Review Article

A Bibliometric Analysis and Visualization of Fractional Order Research in China over Two Decades (2001–2020)

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Fractional order research has interdisciplinary characteristics and has been widely used in the field of natural sciences. Therefore, fractional order research has become an important area of concern for scholars. This paper used 2854 literatures collected from China National Knowledge Infrastructure (CNKI) database from 2001 to 2020 as the data source and used bibliometrics and two visualization methods to conduct bibliometric analysis and visualization research on China's fractional order research. To begin with, this paper analyzed the time series distribution of publications, the distribution of research institutions, the author cooccurrence network, the distribution of important journals, and the distribution of important literature, which explained the research status of the fractional order. Furthermore, this paper used VOSviewer software to analyze the clustering and density distribution of the fractional order research keywords, which revealed the hotspots of the fractional order research. Finally, with the help of CiteSpace software, the burst keywords were analyzed to further explore the frontiers of fractional order research. This paper systematically reveals the research status, research hotspots, and research frontiers of China's fractional order research, which can provide certain theoretical and practical references for related follow-up researchers.

1. Introduction

The fractional calculus is a name for the theory of integrals and derivatives of arbitrary order, which unifies and generalizes the notions of integer-order differential and integral [1]. The idea of fractional calculus is known in developing regular calculation referring to Leibniz and L'hospital's work in 1695 [2]. Fractional calculus was introduced more than 300 years ago [3], and it was introduced by Leibniz in the scope of mathematics by the end of the 17th century [4]. The research on fractional calculus experienced its boom in the past decades [5], and recently this mathematical tool has found its way in various branches of science and engineering [6], such as electrical and electronic engineering [7, 8], automation control systems [9, 10], and computer science information systems [11, 12]. Therefore, the related research of fractional order has attracted the wide attention of international scholars.

In recent years, scholars have conducted reviews around the relevant topics of fractional order research and have

achieved relatively rich research results, such as literatures [2–6]. In addition, other scholars focused on specific areas of fractional order research and reviewed them, such as the research field of fractional order control [13–15] and the research field of image processing [16, 17]. There is no doubt that the research results of the above scholars have contributed to the advancement of fractional order research and can also provide reference and support for subsequent scholars to further deepen the relevant research of fractional order. However, it is difficult to comprehensively extract the common elements and core points of the literature to a certain extent if the comprehensive analysis of the long term and a large amount of literature is conducted by combination, summary, and other methods. Furthermore, in this case, the researchers may find it difficult to avoid the subjective influence in the analysis process, so its scientificity is worth discussing.

Bibliometrics and visualization software are effective tools for analyzing mass literature [18]. VOSviewer is a software tool for constructing and visualizing bibliometric

networks [19], which can be used to construct maps of authors or journals based on cocitation data or to construct maps of keywords based on cooccurrence data [20]. Meanwhile, VOSviewer has an advanced graphic presentation capability that is suitable for large-scale data and locates key points and hotspots in scientific research using density view [21]. In addition, CiteSpace is a knowledge visualization tool developed by Dr. Chen Chaomei [22], which combines citation analysis and cocitation analysis to transform literature data from abstract data into visual and intuitive maps of scientific knowledge through information visualization, helping researchers find research hotspots, and frontiers in a scientific field [23].

In summary, regarding the related research of fractional literature review, many scholars reviewed the related research of fractional order from a micro perspective, that is, focusing on a specific area of the fractional order for exploration. However, there are few related research results of literature analysis on the overall fractional research from a macro perspective. At the same time, from the perspective of literature analysis methods, many scholars used methods such as combination and induction, but, with the help of visual analysis software, the findings of visual research on fractional research are rarely found.

Therefore, this paper collected the research data of China's fractional literature from 2001 to 2020 and used bibliometrics and two visualization research methods to analyze the literature of China's fractional order over two decades. The purpose of this paper is to analyze the research status and explore the hotspots and research frontiers of China's fractional order research. To be specific, this paper used the method of bibliometric analysis to explore the time series distribution of publications, the distribution of research institutions, the author cooccurrence network, the distribution of key journals, and the distribution of important literature, which clarified the research status of fractional order. Furthermore, it used VOSviewer to analyze the clustering and density distribution of the fractional research keyword network and used CiteSpace to analyze the fractional research burst keywords, which can reveal the hotspots and research frontiers of the fractional order research. This paper systematically analyzes and explores the research status, research hotspots, and frontier fields of fractional order research, which enriches the content system of fractional order research to a certain extent and can also provide some references for scholars to explore and further deepen the research on fractional order.

2. Research Methods and Data Sources

The research methods of this paper mainly include bibliometric analysis and visualization research methods. Firstly, this paper used the bibliometric analysis method to analyze the publications, research institutions, important journals, and important literature of fractional order research, which can explore the current status of fractional order research. Secondly, VOSviewer and CiteSpace software were used to visualize the fractional order research. To be specific, VOSviewer was used to analyze the author cooccurrence

network, keyword network, keyword density, and so forth of fractional order research, thus revealing the author cooperation situation and research hotspots of fractional order research. Meanwhile, CiteSpace was used to analyze the burst keywords of fractional order research, thereby further exploring the frontiers of fractional order research.

The data of this paper came from China National Knowledge Infrastructure (CNKI). The literature retrieval conditions were as follows: set the title as fractional order, selected the categories of the sources as SCI source journals, EI source journals, core journals, CSSCI, and CSSCD, selected the period time from 2001 to 2020, and a total of 2854 literatures were retrieved. In view of this, 2854 literatures were retrieved as research data sources, and the retrieval date was May 18, 2021. It should be noted that the bibliometric data of this paper are all from CNKI statistics.

3. Research Status

This study focused on the perspective of the time series distribution of publications, the distribution of research institutions, the author cooccurrence network, the distribution of important journals, and the distribution of important literature and used the bibliometric analysis method to explore the research status of fractional order.

3.1. Time Series Distribution of Publications. The dynamic change of the publications with time evolution can effectively reveal the degree of concern in a certain research field. Therefore, this paper made a quantitative analysis on the time series distribution of 2854 papers on fractional order research in the CNKI database. The results are shown in Figure 1.

It can be seen from Figure 1 that the publications of fractional order research show an overall upward trend during the research period (2001–2020), but there are obvious differences in the distribution of publications in different time series phases. To be specific, the time series distribution of the publications of fractional order research can be divided into the four following phases: The first phase is from 2001 to 2010; the total number of publications in this phase is relatively small, accounting for about 15% of the total number of publications in the past 10 years, especially from 2001 to 2006, and the average annual number of publications in fractional order research is less than 20, so the first phase belongs to the slow development phase of fractional order research. The second phase is from 2010 to 2014; this phase belongs to the rapid growth phase of fractional order research. During this period, the publications of fractional order research continued to emerge, which indicates that Chinese scholars pay more and more attention to fractional order research. Especially in 2011, the number of publications in fractional order research increased by 68.13% compared with the previous year, and the number of publications in that year exceeded 100 for the first time. Therefore, it can be considered that 2011 is the transition period of fractional order research. The third phase is from 2014 to 2019; the average number of annual

FIGURE 1: Time series distribution of publications.

publications in this phase is about 260, and there is a little difference in the number of publications each year, which indicates that the fractional order research has entered a phase of steady development. The fourth phase is from 2019 to 2020; the publications of fractional order research continued to grow, and the publications exceeded 300 for the first time in 2020. This phase belongs to the sustained growth phase of fractional order research, which shows that Chinese scholars' attention to fractional order research continues to advance.

3.2. Distribution of Research Institutions. In order to further explore the distribution of the main research power of fractional order, this paper made a quantitative analysis of the top 25 research institutions in the publications of fractional order research. The results are shown in Figure 2.

Figure 2 shows the distribution of the publications by the main research institutions. These research institutions are an important power of fractional order research and play an important role in promoting the development of fractional order research. Among them, Sichuan University has the largest number of publications on fractional order research, up to 95. Anhui University, Beijing Institute of Technology, Zhengzhou University of Aeronautics, and Yanshan University also published more papers; all of them have published more than 50 papers, which are 71, 66, 62, and 53, respectively. Therefore, it can be considered that these research institutions have an important influence on the development of fractional order research in China. In addition, according to the horizontal comparison of the publications by research institutions in Figure 2, we can see that the overall difference in the number of publications by research institutions is relatively small. Combined with data statistics, it can be seen that the average number of publications by the main research institutions in Figure 2 is about 45, which

indicates that the distribution of fractional order research publications among research institutions is relatively balanced. Additionally, further analysis of the attributes of research institutions shows that the distribution of fractional order research institutions includes comprehensive universities, science, engineering universities, and military academy. In summary, the bibliometric analysis of the distribution of main research institutions in the fractional order can contribute to understanding the publications and attributes of research institutions for scholars.

3.3. Author Cooccurrence Network. This paper used VOSviewer to analyze the author cooccurrence network of fractional order research. The parameter options were set as follows: chose coauthorship for type of analysis, authors for unit of analysis, full counting for counting method, and 3 for the minimum number of documents of an author, thereby generating the author cooccurrence network visualization map, as shown in Figure 3.

Figure 3 shows the author cooccurrence network of fractional order research, in which the node size is related to the publications by authors, and the larger the node, the more publications by authors. The thickness of connecting lines and distance between nodes is significantly related to the cooperative relationship between authors. The thicker the connection and the closer the location between nodes, the closer the cooperation relationship between authors.

As can be seen from Figure 3, based on the global perspective analysis, it can be found that the author cooccurrence network nodes of fractional order research are relatively scattered, the overall connecting lines between nodes are relatively few, and the network distribution and connecting lines between nodes do not show obvious network cooperation relationship. Therefore, it can be considered that the overall cooperation intensity among authors

FIGURE 2: Distribution of main research institutions.

in fractional order is not high. However, based on the local perspective analysis, it can be found that some of the nodes are large and show a relatively close cooperative relationship, such as scholars Mao Beixing, Jiang Wei, Tao Ran, Pu Yifei, and Hu Weimin. These authors are in a relatively central position in the cooccurrence network and form a strong cooperative relationship with other scholars in the research group. Therefore, the global and local perspective analysis shows that the overall cooperation intensity of fractional order research is not high, but there are many research groups with a big number of publications and strong co-operation relationships.

3.4. Distribution of Important Journals. To a certain extent, the publications in a journal can reflect its influence in a certain research field. Statistics on the number of publications in a journal in fractional order research are also of great significance for revealing the distribution of fractional order research disciplines. In view of this, this paper made a quantitative analysis of the top 15 journals in terms of the publications of fractional order research. The results are shown in Table 1.

As can be seen from Table 1, regarding fractional research, *Acta Physica Sinica* has the largest number of

publications, with 123 papers. Meanwhile, *Mathematics in Practice and Theory*, *Journal of Jilin University (Science Edition)*, *Mathematica Applicata*, and *Journal of Shandong University (Natural Science)* also have more publications, which show that these journals have an important influence in the field of fractional order research. From the number of publications and subject areas of journals, we can see that publications in mathematics are the largest, such as *Mathematics in Practice and Theory* and *Mathematica Applicata*. At same the time, some other journals also involve the research publications of mathematics, such as *Applied Mathematics and Mechanics* and *Acta Mathematica Scientia*. In addition, the publications of fractional order research are also involved in physics, computer, control engineering, and other fields, such as *Acta Physica Sinica*, *Computer Engineering and Applications*, and *Control Engineering of China*. Therefore, it indicates the interdisciplinary attribute of fractional order research.

3.5. Distribution of Important Literature. To further explore the highly cited literature of fractional order research, this paper made a bibliometric analysis on the top 10 cited literatures of fractional order research. The results are shown in Table 2.

FIGURE 3: Author cooccurrence network. Note: due to the large number of authors in Figure 3 and limited space, this paper only translates the names of the following authors: 毛北行 = Mao Beixing; 蒋威 = Jiang Wei; 陶然 = Tao Ran; 蒲亦菲 = Pu Yifei; and 胡卫敏 = Hu Weimin.

TABLE 1: Statistics of the top 15 journals publications.

No.	Journal title	Publications
1	Acta Physica Sinica	123
2	Mathematics in Practice and Theory	119
3	Journal of Jilin University (Science Edition)	76
4	Mathematica Applicata	53
5	Journal of Shandong University (Natural Science)	50
6	Journal of Sichuan University (Natural Science Edition)	49
7	Computer Engineering and Applications	46
8	Control Engineering of China	39
9	Applied Mathematics and Mechanics	36
10	Acta Electronica Sinica	36
11	Acta Mathematica Scientia	35
12	Control Theory & Applications	34
13	Journal of Electronics & Information Technology	33
14	Science Technology and Engineering	33
15	Application Research of Computers	31

It can be seen from Table 2 that *Detection and parameter estimation of multi-component LFM signals based on fractional Fourier transform* is the most frequently cited literature in the fractional order research, and this paper proposed a method of multicomponent LFM signal detection and parameter estimation based on fractional Fourier transform [24]. Meanwhile, the frequently cited literatures in Table 2 such as *Dimensional normalization in the digital computation of the fractional Fourier transform* and *Research*

progress of fractional Fourier transform in the field of signal processing are also related to the fractional Fourier transform, so this also reflects the importance of the fractional Fourier transform in the field of fractional order research. Based on the perspective of literature attribute analysis, there are many review literatures about fractional order, such as *Summary of research on fractional-order control*, *Research progress of fractional Fourier transform in the field of signal processing*, and *Summary of research on image processing*

TABLE 2: Statistics of top 10 important literatures cited in frequency.

No.	Literature	First author	Citation frequency	Year
1	Detection and parameter estimation of multi-component LFM signals based on fractional Fourier transform	Qi lin	358	2003
2	Design of fractional PID controller for fractional order system	Xue Dingyu	352	2007
3	Dimensional normalization in the digital computation of the fractional Fourier transform	Zhao Xinghao	263	2005
4	Summary of research on fractional-order control	Zhu Chengxiang	216	2009
5	Research progress of fractional Fourier transform in the field of signal processing	Tao Ran	182	2006
6	Image enhancement based on fractional differentials	Yang Zhuzhong	177	2008
7	Fractional order sliding-mode control for permanent magnet synchronous motor	Zhang Bitao	134	2012
8	Fractional differential masks of digital image and their numerical implementation algorithms	Pu Yifei	130	2007
9	Application of fractional differential approach to digital image processing	Pu Yifei	121	2007
10	Summary of research on image processing using fractional calculus	Huang Guo	117	2012

using fractional calculus. In addition, it can be seen from the year of publication that most of the highly cited literatures of fractional order research were published before 2010, while the highly cited literatures after 2010 were relatively few; both groups of literatures provide important support for the development of fractional order research.

4. Research Hotspots and Frontier Analysis

This study used VOSviewer to analyze the cooccurrence of the keywords, combined with CiteSpace to analyze the burst keywords, thereby revealing the hotspots and research frontiers of the fractional research.

4.1. Research Hotspots. The keyword is the core and essence of a document, which is a high-level summary of the content of the article [25]. After cluster analysis and words frequency count, topics and research hotspots can be known [26]. In view of this, this paper visually analyzed the cooccurrence of keywords in fractional order research using VOSviewer, which can explore the distribution of hotspots in fractional order research. To be specific, it selected type of analysis as cooccurrence, unit of analysis as keywords, and full counting as counting method and set minimum number of occurrences of a keyword to 18. Moreover, the keywords were merged and cleaned; that is, the different expressions of fractional Fourier transform were merged, and the repeated English keywords were cleaned. On this basis, by adjusting the clustering option and running the software to generate the keyword network and density visualization view, as shown in Figure 4 and Figure 5, respectively, it should be noted that this paper also tried to merge the keywords of fractional PID controller and fractional order $PI^{\lambda}D^{\mu}$ controller as well as fractional order and fractional calculus with the same method as above, but it was not successful. However, because fractional PID controller and fractional order $PI^{\lambda}D^{\mu}$ controller are distributed in the same cluster, there is only a slight difference in expression. Meanwhile, the fact that fractional order and fractional calculus are not merged has almost no effect on the cluster distribution, and

the clustering distribution formed by software operation is still clear. Thus, the fact that these keywords were not merged has almost no effect on the analysis results.

In order to further explore the distribution of high-frequency keywords in fractional order research, the occurrences of the top 20 keywords were counted, as shown in Table 4.

According to Table 4, the most frequent keyword of fractional order research is fractional order (342), which is consistent with the retrieval method of this study. Meanwhile, the core keywords of fractional order theory, such as fractional differential equation, fractional Fourier transform, and fractional calculus, have high frequencies of 250, 243, and 228, respectively, which shows that these keywords are the key to the subject of fractional order research. In addition, fixed point theorem, positive solution, stability, chaos synchronization, sliding mode control, image enhancement, and so forth have a relatively high frequency of occurrence and involve multiple disciplines. Therefore, it can be also shown that the research of fractional order presents the characteristic of diversification.

Figure 4 shows the distribution of hot keywords in fractional order research, and different colors in the figure represent different clusters of research hotspots. Figure 4 is mainly composed of nodes and connecting lines. The larger the keyword node, the higher the frequency of occurrence, and the thicker the line between nodes, the higher the cooccurrence frequency. Moreover, the distribution of the distance between nodes is positively correlated with the relevance of keywords.

According to Figure 4, the research hotspots of fractional order are divided into four keyword clusters, and the different clusters represent the distribution of fractional order research in different fields.

Cluster 1 (red region) represents the research of fractional order in the field of mathematics. Cluster 1 has 25 keywords, including fractional differential equation, fixed point theorem, positive solution, boundary value problem, stability, and existence, and it is the largest cluster in the distribution of research hotspots of fractional order. According to the distribution of nodes, the high-frequency

FIGURE 4: Keyword network visualization map. Note: the keywords translation in Figure 4 is shown in Table 3.

keywords are relatively close and the connecting lines are relatively thick, such as fractional differential equation, boundary value problem, fixed point theorem, and positive solution. Scholars took this kind of keyword as the mainline to carry out related research, such as using fixed point theorem to explore the positive solutions of boundary value problems of fractional differential equations with different properties [27–29]. There were also some scholars who proved, respectively, the existence of positive solutions for the boundary value problem of a class of nonlinear fractional differential equations [30] and a class of fractional differential equations on the infinite interval [31] based on the fixed point theorem. Meanwhile, also some keywords have relatively high occurrence frequencies, such as fractional derivatives, stability, and convergence, which also represent the research hotspots of fractional orders in the field of mathematics. Especially for stability and convergence, their nodes are closer and lines are thicker, which means that the cooccurrence degree between them is high. Some scholars proposed a difference scheme for solving the fractional diffusion equation, such as a difference scheme for three-dimensional space fractional advection-diffusion equation [32] and a finite difference scheme for Riesz Space fractional diffusion equation [33]; the stability and convergence were proved by the matrix method. Furthermore, from the node distribution, it can be seen that the keywords in cluster 1 are scattered and compared with those in other clusters; this cluster involves the biggest number of keywords which indicates that the research of fractional order in the field of mathematics covers a wide range, and the research hotspots are relatively scattered.

Cluster 2 (green region) represents the research of fractional order in the field of control engineering. Cluster 2

has 15 keywords, including fractional calculus, sliding mode control, fractional order controller, fractional order PID, permanent magnet synchronous motor, sliding mode, and fractional order control. Compared with cluster 1, this cluster has a shorter distance between nodes, and the network connection is relatively close, which indicates that the research of fractional order in the field of control engineering is relatively concentrated. According to the distribution of keywords in cluster 2, the research hotspots of fractional order in the field of control engineering include the research of fractional order sliding mode control method and strategy and fractional order controller design, and so forth. Among them, regarding the research of fractional order sliding mode control method and strategy, some scholars put forward fractional order sliding mode control method and strategy based on the fractional order calculus theory and neural network, such as ANN-inversion system based fractional order sliding mode control scheme [34], fractional order sliding mode control scheme based on neural network and adaptive control algorithm [35], and fractional order sliding mode control algorithm based on radial basis function (RBF) neural network [36]. Regarding the research of fractional order controller design, it includes fractional order PID controller design [37–40], fractional order internal model controller design [41, 42], fractional order nonsingular terminal sliding mode controller [43], and fractional order PID improved aut disturbance rejection controller (FOPID-IADRC) [44]. In addition, the frequency of permanent magnet synchronous motor is relatively high, which is also one of the research hotspots in the field of fractional order control engineering. Zhou et al. studied the fractional order PI^1 controller parameters design method of permanent magnet synchronous generation wind

TABLE 3: Translation of keywords in Figures 4 and 5.

No.	The keywords	Translation
1	Caputo分数阶导数	Caputo fractional derivative
2	Caputo导数	Caputo derivative
3	Green函数	Green function
4	Laplace变换	Laplace transform
5	p -laplacian算子	p -Laplacian operator
6	Riemann-Liouville分数阶	Riemann-Liouville fractional order
7	Schauder不动点定理	Schauder fixed point theorem
8	不动点定理	Fixed point theorem
9	分数阶导数	Fractional derivative
10	分数阶微分方程	Fractional differential equation
11	唯一性	Uniqueness
12	存在性	Existence
13	收敛性	Convergence
14	数值解	Numerical solution
15	时滞	Time lag
16	正解	Positive solution
17	积分边值问题	Integral boundary value problem
18	稳定性	Stability
19	算子矩阵	Operator matrix
20	精确解	Exact solution
21	脉冲	Pulses per second
22	解的存在性	The existence of solutions
23	边值问题	Boundary value problem
24	非线性	Nonlinear
25	黏弹性	Viscoelasticity
26	内模控制	Internal model control
27	分数阶pid	Fractional order PID
28	分数阶pid控制器	Fractional order PID controller
29	分数阶PI ^{λ} D ^{μ} 控制器	Fractional order PI ^{λ} D ^{μ} controller
30	分数阶微积分	Fractional calculus
31	分数阶控制	Fractional order control
32	分数阶控制器	Fractional order controller
33	分数阶模型	Fractional order model
34	参数整定	Parameter tuning
35	模糊控制	Fuzzy control
36	永磁同步电机	Permanent magnet synchronous motor
37	滑模控制	Sliding mode control
38	神经网络	Neural networks
39	遗传算法	Genetic algorithm
40	鲁棒性	Robustness
41	分数阶	Fractional order
42	分数阶混沌系统	Fractional chaos system
43	分数阶系统	Fractional order system
44	参数辨识	Parameter identification
45	同步	Synchronization
46	图像加密	Image encryption
47	混沌	Chaos
48	混沌同步	Chaos synchronization
49	混沌控制	Chaos control
50	混沌系统	Chaotic system
51	滑模	Sliding mode
52	分数阶傅里叶变换	Fractional Fourier transform
53	分数阶微分	Fractional differential
54	分数阶积分	Fractional integral
55	参数估计	Parameter estimation
56	图像去噪	Image denoising
57	图像增强	Image enhancement
58	小波变换	Wavelet transform
59	线性调频信号	Linear frequency modulation signal
60	自适应	Self-adaptive
61	边缘检测	Edge detection

FIGURE 5: Visualization map of keyword density. Note: the keywords translation in Figure 5 is shown in Table 3.

TABLE 4: Occurrence distribution of the top 20 keywords.

No.	Keyword	Occurrence
1	Fractional order	342
2	Fractional differential equation	250
3	Fractional Fourier transform	243
4	Fractional calculus	228
5	Fractional differential	145
6	Fixed point theorem	138
7	Positive solution	135
8	Boundary value problem	123
9	Fractional derivative	119
10	Fractional order system	115
11	Stability	107
12	Chaos synchronization	72
13	Fractional chaos system	64
14	Convergence	56
15	Synchronization	52
16	Sliding mode control	52
17	Chaos	51
18	Image enhancement	51
19	Existence	47
20	Caputo derivative	45

turbine systems [45], Yu and Huang proposed the fuzzy RBF neural network based fractional order speed control system of permanent magnet synchronous motor [46], and so forth.

Cluster 3 (blue region) represents the research of fractional order in the field of physics. Cluster 3 has 11 keywords, including fractional order, fractional order system, chaos synchronization, fractional chaos system, synchronization, chaos, and sliding mode. Compared with other clusters, the nodes are relatively close in cluster 3, and the

connecting lines between some nodes are relatively thick, such as the connecting lines between fractional order and synchronization, sliding mode, and chaos, as well as the connecting line between chaos synchronization and fractional order system and so forth, which indicates that the research of fractional order in the field of physics is more focused, and the connection of keywords is more closely related. Focusing on the distribution of keyword nodes in cluster 3, the research hotspots of fractional order in the field of physics are focused on the synchronization and control of fractional order chaotic systems and so forth. Regarding the research of synchronization and control of fractional order chaotic systems, some scholars combined Lyapunov stability theory to explore synchronization of fractional order chaotic systems [47–49], and some others studied the synchronization control of fractional order chaotic system [50, 51]. In addition, some scholars analyzed the synchronization and control of fractional order chaotic systems from the perspective of relative subdivision, such as adaptive synchronization or synchronization control of fractional order chaotic systems [52–54], sliding mode synchronization or synchronization control of fractional order chaotic system [55–57], and adaptive sliding mode synchronization or synchronization control of fractional order chaotic system [58–60].

Cluster 4 (yellow region) represents the research of fractional order in the field of computer and information communications. Cluster 4 has 10 keywords: fractional Fourier transform, fractional differential, fractional integral, image enhancement, image denoising, linear frequency modulation (LFM) signal, edge detection, parameter estimation, wavelet transform, and self-adaptive. Compared

with the first three clusters, the keywords around fractional differential and fractional Fourier transform in cluster 4 are relatively concentrated, but the distance between the keywords around them is far and the connecting lines are relatively less, such as LFM signal and image denoising, which indicates that the research hotspots of fractional order in the field of computer and information communication present the characteristics of partly concentration and general dispersion.

According to the distribution of nodes, there are some keyword node distribution forms clusters, such as the fractional differential, fractional integral, image denoising, image enhancement, edge detection, and self-adaptive, which reflects the application of fractional order in the computer field. Focusing on research field segmentation, we can find that image processing direction is the research hotspot of fractional order in the field of computers. To be specific, regarding the combination of fractional order and image enhancement, there are many research results related to image enhancement algorithm. Many scholars proposed fractional differential image enhancement algorithm, where there were relatively many findings related to adaptive fractional differential image enhancement algorithms, such as adaptive fractional differential MR image enhancement algorithm based on nonlocal means value [61], adaptive fractional order differential image enhancement algorithm based on 3 parameters related to an image texture, which were the local gradient of the image, the information entropy, and the variance [62], and image enhancement algorithm combining adaptive threshold fuzzy set enhancement and fractional differential [63]. Regarding the specific application of fractional differential in image enhancement, it includes medical image [64] and haze traffic image [65]. Regarding the combination of fractional order and image denoising, many scholars focused on the fractional integral theory to explore image denoising methods; for example, they studied image denoising based on fractional integral [66, 67] and proposed an improved fractional integration method for laser image denoising [68, 69]. Moreover, some scholars put forward image denoising method by combining fractional integral with wavelet transform [70], and some scholars introduced Gauss curvature into partial differential equation, and the image edge was detected by the image gradient, and they established adaptive image denoising model by combining Gauss curvature and fractional different operator [71]. Through further analysis, it can be found that the research on the application of fractional order in image enhancement focuses more on fractional differentiation, while the research on the application of fractional order in image denoising focuses more on fractional integral.

Combining with Figure 4, it can also be found that the distance between the fractional differential and the image enhancement node is closer, and the distance between the fractional integral and the image denoising node is closer, which indicates that the correlation between the nodes is strong, and the cooccurrence degree is high. Moreover, Huang et al. proposed that the applicable situations of the fractional differential equation, fractional integral equation,

and fractional partial differential equation are image enhancement, image denoising, image enhancement, or denoising, respectively [72]. Therefore, it also provides further support and guidance for the above conclusions. According to the node distribution of cluster 4, some keywords form another cluster, such as fractional Fourier transform, LFM signal, and parameter estimation, which represents the application research of fractional order in the field of information and communication. Based on the perspective of research field segmentation, we can find that the direction of LFM signal detection and parameter estimation are the research hotspots of fractional order in the field of information and communication. To be specific, some scholars proposed joint parameter estimation method or signal detection and parameter estimation method of LFM signals based on Fourier transform, such as a novel method for joint parameter estimation of LFM signals in bistatic Multiple-Input Multiple-Output (MIMO) radar system [73], fast adaptive method of the multi-LFM signal detection, and parameter estimation based on fractional Fourier transform [74]. In addition, regarding the detection of the LFM signal or low parameter estimation accuracy of the LFM signal under low signal-to-noise ratio (SNR), some scholars proposed a concise fractional Fourier transform method for LFM signal detection under low SNR [75]. Some scholars aimed at the problem of low parameter estimation accuracy of LFM signal under low SNR, proposed a Rife interpolation short-time fractional Fourier transform (STFRFT-) variable weight least square fitting (VWSF) algorithm [76]. Through further analysis, it can be found that the research findings of fractional order in the direction of LFM signal detection and parameter estimation focus more on the implementation based on the Fourier transform. Combining with the size of the nodes, it can also be found that the Fourier transform has the biggest nodes in this cluster, which means it has the highest frequency. Moreover, the connecting line between some keyword nodes and Fourier transform is relatively thick, such as LFM signal and parameter estimation, which can indicate that the correlation between these keywords is strong and the cooccurrence degree is high. Thus, it can be seen that fractional Fourier transform is an important method for the application of fractional order in information and communication.

Additionally, cluster 4 represents the research of fractional order in the computer and information communication field; the keywords belong to different disciplines, respectively, but they are distributed in the identical cluster because of the similar discipline direction and obvious cross characteristics. Furthermore, it can also be seen from the node distribution in Figure 4 that there are connecting lines among the keyword nodes of fractional order in the computer and information communication field, which further proves the rationality of their distribution in the identical cluster.

Figure 5 shows a visualization map of the keyword density of fractional order research, in which the keyword distribution shows a feature of a gradual change from the cold color region to the warm color region. Among them, the larger the keyword node and the more biased towards the

warm color series, the higher frequency of its occurrence. On the contrary, the smaller the keyword node and the more biased towards the cold color series, the lower frequency of its occurrence. It can be seen from Figure 5 that some keywords have large nodes and show the distribution characteristics of warm color series, such as fractional order, fractional differential equation, fractional calculus, fractional Fourier transform, fractional differential, fixed point theorem, and positive solution. Especially for the nodes of fractional order, fractional differential equation, fractional calculus, fractional Fourier transform, fractional differential, and so forth, their feature of warm color system is more obvious. Combined with the keyword node distribution in Figure 4, we can see that the keywords are centered on the important nodes such as fractional order, fractional differential equation, fractional calculus, fractional Fourier transform, and fractional differential and radiate to the surrounding keywords of their respective regions. Additionally, combined with the above cluster analysis, it can be seen that these keywords are key points or important methods and tools of fractional order research in the fields of mathematics, control engineering, physics, computer, information communication, and so forth. Therefore, keywords such as fractional order, fractional differential equation, fractional calculus, fractional Fourier transform, and fractional differential are the core points and important support of fractional order research.

In summary, this study analyzed the hotspots of fractional order research from the perspectives of keyword network and density analysis. In addition, it should be noted that some keywords are cross-related with other clusters in the four clusters. For instance, not only is the keyword stability distributed in the research of fractional order in the field of mathematics, but also it involves the fields of physics, control engineering, and so forth. Again, for instance, the keyword chaos not only belongs to the research of fractional order in the field of physics but also is distributed in the fields of mathematics and control engineering. Thus, this also reflects the interdisciplinary characteristics of fractional order research.

4.2. Research Frontiers. Keyword burst not only detects the focus of research on bursts but also realizes the research frontier [77]. Burst detection in CiteSpace is mainly based on the Kleinberg algorithm [78]; burst keyword analysis method shows keywords had rapidly changed in a short period of time or dramatically increased in number and emphasizes the abrupt change of keywords [79]. Therefore, CiteSpace was used to analyze the burst keywords of fractional order research, so as to explore the research frontier of fractional order. The relevant parameters of the software were set as follows: set the time slicing option to be from 2001 to 2020, set the years per slice option to 2, set the node types option to keyword, and use pathfinder to trim the keyword network and other options as default settings. By running the software, the cooccurrence distribution of the fractional research keywords was obtained. After a software trial run, it was found that there were repetitions in the

expression of keywords about the fractional Fourier transform and LFM signals, so these keywords were merged. On this basis, the minimum duration in the burstness option was set to 3, and finally keywords with the strongest citation bursts of fractional order research were generated, as shown in Figure 6. It should be noted that the above keywords of fractional order and fractional calculus were not merged. In order to keep consistency with the above research, this paper also did not merge the two in this part. At the same time, combining with the analysis results in Figure 5, it can also be found that the burst keywords of fractional order and fractional calculus do not appear in Figure 5 at the same time; thus the fact that these two keywords were not merged has almost no effect on the overall results.

Figure 6 shows the top 26 keywords with the strongest citation bursts. To be specific, “Begin” represents the time when a certain keyword begins to burst, and “End” represents the burst end time for a certain keyword. In addition, the red and blue columns together constitute the time distribution (2001–2020) in the study period, and the red column represents the duration of a burst keyword. It can be seen from Figure 6 that the first burst keyword to appear in the research period is the LFM signal, and this keyword has the longest burst duration (2001–2012). According to the burst strength, the burst strength of fractional Fourier transform is the highest, and its burst value is 33.9667, which further reveals the importance of fractional Fourier transform in the field of fractional order research. Meanwhile, the burst value of the LFM signal is also relatively high, and its burst value is 15.3201. In addition, combined with the burst strength, it can also be found that the burst strength distribution of other burst keywords is relatively uniform, except for the significant burst strength of fractional Fourier transform and LFM signal, which is consistent with the multidisciplinary distribution characteristics of fractional order research.

Based on the time distribution of burst keywords and the subdivision of fractional order research, the distribution of fractional order burst keywords can be basically divided into the three following phases: The first phase is from 2001 to 2012; the burst keywords in this phase mainly include LFM signal, parameter estimation, information processing technology, fractional calculus, fractional Fourier transform, and edge detection. Through further analysis of burst keywords, it can be found that the research frontier of this phase focuses on the research of fractional order in the field of information communication and computer. Combined with the above clustering analysis, the keywords such as fractional Fourier transform, LFM signal, and parameter estimation form clusters, which indicates the research of fractional order in the field of information and communication, and the burst keywords such as edge detection and wavelet transform also represent the research of fractional order in the field of computer. Meanwhile, projective synchronization of burst keywords also appears in this phase, which mainly involves the application of fractional order in the field of physics, such as the study of projective synchronization in fractional order chaotic system [80, 81]. In addition, compared with the other two phases, the duration of

FIGURE 6: Top 26 keywords with the strongest citation bursts. Note: the keywords translation in Figure 6 is shown in Table 5.

burst keywords in this phase is longer, such as LFM signal, information processing technology, and fractional Fourier transform, which shows the importance of fractional order in information communication and computer fields. The second phase is from 2012 to 2017; this phase mainly covers burst keywords such as operator matrix, Schauder fixed point theorem, error estimation, uniqueness, fixed point theorem, and integral boundary value problem, which indicates that the research of fractional order in the field of mathematics has become the main frontier direction in this phase. Furthermore, compared with other phases, the number of burst keywords focusing on the identical frontier direction is the biggest in this phase, which further confirms the breadth of fractional order research in the field of mathematics. The third phase is from 2017 to 2020; new burst keywords appeared in 2017, such as exact solution, genetic algorithm, and particle swarm optimization. Among them, the exact solution mainly involves the research of fractional order in the field of mathematics, while genetic algorithm, particle swarm optimization, and so forth mainly

involve the research of fractional order in the fields of automotive engineering, control engineering, and so forth. For example, some scholars used genetic algorithm to optimize the parameters of fractional order controller [82, 83] and particle swarm optimization to explore the optimal design of passive fractional order vehicle suspension parameters [84]. Also some scholars proposed a dynamic constriction factor based particle swarm optimization with fractional order velocity (DFFV-PSO) [85]. In addition, at the intersection of the second and third phases, there also appear some burst keywords, such as internal model control, fractional order PID control, and p-Laplacian operator. These burst keywords are mainly distributed around the fractional order in frontier fields such as control engineering and mathematics.

In conclusion, the priorities of the research frontier of fractional order are different in a different phase, which also reveals the law of the dynamic change of the research frontier of fractional order with time evolution. In particular, the burst time of burst keywords such as exact solution, genetic algorithm, and particle swarm

TABLE 5: Translation of keywords in Figure 6.

No.	The keywords	Translation
1	线性调频信号	Linear frequency modulation signal
2	参数估计	Parameter estimation
3	信息处理技术	Information processing technology
4	分数阶微积分	Fractional calculus
5	分数阶傅里叶变换	Fractional Fourier transform
6	边缘检测	Edge detection
7	投影同步	Projective synchronization
8	信号检测	Signal detection
9	小波变换	Wavelet transform
10	初值问题	Initial value problem
11	算子矩阵	Operator matrix
12	Schauder不动点定理	Schauder fixed point theorem
13	误差估计	Error estimation
14	唯一性	Uniqueness
15	守恒量	Conserved quantity
16	不动点定理	Fixed point theorem
17	鲁棒性	Robustness
18	积分边值问题	Integral boundary value problem
19	内模控制	Internal model control
20	分数阶模型	Fractional order model
21	自适应	Self-adaptive
22	分数阶pid控制	Fractional order PID control
23	p-Laplacian算子	p-Laplacian operator
24	精确解	Exact solution
25	遗传算法	Genetic algorithm
26	粒子群算法	Particle swarm optimization

optimization lasted until the end of the research period (2020), so these burst keywords are to a large extent the frontier direction and method of fractional order research now and in the future.

5. Conclusion

This paper combined bibliometrics and visualization research methods to conduct a quantitative and visual analysis of the 2,854 Chinese fractional order research literatures collected by CNKI from 2001 to 2020, which explored the current status of China's fractional order research and also revealed the hotspots and frontier areas. The main research conclusions of this paper are as follows:

- (1) Research current status aspect: China's fractional order research publications show an overall upward trend, but there are also obvious differences in the distribution of the number of publications in different time series; China's fractional order research publications are relatively evenly distributed among various research institutions, and the attributes of research institutions include comprehensive universities, universities of science and technology, and military academy. The overall intensity of co-operation among Chinese fractional order research authors is not high, but there are also many research groups with a high volume of publications and close cooperation relationships; China's fractional order research disciplines are distributed in a wide range, and the number of publications in the field of

mathematics is the biggest, which had the characteristic of the multidisciplinary distribution.

- (2) Research hotspot aspect: The research hotspots of fractional order in China are divided into 4 keyword clusters, which include the research of fractional order in the fields of mathematics, control engineering, physics, and computer and information communication. Meanwhile, keywords such as fractional order, fractional order differential equations, fractional calculus, fractional Fourier transform, and fractional differential are the core points and important supports of fractional order research in China.
- (3) Research frontier aspect: The focus of China's fractional order research frontier is different at different stages, and it changes dynamically with the evolution of time. Meanwhile, burst keywords such as exact solution, genetic algorithm, and particle swarm optimization are likely to be the frontier directions and methods of China's fractional order research at present and in the future.

Data Availability

The data collected to support the findings of this study come from CNKI and are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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