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Research Article

The Polynomial Solutions of Quadratic Diophantine Equation $X^2 - p(t)Y^2 + 2K(t)X + 2p(t)L(t)Y = 0$

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In this study, we consider the number of polynomial solutions of the Pell equation $x^2 - p(t)y^2 = 2$ is formulated for a nonsquare polynomial $p(t)$ using the polynomial solutions of the Pell equation $x^2 - p(t)y^2 = 1$. Moreover, a recurrence relation on the polynomial solutions of the Pell equation $x^2 - p(t)y^2 = 2$. Then, we consider the number of polynomial solutions of Diophantine equation E: $X^2 - p(t)Y^2 + 2K(t)X + 2p(t)L(t)Y = 0$. We also obtain some formulas and recurrence relations on the polynomial solution (X_n, Y_n) of E.

1. Introduction

A Diophantine equation is an indeterminate polynomial equation that allows the variables to be integers only. Diophantine problems have fewer equations with unknown variables and involve finding integers that work correctly for all equations. The equation $ax + by = 1$ is known as the liner Diophantine equation. In general, the Diophantine equation is the equation given by

$$ax^2 + bxy + cy^2 + dx + ey + f = 0. \tag{1}$$

The equation $x^2 - Dy^2 = N$, with given integers D and N and unknowns x and y, is called Pell's equation. The most interesting case of the equation arises when $D \neq 1$ be a positive nonsquare. Pell's equation $x^2 - Dy^2 = 1$ was solved by Lagrange in terms of simple continued fractions. We recall that there are many studies in which there are different types of Pell's equation. Many authors such as Tekcan [1], Matthews [2], Chandoul [3], and Li [4] have researched. In [5], the equation $x^2 - Dy^2 = 2$ was considered, and some formulas of its integer solutions were obtained. In [6, 7], the number of integer solutions of Diophantine equation $x^2 - (t^2 - 1)y^2 - (4t - 2)x + (4t^2 - 4t)y = 0$ and Diophantine equation $x^2 - (t^2 - 1)y^2 - (16t - 4)x + (16t^2 - 16t)y = 0$ over $\mathbb{Z}$ is considered, where $t \geq 2$. In [3, 8], the number of polynomial solutions of Diophantine equation

$$x^2 - \left(p(t)^2 - p(t)\right)y^2 - (4p(t) - 2)x + \left(4p(t)^2 - 4p(t)\right)y = 0 \tag{2}$$

and Diophantine equation

$$x^2 - p(t)y^2 - 2p'(t)x + 4p(t)y + \left(p'(t)\right)^2 - 4p(t) - 1 = 0 \tag{3}$$

over $\mathbb{Z}$ was considered, where $p(t)$ be a polynomial in $\mathbb{Z}[t]/\{0, 1\}$.

2. Preliminaries

In this section, we introduce the objects we need later and collect some important facts about them.

In [4], Li proved that the Pell equation $x^2 - Dy^2 = 1$ has infinitely positive solutions. If (x_1, y_1) is the fundamental solution, then for $n = 2, 3, \ldots$, $x_n + y_n\sqrt{D} = (x_1 + y_1\sqrt{D})^2$. The pairs (x_n, y_n) are all the positive solutions of the Pell equation. The x_n's and y_n's are strictly increasing to infinity and satisfy the recurrence relations:

$$x_{n+2} = 2x_1 x_{n+1} - x_n,$$
$$y_{n+2} = 2x_1 y_{n+1} - y_n. \quad (4)$$

Theorem 1 (Tekcan (see [5])). *Let $(X_1, Y_1) = (K, L)$ be the fundamental solution of the Pell equation $x^2 - Dy^2 = 2$ and $(x_1, y_1) = (a, b)$ be the fundamental solution of the Pell equation $x^2 - Dy^2 = 1$. Then, the other solutions of the Pell equation $x^2 - Dy^2 = 2$ are (X_n, Y_n):*

(1) For $n \geq 2$,

$$\begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \begin{pmatrix} K & L D \\ L & K \end{pmatrix} \begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix}. \quad (5)$$

(2) For $n \geq 1$,

$$\begin{pmatrix} X_{n+1} \\ Y_{n+1} \end{pmatrix} = \begin{pmatrix} aX_n + b\,DY_n \\ bX_n + aY_n \end{pmatrix}. \quad (6)$$

(3) For $n \geq 4$,

$$\begin{cases} X_{n+1} = \left(2K^2 + 1\right)\left(X_{n-1} - X_{n-2}\right) + X_{n-3}, \\ Y_{n+1} = \left(2K^2 + 1\right)\left(Y_{n-1} - Y_{n-2}\right) + Y_{n-3}. \end{cases} \quad (7)$$

3. New Results

Our principal result is the following.

Theorem 2. *Let $(x_1, y_1) = (a(t), b(t))$ be the fundamental solution of the Pell equation:*

$$x^2 - p(t)y^2 = 1. \quad (8)$$

Then, the other solutions of the Pell equation $x^2 - p(t)y^2 = 1$ are (x_n, y_n), where

(1) For $n \geq 1$,

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} a(t) & b(t)p(t) \\ b(t) & a(t) \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (9)$$

(2) For $n \geq 2$,

$$\begin{cases} x_n = a(t)x_{n-1} + b(t)p(t)y_{n-1}, \\ y_n = b(t)x_{n-1} + a(t)y_{n-1}. \end{cases} \quad (10)$$

(3) For $n \geq 4$,

$$\begin{cases} x_n = (2a(t) - 1)\left(x_{n-1} + x_{n-2}\right) - x_{n-3}, \\ y_n = (2a(t) - 1)\left(y_{n-1} + y_{n-2}\right) - y_{n-3}. \end{cases} \quad (11)$$

Proof

(1) We prove it using the method of mathematical induction. Let $n = 1$, and we get $(x_1, y_1) = (a(t), b(t))$, which is the fundamental solution of equation (8). Now, we assume that (9) is satisfied for n, that is,

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} a(t) & b(t)p(t) \\ b(t) & a(t) \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (12)$$

We try to show that this equation is also satisfied for $n + 1$. Applying (9), we find that

$$\begin{aligned} \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} &= \begin{pmatrix} a(t) & b(t)p(t) \\ b(t) & a(t) \end{pmatrix}^{n+1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} a(t) & b(t)p(t) \\ b(t) & a(t) \end{pmatrix} \begin{pmatrix} a(t) & b(t)p(t) \\ b(t) & a(t) \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} a(t) & b(t)p(t) \\ b(t) & a(t) \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} \\ &= \begin{pmatrix} a(t)x_n + b(t)p(t)y_n \\ b(t)x_n + a(t)y_n \end{pmatrix}. \end{aligned} \quad (13)$$

Hence, we conclude that

$$\begin{aligned} x_{n+1}^2 - p(t)y_{n+1}^2 &= \left(a(t)x_n + b(t)p(t)y_n\right)^2 \\ &\quad - p(t)\left(b(t)x_n + a(t)y_n\right)^2 \\ &= x_n^2 - p(t)y_n^2 \\ &= 1. \end{aligned} \quad (14)$$

So, (x_{n+1}, y_{n+1}) is also solution of equation (8).

(2) Using (13), we find that

$$\begin{cases} x_n = a(t)x_{n-1} + b(t)p(t)y_{n-1}, \\ y_n = b(t)x_{n-1} + a(t)y_{n-1}, \end{cases} \quad (15)$$

For $n \geq 2$.

(3) We prove it using the method of mathematical induction. For $n = 4$, we get

$$\begin{aligned} x_1 &= a(t), \\ x_2 &= 2a(t)^2 - 1, \\ x_3 &= 4a(t)^3 - 3a(t), \\ x_4 &= 8a(t)^4 - 8a(t)^2 + 1. \end{aligned} \quad (16)$$

Hence,

$$\begin{aligned}(2a(t)-1)(x_3+x_2)-x_1 &= (2a(t)-1)\left(4a(t)^3-3a(t)+2a(t)^2-1\right)-a(t)\\ &= 8a(t)^4-8a(t)^2+1\\ &= x_4.\end{aligned} \tag{17}$$

So, $x_n = (2a(t)-1)(x_{n-1}+x_{n-2})-x_{n-3}$ is satisfied for $n=4$. Let us assume that this relation is satisfied for n, that is,

$$x_n = (2a(t)-1)(x_{n-1}+x_{n-2})-x_{n-3}, \tag{18}$$

then, using (13) and (18), we conclude that

$$x_{n+1} = (2a(t)-1)(x_n+x_{n-1})-x_{n-2}, \tag{19}$$

completing the proof.

Similarly, we prove that

$$y_n = (2a(t)-1)(y_{n-1}+y_{n-2})-y_{n-3}, \quad \forall n\geq 4. \tag{20}$$

Now, we give a relation between $(K(t),L(t))$ and $(a(t),b(t))$. □

Theorem 3. *If $(K(t),L(t))$ be the fundamental solution of the equation*

$$x^2-p(t)y^2=2, \tag{21}$$

then $(K(t)^2-1, K(t)L(t))$ be a solution of the equation (8) and

$$\left(\frac{K(t)^2+p(t)L(t)^2}{2}, K(t)L(t)\right) = \left(K(t)^2-1, K(t)L(t)\right). \tag{22}$$

Proof. Hence, it is easily seen that

$$\begin{aligned}\left(K(t)^2-1\right)^2-p(t)(K(t)L(t))^2 &= K(t)^2\left(K(t)^2-p(t)L(t)^2\right)-2K(t)^2+1=1,\\ \left(\frac{K(t)^2+p(t)L(t)^2}{2}, K(t)L(t)\right) &= \left(\frac{K(t)^2+K(t)^2-2}{2}, K(t)L(t)\right)\\ &= \left(K(t)^2-1, K(t)L(t)\right),\end{aligned} \tag{23}$$

since $(K(t),L(t))$ is the fundamental solution of the equation $x^2-p(t)y^2=2$, i.e, $K(t)^2-p(t)L(t)^2=2$. □

Theorem 4. *Let $(U_1,V_1)=(K(t),L(t))$ be the fundamental solution of the equation (21) and $(x_1,y_1)=(a(t),b(t))$ be the fundamental solution of the equation (8). Then,*

(1)

$$\begin{pmatrix}U_n\\ V_n\end{pmatrix} = \begin{pmatrix}K(t) & L(t)p(t)\\ L(t) & K(t)\end{pmatrix}\begin{pmatrix}x_{n-1}\\ y_{n-1}\end{pmatrix}, \quad \forall n\geq 2. \tag{24}$$

(2)

$$\begin{cases}U_{n+1}=a(t)U_n+b(t)p(t)V_n,\\ V_{n+1}=b(t)U_n+a(t)V_n\end{cases}, \quad \forall n\geq 1. \tag{25}$$

(3) The solution (U_n,V_n) satisfies the recurrence relations

$$\begin{cases}U_n=2a(t)U_{n-1}-U_{n-2},\\ V_n=2a(t)V_{n-1}-V_{n-2}\end{cases}, \quad \forall n\geq 3. \tag{26}$$

(4) The solution (U_n,V_n) satisfies the recurrence relations

$$\begin{cases}U_n=\left(2K(t)^2-1\right)(U_{n-1}-U_{n-2})+U_{n-3},\\ V_n=\left(2K(t)^2-1\right)(V_{n-1}-V_{n-2})+V_{n-3}\end{cases}, \quad \forall n\geq 4. \tag{27}$$

Proof

(1) From (24), we get

$$\begin{pmatrix}U_n\\ V_n\end{pmatrix} = \begin{pmatrix}K(t)x_{n-1}+L(t)p(t)y_{n-1}\\ L(t)x_{n-1}+K(t)y_{n-1}\end{pmatrix}. \tag{28}$$

Hence, it is easily seen that

$$\begin{aligned} U_n^2 - p(t)V_n^2 &= \left(K(t)x_{n-1} + L(t)p(t)y_{n-1}\right)^2 - p(t)\left(L(t)x_{n-1} + K(t)y_{n-1}\right)^2 \\ &= K(t)^2\left(x_{n-1}^2 - p(t)y_{n-1}^2\right) - p(t)L(t)^2\left(x_{n-1}^2 - p(t)y_{n-1}^2\right) \\ &= K(t)^2 - p(t)L(t)^2 \\ &= 2. \end{aligned} \tag{29}$$

Since $x_{n-1}^2 - p(t)y_{n-1}^2 = 1$, and $K(t)^2 - p(t)L(t)^2 = 2$.

(2) From (24), we get

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \begin{pmatrix} K(t) & L(t)p(t) \\ L(t) & K(t) \end{pmatrix}\begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix}. \tag{30}$$

Then,

$$\begin{aligned} \begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix} &= \begin{pmatrix} K(t) & L(t)p(t) \\ L(t) & K(t) \end{pmatrix}^{-1}\begin{pmatrix} U_n \\ V_n \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{K(t)L(t)U_n + \left(2 - K(t)^2\right)V_n}{2L(t)} \\ \dfrac{-L(t)U_n + K(t)V_n}{2} \end{pmatrix}. \end{aligned} \tag{31}$$

Then,

$$\begin{cases} x_{n-1} = \dfrac{K(t)L(t)U_n + \left(2 - K(t)^2\right)V_n}{2L(t)}, \\ y_{n-1} = \dfrac{-L(t)U_n + K(t)V_n}{2}. \end{cases} \tag{32}$$

On the other hand, by using (10) and (32), we get

$$\begin{aligned} \begin{pmatrix} U_{n+1} \\ V_{n+1} \end{pmatrix} &= \begin{pmatrix} K(t) & L(t)p(t) \\ L(t) & K(t) \end{pmatrix}\begin{pmatrix} x_n \\ y_n \end{pmatrix} \\ &= \begin{pmatrix} K(t) & L(t)p(t) \\ L(t) & K(t) \end{pmatrix}\begin{pmatrix} a(t)x_{n-1} + b(t)p(t)y_{n-1} \\ b(t)x_{n-1} + a(t)y_{n-1} \end{pmatrix} \\ &= \begin{pmatrix} x_{n-1}(K(t)a(t) + p(t)L(t)b(t)) + y_{n-1}(K(t)b(t) + p(t)L(t)a(t)) \\ x_{n-1}(L(t)a(t) + b(t)K(t)) + y_{n-1}(b(t)L(t)p(t) + a(t)K(t)) \end{pmatrix}. \end{aligned} \tag{33}$$

Applying (32) and (33), we find

$$\begin{aligned} U_{n+1} &= x_{n-1}(K(t)a(t) + p(t)L(t)b(t)) + y_{n-1}(K(t)b(t) + p(t)L(t)a(t)) \\ &= \left(\frac{K(t)L(t)U_n + \left(2K(t)^2 + 1\right)V_n}{2L(t)}\right)(a(t)K(t) + b(t)L(t)p(t)) \\ &\quad + \left(\frac{-L(t)U_n + K(t)V_n}{2}\right)(b(t)L(t)p(t) + a(t)K(t)) \\ &= \frac{2a(t)U_n + 2b(t)p(t)L(t)V_n}{2L(t)} \\ &= a(t)U_n + b(t)p(t)v_n. \end{aligned} \tag{34}$$

Similarly, we prove that

$$V_{n+1} = b(t)U_n + a(t)V_n, \quad \forall n \geq 3. \tag{35}$$

(3) From (25), we get

$$\begin{aligned} U_n &= a(t)U_{n-1} + b(t)p(t)\left(b(t)U_{n-2} + a(t)V_{n-2}\right) \\ &= a(t)U_{n-1} + b(t)^2 p(t)U_{n-2} + a(t)b(t)v_{n-2} \\ &= a(t)U_{n-1} + \left(a(t)^2 - 1\right)U_{n-2} + a(t)b(t)V_{n-2} \\ &= a(t)U_{n-1} + a(t)\left(a(t)U_{n-2} + b(t)p(t)v_{n-2}\right) - U_{n-2} \\ &= 2a(t)U_{n-1} - U_{n-2}. \end{aligned} \tag{36}$$

Similarly, we prove that

$$V_n = 2a(t)V_{n-1} - V_{n-2}, \quad \forall n \geq 3. \tag{37}$$

(4) From (22) and (26), we get

$$\begin{aligned} U_n &= U_n - U_{n-1} + U_{n-1} \\ &= 2a(t)U_{n-1} - U_{n-2} - 2a(t)U_{n-2} + U_{n-3} + U_{n-1} \\ &= (2a(t) + 1)\left(U_{n-1} - U_{n-2}\right) + U_{n-3} \\ &= \left(2K(t)^2 - 1\right)\left(U_{n-1} - U_{n-2}\right) + U_{n-3}. \end{aligned} \tag{38}$$

Similarly, we prove that

$$V_n = \left(2K(t)^2 - 1\right)\left(V_{n-1} - V_{n-2}\right) + V_{n-3}, \quad \forall n \geq 4, \tag{39}$$

and we consider the number of polynomial solutions of Diophantine equation. Now,

$$E\colon x^2 - p(t)y^2 - 2K(t)x + 2p(t)L(t)y = 0, \tag{40}$$

where $(K(t), L(t))$ is the fundamental solution of equation (21).

We have to transform E into an appropriate Diophantine equation which can be easily solved. To get this, let

$$T\colon \begin{cases} x = U + M, \\ y = V + N. \end{cases} \tag{41}$$

be a translation for some M and N.

By applying the transformation T to E, we get

$$T(E) = (U + M)^2 - p(t)(V + N)^2 - 2K(t)(U + M) + 2p(t)L(t)(V + N) = 0. \tag{42}$$

In (42), we obtain $U(2M - 2K(t))$ and $V(-2p(t)N + 2p(t)L(t))$. So we get $M = K(t)$ and $N = L(t)$. Consequently, for $x = U + K(t)$, $y = V + L(t)$, we have the Diophantine equation

$$\widetilde{E}\colon U^2 - p(t)V^2 = 2, \tag{43}$$

which is Pell equation.

Now, we try to find all polynomial solutions (U_n, V_n) of $T(E)$, and then, we can retransfer all results from $T(E)$ to E by using the inverse of T. □

Theorem 5. *Let E be the Diophantine equation in (40), where $(K(t), L(t))$ is the fundamental solution of equation (43); then,*

(1) The fundamental (minimal) solution of E is

$$(X_1, Y_1) = (2K(t), 2L(t)). \tag{44}$$

(2) Define the sequence

$$\{(x_n, y_n)\}_{n\geq 1} = \{(U_n + K(t), V_n + L(t))\}, \tag{45}$$

where $\{(U_n, V_n)\}$ is defined in (24). Then, (X_n, Y_n) is a solution of E. So, it has infinitely many integer solutions $(X_n, Y_n) \in \mathbb{Z} \times \mathbb{Z}$.

(3) The solutions (X_n, Y_n) satisfy the recurrence relations

$$\begin{cases} X_{n+1} = a(t)X_n + b(t)p(t)Y_n + (1 - a(t))K(t) - b(t)p(t)L(t), \\ Y_{n+1} = b(t)X_n + a(t)Y_n - b(t)K(t) + (1 - a(t))L(t), \end{cases} \tag{46}$$

For $n \geq 1$.

(4) The solutions (X_n, Y_n) satisfy the recurrence relations

$$\begin{cases} X_n = 2a(t)X_{n-1} - X_{n-2} + 2(1 - a(t))K(t), \\ Y_n = 2a(t)Y_{n-1} - Y_{n-2} + 2(1 - a(t))L(t), \end{cases} \tag{47}$$

For $n \geq 3$.

(5) *The solutions* (X_n, Y_n) *satisfy the recurrence relations*

$$\begin{cases} X_n = \left(2K(t)^2 - 1\right)\left(X_{n-1} - X_{n-2}\right) + X_{n-3}, \\ Y_n = \left(2K(t)^2 - 1\right)\left(Y_{n-1} - Y_{n-2}\right) + Y_{n-3}. \end{cases} \quad (48)$$

For $n \geq 4$.

Example 1. Let $p(t) = t^2 - 2$; then, $(U_1, V_1) = (t, 1)$ is the fundamental solution of

$$\widetilde{E}\colon U^2 - \left(t^2 - 2\right)V^2 = 2. \quad (49)$$

By Theorem 3, $(x_1, y_1) = (t^2 - 1, t)$ is the fundamental solution of

$$x^2 - \left(t^2 - 2\right)y^2 = 1. \quad (50)$$

By Theorem 4, it is easily seen that some other solutions of equation $\widetilde{E}$ are

$$\begin{aligned} \begin{pmatrix} U_2 \\ V_2 \end{pmatrix} &= \begin{pmatrix} 2t^3 - 3t \\ 2t^2 - 1 \end{pmatrix}, \\ \begin{pmatrix} U_3 \\ V_3 \end{pmatrix} &= \begin{pmatrix} 4t^5 - 10t^3 + 5t \\ 4t^4 - 6t^2 + 1 \end{pmatrix}, \\ \begin{pmatrix} U_4 \\ V_4 \end{pmatrix} &= \begin{pmatrix} 8t^7 - 30t^5 + 29t^3 - 9t \\ 8t^6 - 20t^4 + 12t^2 - 1 \end{pmatrix}, \\ \begin{pmatrix} U_5 \\ V_5 \end{pmatrix} &= \begin{pmatrix} 16t^9 - 76t^7 + 112t^5 - 69t^3 + 11t \\ 16t^8 - 56t^6 + 60t^4 - 20t^2 + 1 \end{pmatrix}. \end{aligned} \quad (51)$$

It can be concluded now that the fundamental solution of

$$E\colon x^2 - \left(t^2 - 2\right)y^2 - 2tx + \left(2t^2 - 4\right)y = 0 \quad (52)$$

is $(2t, 2)$. By Theorem 5, it is easily seen that some other solutions of equation E are

$$\begin{aligned} \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix} &= \begin{pmatrix} 2t^3 - 2t \\ 2t^2 \end{pmatrix}, \\ \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix} &= \begin{pmatrix} 4t^5 - 10t^3 + 6t \\ 4t^4 - 6t^2 + 2 \end{pmatrix}, \\ \begin{pmatrix} X_4 \\ Y_4 \end{pmatrix} &= \begin{pmatrix} 8t^7 - 30t^5 + 29t^3 - 8t \\ 8t^6 - 20t^4 + 12t^2 \end{pmatrix}, \\ \begin{pmatrix} X_5 \\ Y_5 \end{pmatrix} &= \begin{pmatrix} 16t^9 - 76t^7 + 112t^5 - 69t^3 + 12t \\ 16t^8 - 56t^6 + 60t^4 - 20t^2 + 2 \end{pmatrix}. \end{aligned} \quad (53)$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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