Research Article

Data-Driven Repeated-Feedback Adjustment Strategy for Smart Grid Pricing

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1. Introduction

More and more research has focused on environmental protection and energy conservation. Due to the nonrenewable and high pollution of traditional energy, people are more and more inclined to use clean and renewable energy [1]. The rapid development of new energy vehicles may increase the power demand. We need to do more to balance the power supply to ensure the safety and dependability of the power grid.

As the abovementioned problems need to be solved urgently, many countries have extensively researched and applied the smart grid (SG). SG solves energy problems through superior electronic communication technology and power grid system [2]. Smart meter, one of the essential nodes in SG, has a real-time interaction function and can realize interactive communication between consumers and power suppliers at any time, due to the scientific and technological progress of intelligent algorithms and energy networks.

As the demand response mechanism of the smart grid controller, combined with smart meters, shows us a picture of real-time pricing (RTP) [3, 4], RTP can realize real-time price and load interaction between consumers and power suppliers more elastically and cleverly. This is an advantage that other traditional pricing strategies do not have. Hu et al. proposed a distributed algorithm to solve the power dispatching strategy between a power company and multiple power users [5]. Goudarzi et al. put forward a game theory method considering both incentive and price in reducing power consumption and improving users’ benefit [6]. Zhao et al. developed a model to obtain dynamic pricing and lead the consumers’ power consumption mode and update the power load [7].

The prime intent of implementing RTP in SG is to obtain optimal power consumption, power supply, and electricity price. When power suppliers implement the optimal electricity price, users can use electricity according to the optimal electricity consumption, which can run steadily. However, the reality is the real power consumption alters radically and reduces the grid’s steadiness and dependability. Up to now, the researchers have seldom paid attention to how to keep the real load steady. Kb and Ms brought forward...
2. System Model and Preliminary Knowledge

2.1. System Model. Suppose that the smart grid includes a power supplier, a lot of power users with smart meters, and some rules to be followed by both parties. Smart meters monitor and adjust power load through the price adjustment and can harmonize every user with the power supplier and the other users. Power users and the power supplier exchange information such as electricity price and power load every hour through communication facilities such as the network. The notations and variables are summarized in Table 1. The cycle time in our system is separated into \( \mathcal{H} \) periods, and the set is \( \Lambda = \{1, 2, \ldots, \mathcal{H}\} \). Let \( \omega \) denote the number of users, and the set of power users is \( \Xi = \{1, 2, \ldots, \omega\} \). Let \( d_n^\tau \) denote the power consumption used by every user in period \( n \in \Xi \) in period \( \tau \in \Lambda \), \( L_n^\tau \leq d_n^\tau \leq L_n^\tau \), where \( L_n^\tau \) and \( L_n^\tau \) are explained in Table 1.

2.1.1. Profit Function and Benefit Function. Let \( S^\tau \in [S_{\min}^\tau, S_{\max}^\tau] \) denote the power supply of power supplier in period \( \tau \) and \( g(S^\tau) \) represent the cost providing \( S^\tau \) in period \( \tau \). Assuming that supply can meet consumption, \( S^\tau \in [S_{\min}^\tau, S_{\max}^\tau] \), \( S_{\min}^\tau = \sum_{n=1}^\omega l_n^\tau \) and \( S_{\max}^\tau = \sum_{n=1}^\omega L_n^\tau \) are explained in Table 1. According to [10, 11], the power supplier’s power supply cost function in period \( \tau \) is

\[
g(S^\tau) = \alpha (S^\tau)^2 + \beta S^\tau + \gamma, \tag{1}
\]

where \( \alpha > 0, \beta, \gamma \geq 0 \) are shown in Table 1. The power supplier’s profit function in period \( \tau \) is

\[
h(S^\tau) = \rho^\tau S^\tau - g(S^\tau), \tag{2}
\]

where \( \rho^\tau \) is defined in Table 1.

In economics, experts used to characterize the degree of happiness generated from power consumption with a utility function. In this paper, it is assumed that the utility increases with user power consumption and will not increase when it increases to a certain amount and will not grow when it increases to a certain amount. Let \( \delta_n^\tau > 0 \) denote the user \( n \)'s preference. We model a logarithmic function as the utility function [11]:

\[
U(d_n^\tau, \delta_n^\tau) = \begin{cases} 
\delta_n^\tau \ln(d_n^\tau + 100), & \text{if } d_n^\tau \geq 0, \\
0, & \text{if } d_n^\tau < 0, 
\end{cases} \tag{3}
\]

where user \( n \)'s benefit function in period \( \tau \) is

\[
f(d_n^\tau, \delta_n^\tau) = U(d_n^\tau, \delta_n^\tau) - \rho^\tau d_n^\tau. \tag{4}
\]

2.1.2. The Optimal Problem. From (2) and (4), the whole social welfare function is [10]

\[
\Phi(d_n^\tau, S^\tau) = \sum_{\tau=1}^\mathcal{H} \sum_{n=1}^\omega U(d_n^\tau, \delta_n^\tau) - g(S^\tau). \tag{5}
\]

We assume that the power supplier's power supply is not less than power consumption in total in each period \( \tau \),
Table 1: List of notations and variables.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n/\tau )</td>
<td>Index of user/period</td>
</tr>
<tr>
<td>( \omega/\varpi )</td>
<td>Amount of users/periods</td>
</tr>
<tr>
<td>( \Xi/\Lambda )</td>
<td>Set of users/periods</td>
</tr>
<tr>
<td>( \vartheta_{\mu}^{\tau} )</td>
<td>User ( n )'s preference in period ( \tau )</td>
</tr>
<tr>
<td>( \alpha/\beta/\gamma )</td>
<td>Parameters of cost function</td>
</tr>
<tr>
<td>( d_{\mu}^{\tau} )</td>
<td>The amount of user ( n )'s power consumption in period ( \tau )</td>
</tr>
<tr>
<td>( I_{1m}^{\tau} )</td>
<td>User ( n )'s minimum/maximum power consumption in period ( \tau )</td>
</tr>
<tr>
<td>( S_{\min}^{\tau}/S_{\max}^{\tau} )</td>
<td>Power supplier’s minimum/maximum power supply in period ( \tau )</td>
</tr>
<tr>
<td>( \rho^* )</td>
<td>Electricity price in period ( \tau )</td>
</tr>
<tr>
<td>( \theta^* )</td>
<td>Variation of electricity price in period ( \tau )</td>
</tr>
</tbody>
</table>

2.2. Preliminary Knowledge. The optimal power supply \( S_{\tau}^{\star} \), power consumption \( d_{\tau}^{\star} \), and electricity price \( \rho_{\tau}^{\star} \) in period \( \tau \) can be obtained by solving the optimization problems (7) and (8). Ideally, the users' real power consumption is consistent with the optimal power supply. However, the real situation is that there is a particular discrepancy between real consumption \( d^* \) and optimal supply \( S_{\tau}^{\star} \), and even the discrepancy fluctuates widely in some periods. The real power consumption does not have the stability and dependability of the optimal power supply. The power discrepancy in period \( \tau \) is described as [10, 11]

\[
\zeta^\tau = d^{\tau} - S^{\star}_{\tau},
\]

To keep the real power consumption dependable and steady, we want to adjust \( d^{\tau} \) to \( S^{\star}_{\tau} \) by price incentive. Users may reduce the power consumption in peak time when the price increases and increase it in valley time when decreased. Next, we need to incorporate the optimal problems (7) and (8) into the exponentially weighted moving average (EWMA) process to minimize the standard error.

3. Problem Formulation and Solutions

3.1. Problem Formulation. Undependability implies that there is no fixed mean value globally, so we need to study the local average and the deviations. We use an EWMA to represent the local average.

Definition 1 (see [10]). \( \zeta^\tau, t = \tau, \tau - 1, \ldots, \) is the beginning power discrepancy, \( \zeta^\tau, t = \tau, \tau - 1, \ldots, \) is the adjusted one,

\[
\zeta^{\tau+1} = \lambda(\zeta^\tau + \nu\zeta^{\tau-1} + \nu^2\zeta^{\tau-2} + \ldots) = \lambda\zeta^\tau + \nu\zeta^\tau - 1, \quad 0 < \nu < 1,
\]

is called the power discrepancy EWMA in next period \( \tau + 1 + 1 \), where \( \lambda = 1 - \nu \) is the parameter.

Definition 2 (see [11]). \( \rho^\tau, t = \tau, \tau - 1, \ldots, \) is the beginning real-time price, \( \rho^\tau, t = \tau, \tau - 1, \ldots, \) is the adjusted price of \( \rho^\tau \),

\[
\tilde{\rho}^{\tau+1} = \lambda(\rho_a^\tau + \nu\rho_a^{\tau-1} + \nu^2\rho_a^{\tau-2} + \ldots) = \lambda\rho_a^\tau + \nu\rho_a^\tau - 1, \quad 0 < \nu < 1,
\]

is called the real-time price EWMA in next period \( \tau + 1 \).

Definition 3 (see [11]). \( \rho^\tau, t = \tau, \tau - 1, \ldots, \) is the readjusted real-time price of \( \rho^\tau \),

\[
\tilde{\rho}^{\tau+1} = \lambda\rho^\tau + \nu\tilde{\rho}^\tau, \quad 0 < \nu < 1,
\]

is called the adjusted price EWMA in next period \( \tau + 1 \).

Next, we will discuss a data-driven repeated-feedback adjustment strategy and reduce deviations from the objective in power discrepancy utilizing process monitoring and adjustment. If

\[
|\tilde{\zeta}^{\tau+1} - \mu| \geq L
\]

holds at period \( \tau + 1 \), the price will be adjusted, where \( \mu \) is the objective, standard load discrepancy of the optimal problem and \( L \) is the boundary.

As we know, if the real power consumption equals the optimal power supply, it will be best. So, the standard power discrepancy in our model is \( \mu = 0 \). Let \( L_1 > 0 \) be the upper boundary and \( L_2 < 0 \) be the lower boundary; we reformulate (13) as

\[
\tilde{\zeta}^{\tau+1} \geq L_1 \quad \text{or} \quad \tilde{\zeta}^{\tau+1} \leq L_2.
\]

Via (14), the automatic monitoring process is developed. In period \( \tau + 1 \), if \( \tilde{\zeta}^{\tau+1} \geq L_1 \), the prices will be gone up, the power consumption in current period will shift to other periods, thereby reducing the power consumption, and \( \tilde{\zeta}^{\tau+1} \) is adjusted to objective \( e_1 \in [0, L_1] \). If \( \tilde{\zeta}^{\tau+1} \leq L_2 \), to increase the power consumption in the current period, the prices will...
be diminished, and then, $\bar{\zeta}^{r+1}$ is modified to the objective $\varepsilon_2 \in (L_2, 0]$.

### 3.2. Problem Solutions

**Definition 4.** Set an inverse proportional function (after this referred to as “IPF”) as the demand function. Thus, power discrepancy $\text{EWMA} \bar{\zeta}^{r+1}$ is inverse proportional to price $\text{EWMA} \bar{\rho}^{r+1}$ in the form of $\bar{\zeta}^{r+1} = (a/\bar{\rho}^{r+1})$, where $a > 0$ is a constant.

**Theorem 1.** Suppose an IPF with a constant $a (a \geq 0)$ is the demand function. Then, if $\bar{\zeta}^{r+1} \geq L_1 > 0$, we will adjust $\bar{\zeta}^{r+1}$ to $\varepsilon_1 \in [0, L_1)$, and the change of price is

$$\theta^{r+1} = \frac{a}{L_1 - \frac{1}{\bar{\zeta}^{r+1}}}.$$  \hfill (15)

If $\bar{\zeta}^{r+1} \leq L_2 < 0$, we will adjust $\bar{\zeta}^{r+1}$ to $\varepsilon_2 \in (L_2, 0]$, and the change of price is

$$\theta^{r+1} = \frac{a}{L_2 - \frac{1}{\bar{\zeta}^{r+1}}}.$$  \hfill (16)

**Proof.** By condition, in period $r + 1$, if $\bar{\zeta}^{r+1} \geq L_1 > 0$, we will adjust $\bar{\zeta}^{r+1}$ to $\varepsilon_1 \in (0, L_1)$. At this point, the price $\text{EWMA}$ changes from $\bar{\rho}^{r+1}$ to $\bar{\rho}_a^{r+1}$.

According to Definition 4, $\bar{\zeta}^{r+1} = (a/\bar{\rho}^{r+1})$ and $\varepsilon_1 = (a/\bar{\rho}_a^{r+1})$.

$$\bar{\rho}^{r+1} = \frac{a}{\bar{\zeta}^{r+1}},$$  \hfill (17)

$$\bar{\rho}_a^{r+1} = \frac{a}{\varepsilon_1}$$  \hfill (18)

$$\bar{\rho}_a^{r+1} - \bar{\rho}^{r+1} = a \left( \frac{1}{\varepsilon_1} - \frac{1}{\bar{\zeta}^{r+1}} \right).$$  \hfill (19)

$$\lambda \rho^r + \gamma \bar{\rho}^r - (\lambda \rho_a^r + \gamma \bar{\rho}_a^r) = a \left( \frac{1}{\varepsilon_1} - \frac{1}{\bar{\zeta}^{r+1}} \right).$$  \hfill (20)

Denote $\theta^{r+1} = \rho^r - \rho_a^r$ as the change of price, and (20) can be transformed into the following:

$$\lambda \theta^{r+1} = \frac{a}{\varepsilon_1} - \frac{1}{\bar{\zeta}^{r+1}},$$  \hfill (21)

$$\varepsilon^{r+1} = \frac{a}{L_1 - \frac{1}{\zeta^{r+1}}}. $$

Similarly, if $\bar{\zeta}^{r+1} \leq L_2 < 0$, (16) holds.

**3.3. Algorithm.** From Theorem 1, if (14) holds, we will adjust the users’ real power consumption with price variations to reach an equilibrium state.

**Lemma.** Suppose that the adjusted power consumption is inversely proportional to the change of price. Therefore, the revised users’ real power consumption is

$$d_a^{r+1} = d^{r+1} - \frac{a}{\theta^{r+1}}.$$  \hfill (22)

**Proof.** If $\theta^{r+1} > 0$, the users’ real power consumption will decrease, and if $\theta^{r+1} < 0$, it increases. Therefore,

$$d_a^{r+1} - d^{r+1} = -\frac{a}{\theta^{r+1}};$$  \hfill (23)

$$d_a^{r+1} = d^{r+1} - \frac{a}{\theta^{r+1}}.$$  \hfill (24)

**Lemma 2.** Set $\zeta^{r+1}_a$ as the adjusted power discrepancy between the users’ real power consumption $d^{r+1}$ and the optimal power supply $S^{r+1}_a$; therefore,

$$\zeta^{r+1}_a = \zeta^{r+1} - \frac{a}{\theta^{r+1}}.$$  \hfill (25)

**Proof.** By condition,

$$\zeta^{r+1}_a = d_a^{r+1} - S^{r+1}_a.$$  \hfill (26)

According to Lemma 1 and (25), the adjusted power discrepancy is

$$\zeta^{r+1}_a = d_a^{r+1} - S^{r+1}_a = d^{r+1} - \frac{a}{\theta^{r+1}} - S^{r+1}_a = d^{r+1} - \zeta^{r+1} - \frac{a}{\theta^{r+1}} - \frac{a}{\theta^{r+1}}.$$  \hfill (27)

Proof is completed.

### 4. Numerical Tests

In this section, we conduct simulations to demonstrate the effectiveness of the proposed data-driven IPF-based repeated-feedback adjustment strategy through realistic data from [21]. Our simulated smart grid comprises a power supplier and 1000 users ($a = 1000$). Divide the period of two days into 48 hours ($\mathcal{H} = 48$). The parameters $\alpha, \beta, \gamma$ in (1)
are 0.01, 0, and 0. Each user’s utility function is defined as (3), where the preference parameter $\vartheta$ is selected randomly from the interval [1,4].

4.1. Results’ Analysis. The profiles of power consumption load, power supply, and price data are obtained from Singapore [21]. The data used in the simulation are listed in Table 2. Figure 1 shows the electric energy loads before adjustment.

From Figure 1, we can see that the power supplier’s optimal power supply runs stably, but the discrepancy between the users’ real power consumption and optimal power supply is relatively large. For the sake of shortening the gap, the power supplier should take action to encourage users to adjust the real power consumption. Our purpose is to reduce power consumption in peak hours and increase it in low hours, so as to achieve the energy-saving effect by the data-driven IPF-based repeated-feedback adjustment strategy, which can lead to steady real power consumption.

Set $\bar{F}_1 = 75$ and list the parameters in Algorithm 1 in Table 3. The process of repeated-feedback adjustment from the performance of Algorithm 1 in 48 periods is illustrated in Figure 2. Figure 2 shows that the total adjustments times are 12. The standard error of residuals $\sigma_\delta$ is

$$\sigma_\delta = \sqrt{\frac{\sum_{t=1}^{48} \delta_t^2}{47}} = 89.66. \quad (27)$$

Figure 2 shows that, after applying the algorithm, all points are within the $3\sigma_\delta$ boundary, so there is no anomaly, and we can get steady power consumption.

We can learn from Figure 3 that the electricity price is changed slightly to balance the power discrepancies. The most remarkable change of price is $1.929 \times 10^{-3}$ $$/\text{kWh}.$ Figure 3 also shows that compared with the original users’

<table>
<thead>
<tr>
<th>Data used in Algorithm 1</th>
<th>The period in [21]</th>
</tr>
</thead>
<tbody>
<tr>
<td>The real original price $[p^*_t]$</td>
<td>Price series from Sep 23, 2017, to Sep 24, 2017</td>
</tr>
<tr>
<td>The real power consumption $[d^t]$</td>
<td>The power load data of 2 days from Sep 23, 2017, to Sep 24, 2017</td>
</tr>
<tr>
<td>The optimal power supply $[S^*_t]$</td>
<td>Power load data at the same hour a week ago from Sep 16, 2017, to Sep 17, 2017</td>
</tr>
</tbody>
</table>

**Algorithm 1:** Data-driven IPF-based repeated-feedback adjustment strategy.
power consumption, the adjusted users’ power consumption is closer to the optimal power supply, achieving the expected effect.

Table 3 and Figure 4 illustrate the whole social welfare computed according to (5) and the profits computed according to (2). We can learn from them that the whole social welfare in the proposed repeated-feedback adjustment strategy is significantly higher than the original one because the users’ real adjusted power consumption is much closer to the optimal power supply. The profit is slightly higher than the original one because the power consumption is more stable.

Table 3: Simulation results.

<table>
<thead>
<tr>
<th>Index</th>
<th>Original pricing</th>
<th>Adjusted pricing</th>
<th>Fixed pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjustments’ times</td>
<td>/</td>
<td>12</td>
<td>/</td>
</tr>
<tr>
<td>$\sigma_{\delta}$</td>
<td>/</td>
<td>89.66</td>
<td>/</td>
</tr>
<tr>
<td>Whole social welfare ($\times 10^8$)</td>
<td>6.32</td>
<td>8.68</td>
<td>3.34</td>
</tr>
<tr>
<td>Power supplier’s profit ($\times 10^7$)</td>
<td>3.99</td>
<td>4.17</td>
<td>3.69</td>
</tr>
</tbody>
</table>

Figure 2: Adjustment effects (black arrows mark the adjustment process, black solid lines with circle markers denote EWMA forecast, blue solid lines with plus sign markers indicate original power discrepancy, red solid lines with asterisks depict adjusted power discrepancy, and green dash-dot lines denote residual error).

Figure 3: Power load after adjustment (red solid lines with asterisks denote real original power consumption, magenta solid lines with triangle markers indicate optimal power supply, and blue solid lines with point markers depict adjusted power consumption) and the change of price (black solid lines with circle markers denote the change of price).
By running Algorithm 1, we not only can obtain the optimal adjustment strategy and steady power consumption but also can improve the economic benefits of both sides.

### 4.2. Comparison of Adjustment Effect

To verify the proposed IPF-based repeated-feedback adjustment strategy is more reasonable to obtain steady and dependable power

![Graph showing economic benefits of both sides](image)

**Figure 4:** Economic benefits of both sides (histogram depicts whole social welfare and red solid lines with point markers denote profit).

<table>
<thead>
<tr>
<th>Table 4: Comparison results under two adjustments.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Index</strong></td>
</tr>
<tr>
<td>Adjustments’ times</td>
</tr>
<tr>
<td>$\sigma_\delta$</td>
</tr>
<tr>
<td>Whole social welfare ($\times 10^8$)</td>
</tr>
<tr>
<td>Power supplier’s profit ($\times 10^7$)</td>
</tr>
</tbody>
</table>

![Graph showing comparison of IPF adjustments](image)

**Figure 5:** Comparison of IPF adjustments (black arrows mark the IPF adjustment process, red arrows mark the LF adjustment process, black solid lines denote original power discrepancy, blue solid lines with circle markers depict IPF EWMA forecast, and magenta solid lines with asterisks indicate IPF adjusted power discrepancy) and LF adjustments (red solid lines with plus sign markers denote LF EWMA forecast and green dash-dot lines indicate LF adjusted power discrepancy).

By running Algorithm 1, we not only can obtain the optimal adjustment strategy and steady power consumption but also can improve the economic benefits of both sides.
consumption, we compare it with the linear function (after this referred to as “LF”) -based strategy model, such as [10], which adjusts the power consumption with the LF. The detailed method is provided in Appendix. The parameters are listed in Table 4, and the comparison is illustrated in Table 4 and Figures 5-7.

It is shown in Table 4 and Figure 5 that the adjustments times and $\sigma_d$ under IPF adjustment strategy are less than that under LF scenario, and the IPF adjusted power discrepancy is steadier than that of the LF scenario.

Table 4 and Figure 6 show that the economic benefits of both sides under the IPF adjustment pricing are more than that under the LF adjustment pricing.

It can be observed in Figure 7 that the change of price under the IPF adjustment pricing is less than in the LF scenario.

The test results mean that it is more reasonable to consider the proposed data-driven IPF-based repeated-feedback adjustment strategy than the LF scenario.

5. Conclusion

A data-driven inverse proportional function-based repeated-feedback adjustment strategy that the power supplier may adopt to monitor the consumption power discrepancy between the real power consumption and the optimal power supply is proposed. The power supplier can get the optimal power supply by the optimal problem in the smart grid. Only when the trend of the power discrepancy is identified as an anomaly by exponentially weighted moving average, we will adjust the real-time price. That is to say, the data-driven inverse proportional function-based repeated-feedback

Figure 6: Comparison of economic benefits (histogram denotes whole social welfare and red solid lines with point markers indicate profit).

Figure 7: Comparison of price changes (blue solid lines denote IPF price adjustment and red solid lines depict LF price adjustment).
adjustment strategy is conducted only when the power discrepancy deviates far from the standard power discrepancy exceeding either the upper or lower boundary. Applying the data-driven inverse proportional function-based repeated-feedback adjustment strategy, we are able to get the least adjustments times and the steady and dependable power consumption close to the optimal power supply and more whole social welfare and profit. The proposed strategy can be applied to enrich the power grid monitoring and adjustment theory. At the same time, in practical application, it can help power suppliers formulate reasonable electricity price and manage the consumption power.

Appendix

Linear Function (LF)-Based Adjustment Strategy

Theorem A.1. If $\bar{c}^{\tau+1} \geq L_1 > 0$ and $\bar{c}^{\tau+1}$ is adjusted to $\chi_1 \in [0, L_1)$, the effect $\theta^{\tau+1}$ of price adjustment is [10]

$$\theta^{\tau+1} = \frac{(S^{\tau+1} - \chi_1)}{(\lambda_2 a)}.$$ (A.1)

If $\bar{c}^{\tau+1} \leq L_2 < 0$ and $\bar{c}^{\tau+1}$ is adjusted to $\chi_2 \in (L_2, 0]$, the effect $\theta^{\tau+1}$ of price adjustment is [10]

$$\theta^{\tau+1} = \frac{(S^{\tau+1} - \chi_2)}{(\lambda_2 a)},$$ (A.2)

where $\lambda_2$ is defined in (10).

Thus, when $\bar{c}^{\tau+1}$ exceeds the boundary $L_1$ or $L_2$, the users’ real adjusted consumption load $d_a^{\tau+1}$ and the power discrepancy $c_a^{\tau+1}$ in period $\tau + 1$ will change as follows [10]:

$$d_a^{\tau+1} = d^{\tau+1} - a \theta^{\tau+1}.$$ (A.3)

Data Availability

The data used to support the findings of this study are included in the references within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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