

Research Article

Lacunary \mathcal{F} -Invariant Convergence of Sequence of Sets in Intuitionistic Fuzzy Metric Spaces

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The concepts of invariant convergence, invariant statistical convergence, lacunary invariant convergence, and lacunary invariant statistical convergence for set sequences were introduced by Pancaroğlu and Nuray (2013). We know that ideal convergence is more general than statistical convergence for sequences. This has motivated us to study the lacunary \mathcal{F} -invariant convergence of sequence of sets in intuitionistic fuzzy metric spaces (briefly, IFMS). In this study, we examine the notions of lacunary \mathcal{F} -invariant convergence ($W\mathcal{F}_{\sigma\theta}^{(\eta,\nu)}$) (Wijsman sense), lacunary \mathcal{F}^* -invariant convergence ($W\mathcal{F}_{\sigma\theta}^{*(\eta,\nu)}$) (Wijsman sense), and q -strongly lacunary invariant convergence ($[WN_{\sigma\theta}^{(\eta,\nu)}]_q$) (Wijsman sense) of sequences of sets in IFMS. Also, we give the relationships among Wijsman lacunary invariant convergence, $[WN_{\sigma\theta}^{(\eta,\nu)}]_q$, $W\mathcal{F}_{\sigma\theta}^{(\eta,\nu)}$, and $W\mathcal{F}_{\sigma\theta}^{*(\eta,\nu)}$ in IFMS. Furthermore, we define the concepts of $W\mathcal{F}_{\sigma\theta}^{(\eta,\nu)}$ -Cauchy sequence and $W\mathcal{F}_{\sigma\theta}^{*(\eta,\nu)}$ -Cauchy sequence of sets in IFMS. Furthermore, we obtain some features of the new type of convergences in IFMS.

1. Introduction and Background

Fast [1] investigated the concept of statistical convergence. The publication of the study is affected deeply all the scientific fields. Nuray and Ruckle [2] redefined this concept which is known as generalized statistical convergence. A lot of development has been made in area about statistical convergence. Kostyrko et al. [3] defined ideal convergence, as a generalization of statistical convergence and worked some features of this convergence. Ideal convergence became a remarkable topic in summability theory after the studies of [4–8]. Fridy and Orhan [9] worked the notion of lacunary statistical convergence by using lacunary sequence.

Various authors involving Raimi [10], Schaefer [11], and Mursaleen [12] worked invariant convergent sequences. Nuray et al. [13] investigated \mathcal{F}_σ -convergence with the help of σ -uniform density. Mursaleen [14] put forward the idea of strongly σ -convergence. Savaş and Nuray [15] presented the opinion of σ -statistical convergence and lacunary σ -statistical convergence and proved some correlation theorems. Nuray and Ulusu [16] defined lacunary \mathcal{F} -invariant

convergence and lacunary \mathcal{F} -invariant Cauchy sequence of real numbers.

After the original study of Zadeh [17], a huge number of research works have appeared on fuzzy theory and its applications as well as fuzzy analogues of the classical theories. Fuzzy sets (FSs) have been extensively applied in different disciplines and technologies. The theory of intuitionistic fuzzy sets (IFS) was presented by Atanassov [18]. The fuzzy sets and intuitionistic fuzzy sets have been widely used to solve many complex problems connected to different areas, especially in decision-making [19–22]. Kramosil and Michalek [23] worked fuzzy metric space (FMS) using the concepts fuzzy and probabilistic metric space. Park [24] rethought FMSs and investigated intuitionistic fuzzy metric space (IFMS). Park utilized George and Veeramani's [25] opinion of using t -norm and t -conorm to the FMS meantime describing IFMS and investigating its fundamental properties. In [26], motivated by Park's definition of an IF-metric, Lael and Nourouzi first defined an IF-normed space and then investigated, among other results, the fundamental theorems: open mapping, closed graph, and uniform

boundedness in IF-normed spaces. In order to have a different topology from the topology generated by the F -norm ψ , the condition $\psi + \phi \leq 1$ was omitted from Park's definition. Statistical convergence, ideal convergence, and different features of sequences in INFS were examined by several authors [27–31]. For the extraction of information by reflecting and modeling the hesitancy present in real-life situation, intuitionistic fuzzy set theory has been playing a significant role. The implementation of IF sets in place of fuzzy sets means the introduction of another degree of freedom into set description. IF fixed point theory has become a subject of great interest for expert in fixed point theory because this branch of mathematics has covered new possibilities for summability theory.

Convergence of sequences of sets has been examined by several authors. Nuray and Rhoades [32] presented a new convergence concept for sequences of sets called Wijsman statistical convergence. Ulusu and Nuray [33] examined the lacunary statistical convergence of sequence of sets. Kişi and Nuray [34] investigated ideal convergence for sequences of sets (Wijsman sense) and established some essential theorems. Convergence for sequences of sets became a notable topic in summability theory after the studies of [35–40].

Lacunary statistical convergence and lacunary strongly convergence for sequence of sets in IFMS were examined by Kişi [41]. Furthermore, Wijsman \mathcal{I} -convergence and Wijsman \mathcal{I}^* -convergence for sequence of sets in IFMS were investigated by Esi et al. [42].

The purpose of this study is to present some recent development in IFMS. The aim of the study is to examine some features of this new kind of convergence in IFMS. Also,

it is demonstrated that the new kind of convergence in IFMS is generally different from the known convergence in classical metric space. However; it is indicated that if certain conditions are met, every classical metric space can be a IFMS at the same time.

Throughout this work, we indicate \mathcal{I} to be the admissible ideal in \mathbb{N} , θ be a lacunary sequence, $(\mathcal{X}, \psi, \phi, *, \diamond)$ to be the IFMS, and $Y, \{Y_k\}$ to be nonempty closed subsets of \mathcal{X} .

2. Main Results

Definition 1. A sequence $\{Y_k\}$ of nonempty closed subsets of \mathcal{X} is called to be lacunary invariant convergent (Wijsman sense) to Y with regards to IFM (ψ, ϕ) , if for every $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, such that

$$\lim_{r \rightarrow \infty} \left| \frac{1}{h_r} \sum_{k \in I_r} \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| = 1, \quad (1)$$

$$\lim_{r \rightarrow \infty} \left| \frac{1}{h_r} \sum_{k \in I_r} \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| = 0, \quad (2)$$

uniformly in m .

Definition 2. A sequence $\{Y_k\}$ of nonempty closed subsets of \mathcal{X} is known as lacunary \mathcal{I} -invariant convergent or $W\mathcal{I}_{\sigma\theta}^{(\eta, \nu)}$ -convergent (Wijsman sense) to Y with regards to IFM (ψ, ϕ) , if for every $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, the set

$$P(\xi, \alpha, \tau) = \{k \in \mathbb{N} : |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \xi\} \in \mathcal{I}_{\sigma\theta}, \quad (3)$$

that is, $V_\theta(P(\xi, \alpha, \tau)) = 0$. We demonstrate this symbolically by $Y_k \longrightarrow Y (W\mathcal{I}_{\sigma\theta}^{(\psi, \phi)})$.

Theorem 1. Let $\{Y_k\}$ be a bounded sequence. If $\{Y_k\}$ is $W\mathcal{I}_{\sigma\theta}^{(\psi, \phi)}$ -convergent to Y , then $\{Y_k\}$ is lacunary invariant convergent (Wijsman sense) to Y with regards to IFM (ψ, ϕ) .

Proof. Let $m \in \mathbb{N}$ be arbitrary and $\xi \in (0, 1)$. For each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, we estimate

$$s(m, r, \alpha) := \left| \frac{1}{h_r} \sum_{k \in I_r} \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|. \quad (4)$$

Then, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, we get

$$s(m, r, \alpha) \leq s^1(m, r, \alpha) + s^2(m, r, \alpha), \quad (5)$$

where

$$s^1(m, r, \alpha) := \frac{1}{h_r} \sum_{k \in I_r, \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \geq 1 - \xi} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|, \quad (6)$$

$$s^2(m, r, \alpha) := \frac{1}{h_r} \sum_{k \in I_r, \left| \eta(\alpha, Y_{\sigma^k(m)}, \tau) - \eta(\alpha, Y, \tau) \right| < 1 - \xi} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|. \quad (7)$$

For every $m \geq 1$ and for every $\alpha \in \mathcal{X}$, it is obvious that $s^2(m, r, \alpha) < 1 - \xi$. Since $\{Y_k\}$ is bounded sequence, there is a $M > 0$, such that

$$|\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)| \leq M, \quad (k \in I_r, m \geq 1), \quad (8)$$

and so, we have

$$\begin{aligned} s^1(m, r, \alpha) &= \frac{1}{h_r} \sum_{k \in I_r, |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)| \geq 1 - \xi} |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)| \\ &\leq \frac{M}{h_r} \left| \left\{ k \in I_r : |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)| \geq 1 - \xi \right\} \right| \\ &\leq M \frac{\max_m \left| \left\{ k \in I_r : |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)| \geq 1 - \xi \right\} \right|}{h_r} = M \frac{S_r}{h_r}. \end{aligned} \quad (9)$$

Hence, we obtain

$$\lim_{r \rightarrow \infty} \left| \frac{1}{h_r} \sum_{k \in I_r} \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| = 1. \quad (10)$$

Similarly, we have

$$\lim_{r \rightarrow \infty} \left| \frac{1}{h_r} \sum_{k \in I_r} \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| = 0. \quad (11)$$

Hence, $\{Y_k\}$ is lacunary invariant convergent to Y (Wijsman sense) with regards to IFM (ψ, ϕ) . \square

Definition 3. Let $(\mathcal{X}, \psi, \phi, *, \diamond)$ be a separable IFMS, and \mathcal{I} be a proper ideal in \mathbb{N} . The sequence $\{Y_k\}$ is known as lacunary \mathcal{I}^* -invariant convergent or $W\mathcal{I}^*(\psi, \phi)$ -convergent (Wijsman sense) to Y with regards to IFM (ψ, ϕ) , if there is a set

$$M = \{m = (m_j) : m_j < m_{j+1}, j \in \mathbb{N}\} \in \mathcal{F}(\mathcal{I}_{\sigma\theta}). \quad (12)$$

such that for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$,

$$\lim_{r \rightarrow \infty} \psi(\alpha, Y_{m_k}, \tau) = \psi(\alpha, Y, \tau), \quad (13)$$

$$\lim_{r \rightarrow \infty} \phi(\alpha, Y_{m_k}, \tau) = \phi(\alpha, Y, \tau). \quad (14)$$

In that case, we write $Y_k \longrightarrow Y (W\mathcal{I}^*(\psi, \phi))$.

Theorem 2. If a sequence $\{Y_k\}$ is $W\mathcal{I}^*(\psi, \phi)$ -convergent to Y , then $\{Y_k\}$ is $W\mathcal{I}^*(\eta, \nu)$ -convergent to Y with regards to IFM (ψ, ϕ) .

Proof. Presume that $Y_k \longrightarrow Y (W\mathcal{I}^*(\psi, \phi))$. Then, $M = \{m = (m_j) : m_j < m_{j+1}, j \in \mathbb{N}\} \in \mathcal{F}(\mathcal{I}_{\sigma\theta})$ (i.e., $\mathbb{N} \setminus M = H(\text{say}) \in \mathcal{I}_{\sigma\theta}$), such that

$$\lim_{r \rightarrow \infty} \psi(\alpha, Y_{m_k}, \tau) = \psi(\alpha, Y, \tau), \quad (15)$$

$$\lim_{r \rightarrow \infty} \phi(\alpha, Y_{m_k}, \tau) = \phi(\alpha, Y, \tau). \quad (16)$$

But then, for each $\xi \in (0, 1)$ and $\tau > 0$, there is $N > 0$, such that

$$|\psi(\alpha, Y_{m_k}, \tau) - \psi(\alpha, Y, \tau)| > 1 - \xi, \quad (17)$$

$$|\phi(\alpha, Y_{m_k}, \tau) - \phi(\alpha, Y, \tau)| < \xi. \quad (18)$$

for all $k > N$. Since

$$\left\{ (m_k) \in M : |\psi(\alpha, Y_{m_k}, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_{m_k}, \tau) - \phi(\alpha, Y, \tau)| \geq \xi \right\} \quad (19)$$

is included in $\{m_1 < m_2 < \dots < m_{N-1}\}$ and the ideal $\mathcal{I}_{\sigma\theta}$ is admissible, we get

$$\left\{ (m_k) \in M : |\psi(\alpha, Y_{m_k}, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_{m_k}, \tau) - \phi(\alpha, Y, \tau)| \geq \xi \right\} \in \mathcal{I}_{\sigma\theta}. \quad (20)$$

Hence,

$$\begin{aligned} & \{k \in \mathbb{N}: |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \xi\} \\ & \subseteq H \cup \{m_1 < m_2 < \dots < m_{N-1}\} \in \mathcal{F}_{\sigma\theta}, \end{aligned} \tag{21}$$

for all $\xi \in (0, 1)$ and $\tau > 0$. Therefore, we conclude that $Y_k \rightarrow Y(W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)})$. \square

Theorem 3. Let the ideal $\mathcal{F}_{\sigma\theta}$ fulfill the property (AP). If $\{Y_k\}$ is a sequence in \mathcal{X} , such that $Y_k \rightarrow Y(W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)})$, then $Y_k \rightarrow Y(W\mathcal{F}_{\sigma\theta}^{*(\psi, \phi)})$.

Proof. Assume that $\mathcal{F}_{\sigma\theta}$ provides the feature (AP) and $Y_k \rightarrow Y(W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)})$. Then, for every $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$,

$$P(\xi, \alpha, \tau) = \{k \in \mathbb{N}: |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \xi\} \in \mathcal{F}_{\sigma\theta}. \tag{22}$$

We define the set Q_n for $n \in \mathbb{N}$ and $\tau > 0$ as

$$Q_n := \left\{ \begin{array}{l} k \in \mathbb{N}: 1 - \frac{1}{n} \leq |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| < 1 - \frac{1}{n+1} \\ \text{or } \frac{1}{n+1} < |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \leq \frac{1}{n} \end{array} \right\}. \tag{23}$$

Clearly, $\{Q_1, Q_2, \dots\}$ is countable and belongs to $\mathcal{F}_{\sigma\theta}$ and $Q_i \cap Q_j = \emptyset$ for $i \neq j$. By the feature (AP), there is a sequence of $\{F_n\}_{n \in \mathbb{N}}$, such that the symmetric differences $Q_j \Delta F_j$ are finite sets for $j \in \mathbb{N}$ and $F = (\cup_{j=1}^{\infty} F_j) \in \mathcal{F}_{\sigma\theta}$. Now, to conclude the proof, it is enough to show that for $M = \mathbb{N} \setminus F$ and for each $\alpha \in \mathcal{X}$, we get

$$\lim_{\tau \rightarrow \infty} \psi(\alpha, Y_k, \tau) = \psi(\alpha, Y, \tau), \tag{24}$$

$$\lim_{\tau \rightarrow \infty} \phi(\alpha, Y_k, \tau) = \phi(\alpha, Y, \tau), \tag{25}$$

for $k \in M$. Let $\rho > 0$. Select $n \in \mathbb{N}$, such that $(1/n + 1) < \rho$. For each $\alpha \in \mathcal{X}$, we acquire

$$\begin{aligned} & \{k \in \mathbb{N}: |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \rho \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \rho\} \\ & \subset \left\{ k \in \mathbb{N}: |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \frac{1}{n+1} \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \frac{1}{n+1} \right\} \subset \bigcup_{j=1}^{n+1} Q_j. \end{aligned} \tag{26}$$

Since $Q_j \Delta F_j$ ($j = 1, 2, \dots, n+1$) are finite sets, there is a $k_0 \in \mathbb{N}$, such that

$$\left(\bigcup_{j=1}^{n+1} F_j \right) \cap \{k \in \mathbb{N}: k > k_0\} = \left(\bigcup_{j=1}^{n+1} Q_j \right) \cap \{k \in \mathbb{N}: k > k_0\}. \tag{27}$$

If $k > k_0$ and $k \notin F$, then

$$k \notin \bigcup_{j=1}^{n+1} F_j \text{ and by (27) } k \notin \bigcup_{j=1}^{n+1} Q_j. \tag{28}$$

Hence, for each $k > k_0$ and $k \in M$, we have

$$|\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| > 1 - \rho \text{ and } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| < \rho. \tag{29}$$

Since $\rho > 0$ is arbitrary, we obtain $T_k \rightarrow T(W\mathcal{F}_{\sigma\theta}^{*(\psi, \phi)})$. \square

Definition 4. A sequence $\{Y_k\}$ is known as lacunary \mathcal{F} -invariant Cauchy sequence or $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence

(Wijsman sense) with regards to IFM (ψ, ϕ) if for each $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, there is $N = N(\xi, \alpha) \in \mathbb{N}$, such that

$$A(\xi, \alpha, \tau) = \{k \in \mathbb{N} : |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_N, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_N, \tau)| \geq \xi\} \in \mathcal{F}_{\sigma\theta}, \quad (30)$$

that is, $V_\theta(A(\xi, \alpha, \tau)) = 0$.

Definition 5. A sequence $\{Y_k\}$ is known as lacunary \mathcal{F}^* -invariant Cauchy sequence or $W\mathcal{F}_{\sigma\theta}^{*(\eta, \nu)}$ -Cauchy sequence (Wijsman sense) with regards to IFM (ψ, ϕ) provided that there is a set

$$M = \{m = (m_j) : m_j < m_{j+1}, j \in \mathbb{N}\} \in \mathcal{F}(\mathcal{F}_{\sigma\theta}), \quad (31)$$

such that

$$\lim_{k,l \rightarrow \infty} |\psi(\alpha, Y_{m_k}, \tau) - \psi(\alpha, Y_{m_l}, \tau)| = 1, \quad (32)$$

$$\lim_{k,l \rightarrow \infty} |\phi(\alpha, Y_{m_k}, \tau) - \phi(\alpha, Y_{m_l}, \tau)| = 0, \quad (33)$$

for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$.

We give following theorems which indicate relationships among $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -convergence, $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence, and $W\mathcal{F}_{\sigma\theta}^{*(\psi, \phi)}$ -Cauchy sequence with regards to IFM (ψ, ϕ) .

Theorem 4. *If a sequence $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -convergent, then $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence with regards to IFM (ψ, ϕ) .*

Proof. Presume that $Y_k \rightarrow Y(W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)})$. Then, for every $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, the set

$$P(\xi, \alpha, \tau) = \{k \in \mathbb{N} : |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \xi\}, \quad (34)$$

belongs to $\mathcal{F}_{\sigma\theta}$. Since $\mathcal{F}_{\sigma\theta}$ is an admissible ideal, then there is $k_0 \in \mathbb{N}$ with the result that $k_0 \notin P(\xi, \alpha, \tau)$. Now, assume that

$$R(\xi, \alpha, \tau) = \{k \in \mathbb{N} : |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_{k_0}, \tau)| \leq 1 - 2\xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_{k_0}, \tau)| \geq 2\xi\}. \quad (35)$$

Thinking the inequality

$$|\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_{k_0}, \tau)| \leq |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| + |\psi(\alpha, Y_{k_0}, \tau) - \psi(\alpha, Y, \tau)|, \quad (36)$$

$$|\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_{k_0}, \tau)| \leq |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| + |\phi(\alpha, Y_{k_0}, \tau) - \phi(\alpha, Y, \tau)|. \quad (37)$$

Observe that if $k \in R(\xi, \alpha, \tau)$, therefore,

$$|\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y, \tau)| + |\psi(\alpha, Y_{k_0}, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - 2\xi, \quad (38)$$

$$|\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| + |\phi(\alpha, Y_{k_0}, \tau) - \phi(\alpha, Y, \tau)| \geq 2\xi. \quad (39)$$

From another standpoint, since $k_0 \notin P(\xi, \alpha, \tau)$, we obtain

$$|\psi(\alpha, Y_{k_0}, \tau) - \psi(\alpha, Y, p)| > 1 - \xi \text{ and } |\phi(\alpha, Y_{k_0}, \tau) - \phi(\alpha, Y, \tau)| < \xi. \quad (40)$$

We reach that

$$|\psi(\alpha, Y_k, p) - \psi(\alpha, Y, p)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y, \tau)| \geq \xi. \quad (41)$$

Hence, $k \in P(\xi, \alpha, \tau)$. This gives that $R(\xi, \alpha, \tau) \subset P(\xi, \alpha, \tau) \in \mathcal{F}_{\sigma\theta}$ for every $\xi \in (0, 1)$ and $x \in \mathcal{X}$. Therefore, $R(\xi, \alpha, \tau) \in \mathcal{F}_{\sigma\theta}$, so $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence with regards to IFM (ψ, ϕ) . \square

Theorem 5. Let $(\mathcal{X}, \psi, \phi, *, \diamond)$ be a separable IFMS and \mathcal{F} be an admissible ideal. If a sequence $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence, then $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence with regards to IFM (ψ, ϕ) .

Proof. Assume that sequence $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy with regards to IFM (ψ, ϕ) . Then, for each $\alpha \in \mathcal{X}$ and for each $\xi \in (0, 1)$, there is $M \in \mathcal{F}(\mathcal{F}_{\sigma\theta})$, where $M = \{m = (m_j) : m_j < m_{j+1}, j \in \mathbb{N}\}$, such that

$$\begin{aligned} Q(\xi, \alpha, \tau) &= \{k \in \mathbb{N} : |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_N, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_N, \tau)| \geq \xi\} \\ &\subset H \cup \{m_1, m_2, \dots, m_{k_0}\} \in \mathcal{F}_{\sigma\theta}. \end{aligned} \quad (46)$$

As a consequence, for all $\tau > 0$ and for each $\xi \in (0, 1)$, one can identify $N = N(\xi)$, such that $Q(\xi, \alpha, \tau) \in \mathcal{F}_{\sigma\theta}$, that is, sequence $\{Y_k\}$ is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy with regards to IFM (ψ, ϕ) . \square

Lemma 1 (see [4]). Assume \mathcal{F} be an admissible ideal with the feature (AP). Let there be a countable collection of subsets $\{Y_k\}_{k=1}^{\infty}$ of \mathbb{N} in such a way that $Y_k \in \mathcal{F}(\mathcal{F})$. As a result, there is a set $Y \subset \mathbb{N}$, such that $Y \setminus Y_k$ is finite for all $k \in \mathbb{N}$ and $Y \in \mathcal{F}(\mathcal{F})$.

$$A(\xi, \alpha, \tau) = \{k \in \mathbb{N} : |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_N, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_N, \tau)| \geq \xi\} \in \mathcal{F}_{\sigma\theta}. \quad (47)$$

Now, presume that

$$Q_j(\xi, \alpha, \tau) = \left\{ k \in \mathbb{N} : \left| \psi(\alpha, Y_k, p) - \psi(\alpha, Y_{m_j}, \tau) \right| > 1 - \frac{1}{j} \text{ and } \left| \phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_{m_j}, \tau) \right| < \frac{1}{j} \right\}, \quad (48)$$

$$\left| \psi(\alpha, Y_{m_k}, \tau) - \psi(\alpha, Y_{m_l}, \tau) \right| \leq 1 - \xi, \quad (42)$$

$$\left| \phi(\alpha, Y_{m_k}, \tau) - \phi(\alpha, Y_{m_l}, \tau) \right| \geq \xi, (\forall k, l > k_0 = k_0(\xi)). \quad (43)$$

Presume that $N = N(\xi) = m_{k_0+1}$. Therefore, for each $\xi \in (0, 1)$, one gets

$$\left| \psi(\alpha, Y_{m_k}, \tau) - \psi(\alpha, Y_N, \tau) \right| \leq 1 - \xi, \quad (44)$$

$$\left| \phi(\alpha, Y_{m_k}, \tau) - \phi(\alpha, Y_N, \tau) \right| \geq \xi. \quad (45)$$

for all $k > k_0$. Now, assume that $H = \mathbb{N} \setminus M$. Obviously, $H \in \mathcal{F}_{\sigma\theta}$ and

Theorem 6. Let $\mathcal{F}_{\sigma\theta}$ provides the feature (AP). Then, the notions $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence and $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence with regards to IFM (ψ, ϕ) coincide.

Proof. The direct part has been proved in Theorem 5.

Now, assume that the sequence is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -Cauchy sequence with regards to IFM (ψ, ϕ) . Then, for each $\xi \in (0, 1)$, for each $x \in \mathcal{X}$, and for all $\tau > 0$, there is a $N = N(\xi, x) \in \mathbb{N}$, such that

where $m_j = m(1/j)$, $j = 1, 2, \dots$. Clearly, for $j = 1, 2, \dots$, $Q_j(\xi, \alpha, \tau) \in \mathcal{F}(\mathcal{I}_{\sigma\theta})$. Using Lemma 1, there is $Q \subset \mathbb{N}$, so that $Q \in \mathcal{F}(\mathcal{I}_{\sigma\theta})$ and $Q \setminus Q_j$ are finite for all j .

Now, we denote that

$$\lim_{k,l \rightarrow \infty} |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_l, \tau)| = 1, \quad (49)$$

$$\lim_{k,l \rightarrow \infty} |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_l, \tau)| = 0. \quad (50)$$

To demonstrate the above equations, let $\xi \in (0, 1)$ and $r \in \mathbb{N}$, such that $r > (2/\xi)$. If $k, l \in Q$, then $Q \setminus Q_j$ is a finite set; so, there is $\nu = \nu(r)$ in order that

$$|\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_l, \tau)| > 1 - \frac{1}{r}, \quad |\psi(\alpha, Y_l, \tau) - \psi(\alpha, Y_l, \tau)| > 1 - \frac{1}{r}, \quad (51)$$

$$|\phi(\alpha, Y_k, p) - \phi(\alpha, Y_l, p)| < \frac{1}{r}, \quad |\phi(\alpha, Y_l, \tau) - \phi(\alpha, Y_l, \tau)| < \frac{1}{r}, \quad (52)$$

for all $k, l > \nu(r)$. From the above inequalities, we obtain

$$\begin{aligned} |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_l, \tau)| &\leq |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_l, \tau)| \\ + |\psi(\alpha, Y_l, \tau) - \psi(\alpha, Y_l, \tau)| &> \left(1 - \frac{1}{r}\right) + \left(1 - \frac{1}{r}\right) > 1 - \xi, \end{aligned} \quad (53)$$

$$\begin{aligned} |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_l, \tau)| &\leq |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_l, \tau)| \\ + |\phi(\alpha, Y_l, p) - \phi(\alpha, Y_l, p)| &< \frac{1}{r} + \frac{1}{r} < \xi, \end{aligned} \quad (54)$$

for all $k, l > \nu(r)$.

Therefore, for every $\xi \in (0, 1)$, $\exists \nu = \nu(\xi)$, and $k, l \in Q \in \mathcal{F}(\mathcal{I}_{\sigma\theta})$, we acquire

$$\{k \in \mathbb{N}: |\psi(\alpha, Y_k, \tau) - \psi(\alpha, Y_l, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_k, \tau) - \phi(\alpha, Y_l, \tau)| \geq \xi\} \in \mathcal{I}_{\sigma\theta}. \quad (55)$$

This give that the sequence $\{Y_k\}$ is $W_{\mathcal{I}_{\sigma\theta}}^{*(\psi, \phi)}$ -Cauchy. \square

uniformly in m , where $0 < q < \infty$. We indicate this symbolically by $Y_k \longrightarrow Y([WN_{\sigma\theta}^{(\psi, \phi)}]_q)$.

Definition 6. The sequence $\{Y_k\}$ is named to be q -strongly lacunary invariant convergent (Wijsman sense) to Y , provided that for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$,

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} \sum_{k \in I_r} |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)|^q = 1, \quad (56)$$

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} \sum_{k \in I_r} |\phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau)|^q = 0, \quad (57)$$

Theorem 7. Let $\mathcal{I}_{\sigma\theta} \subset 2^{\mathbb{N}}$ be an admissible ideal and $0 < q < \infty$.

- (i) If $Y_k \longrightarrow Y([WN_{\sigma\theta}^{(\psi, \phi)}]_q)$, then $Y_k \longrightarrow Y(W_{\mathcal{I}_{\sigma\theta}}^{(\psi, \phi)})$
- (ii) If $\{Y_k\}$ is bounded and $Y_k \longrightarrow Y(W_{\mathcal{I}_{\sigma\theta}}^{(\psi, \phi)})$, then $Y_k \longrightarrow Y([WN_{\sigma\theta}^{(\psi, \phi)}]_q)$
- (iii) If $\{Y_k\} \in L_{\infty}$, then $Y_k \longrightarrow Y(W_{\mathcal{I}_{\sigma\theta}}^{(\psi, \phi)})$ iff $Y_k \longrightarrow Y([WN_{\sigma\theta}^{(\psi, \phi)}]_q)$

Proof.

(i) If $Y_k \longrightarrow Y ([WN_{\sigma\theta}^{(\psi,\phi)}]_q)$, then for every $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$ and for all $\tau > 0$, we obtain

$$\begin{aligned}
 & \sum_{k \in I_r} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|^q \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right|^q \\
 \geq & \sum_{\substack{k \in I_r, \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \leq 1 - \xi \\ \text{or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| \geq \xi}} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|^q \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right|^q \\
 \geq & \xi^q \left| \left\{ k \in I_r : \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \leq 1 - \xi \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| \geq \xi \right\} \right| \\
 \geq & \xi^q \max_m \left| \left\{ k \in I_r : \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \leq 1 - \xi \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| \geq \xi \right\} \right|.
 \end{aligned} \tag{58}$$

And so,

$$\begin{aligned}
 & \frac{1}{h_r} \sum_{k \in I_r} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|^q \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right|^q \\
 \geq & \xi^q \frac{\max_m \left| \left\{ k \in I_r : \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \leq 1 - \xi \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| \geq \xi \right\} \right|}{h_r} \\
 = & \xi^q \frac{S_r}{h_r}.
 \end{aligned} \tag{59}$$

For every $m = 1, 2, \dots$. This gives that $\lim_{r \rightarrow \infty} (S_r/h_r) = 0$, and hence, $T_k \longrightarrow T(W_{\mathcal{F}_{\sigma\theta}^{(\psi,\phi)}})$.

(ii) Presume that $\{Y_k\} \in L_\infty$, then $Y_k \longrightarrow Y (W_{\mathcal{F}_{\sigma\theta}^{(\psi,\phi)}})$. Let $\xi > 0$. By supposition, we get $V_\theta(P(\xi, \alpha, \tau)) = 0$. Since $\{Y_k\}$ is bounded, there is $M > 0$, such that for each $\alpha \in \mathcal{X}$ and for all k, m . Then, we acquire

$$\begin{aligned}
 & \frac{1}{h_r} \sum_{k \in I_r} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|^q \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right|^q \\
 = & \frac{1}{h_r} \sum_{\substack{k \in I_r, \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \leq 1 - \xi \\ \text{or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| \geq \xi}} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|^q \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right|^q \\
 + & \frac{1}{h_r} \sum_{\substack{k \in I_r, \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| > 1 - \xi \\ \text{or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| < \xi}} \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right|^q \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right|^q \\
 \leq & M \frac{\max_m \left| \left\{ k \in I_r : \left| \psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau) \right| \leq 1 - \xi \text{ or } \left| \phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau) \right| \geq \xi \right\} \right|}{h_r} + \xi^q \\
 \leq & M \frac{S_r}{h_r} + \xi^q,
 \end{aligned} \tag{60}$$

For each $\alpha \in \mathcal{X}$. Hence, we obtain

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} \sum_{k \in I_r} |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)|^q = 1, \quad (61)$$

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} \sum_{k \in I_r} |\phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau)|^q = 0, \quad (62)$$

Uniformly in m . As a result, we get $Y_k \rightarrow Y ([WN_{\sigma\theta}^{(\psi, \phi)}]_q)$.

(iii) It is clear from the consequence of (i) and (ii) \square

Definition 7. A sequence $\{Y_k\}$ is named to be lacunary invariant statistical convergent or $WS_{\sigma}^{\theta}(\psi, \phi)$ -convergent (Wijsman sense) to Y , if for each $\xi \in (0, 1)$, for each $\alpha \in \mathcal{X}$, and for all $\tau > 0$,

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} \left| \left\{ k \in I_r : |\psi(\alpha, Y_{\sigma^k(m)}, \tau) - \psi(\alpha, Y, \tau)| \leq 1 - \xi \text{ or } |\phi(\alpha, Y_{\sigma^k(m)}, \tau) - \phi(\alpha, Y, \tau)| \geq \xi \right\} \right| = 0, \quad (63)$$

uniformly in m .

Theorem 8. A sequence $\{Y_k\}$ is $WS_{\sigma}^{\theta}(\psi, \phi)$ -convergent to Y iff it is $W\mathcal{F}_{\sigma\theta}^{(\psi, \phi)}$ -convergent to Y .

3. Conclusion

Fuzzy set theory is based on the assumption that reasoning is not crystal clear. This theory has a significant role in the areas of technology and science. Intuitionistic fuzzy set has many application areas; for example, sale analysis, new product marketing, and financial services. This study aims to find out the use of the notion of lacunary \mathcal{F} -invariant convergence of sequence of sets for demonstrating some results in the area of intuitionistic fuzzy metric space. With the help of its applications, we give the notions of Wijsman lacunary \mathcal{F} -invariant convergent, Wijsman lacunary \mathcal{F}^* -invariant convergent, and Wijsman q -strongly lacunary invariant convergent sequences in IFMS and acquired meaningful results for these notions. Also, we have examined the notions of Wijsman lacunary \mathcal{F} -invariant Cauchy and Wijsman \mathcal{F}^* -invariant Cauchy sequence in IFMS. The elements of IFMS have been studied. The results acquired here are more common than corresponding results for metric spaces. It is expected that new results will help to understand deeply the concept of this new type of convergence on IFMS. It could also be possible to work with the opinion of "Lacunary \mathcal{F} -invariant convergence of sequence of sets in probabilistic metric space" utilizing intuitionistic probability theory in the prospective studies.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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