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Research Article

Predicting the Population Growth and Structure of China Based on Grey Fractional-Order Models

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Scientific prediction and accurate grasp of the future trend of population change are conducive to the formulation of different population policies at different stages, so as to alleviate the adverse effects of the aging population on society and provide scientific theoretical reference for controlling the population size and making policy. Considering that the population system is affected by many complex factors and the structural relationship among these factors is complex, it can be regarded as a typical dynamic grey system. In this paper, the fractional-order GM (1, 1) model and the fractional-order Verhulst model are established, respectively, based on the statistical data of China's population indices from 2015 to 2019 to forecast the population size and the change trend of population structure of China from 2015 to 2050 in the short-term and medium- to long-term. The forecast results show that China's population will grow in an inverse S shape from 2015 to 2050, when the total population will reach 1.43 billion. Moreover, during this period, the birth rate and natural growth rate of population will decrease year by year, and the proportion of aging population and the dependency ratio of population will increase year by year. Besides, the problem of aging population is going to become increasingly serious. The application of grey system method to population prediction can mine the complex information contained in the population number series. Meanwhile, the fractional-order accumulation can weaken the randomness of the original data series and reduce the influence of external disturbance factors, so it is a simple and effective population prediction method.

1. Introduction

China is the country with the largest population in the world. From the perspective of the process of population development, China's population grew at a relatively high speed from the early years after the founding of the People's Republic of China to the end of the 1970s. After the reform and opening up, despite the implementation of a strong family planning policy and efforts to control population growth, the growth rate was still relatively fast in the early 1980s due to the large population base and the effect of growth inertia. As time goes on, the growth rate of population slows down and the natural growth rate slowly decreases. By 2018, the natural growth rate of China's population has been below 4‰, achieving the target of low population growth, and the population growth is in an important period of transition to a low rate of birth, death, and growth. The number and structure of the population can reflect the level of economic development of a region, and it is also the indicator to measure social progress. The change of population will affect the formulation of basic state policies, the arrangement of employment, the development of social welfare, and even the standardization of national economic and social development strategies. Only by correctly dealing with the relationship between population, resources, and economy can we promote the sustainable development of society and construct a harmonious society. Population prediction is to infer the future population development process; its purpose is to be able to predict the future population development trend, and put forward the corresponding solutions and effective suggestions. It is of great significance to the formulation of population planning and policies, the strengthening of population management, and the formulation of national economic and social development plans.

The method of population prediction is to establish a mathematical model on the basis of understanding the objective law of population development and change, the characteristics of population variables, and their internal relations. Many scholars usually use Leslie model [1, 2], Markov model [3, 4], combination model [5], random prediction [6], and other model methods to predict and analyze the population size and structure. However, since the index parameters of the model are set on the basis of various assumptions and conjectures, the combined operation will bring cumulative errors to the prediction inevitably, resulting in a decline in the prediction accuracy. At the same time, there are many factors affecting the population system, including social and economic factors, natural environment factors, traditional customs, and thinking mode factors. The structural relationship among these factors is quite complex and in dynamic change, and its operation mechanism, change rule, and effect on population change cannot be accurately expressed, which is exactly the difficulty of population prediction. However, the grey system theory holds that the population system is a grey system containing both many known information and many unknown or uncertain information [7]. The quantity which is affected by many factors and cannot be accurately determined is called grey quantity. The grey system method is to dig and find its change rule from the time series of the grey quantity itself. The dynamic change of population time series data is the result of the interaction between the main, secondary, direct, indirect, known, unknown, obvious, and implicit factors. The interaction of these factors with known information and unknown information determines the actual grey quantity, i.e., the total population.

Grey system theory is a borderline subject with large cross-section, strong permeability, and wide application. It takes the uncertain system with small samples and poor information with "partial information known and partial information unknown" as the research object, mainly through the generation and development of some known information to extract valuable information, achieving the effective control on the system operation law [8, 9]. The essence of the core grey GM (1, 1) model is an exponential model, while the economic system, ecological system, and agricultural system can all be regarded as a generalized energy system, and the accumulation and release of energy generally have an exponential rule [10–14]. Population growth in a certain period of time in line with the exponential growth law, so the use of GM (1, 1) model for short-term prediction under the condition of low development coefficient has the higher prediction accuracy. However, the limitation of living resources and space, as well as the competition and conflict between people, will restrict the population growth, so the population growth is not an exponential model in the long run. Dutch scientist Verhulst put forward the Logistic Curve for the study of population development. The Verhulst model took into account the finiteness of the total population growth and proposed the law of the total population growth: the population growth rate gradually decreases with the total population growth, so it is applicable to the long-term population prediction [15, 16].

However, the traditional grey prediction models are all integer-order derivative models and belong to ideal memory models. The actual phenomenon is often irregular. Based on the idea of "in between," the fractional order is usually used to replace the integer order. There must be a fractional order between the 0 order and the 1 order. The order of magnitude between the accumulations can be adjusted accurately through the fractional order, and the target sequence can be generated by adjusting the order to improve the fitting accuracy of the prediction model. As an important branch of the grey system theory, Wu proposed the fractional-order accumulation grey model for the first time, which transformed the traditional first-order accumulation into fractional-order accumulation, and used the data after fractional-order accumulation to make predictions [17, 18]. It is proved that fractional-order accumulation can not only weaken the randomness of the original data series, but also make the perturbation bound of the solution of the grey prediction model smaller, which improves the priority of new information to a certain extent and indeed enhances the prediction stability of the model, thus achieving fruitful research results in many fields.

Fang proposed FGM (1, 1) to predict the maintenance cost of weapon system with small sample and improved the prediction performance [19]. Wu proposed a novel nonlinear grey Bernoulli model with fractional-order accumulation (FANGBM (1, 1)) to forecast short-term renewable energy consumption of China during the 13th Five-Year Plan [20]. Yan put forward fractional Hausdorff grey model to predict natural gas consumption, quarterly hydropower production, etc. [21]. Şahin proposed a novel optimized fractional nonlinear grey Bernoulli model (OFANGBM (1, 1)) to forecast the gross final energy consumption, energy consumption of renewable energy sources, and its share in France, Germany, Italy, Spain, Turkey, and the United Kingdom [22]. Liu developed a novel fractional grey polynomial model with time power term (FPGM (1, 1, t^{α})) for forecasting electricity consumption of India and China [23]. Based on the grey prediction model GM (1, 1), Meng proposed a novel fractional-order grey prediction model and systematically studied its modeling error [24].

The paper is organized as follows. Section 2 provides an overview on modeling process of fractional-order GM (1, 1) model, fractional-order Verhulst model, and particle swarm optimization algorithm. In Section 3, the fractional-order accumulation grey prediction models are adopted to forecast not only the total population of China, but also the birth rate, death rate, natural growth rate, and the changing trend of the age structure of the population. Finally, some conclusions are drawn and suggestions proposed in Section 4.

2. Modeling Methodologies

In fractional-order accumulation grey model, the traditional first-order accumulation is transferred into fractional-order accumulation. And the data after fractional-order accumulation are used to forecast. This method can weaken randomness of the original data sequence, making smaller disturbance of solution of grey prediction model, and to some extent improve the priority of the new information to gain higher prediction precision.

2.1. Fractional-Order GM (1, 1) Model. The modeling process of fractional-order GM (1, 1) model (abbreviated as FGM (1, 1)) is as follows:

Step 1: set the nonnegative sequence of the original data as $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$, $r \in R^{+}$, and the corresponding r-order cumulative generation sequence is $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}$, where

$$x^{(r)}(k) = \sum_{i=1}^{k} \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \tag{1}$$

Also, $\Gamma(r+k-i) = (r+k-i-1)!$ and $\Gamma(k-i+1) = (k-i)!$, $\Gamma(r) = (r-1)!$. For background value generation of $X^{(r)}$, the sequence adjacent to the mean value generation is $Z^{(r)} = \{z^{(r)}(2), z^{(r)}(3), \ldots, z^{(r)}(n)\}$, where

$$z^{(r)}(k) = \frac{x^{(r)}(k) + x^{(r)}(k-1)}{2}, \quad k = 2, 3, \ldots, n. \tag{2}$$

Step 2: let $X^{(0)}$, $X^{(r)}$, and $Z^{(r)}$ be as described in Step 1, and r be nonnegative real numbers; then the grey differential equation of the r-order cumulative grey GM (1, 1) model (abbreviated as FGM (1, 1)) is

$$x^{(r)}(k) - x^{(r)}(k-1) + az^{(r)}(k) = b. \tag{3}$$

Here, a is the development coefficient and b is the grey action. The whitening differential equation can be expressed as

$$\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t} + ax^{(r)}(t) = b. \tag{4}$$

Solve differential equation (4), and the time response function can be obtained as

$$x^{(r)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}. \tag{5}$$

Step 3: set parameters a, b as described in Step 2, the parameter sequence $\begin{bmatrix} \widehat{a} \\ \widehat{b} \end{bmatrix}$ of FGM (1, 1) model can be based on the principle of minimum error sum of squares, and be obtained by using the least squares estimation method

$$\begin{bmatrix} \widehat{a} \\ \widehat{b} \end{bmatrix} = \left(B^{T}B\right)^{-1}B^{T}Y. \tag{6}$$

Here, $Y = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}$, $B = \begin{bmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ \vdots & \vdots \\ -z^{(r)}(n) & 1 \end{bmatrix}$.

Step 4: substitute the parameters $\widehat{a}$ and $\widehat{b}$ into the time response function (5), and set $\widehat{x}^{(r)}(1) = x^{(r)}(1)$ to obtain the time response function of the original sequence

$$\widehat{x}^{(r)}(k+1) = \left(x^{(0)}(1) - \frac{\widehat{b}}{\widehat{a}}\right)e^{-\widehat{a}k} + \frac{\widehat{b}}{\widehat{a}}. \tag{7}$$

Here, $k = 1, 2, \ldots, n, \ldots$, $\widehat{x}^{(r)}(k+1)$ is the fitting value at the time-point $k+1$, and the sequence is obtained as

$$\widehat{X}^{(r)} = \left\{\widehat{x}^{(r)}(1), \widehat{x}^{(r)}(2), \ldots, \widehat{x}^{(r)}(n), \ldots\right\}. \tag{8}$$

Step 5: r-order reduction is made for sequence $\widehat{X}^{(r)}$, and the fitting sequence of original data can be obtained as $\widehat{X}^{(0)} = \left\{\widehat{x}^{(0)}(1), \widehat{x}^{(0)}(2), \ldots, \widehat{x}^{(0)}(n)\right\}$, where the reducing value is $\widehat{x}^{(0)}(1) = x^{(0)}(1)$, and

$$\widehat{x}^{(0)}(k) = \left(\widehat{x}^{(r)}\right)^{(-r)}(k) = \sum_{i=0}^{k-1} (-1)^{i} \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \widehat{x}^{(r)}(k-i), \quad k = 2, 3, \ldots, n, \tag{9}$$

where $\Gamma(r+1) = r!$, $\Gamma(i+1) = i!$, $\Gamma(r-i+1) = (r-i)!$. And the predicted value is $\widehat{x}^{(0)}(n+1)$, $\widehat{x}^{(0)}(n+2)$, ….

2.2. Fractional-Order Verhulst Model. The modeling process of fractional-order Verhulst model (abbreviated as FVGM) is as follows:

Step 1: let the nonnegative sequence of the original data be $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$, $r \in R^{+}$, and the corresponding r-order cumulative generation sequence $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}$ and the sequence adjacent to the mean value generation $Z^{(r)} = \{z^{(r)}(2), z^{(r)}(3), \ldots, z^{(r)}(n)\}$ are the same as Step 1 in Section 2.1.

Step 2: suppose $X^{(0)}$, $X^{(r)}$, and $Z^{(r)}$ as described in Step 1, and r is a nonnegative real number; then the grey differential equation of the r-order accumulation grey Verhulst model (abbreviated as FVGM) is

$$x^{(r)}(k) - x^{(r)}(k-1) + az^{(r)}(k) = b\left(z^{(r)}(k)\right)^{2}. \tag{10}$$

Here, a is the development coefficient and b is the gray action. The whitening differential equation can be expressed as

$$\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t}+ax^{(r)}(t)=b\left(x^{(r)}(t)\right)^2. \tag{11}$$

Solve differential equation (11), and the time response function can be obtained as

$$x^{(r)}(k+1)=\frac{ax^{(0)}(1)}{bx^{(0)}(1)+\left(a-bx^{(0)}(1)\right)e^{ak}}. \tag{12}$$

Step 3: same as Step 3 in Section 2.1; the parameter sequence $\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix}$ of FVGM model is obtained by least square estimation, where $Y=\begin{bmatrix}x^{(r-1)}(2)\\ x^{(r-1)}(3)\\ \vdots\\ x^{(r-1)}(n)\end{bmatrix}$ and

$$B=\begin{bmatrix}-z^{(r)}(2) & (z^{(r)}(2))^2\\ -z^{(r)}(3) & (z^{(r)}(3))^2\\ \vdots & \vdots\\ -z^{(r)}(n) & (z^{(r)}(n))^2\end{bmatrix}.$$

Step 4: substitute the parameters $\hat{a}$ and $\hat{b}$ into the time response function (12), and let $\hat{x}^{(r)}(1)=x^{(r)}(1)$; the time response function of the original sequence can be obtained as

$$\hat{x}^{(r)}(k+1)=\frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1)+\left(\hat{a}-\hat{b}x^{(0)}(1)\right)e^{\hat{a}k}}, \tag{13}$$

Here, $k=1,2,\ldots,n,\ldots$, $\hat{x}^{(r)}(k+1)$ is the fitting value at the $k+1$ time-point, thus obtaining the sequence $\hat{X}^{(r)}$.

Step 5: same as Step 5 in Section 2.1; r-order reduction is made for sequence $\hat{X}^{(r)}$. Thus, the fitting sequence of the original data is $\hat{X}^{(0)}=\left\{\hat{x}^{(0)}(1),\ \hat{x}^{(0)}(2),\ldots,\ \hat{x}^{(0)}(n)\right\}$, and predicted values are $\hat{x}^{(0)}(n+1)$, $\hat{x}^{(0)}(n+2),\ldots$.

2.3. Particle Swarm Optimization Algorithm. For fractional-order grey prediction model, the value of order r has a great influence on the prediction accuracy of the model, and the results of the model are also different with the different value of r. The main idea of finding the optimal value of order r is to minimize the error of the prediction model, which is generally expressed by the mean absolute relative error (MAPE):

$$\mathrm{MAPE}=\frac{1}{n}\sum_{k=1}^{n}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%. \tag{14}$$

To determine the optimal order r, a number of operations need to be repeated. It is difficult to achieve that by using the traditional method, but the particle swarm optimization algorithm can provide a good way for the determination of the optimal order r. Particle swarm optimization (abbreviated as PSO) was proposed by Dr. Kennedy and Dr. Eberhart in 1995 [25], which is a simulation of a simple social model. It originates from the artificial life theory and the clustering phenomenon of birds and fish and is mainly inspired by the behavior of animals.

The basic process of PSO algorithm is to assume that there are m particles in a D-dimensional target search space, and the position of each particle represents a potential solution. The position vector of the i particle is $X_i=(x_i^1,x_i^2,\ldots,x_i^D)$, the velocity vector is $V_i=(v_i^1,v_i^2,\ldots,v_i^D)$, and the best position it passes through is the individual extreme value denoted by p_{best}, and the optimal position searched by the whole particle swarm so far is denoted by g_{best}. In each iteration, the velocity of the particle is updated by the individual extremum and the global extremum, and the formula for calculating the change in the velocity of the particle is

$$V_{i+1}=wv_i^d=c_1r_1\left(p_i^d-x_i^d\right)+c_2r_2\left(p_g^d-x_i^d\right). \tag{15}$$

where V_{i+1} is the velocity of the updated particle, w is the inertial vector, r_1 and r_2 are the random numbers that vary within the range $[0,1]$, c_1 and c_2 are the acceleration constants (usually $c_1=c_2=2$), and v_i is limited by a maximum velocity $v_{\max}$. In each iteration, the position of each particle is modified by the velocity vector plus the position vector, and the formula to determine the position of the particle is

$$x_{i+1}=x_i+v_i, \tag{16}$$

where x_{i+1} is the position of the updated particle.

The termination condition of iteration is determined according to the specific problem. It is generally selected as the best position found by the particle swarm so far, satisfying the preset minimum adaptive threshold or reaching the maximum number of iterations.

2.4. Model Verification. Error analysis is an important criterion to judge the prediction model. Before extrapolation and application, the prediction model must be verified, so as to judge the reliability and robustness of the prediction model. In practice, a variety of error analysis methods can be used to verify the model. Considering the typical grey uncertainty features of grey fractional-order prediction model, this paper uses residual, relative error, average relative error, posterior error ratio, and small error probability to verify the model [9].

Let the original data sequence be $X^{(0)}=\left\{x^{(0)}(k)\right\}$, $k=1,2,\ldots,n$, and its corresponding fitting sequence be $\hat{X}^{(0)}=\left\{\hat{x}^{(0)}(k)\right\}$, $k=1,2,\ldots,n$:

Error criterion 1: calculate the residual $e(k)$, relative error Δ_k and average relative error $\overline{\Delta}$ between the original value $x^{(0)}(k)$ and the fitting value $\hat{x}^{(0)}(k)$ at the time-point k, as follows:

$$\begin{aligned} e(k)&=x^{(0)}(k)-\hat{x}^{(0)}(k),\\ \Delta_k&=\left|\frac{e(k)}{x^{(0)}(k)}\right|, \qquad \Delta_k. \\ \overline{\Delta}&=\frac{1}{n}\sum_{k=2}^{n} \end{aligned} \tag{17}$$

Error criterion 2: calculate the mean value $\overline{x}$ and the mean residual $\overline{e}$ of the original data series $X^{(0)}$ as follows:

$$\begin{aligned}\overline{x} &= \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k),\\ \overline{e} &= \frac{1}{n-1}\sum_{k=2}^{n} e^{(0)}(k).\end{aligned} \tag{18}$$

Error criterion 3: calculate the posterior error ratio C between variance S_1^2 and residual variance S_2^2 of the original data series $X^{(0)}$, and the small error probability P as follows:

$$\begin{aligned}S_1^2 &= \frac{1}{n}\sum_{k=1}^{n}\left[x^{(0)}(k) - \overline{x}\right]^2,\\ S_2^2 &= \frac{1}{n-1}\sum_{k=2}^{n}\left[e^{(0)}(k) - \overline{e}\right]^2,\\ C &= \frac{S_2}{S_1},\\ P &= P\left\{\left|e^{(0)}(k) - \overline{e}\right| < 0.6745 S_1\right\}.\end{aligned} \tag{19}$$

In general, the smaller the value of residual $e(k)$, relative error Δ_k, average relative error $\overline{\Delta}$ and posterior error ratio C is, the larger the value of small error probability P is, and the higher the prediction accuracy of the model is. If $\Delta_k < 0.01$, and $\overline{\Delta} < 0.01, C < 0.35, P > 0.95$, then the prediction accuracy of the model is first-level. According to the grey system theory, when the development coefficient is $a \in (-2, 2)$ and $a \geq -0.3$, the grey fractional-order model can be used for medium- to long-term prediction.

3. Population Forecasting and Empirical Analysis

According to the statistical data of population indicators from 2015 to 2019 in China Statistical Yearbook 2020 (shown in Table 1) [26], the grey fractional-order models are used to predict the total population, the factors of population change, and the age structure of the population, respectively, in the short-term and medium- to long-term. Among them, the FGM (1, 1) model is used for the short-term prediction in the period of 2020–2025, while the FVGM model is used for the medium- to long-term prediction in the period of 2026–2050.

The calculation is performed as mentioned above with the help of MATLAB software. And the modeling flowsheet of FGM (1, 1) model for population prediction is taken as an example to illustrate (shown in Figure 1).

3.1. Predicting Gross Population. Based on the statistical data of China's total population from 2015 to 2019, a 5-dimensional grey dynamic prediction model is established. The short-term prediction of the total population from 2020 to 2025 is firstly made by using the 5-year data. The FGM (1, 1) model of the total population is

$$\widehat{x}^{(r)}(k+1) = \left(x^{(0)}(1) + \frac{13.689211}{0.001180}\right)e^{0.001180k} + \frac{13.689211}{0.001180}, \tag{20}$$

where the optimal value of the accumulation order r determined by the PSO algorithm is $r = 0.9917$. Through the model test, the values of the corresponding relative error Δ_k and average relative error $\overline{\Delta}$ are all <0.01, and posteriori error ratio, $C < 0.35$, small error probability $P > 0.95$; the model meets the first-level accuracy requirements, so it can be used for short-term prediction. The medium- to long-term prediction takes 5 years as the interval, and the FVGM model is established as

$$\widehat{x}^{(r)}(k+1) = \frac{-0.146298x^{(0)}(1)}{-0.010206x^{(0)}(1) + \left(-0.146298 + 0.010206x^{(0)}(1)\right)e^{-0.146298k}}, \tag{21}$$

where the order r is determined by the PSO algorithm and its optimal value is $r = 1.0292e-07$. The prediction results of total population and gender composition of China are shown in Table 2.

As can be seen from Table 2, the total population of China will reach 1.42 billion in 2025 and 1.433 billion in the middle of this century. However, the gender ratio of males to females will continue to decrease since 2015 and will be as low as 1.0134 in 2050. In addition, it can be seen from Figure 2 that the growth trend of China's total population from 2015 to 2050 is in the shape of an anti-S curve.

3.2. Predicting Indexes of Population Change Factor. Birth rate and death rate are the main indicators to directly measure the population change. If we only know the general trend of population change, it is far from enough to grasp the specific situation of the population change in a country or region. Therefore, the change situation of birth rate and death rate must be analyzed accordingly.

According to the statistical data of birth rate from 2015 to 2019, a 5-dimensional grey dynamic prediction model was established, and a FGM (1, 1) model for short-term prediction of birth rate was established as

Table 1: Demographic indicators statistics.

Population indicators		2015	2016	2017	2018	2019
Population size and composition (unit: 10^8people)	Gross population	13.7462	13.8271	13.9008	13.9538	14.0005
	Male population	7.0414	7.0815	7.1137	7.1351	7.1527
	Female population	6.7048	6.7456	6.7871	6.8187	6.8478
Birth rate, death rate, and natural growth rate (unit: ‰)	Birth rate	12.07	12.95	12.43	10.94	10.48
	Death rate	7.11	7.09	7.11	7.13	7.14
	Natural growth rate	4.96	5.86	5.32	3.81	3.34
Population age structure (unit: ‰)	Proportion of aged 0–14	16.5	16.7	16.8	16.9	16.8
	Proportion of aged 15–64	73.0	72.5	71.8	71.2	70.6
	Proportion of age over 65	10.5	10.8	11.4	11.9	12.6

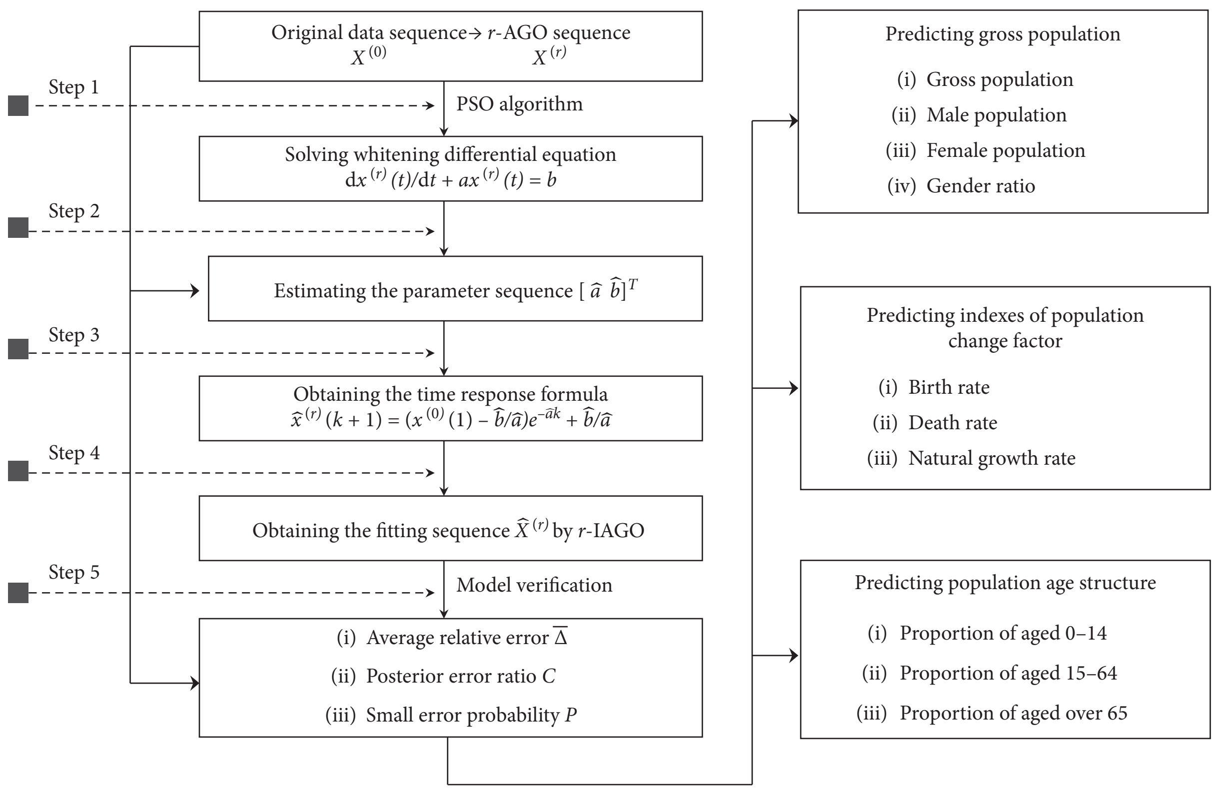

Figure 1: Modeling flowsheet of FGM (1, 1) model for population prediction.

$$\hat{x}^{(r)}(k+1) = \left(x^{(0)}(1) - \frac{13.940994}{0.147828}\right)e^{-0.147828k} + \frac{13.940994}{0.147828}, \tag{22}$$

where the optimal accumulation order is $r = 0.8636$. Through the model test, all values of error criteria $\Delta_k, \overline{\Delta}, C, P$ all meet the first-level accuracy requirements. Then the FVGM model for medium- to long-term prediction was established with the interval of 5 years as

$$\hat{x}^{(r)}(k+1) = \frac{-0.777432x^{(0)}(1)}{-0.025749x^{(0)}(1) + \left(-0.777432 + 0.025749x^{(0)}(1)\right)e^{-0.777432k}}, \tag{23}$$

where the optimal accumulation order is $r = 0.5019$. The same forecasting method can be used to forecast the death rate and natural growth rate in the short-term and medium- to long-term. The predicted results of the corresponding

Table 2: Predictions of population size and gender composition.

	Year	Gross population predicted values (unit: 10^8people)	Male population predicted values (unit: 10^8people)	Female population predicted values (unit: 10^8people)	Gender ratio
The fitting samples area	2015	13.7462	7.0414	6.7048	1.0502
	2016	13.8271	7.0815	6.7456	1.0498
	2017	13.9003	7.1135	6.7868	1.0481
	2018	13.9548	7.1355	6.8193	1.0464
	2019	14.0000	7.1525	6.8475	1.0445
Short-term forecast area	2020	14.0395	7.1665	6.8731	1.0427
	2021	14.0753	7.1784	6.8970	1.0408
	2022	14.1084	7.1889	6.9196	1.0389
	2023	14.1395	7.1983	6.9414	1.0370
	2024	14.1691	7.2069	6.9624	1.0351
	2025	14.1974	7.2147	6.9829	1.0332
Medium- to long-term forecast area	2030	14.2669	7.2260	7.0482	1.0252
	2035	14.3022	7.2304	7.0886	1.0200
	2040	14.3192	7.2319	7.1131	1.0167
	2045	14.3274	7.2323	7.1279	1.0146
	2050	14.3314	7.2325	7.1369	1.0134

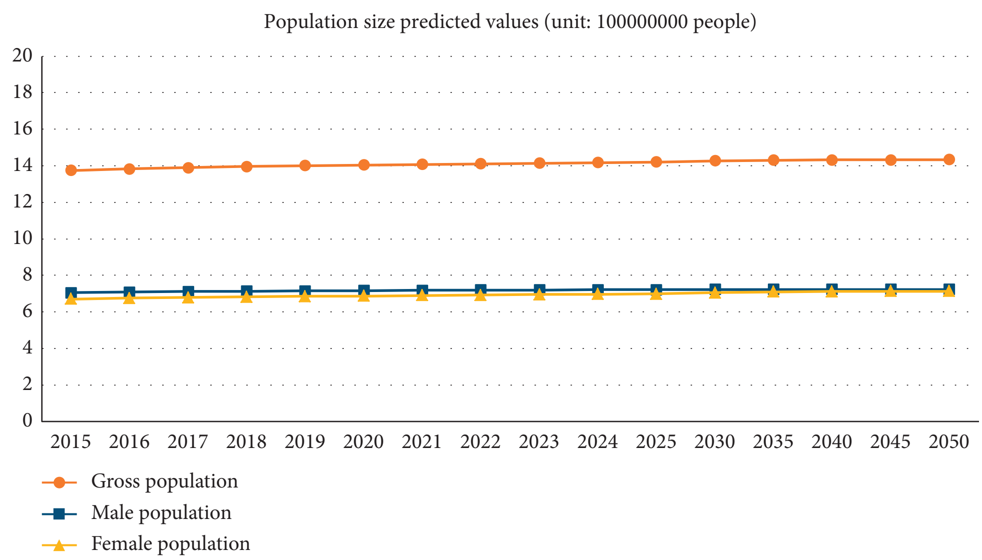

Figure 2: Predicted trend curves of total population and sex composition.

factors of population change are shown in Table 3, and the predicted trend curves of birth rate, death rate, and natural growth rate are shown in Figure 3.

3.2.1. Birth Rate Prediction. The changes in China's birth rate during the forecast period are as follows: the birth rate has been decreasing year by year. In 2015, the birth rate was 12.07‰. It is predicted to drop to 5.65‰ in 2025, below 5‰ in 2030 and 2.90‰ in 2050. Due to China's long-term implementation of family planning policy after the 1970s, and with the continuous improvement of the population quality and the change in fertility attitudes, the birth rate in China has declined accordingly.

3.2.2. Death Rate Prediction. China's death rate will remain between 7.10‰ and 7.20‰ from 2015 to 2050. This is because living standards and living environment have been relatively stable during this period, so the death rate has not changed much. However, from 2040, there will be a small decrease in the death rate because of demographic changes, especially the increase in the aged population, which will lead to a slow decline in the death rate.

3.2.3. Natural Growth Rate Prediction. During the forecast period, the natural growth rate will be on a downward trend due to a decline in the birth rate and little change in the death rate. The natural growth rate reached the peak of 5.86‰ in 2016 and will drop to 1.60‰ in 2025 and 1.06‰ in 2035. By 2040, the natural population growth rate will be below 1.00‰, which means that the death rate will be far greater than the birth rate, and the total population will tend to decrease in the future.

3.3. Predicting Population Age Structure. Population age structure refers to the proportion of the population of each age group in the whole population at a certain point in time

Table 3: Predictions of birth rate, death rate and natural growth rate.

	Year	Birth rate predicted values (unit: ‰)	Death rate predicted values (unit: ‰)	Natural growth rate predicted values (unit: ‰)
The fitting samples area	2015	12.07	7.11	4.96
	2016	12.95	7.09	5.86
	2017	12.22	7.11	4.96
	2018	11.28	7.13	4.05
	2019	10.30	7.14	3.34
Short-term forecast area	2020	9.36	7.15	2.81
	2021	8.47	7.16	2.42
	2022	7.66	7.18	2.13
	2023	6.92	7.19	1.91
	2024	6.25	7.19	1.74
	2025	5.65	7.20	1.60
Medium- to long-term forecast area	2030	4.59	7.21	1.27
	2035	3.91	7.20	1.06
	2040	3.47	7.19	0.93
	2045	3.15	7.18	0.83
	2050	2.90	7.17	0.76

and in a certain area, which is one of the important indicators of population research. It is the result of the combined effect of past and present birth, death, and migration changes on population development. The age structure of population has a great influence on the type, speed, and trend of future population development, so the prediction of its development trend plays a very important role in the formulation of population policy.

According to the statistical data of different population age groups from 2015 to 2019, a 5-dimensional grey dynamic prediction model was established, and a FGM (1, 1) model was established for short-term prediction of the proportion of 0–14-year-old population as

$$\widehat{x}^{(r)}(k+1) = \frac{-0.615838x^{(0)}(1)}{-0.034323x^{(0)}(1) + \left(-0.615838 + 0.034323x^{(0)}(1)\right)e^{-0.615838k}}, \tag{25}$$

where the optimal accumulation order is $r = 0.0281$. The same forecasting method can be used to predict the proportion of the population aged 15–64 and over 65 in the short-term and medium- to long-term. The prediction results of the proportion of the population of different age groups are shown in Table 4, and the column chart of the prediction of the population age structure is shown in Figure 4.

3.3.1. Population Ageing Trend. It can be seen from Table 4 that the aged population in China is on the rise from 2015 to 2050. In 2015, the aged population has exceeded 10%, and China has entered the stage of aging society. Especially in 2030, the pace of aging has accelerated significantly. By 2050, the proportion of the aged population will reach 24.83%; that is, about one in every four people will be aged. The problem of population aging is very serious, and it will bring a series of social and economic problems.

$$\widehat{x}^{(r)}(k+1) = \left(x^{(0)}(1) - \frac{10.839485}{0.605343}\right)e^{-0.605343k} + \frac{10.839485}{0.605343}, \tag{24}$$

where the optimal accumulation order is $r = 0.0273$. Through the model test, all values of error criteria Δ_k, $\overline{\Delta}$, C, and P are in line with the first-level accuracy requirements. Then, the FVGM model for medium- to long-term prediction of the proportion of 0–14-year-old population was established with the interval of 5 years as

3.3.2. Change Trend of Labor Force. The labor population is an important part of the population age structure, and the scale of the labor population will have a crucial impact on the social and economic development. During the period of 2015–2025, the overall change of China's labor population will fluctuate little, but during the period of 2025–2050, the labor population will show a rapid decline trend, mainly due to the serious aging problem.

3.3.3. Dependency Ratio Analysis. According to the above analysis data, from 2015 to 2025, China's population dependency ratio will rise slowly year by year, but will continue to grow rapidly in the next 25 years, which is largely related to China's aging population and will bring great pressure to the society and family.

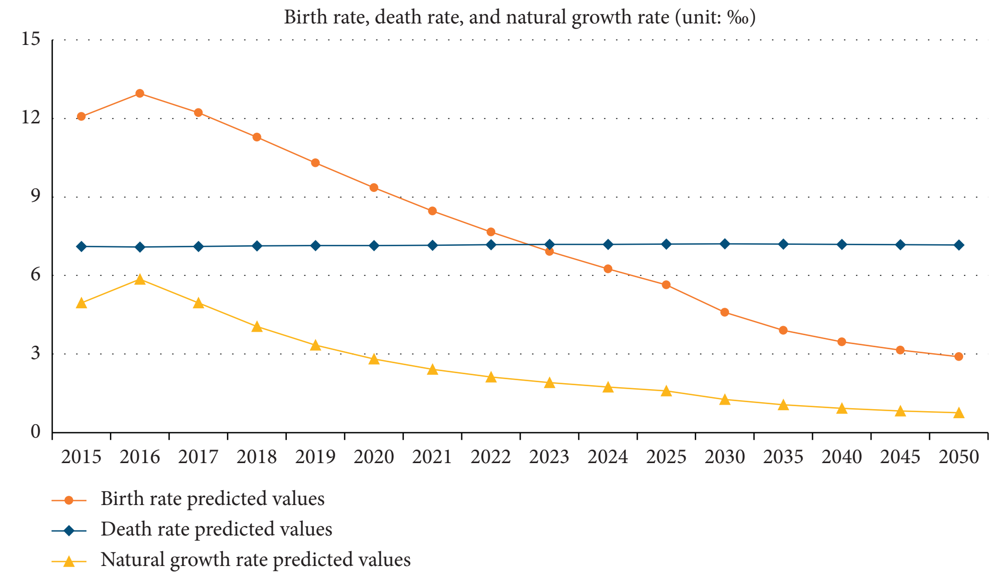

Figure 3: Predicted trend curves of birth rate, death rate, and natural growth rate.

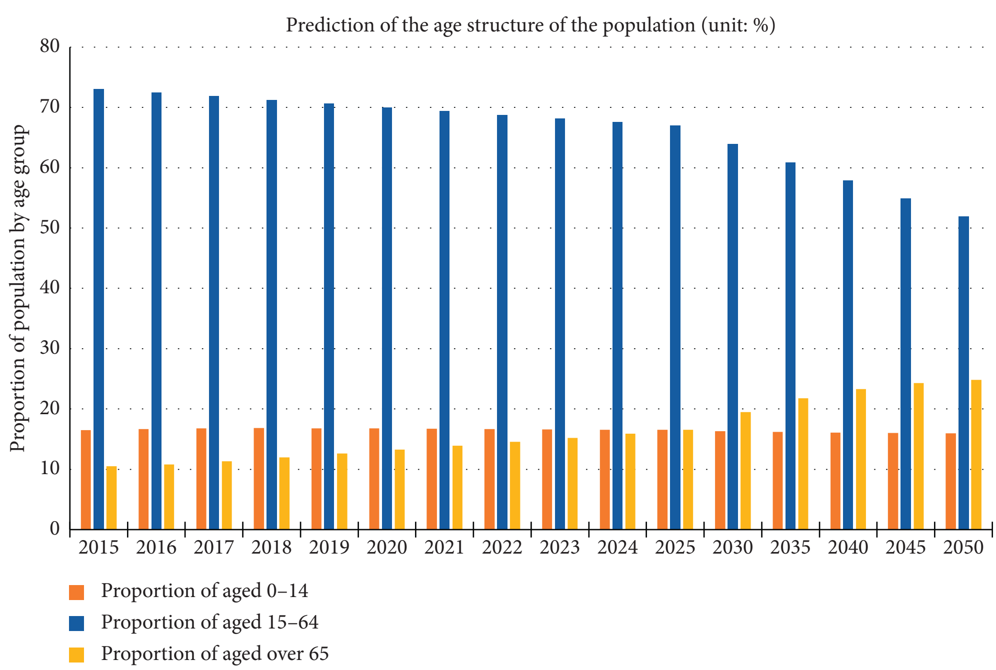

Figure 4: Column chart of predicted trends in the age structure of the population.

Table 4: Prediction of the age structure of the population.

	Year	Proportion of aged 0–14 (unit: %)	Proportion of aged 15–64 (unit: %)	Proportion of aged over 65 (unit: %)	Dependency ratio (unit: %)
The fitting samples area	2015	16.50	73.00	10.50	0.37
	2016	16.69	72.47	10.80	0.38
	2017	16.80	71.84	11.35	0.39
	2018	16.83	71.21	11.96	0.40
	2019	16.81	70.58	12.60	0.42

Table 4: Continued.

	Year	Proportion of aged 0–14 (unit: %)	Proportion of aged 15–64 (unit: %)	Proportion of aged over 65 (unit: %)	Dependency ratio (unit: %)
Short-term forecast area	2020	16.77	69.96	13.25	0.43
	2021	16.72	69.35	13.91	0.44
	2022	16.67	68.74	14.57	0.45
	2023	16.62	68.14	15.23	0.47
	2024	16.57	67.54	15.89	0.48
	2025	16.53	66.95	16.55	0.49
Medium- to long-term forecast area	2030	16.34	63.91	19.49	0.56
	2035	16.21	60.87	21.76	0.62
	2040	16.11	57.86	23.31	0.68
	2045	16.03	54.87	24.28	0.73
	2050	15.96	51.94	24.83	0.79

4. Conclusions

Based on the statistical data of China's population indices from 2015 to 2019, this paper establishes 5-dimensional grey fractional-order prediction models and makes short-term and medium- to long-term prediction of China's population and its structure. The models considered not only the total population and natural growth rate, but also the age structure of population, so they are more reliable and practical, which can describe and predict the population evolution process in a long period. The application of grey system method to population prediction has its unique advantages, which can mine the information contained in the population number series; meanwhile, the fractional-order accumulation can weaken the randomness of the original data series and reduce the influence of external disturbance factors, so it is a simple and effective method for population prediction. Scientific prediction and accurate grasp of the future trend of population change are conducive to the formulation of different population policies at different stages, so as to alleviate the adverse effects of an increasingly aging population on society, and provide scientific theoretical reference for controlling the population size and making policy.

First of all, we need to vigorously develop education and comprehensively improve the quality of the population. In order to change the present situation of unreasonable occupation composition, difficult popularization of science and technology, lack of scientific and technological personnel, and low management level, we must develop education cause actively. Therefore, we must strengthen the basic education, strengthen the pre-job training and technical training, and strive to improve the quality of labor resources. Secondly, it is necessary to solve the employment problem of working-age population through multiple channels. In the future, the increase of the working-age population in China will provide a large number of labor resources, and at the same time, it will bring the problem about employment of labor force. Accordingly, various measures should be taken to provide employment opportunities for labor force. For example, with the aging of the labor force, the proportion of the elderly labor force will continue to rise. According to the physiological characteristics and physical conditions of the labor force, the problem of employment for the elderly labor force should be solved. In addition, the aging of the population needs to be addressed. Population aging is the inevitable result of fertility decline. With the gradual increase of the proportion of the aged population, the problem of population aging is becoming more and more obvious. Consequently, it is necessary to implement the national strategy of actively coping with the aging of the population, promote the modernization of the social governance system and governance capacity for the aged, and grasp the development trend of the aging of the population comprehensively.

Data Availability

All data generated or used during the study are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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