordered set and (X, τ) is a topological space.

Recall that $(X, \tau, \prec)$ is said to be monotonic compact if every monotonic open cover of X has a finite subcover.

Theorem 11. *Let $(X, \tau_e, \prec)$ be monotonic compact for each $e \in E$. Then, $(X, \tau, E, \prec)$ is a monotonic soft compact space if E is finite.*

Proof. Let $\left\{(G_j, E): j \in J\right\}$ be a monotonic soft open cover of $(X, \tau, E, \prec)$. Without loss of generality, suppose that $E = \{e_1, e_2\}$. Then, the collections $\left\{G_j(e_1): j \in J\right\}$ and $\left\{G_j(e_2): j \in J\right\}$ are monotonic open covers of $(X, \tau_{e_1}, \prec)$ and $(X, \tau_{e_2}, \prec)$, respectively. By hypothesis, there exist finitely two subsets M and N of J such that $X = \cup_{j \in M} G_j(e_1)$ and $X = \cup_{j \in N} G_j(e_1)$. Therefore, $\widetilde{X} = \widetilde{\cup}_{j \in M \cup N}(G_j, E)$. Thus, $(X, \tau, E, \prec)$ is monotonic soft compact.

To show that a finite condition of E is necessary, we give the following example. $\square$

Example 5. Let a set of parameters be the set of natural numbers $\mathcal{N}$, and let τ be a soft discrete topology on $X = \{1, 2\}$. If $\prec = \triangle \cup \{(1, 2)\}$ is a partial order relation on X, then the collection Λ of all soft points of $\widetilde{X}$ is a monotonic soft open cover of $\widetilde{X}$. Obviously, Λ has no finite subcover. Therefore, $(X, \tau, \mathcal{N}, \prec)$ is not monotonic soft compact. On the contrary, $(X, \tau_{e_n}, \prec)$ is soft compact for each $n \in \mathcal{N}$.

Theorem 12. *Let $(X, \tau, E, \prec)$ be an extended soft topological space. If $(X, \tau, E, \prec)$ is a monotonic soft compact space, then $(X, \tau_e, \prec)$ is monotonic compact for each $e \in E$.*

Proof. Let $\left\{H_j(e): j \in J\right\}$ be a monotonic open cover of (X, τ_e). Since $(X, \tau, E, \prec)$ is extended, we choose all monotonic soft open sets (F_j, E) such that $F_j(e) = H_j(e)$ and $F_j(e_i) = X$ for all $e_i \neq e$. Obviously, $\left\{(F_j, E): j \in J\right\}$ is a monotonic soft open cover of $(X, \tau, E, \prec)$. By hypothesis, it follows that $\widetilde{X} = \widetilde{\cup}_{j=1}^{n}(F_j, E)$. Thus, $X = \cup_{j=1}^{n} F_j(e) = \cup_{j=1}^{n} H_j(e)$. Hence, $(X, \tau_e, \prec)$ is monotonic compact.

Now, we give a condition which guarantees the converse of the above theorem holds. $\square$

Proposition 10. *Let $(X, \tau, E, \prec)$ be an extended soft topological space such that E is finite. Then, $(X, \tau, E, \prec)$ is a monotonic soft compact space iff $(X, \tau_e, \prec)$ is monotonic compact for some $e \in E$.*

Proof. The proof follows from Theorems 11 and 12. $\square$

5. Ordered Soft Compact Spaces and Applications on the Information System

This section presents a concept of ordered compact spaces and shows their relationships with monotonic compact and compact spaces. Also, it investigates their relationships with T_i-ordered spaces $(i = 2, 3, 4)$ and bicontinuous maps and shows that the product of ordered compact spaces need not be ordered compact. Finally, it gives an interesting application of ordered compact spaces on the information system.

5.1. Ordered Soft Compact Spaces

Definition 32. $(X, \tau, E, \prec)$ is said to be ordered soft compact if every soft open cover of $\widetilde{X}$ has a finitely monotonic subcover.

Proposition 11. *Every ordered soft compact space is soft compact.*

Proof. Straightforward. $\square$

Corollary 5. *Every ordered soft compact space is monotonic soft compact.*

To see that the converse of the above two results need not be true, we give the next example.

Example 6. Consider τ is the discrete topology on $X = \{a, b, c, d\}$, and let $\prec = \triangle \cup \{(a, b)(c, d)\}$ be a partial order relation on X. Then, $(X, \tau, E, \prec)$ is compact because X is finite. Moreover, it is monotonic compact. On the contrary, the collection $\{\{a\}, \{b, c\}, \{d\}\}$ is an open cover of X. Since this collection has no finitely monotonic subcover, $(X, \tau, E, \prec)$ is not ordered compact.

The above example also shows that a finite topological space need not be ordered compact.

Definition 33. The collection $\Lambda = \left\{(F_j, E): j \in J\right\}$ of $S(X_E)$ is said to be minimal if every member of Λ covers some soft points of $\widetilde{X}$ which do not cover by any other members of Λ. In other words, removing any member (F_{j_0}, E) of Λ implies that $\Lambda \smallsetminus (F_{j_0}, E)$ is not a cover of $\widetilde{X}$.

Definition 34. The collection $\Lambda = \left\{(F_j, E): j \in J\right\}$ of $P(X)$ is said to have the finite monotone property (FMP, in short) if all the minimal subcollections of Λ which covers $\widetilde{X}$ are monotonic.

Example 7. Consider τ is the discrete topology on the set of real numbers $\mathcal{R}$. Then, the two collections $\{\mathcal{R} \smallsetminus \{1\}, \mathcal{R} \smallsetminus \{2\}, \mathcal{R} \smallsetminus \{3\}\}$ and $\{\{x, y\}: x, y \in \mathcal{R}\}$ are not minimal. On the contrary, the two collections $\{\mathcal{R} \smallsetminus \{1\}, \mathcal{R} \smallsetminus \{2\}\}$ and $\{\{x\}: x \in \mathcal{R}\}$ are minimal.

Proposition 12. *If every collection of the soft open cover of $(X, \tau, E, \prec)$ satisfies the FMP, then $(X, \tau, E, \prec)$ is ordered soft compact iff it is soft compact.*

Proof. Necessity: it follows from Proposition 11.

Sufficiency: let $\left\{(G_j, E): j \in J\right\}$ be a soft open cover of $(X, \tau, E, \prec)$. By compactness, we have $\widetilde{X} = \widetilde{\cup}_{j=1}^{n}(G_j, E)$. Now, $\left\{(G_j, E): j = 1, 2, \ldots, n\right\}$ is a minimal collection covering $\widetilde{X}$. Since $(X, \tau, E, \prec)$ has the FMP, (G_j, E) is monotonic for each $j = 1, 2, \ldots, n$. Hence, $(X, \tau, E, \prec)$ is ordered soft compact. $\square$

Proposition 13. *Every soft closed subset F of an ordered soft compact space $(X, \tau, E, \prec)$ is ordered soft compact.*

Proof. Let $\{(G_j, E): j \in J\}$ be a soft open cover of a soft closed subset (F, E) of $\widetilde{X}$. Then, F^c, E is soft open. Therefore, $\widetilde{\cup}_{j \in J} (G_j, E) \widetilde{\cup} (F^c, E)$ is a soft open cover of $\widetilde{X}$. By hypothesis, $\widetilde{X}$ is ordered soft compact; then, $\widetilde{X} = \widetilde{\cup}_{j=1}^n (G_j, E) \widetilde{\cup} (F^c, E)$, where (G_j, E) is a monotonic soft open set for each $j = 1, 2, \ldots, n$. Thus, $(F, E) \subseteq \widetilde{\cup}_{j=1}^n (G_j, E)$. Hence, (F, E) is ordered soft compact. $\square$

Corollary 6. *The intersection of soft closed and ordered soft compact sets is ordered soft compact.*

Corollary 7. *Let (F, E) be a soft closed subset of an ordered soft compact space $(X, \tau, E, \prec)$ such that (F, E) is not monotonic. Then, every monotonic cover of (F, E) is also a monotonic cover of $\widetilde{X}$.*

Lemma 2. *Let (F, E) be a monotonic and ordered soft compact subset of a p-soft T_2-ordered space $(X, \tau, E, \prec)$. If $x \widetilde{\notin} (F, E)$, then there are two monotonic soft neighbourhoods (U, E) and (V, E) of x and (F, E), respectively, such that $(U, E) \widetilde{\cap} (V, E) = \widetilde{\varnothing}$.*

Proof. Let (F, E) be a monotonic ordered soft compact set such that $x \widetilde{\notin} (F, E)$ and $y \widetilde{\in} (F, E)$. Without loss of generality, suppose that (F, E) is increasing. Then, $y \prec x$. Since $(X, \tau, E, \prec)$ is $\underline{p}$-soft T_2-ordered, there are an increasing soft neighbourhood (W_i, E) of y and a decreasing soft neighbourhood (V_i, E) of x such that (W_i, E) and (V_i, E) are disjoint. Therefore, there is a soft open set (M_j, E) containing y such that $(M_j, E) \subseteq (W_j, E)$. Now, $\{(M_j, E): j \in J\}$ is a soft open cover of (F, E). By hypothesis, $(F, E) \subseteq \widetilde{\cup}_{j=1}^n (M_j, E)$, where (M_j, E) is a monotonic soft open set for each $j = 1, 2, \ldots, n$. Thus, $(F, E) \subseteq \widetilde{\cup}_{j=1}^n (W_j, E)$. Obviously, $\widetilde{\cup}_{j=1}^n (W_j, E)$ is a decreasing neighbourhood of (F, E), and $\widetilde{\cap}_{j=1}^n (V_j, E)$ is an increasing neighbourhood of x such that $\widetilde{\cup}_{j=1}^n (W_j, E)$ and $\widetilde{\cap}_{j=1}^n (V_j, E)$ are disjoint. Hence, the proof is completed. $\square$

Theorem 13. *Every ordered soft compact and p-soft T_2-ordered space $(X, \tau, E, \prec)$ is p-soft regularly ordered.*

Proof. The proof is similar to that of Theorem 6. $\square$

Corollary 9. *Every ordered soft compact and p-soft T_2-ordered space $(X, \tau, E, \prec)$ is p-soft T_3-ordered.*

Theorem 14. *$(X, \tau, E, \prec)$ is an ordered soft compact space iff every collection of soft closed subsets of $\widetilde{X}$, satisfying the FIP for the monotonic soft sets in this collection, has a nonempty intersection.*

Proof. Necessity: let $\Omega = \{(F_j, E): j \in J\}$ be the collection of soft closed subsets $\widetilde{X}$ which has the FIP for the monotonic soft sets in this collection. Suppose that $\widetilde{\cap}_{j \in J} (F_j, E) = \widetilde{\varnothing}$. Then, $\widetilde{X} = \widetilde{\cup}_{j \in J} (F_j, E)^c$. Since $\widetilde{X}$ is ordered soft compact, $\widetilde{X} = \widetilde{\cup}_{j=1}^n (F_j, E)^c$, where $(F_j, E)^c$ is a monotonic soft set for each $j = 1, 2, \ldots, n$. Therefore, $\widetilde{\cap}_{j=1}^n (F_j, E) = \widetilde{\varnothing}$, but this

contradicts that Ω has the FIP for every monotonic soft set in this collection. Hence, Ω has a nonempty intersection.

Sufficiency: let $\{(G_j, E): j \in J\}$ be a soft open cover of $\widetilde{X}$. Suppose that $\{(G_j, E): j \in J\}$ has no finitely monotonic subcover. Then, $\widetilde{X} \backslash \widetilde{\cup}_{j=1}^n (G_j, E) \neq \widetilde{\varnothing}$ for each $n \in \mathcal{N}$, where (G_j, E) is a monotonic soft set for $j = 1, 2, \ldots, n$. Therefore, $\widetilde{\cap}_{j=1}^n (G_j, E)^c \neq \widetilde{\varnothing}$. This implies that $\{(G_j, E)^c: j \in J\}$ is the collection of monotonic soft closed subsets of $\widetilde{X}$ which has the FIP. Thus, $\widetilde{\cap}_{j \in J} (G_j, E)^c \neq \widetilde{\varnothing}$. Thus, $\widetilde{X} \neq \widetilde{\cup}_{j \in J} (G_j, E)$, but this contradicts that Ω is a soft open cover of X. Hence, X is ordered soft compact. $\square$

Proposition 14. *If $\widetilde{Y}$ is an ordered soft compact set in $(X, \tau, E, \prec)$, then a subspace $(Y, \tau_Y, \prec_Y)$ of $(X, \tau, E, \prec)$ is ordered soft compact.*

Proof. Let $\{(H_j, E): j \in J\}$ be the collection of soft open subsets of $(Y, \tau_Y, \prec_Y)$ which covers $\widetilde{Y}$. Since $(H_j, E) = \widetilde{Y} \widetilde{\cap} (G_j, E)$ for some $(G_j, E) \in \tau$, $\{(G_j, E): j \in J\}$ is a soft open cover of $\widetilde{Y}$ in $(X, \tau, E, \prec)$. By hypothesis, we have $\widetilde{Y} = \widetilde{\cup}_{j=1}^n (G_j, E)$, where (G_j, E) is a monotonic soft set for $j = 1, 2, \ldots, n$. Thus, $\widetilde{Y} = \widetilde{\cup}_{j=1}^n ((G_j, E) \widetilde{\cap} \widetilde{Y})$, where $(G_j, E) \widetilde{\cap} \widetilde{Y}$ is a monotonic soft set for $j = 1, 2, \ldots, n$ in $(Y, \tau_Y, \prec_Y)$. Hence, $(Y, \tau_Y, \prec_Y)$ is ordered soft compact.

The converse of the above proposition fails as illustrated in the following example. $\square$

Example 8. Let $(X, \tau, E, \prec)$ be the same as in Example 6. If $Y = \{a, b, c\}$, then τ_Y is the discrete topology on Y, and $\prec_Y = \triangle \cup \{(a, b)\}$ is a partial order relation on Y. It is clear that $(Y, \tau_Y, \prec_Y)$ is an ordered compact space. On the contrary, the collection $\{\{a\}, \{b, c\}\}$ is an open cover of Y in X. Since $i(\{b, c\}) = \{b, c, d\}$ and $d(\{b, c\}) = \{a, b, c\}$, this collection has no finitely monotonic subcover of Y. Hence, Y is not an ordered compact subset of $(X, \tau, E, \prec)$.

Theorem 15. *Every infinite subset of an ordered soft compact space has a soft limit point.*

Proof. The proof is similar to that of Theorem 10. $\square$

Theorem 16. *The property of being an ordered soft compact set is preserved under a soft bicontinuous surjective and soft ordered embedding map.*

Proof. Let $g: (X, \tau, \prec_1) \longrightarrow (Y, \theta, \prec_2)$ be a soft bicontinuous map, and let (F, E) be an ordered soft compact subset of $\widetilde{X}$. Suppose that $\{(H_j, E): j \in J\}$ is a soft open cover of $g(F, E)$. Then, $(F, E) \subseteq \widetilde{\cup}_{j \in J} g^{-1}(H_j, E)$. Since g is soft continuous, $g^{-1}(H_j, E)$ is a soft open set for all $j \in J$. By hypothesis, (F, E) is ordered soft compact; then, $(F, E) \subseteq \widetilde{\cup}_{j=1}^n g^{-1}(H_j, E)$, where $g^{-1}(H_j, E)$ is a monotonic soft set for each $j = 1, 2, \ldots, n$. Since g is soft open and soft ordered embedding, $g[g^{-1}(H_j, E)]$ is a monotonic soft open set for each $j = 1, 2, \ldots, n$. Now, $g(F, E) \subseteq \widetilde{\cup}_{j=1}^n g[g^{-1}(H_j, E)]$. Since g is surjective, $g[g^{-1}(H_j, E)] = (H_j, E)$ for each $j \in J$. Thus, $g(F, E) \subseteq \cup_{j=1}^n (H_j, E)$ where (H_j, E) is a monotonic soft

open set for each $j = 1, 2, \ldots, n$. Hence, $g(F, E)$ is an ordered soft compact set.

We complete this section with the next example which shows that the product of ordered compact spaces need not be ordered compact. □

Example 9. Let τ and θ be the discrete topologies on $X = \{a, b\}$ and $Y = \{1, 2\}$, respectively. Let $\prec_1 = \triangle \cup \{(a, b)\}$ and $\prec_2 = \triangle \cup \{(1, 2)\}$ be the partial order relations on X and Y, respectively. One can check that $(X, \tau, \prec_1)$ and $(Y, \theta, \prec_2)$ are ordered compact spaces. Now, the product topology T on $X \times Y = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$ is the discrete topology. From Definition 12, the partial order relation $\prec$ on $X \times Y$ is given as follows: $\prec = \triangle \cup \{((a, 1), (a, 2)), ((a, 1), (b, 1)), ((a, 1), (b, 2)), ((a, 2), (b, 2))\}$. It is clear that the collection $\Omega = \{\{(a, 1)\}, \{(a, 2)\}, \{(b, 1)\}, \{(b, 2)\}\}$ forms an open cover of $X \times Y$. Since $i(\{(a, 2)\}) = \{(a, 2), (b, 2)\}$ and $d(\{(a, 2)\}) = \{(a, 1), (a, 2)\}, \{(a, 2)\}$ is not a monotonic subset of $X \times Y$. Since Ω is minimal, $(X \times Y, T, \prec)$ is not an ordered compact space.

Recall that $(X, \tau, \prec)$ is said to be ordered compact if every open cover of X has a finitely monotonic subcover.

Theorem 17. *Let $(X, \tau, E, \prec)$ be an extended soft topological space. If $(X, \tau, E, \prec)$ is an ordered soft compact space, then $(X, \tau_e, \prec)$ is ordered compact for each $e \in E$.*

Proof. Let $\{H_j(e): j \in J\}$ be an open cover of (X, τ_e). Since $(X, \tau, E, \prec)$ is extended, we choose all soft open sets (F_j, E) such that $F_j(e) = H_j(e)$ and $F_j(e_i) = X$ for all $e_i \neq e$. Obviously, $\{(F_j, E): j \in J\}$ is a soft open cover of $(X, \tau, E, \prec)$. By hypothesis, it follows that $\widetilde{X} = \widetilde{\cup}_{j=1}^{n}(F_j, E)$, where (F_j, E) is monotonic for each $j = 1, 2, \ldots, n$. Thus, $X = \cup_{j=1}^{n} F_j(e) = \cup_{j=1}^{n} H_j(e)$ such that $H_j(e)$ is monotonic for each $j = 1, 2, \ldots, n$. Hence, $(X, \tau_e, \prec)$ is ordered compact. □

5.2. An Application of Ordered Compactness on the Information System.

In this part, we investigate an application of ordered compact spaces on the information system. It is well known that study compactness on the information system is meaningless because it is defined on a finite set so that this study is the first attempt of discussing a new type of compactness on the information system.

Definition 35 (see [31]). The information system is a pair of two nonempty sets (X, A) such that X is a finite set of objects and A is a finite set of attributes.

To start this part, consider Table 1 which represents a decision system.

It is well known that the equivalence classes form a soft basis for a soft topology on the universe set X such that every member of this basis is a soft clopen set.

We refer to a soft topology which was generated by the attributes a_i and $a_{i,j}$ by τ_{a_i} and $\tau_{a_{i,j}}$, respectively; and we refer to a soft topology which was generated by an attribute of decision d by τ_d.

TABLE 1: Information system.

X	A			
	a_1	a_2	a_3	Decision
(G_1, E)	2	2	4	Accept
(G_2, E)	3	1	3	Accept
(G_3, E)	2	2	1	Reject
(G_4, E)	3	1	3	Accept
(G_5, E)	1	1	2	Reject
(G_6, E)	4	3	2	Accept

From Table 1, we generate a soft topology on X with respect to an attribute a_2 as follows.

First, we find the basis: $B = \{\{(G_6, E)\}, \{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E)\}\}$.

Second, we find a soft topology on X from this soft basis: $\tau_{a_2} = \{\widetilde{\varnothing}, \widetilde{X}, \{(G_6, E)\}, \{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E)\}, \{(G_1, E), (G_3, E), (G_6, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, (G_5, E), (G_6, E)\}, \{(G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E)\}$.

In a similar way, one can generate soft topologies on X with respect to the attributes a_1, a_3, and d.

Theorem 18. *If $(X, \tau_{a_i}, E, \prec)$ is an ordered soft compact space, then its soft basis β_{a_i} is monotonic.*

Proof. From the definition of a soft basis β_{a_i}, we find that the members of it are disjoint soft clopen sets such that $\widetilde{X} = \cup_{j \in J}(B_j, E)$. Since X is finite, J must be finite, and since $(X, \tau_{a_i}, \prec)$ is an ordered soft compact space, all (B_j, E) are monotonic soft sets.

The converse of the above theorem is not always true as illustrated in the following example. □

Example 10. From Table 1, a soft basis with respect to an attribute a_3 is $B = \{\{(G_1, E)\}, \{(G_3, E)\}, \{(G_2, E), (G_4, E)\}, \{(G_5, E), (G_6, E)\}\}$. A soft topology on X from this soft basis is $\tau_{a_3} = \{\widetilde{\varnothing}, \widetilde{X}, \{(G_1, E)\}, \{(G_3, E)\}, \{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E)\}, \{(G_1, E), (G_2, E), (G_4, E)\}, \{(G_2, E), (G_3, E), (G_4, E)\}, \{(G_1, E), (G_2, E), (G_3, E), (G_4, E)\}, \{(G_1, E), (G_5, E), (G_6, E)\}, \{(G_3, E), (G_5, E), (G_6, E)\}, \{(G_1, E), (G_3, E), (G_5, E), (G_6, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_1, E), (G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E)\}\}$.

Let $\prec = \triangle \bigcup \{(x_1, x_5), (x_2, x_3)\}$ be a partial order relation on X. Then, the basis $B = \{\{(G_1, E)\}, \{(G_3, E)\}, \{(G_2, E), (G_4, E)\}, \{(G_5, E), (G_6, E)\}\}$ is monotonic. On the contrary, the collection $\{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E)\}, \{(G_5, E), (G_6, E)\}\}$ is a soft open cover of $(X, \tau_{a_3}, E, \prec)$. Since this a soft open cover does not have a finitely monotonic subcover, $(X, \tau_{a_3}, E, \prec)$ is not ordered soft compact.

Theorem 19. *Let $a_i: i = 1, 2, \ldots, n$ be superfluous attributes. Then, a soft topology generated by other attributes, which are not superfluous, is ordered soft compact iff a soft topology generated by a decision attribute is ordered soft compact.*

Proof. Without loss of generality, suppose that a_1 is a superfluous attribute. Then, a soft topology generated by all attributes a_j such that $j \neq 1$ is identical with a soft topology generated by a decision attribute. Hence, the desired result is proved.

In the rest of this part, we study the importance of ordered soft compactness to expect the missing values of the table of the information system. To this end, we present an algorithm for expecting the missing values on the information system, and then we provide two illustrative examples.

An algorithm for expecting the missing values on the information system is given.

(1) Determine the objects (which are soft sets here) with a missing value with respect to the given attribute. Say, these objects are $\{(G_j, E) : j \in \{1, 2, \ldots, m\}$, where $m < |X|\}$.

(2) Classify these objects in terms of increasing and decreasing with respect to the given partial order relation $\prec$ in the following three cases:

   (i) Neither increasing nor decreasing
   (ii) Increasing, but not decreasing, or decreasing, but not increasing
   (iii) Increasing and decreasing

(3) Write the soft basis B which generated a soft topology $\tau \smallsetminus \{(G_j, E) : j \in \{1, 2, \ldots, m\}$, where $m < |X|\}$.

(4) If there is a nonmonotonic member of B, then add the sufficient objects with the missing value to make it monotonic. Otherwise, go to the next step.

(5) Examine the objects with the missing value according to the order of them in Step 2.

(6) Write the expected values for each missing value object.

The following two examples are the application of carrying out this algorithm.   □

Example 11. Consider $(X, \tau, E, \prec)$ is an ordered compact space, where τ is a topology generated by an attribute a, which is illustrated in Table 2, on the universe set $X = \{(G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E)\}$, and let $\prec = \triangle \bigcup \{((G_1, E), (G_3, E)), ((G_3, E), (G_4, E)), ((G_1, E), (G_4, E)), ((G_5, E), (G_6, E))\}$ be a partial order relation on X.

According to Table 2, the basis of a soft topology on $X \smallsetminus \{(G_3, E)\}$ is $\{\{(G_1, E), (G_4, E)\}, \{(G_2, E), (G_6, E)\}, \{(G_5, E), (G_7, E)\}\}$. It is clear that the only object with the missing value is (G_3, E). This object is neither increasing nor decreasing. Now, the ordered soft compactness of $(X, \tau, E, \prec)$ implies that a basis must be monotonic. However, a member $\{(G_1, E), (G_4, E)\}$ of B is nonmonotonic. Thus, (G_3, E) must belong to the member $\{(G_1, E), (G_4, E)\}$. In other words, the expected value of (G_3, E) is 3. Hence, Table 2 shall take the following form.

TABLE 2: The value of (G_i, E): $i = 1, 2, \ldots, 7$ with respect to attribute a.

X	(G_1, E)	(G_2, E)	(G_3, E)	(G_4, E)	(G_5, E)	(G_6, E)	(G_7, E)
a	3	2	×	3	4	2	1
X	(G_1, E)	(G_2, E)	(G_3, E)	(G_4, E)	(G_5, E)	(G_6, E)	(G_7, E)
a	3	2	3	3	4	2	1

Example 12. Consider $(X, \tau, E, \prec)$ is an ordered soft compact space, where τ is a soft topology generated by an attribute a, which is illustrated in Table 3, on the universe set $X = \{(G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E), (G_7, E), (G_8, E), (G_9, E), (G_{10}, E)\}$, and let $\prec = \triangle \bigcup \{((G_2, E), (G_4, E)), ((G_4, E), (G_5, E)), ((G_2, E), (G_5, E)), ((G_5, E), (G_6, E)), ((G_2, E), (G_6, E)), ((G_4, E), (G_6, E)), ((G_8, E), (G_7, E))\}$ be a partial order relation on X.

According to Table 3, the basis B of a soft topology on $X \smallsetminus \{(G_2, E), (G_5, E), (G_{10}, E)\}$ is $\{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$. Since $(X, \tau, E, \prec)$ is ordered soft compact, a basis must be monotonic, but a member $\{(G_4, E), (G_6, E)\}$ of B is neither increasing nor decreasing so that we add (G_5, E) to $\{(G_4, E), (G_6, E)\}$. Thus, the expected value of (G_5, E) is 3. Hence, we have a new basis $B' = \{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$ on $X \smallsetminus \{(G_2, E), (G_{10}, E)\}$.

Now, we process the remaining two missing values separately. Since (G_2, E) is a decreasing singleton, but not increasing, and (G_{10}, E) is an increasing and decreasing singleton, we choose (G_2, E) secondly according to item 2 of the algorithm given above. It is clear that if we add (G_2, E) to any decreasing member of B', we obtain a monotonic basis so that we have the following three cases for the new basis B'':

   (i) $\{\{(G_1, E), (G_2, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$
   (ii) $\{\{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$
   (iii) $\{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_2, E), (G_8, E), (G_9, E)\}\}$

Hence, the expected values of (G_2, E) are 1, 2, or 3.

Finally, (G_{10}, E) is a decreasing and an increasing singleton; it can be added to any member of $B'' \smallsetminus \{(G_{10}, E)\}$ so that we have the following cases for the new basis:

   (i) $\{\{(G_1, E), (G_2, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}, \{(G_{10}, E)\}\}$
   (ii) $\{\{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}, \{(G_{10}, E)\}\}$
   (iii) $\{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_2, E), (G_7, E)\}, \{(G_8, E), (G_9, E)\}, \{(G_{10}, E)\}\}$
   (iv) $\{\{(G_1, E), (G_2, E), (G_3, E), (G_{10}, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$
   (v) $\{\{(G_1, E), (G_3, E), (G_{10}, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$

TABLE 3: The value of (G_i, E): i =1, 2, ..., 10 with respect to attribute a.

X	(G_1, E)	(G_2, E)	(G_3, E)	(G_4, E)	(G_5, E)	(G_6, E)	(G_7, E)	(G_8, E)	(G_9, E)	(G_{10}, E)
a	1	×	1	3	×	3	2	5	5	×

(vi) $\{\{(G_1, E), (G_3, E), (G_{10}, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_2, E), (G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$

(vii) $\{\{(G_1, E), (G_2, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E), (G_{10}, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$

(viii) $\{\{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E), (G_{10}, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$

(ix) $\{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E), (G_{10}, E)\}, \{(G_2, E), (G_7, E)\}, \{(G_8, E), (G_9, E)\}\}$

(x) $\{\{(G_1, E), (G_2, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E), (G_{10}, E)\}, \{(G_8, E), (G_9, E)\}\}$

(xi) $\{\{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E), (G_{10}, E)\}, \{(G_8, E), (G_9, E)\}\}$

(xii) $\{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_2, E), (G_7, E), (G_{10}, E)\}, \{(G_8, E), (G_9, E)\}\}$

(xiii) $\{\{(G_1, E), (G_2, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E), (G_{10}, E)\}\}$

(xiv) $\{\{(G_1, E), (G_3, E)\}, \{(G_2, E), (G_4, E), (G_5, E), (G_6, E)\}, \{(G_7, E)\}, \{(G_8, E), (G_9, E), (G_{10}, E)\}\}$

(xv) $\{\{(G_1, E), (G_3, E)\}, \{(G_4, E), (G_5, E), (G_6, E)\}, \{(G_2, E), (G_7, E)\}, \{(G_8, E), (G_9, E), (G_{10}, E)\}\}$

Hence, the expected values of (G_{10}, E) are 1, 2, 3, or 5, or any value different than them.

6. Conclusion

This research paper has studied the concepts of monotonic soft compact and ordered soft compact spaces using monotonic soft sets. These two concepts are considered as an extension of soft compact spaces. We have described them using the finite intersection property and have showed the relationship between them with the help of examples. Also, we have established some results related to soft ordered separation axioms and the finite product space. Furthermore, we have discussed preserving these concepts between the soft topological ordered space and its parametric soft topological ordered spaces. Finally, we give an interesting application of ordered soft compact spaces on the information system. In the upcoming works, we plan to investigate the following schemes:

(i) Study the concepts of almost soft compact and mildly soft compact spaces on ordered setting

(ii) Establish the concepts introduced in this work on some generalizations of soft topological ordered spaces such as soft bitopological ordered and supra soft topological spaces

(iii) Carry out further studies concerning the applications of ordered soft compactness on the information system by making use of the application presented in [32, 33]

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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