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Research Article

Convergence Classes of L-Filters in (L, M)-Fuzzy Topological Spaces

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An (L, M)-fuzzy topological convergence structure on a set X is a mapping which defines a degree in M for any L-filter (of crisp degree) on X to be convergent to a molecule in L^X. By means of (L, M)-fuzzy topological neighborhood operators, we show that the category of (L, M)-fuzzy topological convergence spaces is isomorphic to the category of (L, M)-fuzzy topological spaces. Moreover, two characterizations of L-topological spaces are presented and the relationship with other convergence spaces is concretely constructed.

1. Introduction

A convergence space of filters (as a generalization of a topological space based on the concept of convergence of filters as fundamental) is a pair (X, Conv), where X is a set, Conv is a subset of $\mathbb{F}(X) \times X$, and $\mathbb{F}(X)$ is the set of filters on X. The pair $(\mathcal{F}, x) \in \mathrm{Conv}$ means that $\mathcal{F}$ is said to converge to $x (\forall \mathcal{F} \in \mathbb{F}(X))$. The convergence theory of filters is an important part in topology. It was proved that the category Conv of convergence spaces is a quasitopos which may be thought of as a nice category of spaces that includes Top (the category topological spaces) as a full subcategory (see [1–3] for details).

The corresponding convergence theory of filters in the fuzzy setting is also studied by many authors. E. Lowen and R. Lowen [4] gave a one-to-one correspondence between the set of limit functions on X and the set of stratified I-topologies on X based on I-filters (filters in the lattice $[0, 1]^X$ [5]). Jäger [6] introduced another fuzzy convergence structure (called stratified L-generalized convergence structure for L, a frame which was extended to the case of complete residual lattices by Yao [7] later on). Here, the degree of convergence of a stratified L-fuzzy filter (called also stratified L-filter in [8]) is from L but it converges to a crisp point.

Jäger also gave some kinds of characterizations of stratified L-topological spaces (see [6, 9–11]). With the help of Jäger's work, Li [12] proved that there is a one-to-one correspondence between the set of all convergence functions of specific L-fuzzy filters on X and the set of all specific L-topologies on X for L, a complete residual lattice. Following papers [13, 14], Güloğlu and Çoker [15] proved that there exists a one-to-one correspondence between the set of I-fuzzy topological convergence structures on X and the set of I-fuzzy topologies on X. As a generalization, Pang and Fang [16] proved that there is a one-to-one correspondence between the set of topological L-fuzzy Q-convergence structures on X and the set of L-fuzzy topologies on X when L is a completely distributive complete lattice with an order-reversing involution $'$. Here, the L-fuzzy filter converges to a fuzzy point but the degree of convergence is from $\{0, 1\}$. Furthermore, Pang [17, 18] discussed categorical properties in some fuzzy convergence spaces, such as L-fuzzifying convergence spaces and stratified L-convergence tower spaces.

In [19], the convergence theory of molecular nets in (L, M)-fuzzy topological spaces was discussed. As a generalization, Yao [20] presented a definition of (L, M)-fuzzy nets and established a Moore–Smith convergence in (L, M)-fuzzy topology. It is well known that every molecular net can induce an L-filter (a filter in lattice L^X) and an L-filter can induce a molecular net (see [21]). In addition, the degree of fuzzy topology, openness degree, and quotient degree [22] are interesting. Taking these in mind, we will use L-filters different from that of [6, 8, 16] to give some axioms of filter-theoretical convergence classes in (L, M)-fuzzy topological spaces. Such a convergence class on a set X will be called an (L, M)-fuzzy topological convergence structure, where the L-filter converges to a molecular in L^X and the degree of convergence is an element in M. This kind of convergence class is proved to have nice properties.

The present paper is arranged as follows. The rest of this section contains some basic definitions and notions which will be used in this paper. In Section 2, we prove that there exists a one-to-one correspondence between (L, M)-fuzzy topological convergence structures on a set X and (L, M)-fuzzy topologies on X. In Section 3, we give two approaches to $(L, 2)$-fuzzy topological convergence structures. In Section 4, we discuss the relation between $(L, 2)$-fuzzy topological convergence structures and Li's structures in [12].

2. Preliminaries

Now, we review some basic notions and results which will be used in this paper. Unless other explanations are given, L always stands for a complete lattice with a smallest element 0 and a largest element 1 in this paper. Obviously, for every set X, L^X (the set of all L-subsets of X) is also a complete lattice with the pointwise order. We will denote the L-set taking constant value a on X by a_X, and we write $\uparrow\mu = \{\nu \in L^X \mid \mu \le \nu\}$ for each $\mu \in L^X$ ($\downarrow\mu$, $\uparrow a$, and $\downarrow a$ can be defined analogously for $a \in L$). For the sake of convenience, we denote the orders of L and L^X and their restrictions by the same symbol $\le$. $a \in L - \{0\}$ is called a join irreducible element if, for any finite subset $J \subseteq L$ satisfying $a = \vee J$ (the supremum of J), there exists a $j \in J$ such that $a = j$. $a \in L - \{0\}$ is called a coprime element if, for any finite subset $J \subseteq L$ satisfying $a \le \vee J$, there exists a $j \in J$ such that $a \le j$. $p \in L - \{1\}$ is called a prime element of L iff it is a coprime element of L^{op}, where L^{op} denotes the opposite lattice of L. The set of all join irreducible elements (resp., all coprime elements and all prime elements) of L will be denoted by $J(L)$ (resp., $\mathrm{Copr}(L)$ and $\mathrm{Pr}(L)$). Clearly, $\mathrm{Copr}(L^X) = \{x_a \mid x \in X, a \in \mathrm{Copr}(L)\}$, where $x_a \in L^X$ is the L-subset taking value a at x and 0 elsewhere, and we call the members of $Pt(L^X) = \{x_a \mid x \in X, a \in L - \{0\}\}$ L-points. It is well known that $J(L) = \mathrm{Copr}(L)$ if L is a distributive lattice. L is called a completely distributive complete lattice if it satisfies the following completely distributive laws:

(i) (CD1) $\vee_{i\in I}\wedge_{j\in J_i} a_{i,j} = \wedge_{f\in\prod_{i\in I}J_i}\vee_{i\in I}a_{i,f(i)}$, $(\forall\{a_{i,j} \mid i \in I,\ j \in J_i\} \in 2^L)$

(ii) (CD2) $\wedge_{i\in I}\vee_{j\in J_i} a_{i,j} = \vee_{f\in\prod_{i\in I}J_i}\wedge_{i\in I}a_{i,f(i)}$ $(\forall\{a_{i,j} \mid i \in I,\ j \in J_i\} \in 2^L)$

It is also well known that $a = \vee(\Downarrow a \cap \mathrm{Copr}(L))$ $(\forall a \in L)$ when L is a completely distributive complete lattice, where $\Downarrow a = \{b \in L \mid b \lhd a\}$ ($\Uparrow a$, $\Downarrow\mu$, and $\Uparrow\mu$ can be defined analogously for $\mu \in L^X$), and $\lhd \subseteq L \times L$ is called the wedge-below relation on L which is defined as follows: $a \lhd b$ iff for any $S \subseteq L$ satisfying $b \le \vee S$, there exists an $s \in S$ such that $a \le s$ ($\rhd$ can be defined analogously).

An order-reversing involution $'$ on L is a self-map on L such that, for any $a, b \in L$, the following hold: (1) $a \le b$ implies $b' \le a'$; (2) $a'' = a$. The following hold for any subset $\{a_i \mid i \in I\} \subseteq L$:

(1) $(\vee_{i\in I}a_i)' = \wedge_{i\in I}a_i'$

(2) $(\wedge_{i\in I}a_i)' = \vee_{i\in I}a_i'$

For each mapping $f\colon X \longrightarrow Y$, we have a mapping (called L-forward powerset operator) $f_L^{\longrightarrow}\colon L^X \longrightarrow L^Y$ which is defined by $f_L^{\longrightarrow}(\mu)(y) = \vee\{\mu(x) \mid f(x) = y\}$ $(\forall\mu \in L^X, \forall y \in Y)$. The right adjoint to $f_L^{\longrightarrow}$ (called L-backward powerset operator) is denoted by $f_L^{\leftarrow}$ (therefore, the pair $(f_L^{\longrightarrow}, f_L^{\leftarrow})$ is a Galois connection on $(L^X, \le)$ and $(L^Y, \le)$). It can be easily verified that $f_L^{\longrightarrow}$ preserves arbitrary supremal, $f_L^{\leftarrow}$ preserves arbitrary supremal and arbitrary infimal (and also complements if L has an order-reversing involution $'$), and $f_L^{\leftarrow}(\nu) = \vee\{\mu \mid f_L^{\longrightarrow}(\mu) \le \nu\} = \nu \circ f$ $(\forall\nu \in L^Y)$.

For other undefined notions, refer to [8, 23].

3. Relationship between (L, M)-Fuzzy Topological Spaces and (L, M)-Fuzzy Topological Convergence Spaces

In this section, we assume that L (resp., M) is a completely distributive complete lattice with an order-reversing involution $'$ (resp., *); in this case, $J(L) = \mathrm{Copr}(L)$, $J(M) = \mathrm{Copr}(M)$, and $J(L^X) = \mathrm{Copr}(L^X)$. Our main task is to give a one-to-one correspondence between (L, M)-fuzzy topological convergence structures (resp., (L, M)-fuzzy principle convergence structures) on a set X and (L, M)-fuzzy topologies (resp., (L, M)-fuzzy preinterior operators) on X, which gives rise to an isomorphism between the category of (L, M)-fuzzy topological convergence spaces (resp., the category of (L, M)-fuzzy principle convergence spaces) and the category of (L, M)-fuzzy topological spaces (resp., the category of (L, M)-fuzzy preinterior spaces).

Definition 1 (See [13, 14])

(1) (See [13, 14]). An L-fuzzy filter on X is a mapping $\mathrm{F}\colon L^X \longrightarrow L$, which satisfies the following conditions:

(i) (FF1) $\mathrm{F}(1_X) = 1$, $\mathrm{F}(0_X) = 0$

(ii) (FF2) If $A, B \in L^X$ and $A \leq B$, then $\mathrm{F}(A) \leq \mathrm{F}(B)$
(iii) (FF3) $\mathrm{F}(A) \wedge \mathrm{F}(B) \leq \mathrm{F}(A \wedge B)(\forall A, B \in L^X)$
(iv) An L-fuzzy filter is said to be stratified [8] if it satisfies the following condition: (Fs) $a \leq \mathrm{F}(a_X)(\forall a \in L)$.

The set of all L-fuzzy filters (resp., stratified L-fuzzy filters) on X is denoted as $\mathbb{F}(X, L, L)$ (resp., $\mathbb{F}^S(X, L, L)$). Apparently, the mapping $[x]\colon L^X \longrightarrow L$, defined by $[x](A) = A(x)$, is a stratified L-fuzzy filter $(\forall x \in X)$.

(2) (See [5]). An L-filter on X is a family $\mathrm{F} \subseteq L^X$ which satisfies the following conditions:

(i) (F1) $1_X \in \mathrm{F}$, $0_X \notin \mathrm{F}$
(ii) (F2) If $\mu \in \mathrm{F}$ and $\mu \leq \nu$, then $\nu \in \mathrm{F}$
(iii) (F3) If $\mu, \nu \in \mathrm{F}$, then $\mu \wedge \nu \in \mathrm{F}$

The set of all L-filters on X is denoted as $\mathbb{F}(X, L, 2)$ (where $2 = \{0, 1\}$). Obviously, every L-filter may be looked upon as an L-fuzzy filter.

Remark 1. Apparently, $\uparrow x_a \in \mathbb{F}(X, L, 2)$, $(\forall x \in X, \forall a \in L)$. Moreover, for every mapping $f\colon X \longrightarrow Y$ and every L-filter F on X, define

$$f_L^{\Longrightarrow}(\mathrm{F}) = \cap\{\mathrm{G} \in \mathbb{F}(X, L, 2) \mid f_L^{\longrightarrow}(\mathrm{F}) \subseteq \mathrm{G}\}, \tag{1}$$

and then $f_L^{\Longrightarrow}(\mathrm{F}) \in \mathbb{F}(X, L, 2)$ and $f_L^{\Longrightarrow}(\mathrm{F}) = \{\mu \in L^Y \mid f_L^{\leftarrow}(\mu) \in \mathrm{F}\} = \{\mu \in L^Y \mid \mu \geq f_L^{\longrightarrow}(\nu) \text{ for some } \nu \in \mathrm{F}\}$, where $f_L^{\longrightarrow}(\mathrm{F}) = \{f_L^{\longrightarrow}(\nu) \mid \nu \in \mathrm{F}\}$.

Definition 2 (see [24])

(1) An (L, M)-fuzzy topology on X is a mapping $\mathrm{T}\colon L^X \longrightarrow M$, which satisfies the following conditions:

(i) (LMFT1) $\mathrm{T}(0_X) = \mathrm{T}(1_X) = 1$
(ii) (LMFT2) $\mathrm{T}(\mu) \wedge \mathrm{T}(\nu) \leq \mathrm{T}(\mu \wedge \nu)(\forall \mu, \nu \in L^X)$
(iii) (LMFT3) $\wedge_{j \in J}\mathrm{T}(\mu_j) \leq \mathrm{T}(\vee_{j \in J}\mu_j)$, $(\forall\{\mu_j\}_{j \in J} \subseteq L^X)$

In this case, $\mathrm{T}(\mu)$ can be interpreted as the degree for μ to be an open set $(\forall \mu \in L^X)$, and the pair (X, T) is called an (L, M)-fuzzy topological space. An $(L, 2)$-fuzzy topology is also called L-topology.

(2) A continuous mapping from one (L, M)-fuzzy topological space (X, T_X) to another (L, M)-fuzzy topological space (Y, T_Y) is a mapping $f\colon X \longrightarrow Y$, which satisfies $\mathrm{T}_Y(\mu) \leq \mathrm{T}_X(f_L^{\leftarrow}(\mu))(\forall \mu \in L^Y)$. The category of (L, M)-fuzzy topological spaces and continuous mappings between them is denoted by (L, M)-FTop.

Definition 3 (see [25])

(1) An (L, M)-fuzzy neighborhood operator on a set X is a mapping $\mathrm{N}\colon J(L^X) \longrightarrow M^{L^X}$, which satisfies the following conditions:

(i) (LMFN1) $\mathrm{N}(x_a)(1_X) = 1$, $\mathrm{N}(x_a)(0_X) = 0$, $(\forall x_a \in J(L^X))$
(ii) (LMFN2) $\mathrm{N}(x_a)(\mu) = 0\,(x_a \leq \mu)$
(iii) (LMFN3) $\mathrm{N}(x_a)(\mu \wedge \nu) = \mathrm{N}(x_a)(\mu) \wedge \mathrm{N}(x_a)(\nu)$, $(\forall \mu, \nu \in L^X, \forall x_a \in J(L^X))$
(iv) (LMFN4) $\mathrm{N}(x_a)(\mu) = \vee_{\nu \in \downarrow\mu \cap \uparrow x_a} \wedge_{y_b \in J(L^X) \cap \Downarrow\nu} \mathrm{N}(y_b)(\nu)(\forall \mu \in L^X, \forall x_a \in J(L^X))$

In this case, (X, N) is called an (L, M)-fuzzy neighborhood space.

(2) A mapping $f\colon (X, \mathrm{N}_X) \longrightarrow (Y, \mathrm{N}_Y)$ (where (X, N_X) and (Y, N_Y) are both (L, M)-fuzzy neighborhood spaces) is said to be continuous if $\mathrm{N}_Y(f(x)_a)(\mu) \leq \mathrm{N}_X(x_a)(f_L^{\leftarrow}(\mu))$ holds for any $\mu \in L^Y$ and any $x_a \in J(L^X)$. The category of (L, M)-fuzzy neighborhood spaces and continuous mappings between them will be denoted by (L, M)-**FNS.** In [25], Shi proved that (L, L)-**FNS** is isomorphic to (L, L)-**FTop.** Furthermore, it is easily proved that (L, M)-**FNS** is isomorphic to (L, M)-**FTop.**

Definition 4

(1) An (L, M)-fuzzy preinterior operator on X is a mapping $\mathrm{I}\colon L^X \longrightarrow M^{J(L^X)}$, which satisfies the following conditions:

(i) (LMPI1) $\mathrm{I}(1_X)(x_a) = 1$, $(\forall x_a \in J(L^X))$
(ii) (LMPI2) $\mathrm{I}(\mu)(x_a) = 0$, $(x_a \leq \mu)$
(iii) (LMPI3) $\mathrm{I}(\mu) \wedge \mathrm{I}(\nu) = \mathrm{I}(\mu \wedge \nu)$, $(\forall \mu, \nu \in L^X)$

In this case, (X, I) is called an (L, M)-fuzzy preinterior space.

(2) A mapping $f\colon (X, \mathrm{I}_X) \longrightarrow (Y, \mathrm{I}_Y)$ (where (X, I_X) and (Y, I_Y) are both (L, M)-fuzzy preinterior spaces) is said to be continuous if $\mathrm{I}_Y(\mu)(f(x)_a) \leq \mathrm{I}_X(f_L^{\leftarrow}(\mu))(x_a)$ holds for each $\mu \in L^Y$ and each $x_a \in J(L^X)$. The category of (L, M)-fuzzy preinterior spaces and continuous mappings between them will be denoted by (L, M)-**FPIS.**

Definition 5

(1) An (L, M)-fuzzy convergence structure on X is a mapping $\mathrm{Conv}\colon \mathbb{F}(X, L, 2) \longrightarrow M^{J(L^X)}$, which satisfies the following conditions:

(i) (LM1) $\mathrm{Conv}(\uparrow x_a)(x_a) = 1 \forall x_a \in J(L^X)$
(ii) (LM2) If $\mathrm{F} \subseteq \mathrm{G}$, then $\mathrm{Conv}(\mathrm{F}) \leq \mathrm{Conv}(\mathrm{G})$

In this case, (X, Conv) is called an (L, M)-fuzzy convergence space.

(2) A mapping $f\colon (X, \mathrm{Conv}_X) \longrightarrow (Y, \mathrm{Conv}_Y)$ (where (X, Conv_X) and (Y, Conv_Y) are both (L, M)-fuzzy convergence spaces) is said to be continuous if $\mathrm{Conv}_X(\mathrm{F})(x_a) \leq \mathrm{Conv}_Y(f_L^{\Longrightarrow}(\mathrm{F}))(f(x)_a)$ for any $\mathrm{F} \in \mathbb{F}(X, L, 2)$ and any $x_a \in J(L^X)$. The category of (L, M)-fuzzy convergence spaces and continuous

mappings between them will be denoted by (L, M)-**FCS.**

(3) For an (L, M)-fuzzy convergence space (X, Conv), we define a mapping $\mathrm{N}_{\mathrm{Conv}}: J(L^X) \longrightarrow M^{L^X}$ as follows:

$$\mathrm{N}_{\mathrm{Conv}}(x_a)(\mu) = \bigwedge_{\mu \notin \mathrm{F}} (Conv(\mathrm{F})(x_a))^*, \quad \left(\forall x_a \in J\left(L^X\right), \forall \mu \in L^X\right). \tag{2}$$

Definition 6

(1) An (L, M)-fuzzy convergence structure Conv on X is said to be an (L, M)-fuzzy principle convergence structure if it satisfies the following condition:

(LM3)$\mathrm{Conv}(\mathrm{F})(x_a) = \wedge_{\mu \notin \mathrm{F}} (\mathrm{N}_{\mathrm{Conv}}(x_a)(\mu))^*, (\forall x_a \in J(L^X))$

In this case, (X, Conv) is called an (L, M)-fuzzy pretopological convergence space.

(2) The category of (L, M)-fuzzy pretopological convergence spaces and continuous mappings (see Definition 5) between them will be denoted by (L, M)-**FPCS.**

Definition 7

(1) An (L, M)-fuzzy topological convergence structure on X is an (L, M)-fuzzy principle convergence structure Conv, which satisfies the following condition:

(LM4)$\mathrm{N}_{\mathrm{Conv}}(x_a)(\mu) = \vee_{\nu \in \downarrow\mu \cap \uparrow x_a} \wedge_{y_b \in \Downarrow\nu \cap J(L^X)} \mathrm{N}_{\mathrm{Conv}}(y_b)(\nu), (\forall \mu \in L^X)$

In this case, (X, Conv) is called an (L, M)-fuzzy topological convergence space.

(2) The category of (L, M)-fuzzy topological convergence spaces and continuous mappings (see Definition 5) between them will be denoted by (L, M)-**FTCS.**

In the rest of this section, we will show that (L, M)-**FNS** is isomorphic to (L, M)-**FTCS** (thus, (L, M)-**FTop** is isomorphic to (L, M)-**FTCS** by [25]) and that (L, M)-**FPIS** is isomorphic to (L, M)-**FPCS**.

Proposition 1. *For an (L, M)-fuzzy topological convergence structure Conv, the mapping $\mathrm{N}_{Conv}: J(L^X) \longrightarrow M^{L^X}$ is an (L, M)-fuzzy neighborhood operator on X.*

Proof. Since $\{\mathrm{F} \in \mathbb{F}(X, L, 2) \mid 1_X \notin \mathrm{F}\} = \varnothing$, we have $\mathrm{N}_{\mathrm{Conv}}(x_a)(1_X) = \wedge\varnothing = 1$. $\uparrow x_a$ is an L-filter on X and $0_X \notin \uparrow x_a$, $\mathrm{N}_{\mathrm{Conv}}(x_a)(0_X) \leq (Conv(\uparrow x_a)(x_a))^* = 0$ by (LM1). Thus, (LMFN1) is true. Again for each $\mu \in L^X$ and each $x_a \in J(L^X) - J(\mu)$, we have $\mathrm{N}_{\mathrm{Conv}}(x_a)(\mu) \leq (Conv(\uparrow x_a)(x_a))^* = 0$ since $\mu \notin \uparrow x_a$. (LMFN2) is also true.

Let $\mu, \nu \in L^X$ and $x_a \in J(L^X)$. Then, $\{\mathrm{F} \in \mathbb{F}(X, L, 2) \mid \mu\wedge\nu \notin \mathrm{F}\} = \{\mathrm{F} \in \mathbb{F}(X, L, 2) \mid \mu \notin \mathrm{F}\} \cup \{\mathrm{F} \in \mathbb{F}(X, L, 2) \mid \nu \notin \mathrm{F}\}$ holds. Thus, by definition of $\mathrm{N}_{\mathrm{Conv}}(x_a)$,

$$\begin{aligned} \mathrm{N}_{\mathrm{Conv}}(x_a)(\mu\wedge\nu) &= \wedge\left\{(\mathrm{Conv}(\mathrm{F})(x_a))^* \mid \mathrm{F} \in \mathbb{F}(X, L, 2), \mu\wedge\nu \notin \mathrm{F}\right\} \\ &= \wedge\left\{(\mathrm{Conv}(\mathrm{F})(x_a))^* \mid \mu \notin \mathrm{F}\right\} \wedge \wedge\left\{(\mathrm{Conv}(\mathrm{F})(x_a))^* \mid \nu \notin \mathrm{F}\right\} \\ &= \mathrm{N}_{\mathrm{Conv}}(x_a)(\mu) \wedge \mathrm{N}_{\mathrm{Conv}}(x_a)(\nu), \end{aligned} \tag{3}$$

which means that (LMFN3) is true. (LMFN4) follows from (LM4). □

Proposition 2. *For each (L, M)-fuzzy neighborhood operator* N *on X, the mapping $Conv_{\mathrm{N}}: \mathbb{F}(X, L, 2) \longrightarrow M^{J(L^X)}$, defined by*

$$\mathrm{Conv}_{\mathrm{N}}(\mathrm{F})(x_a) = \wedge\left\{(\mathrm{N}(x_a)(\mu))^* \mid \mu \in L^X, \mu \notin \mathrm{F}\right\}, \quad \left(\forall \mathrm{F} \in \mathbb{F}(X, L, 2), \forall x_a \in J\left(L^X\right)\right), \tag{4}$$

is an (L, M)-fuzzy topological convergence structure on X.

Proof. For any $\mu \in L^X$ and any $x_a \in J(L^X) - J(\mu)$, we have $\mathrm{N}(x_a)(\mu) = 0$ by (LMFN2). Hence, $\mathrm{Conv}_{\mathrm{N}}(\uparrow x_a)(x_a) = 1$, which means that (LM1) is true.

Let $\mathrm{F}, \mathrm{G} \in \mathbb{F}(X, L, 2)$ and $\mathrm{F} \subseteq \mathrm{G}$. It can be easily checked that $\{\mu \in L^X \mid \mu \notin \mathrm{G}\} \subseteq \{\mu \in L^X \mid \mu \notin \mathrm{F}\}$. Thus, $\mathrm{Conv}_{\mathrm{N}}(\mathrm{F})(x_a) = \wedge\{(\mathrm{N}(x_a)(\mu))^* \mid \mu \in L^X, \mu \notin \mathrm{F}\} \leq \wedge\{(\mathrm{N}(x_a)(\mu))^* \mid \mu \in L^X, \mu \notin G\} = \mathrm{Conv}_{\mathrm{N}}(\mathrm{G})(x_a)$ $\mu \notin G$ for any $x_a \in J(L^X)$, which means that (LM2) is true.

Since $\mathrm{Conv}_{\mathrm{N}}$ satisfies (LM1) and (LM2), the mapping $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}: J(L^X) \longrightarrow M^{L^X}$ is well-defined (see Definition 5). In order to prove (LM3), it suffices to prove the equality $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu) = \mathrm{N}(x_a)(\mu), (\forall x_a \in J(L^X), \forall \mu \in L^X)$ by definition of $\mathrm{Conv}_{\mathrm{N}}$. By Definition 5 (3),

$$\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu) = \bigwedge_{\mu \notin \mathrm{F}} (\mathrm{Conv}_{\mathrm{N}}(\mathrm{F})(x_a))^* = \bigwedge_{\mu \notin \mathrm{F}} \bigvee_{\nu \notin \mathrm{F}} \mathrm{N}(x_a)(\nu). \tag{5}$$

For each $\mathrm{F} \in \mathbb{F}(X, L, 2)$ satisfying $\mu \notin \mathrm{F}$, we have $\vee\{\mathrm{N}(x_a)(\nu) \mid \nu \in L^X, \nu \notin \mathrm{F}\} \geq \mathrm{N}(x_a)(\nu)$. As F is arbitrary, the inequality $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu) \geq \mathrm{N}(x_a)(\mu)$ holds. To prove the other inequality $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu) \leq \mathrm{N}(x_a)(\mu)$, let $I = \{\mathrm{F} \in \mathbb{F}(X, L, 2) \mid \mu \notin \mathrm{F}\}$ and $J_{\mathrm{F}} = \{\nu \in L^X \mid \nu \notin \mathrm{F}\}$. As M is a completely distributive complete lattice,

$$\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu)=\bigvee_{g\in\prod_{\mathrm{F}\in I}J_{\mathrm{F}}}\bigwedge_{\mathrm{F}\in I}\mathrm{N}(x_a)(g(\mathrm{F})). \tag{6}$$

First, we show that $\mathrm{F}_b\in I$, where $\mathrm{F}_b=\{\gamma\in L^X\mid \mathrm{N}(x_a)(\gamma)\nleq b\}(\forall b\in\Uparrow(\mathrm{N}(x_a)(\mu))\cap\mathrm{Pr}(M))$. As $\mu\notin\mathrm{F}_b$ is obvious, we only need to show that F_b is an L-filter.

(i) (F1) As $\mathrm{N}(x_a)(1_X)=1\nleq b$ and $\mathrm{N}(x_a)(0_X)=0\leq b$, $1_X\in\mathrm{F}_b$, and $0_X\notin\mathrm{F}_b$.

(ii) (F2) Assume that $\gamma\in\mathrm{F}_b$ and $\gamma\leq\nu$; then $\mathrm{N}(x_a)(\gamma)\leq\mathrm{N}(x_a)(\nu)$. Further, $\mathrm{N}(x_a)(\gamma)\nleq b$, so we have $\mathrm{N}(x_a)(\nu)\nleq b$, and thus, $\nu\in\mathrm{F}_b$.

(iii) (F3) Let $\gamma,\nu\in\mathrm{F}_b$. Then we declare that $\mathrm{N}(x_a)(\gamma\wedge\nu)=\mathrm{N}(x_a)(\gamma)\wedge\mathrm{N}(x_a)(\nu)\nleq b$ (i.e., $\gamma\wedge\nu\in\mathrm{F}_b$). Otherwise, $\mathrm{N}(x_a)(\gamma)\leq b$ or $\mathrm{N}(x_a)(\nu)\leq b$ since $b\in\mathrm{Pr}(M)$, which is a contradiction.

Next, it follows from $\mathrm{F}_b\in I$ that $g(\mathrm{F}_b)\notin\mathrm{F}_b(\forall b\in\Uparrow(\mathrm{N}(x_a)(\mu))\cap\mathrm{Pr}(M),\forall g\in\prod_{\mathrm{F}\in I}J_{\mathrm{F}})$, and thus, $\mathrm{N}(x_a)(g(\mathrm{F}_b))\leq b$ (particularly, $\wedge\{\mathrm{N}(x_a)(g(\mathrm{F}))\mid\mathrm{F}\in I\}\leq b$). As b is arbitrary and M is a completely distributive complete lattice, $\wedge\{\mathrm{N}(x_a)(g(\mathrm{F}))\mid\mathrm{F}\in I\}\leq\mathrm{N}(x_a)(\mu)$. As g is arbitrary, $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu)\leq\mathrm{N}(x_a)(\mu)$.

Since $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}}(x_a)(\mu)=\mathrm{N}(x_a)(\mu)$ and N is an (L,M)-fuzzy neighborhood operator, (LM4) holds. □

Proposition 3. *If $f\colon(X,\mathrm{N}_X)\longrightarrow(Y,\mathrm{N}_Y)$ is an (L,M)-**FNS** morphism, then $f\colon(X,Conv_{\mathrm{N}_X})\longrightarrow(Y,Conv_{\mathrm{N}_Y})$ is an (L,M)-**FTCS** morphism.*

Proof. For any $\mathrm{F}\in\mathbb{F}(X,L,2)$ and any $x_a\in J(L^X)$, as f is an (L,M)-FNS morphism, $\mathrm{N}_Y(f(x)_a)(\nu)\leq\mathrm{N}_X(x_a)(f_L^{\leftarrow}(\nu))$. Further, * is an order-reversing mapping; $(\mathrm{N}_X(x_a)(f_L^{\leftarrow}(\nu)))^*\leq(\mathrm{N}_Y(f(x)_a)(\nu))^*(\forall\nu\in L^Y)$. It follows that

$$\begin{aligned}\mathrm{Conv}_{\mathrm{N}_X}(\mathrm{F})(x_a)&=\bigwedge_{\mu\notin F}(\mathrm{N}_X(x_a)(\mu))^*\\&\leq\bigwedge_{\nu\notin f_L^{\Rightarrow}(\mathrm{F})}(\mathrm{N}_X(x_a)(f_L^{\leftarrow}(\nu)))^*\\&\leq\bigwedge_{\nu\notin f_L^{\Rightarrow}(\mathrm{F})}(\mathrm{N}_Y(f(x)_a)(\nu))^*\\&=\mathrm{Conv}_{\mathrm{N}_Y}(f_L^{\Rightarrow}(\mathrm{F}))(f(x)_a).\end{aligned} \tag{7}$$

Therefore, $f\colon(X,\mathrm{Conv}_{\mathrm{N}_X})\longrightarrow(Y,\mathrm{Conv}_{\mathrm{N}_Y})$ is an (L,M)-**FTCS** morphism. □

Proposition 4. *If $f\colon(X,Conv_X)\longrightarrow(Y,Conv_Y)$ is an (L,M)-**FTCS** morphism, then $f\colon(X,\mathrm{N}_{Conv_X})\longrightarrow(Y,\mathrm{N}_{Conv_Y})$ is an (L,M)-**FNS** morphism.*

Proof. For each $\mathrm{G}\in\mathbb{F}(X,L,2)$ and $x_a\in J(L^X)$, as f is an (L,M)-**FTCS** morphism, $\mathrm{Conv}_X(\mathrm{G})(x_a)\leq\mathrm{Conv}_Y(f_L^{\Rightarrow}(\mathrm{G}))(f(x)_a)$. Further, * is an order-reversing mapping; $(\mathrm{Conv}_Y(f_L^{\Rightarrow}(\mathrm{G}))(f(x)_a))^*\leq(\mathrm{Conv}_X(\mathrm{G})(x_a))^*$. For all $\mu\in L^Y$, it follows that

$$\begin{aligned}\mathrm{N}_{\mathrm{Conv}_Y}(f(x)_a)(\mu)&=\bigwedge_{\mu\notin\mathrm{F}}(\mathrm{Conv}_Y(\mathrm{F})(f(x)_a))^*\\&\leq\bigwedge_{f_L^{\leftarrow}(\mu)\notin\mathrm{G}}(\mathrm{Conv}_Y(f_L^{\Rightarrow}(\mathrm{G}))(f(x)_a))^*\\&\leq\bigwedge_{f_L^{\leftarrow}(\mu)\notin\mathrm{G}}(\mathrm{Conv}_X(\mathrm{G})(x_a))^*\\&=\mathrm{N}_{\mathrm{Conv}_X}(x_a)(f_L^{\leftarrow}(\mu)).\end{aligned} \tag{8}$$

The first inequality holds since $\{\mathrm{F}\in\mathbb{F}(Y,L,2)\mid\mu\notin\mathrm{F}\}\supseteq\{f_L^{\Rightarrow}(\mathrm{G})\mid\mathrm{G}\in\mathbb{F}(X,L,2),f_L^{\leftarrow}(\mu)\notin\mathrm{G}\}$. Therefore, $f\colon(X,\mathrm{N}_{\mathrm{Conv}_X})\longrightarrow(Y,\mathrm{N}_{\mathrm{Conv}_Y})$ is an (L,M)-**FNS** morphism. □

Proposition 5. *For any (L,M)-fuzzy topological convergence structure Conv on X, one has $Conv_{\mathrm{N}_{Conv}}=Conv$.*

Proof. Let $\mathrm{F}\in\mathbb{F}(X,L,2)$ and $x_a\in J(L^X)$. We need to prove $\mathrm{Conv}_{\mathrm{N}_{\mathrm{Conv}}}(\mathrm{F})(x_a)=\mathrm{Conv}(\mathrm{F})(x_a)$. By Proposition 2 and Definition 5,

$$\mathrm{Conv}_{\mathrm{N}_{\mathrm{Conv}}}(\mathrm{F})(x_a)=\bigwedge_{\mu\notin\mathrm{F}}\bigvee_{\mu\notin G}\mathrm{Conv}(\mathrm{G})(x_a). \tag{9}$$

For each $\mu\in L^X$ with $\mu\notin\mathrm{F}$, we have

$$\bigvee_{\mu\notin\mathrm{G}}\mathrm{Conv}(\mathrm{G})(x_a)\geq\mathrm{Conv}(\mathrm{F})(x_a), \tag{10}$$

and then the inequality $\mathrm{Conv}_{\mathrm{N}_{\mathrm{Conv}}}(\mathrm{F})(x_a)\geq\mathrm{Conv}(\mathrm{F})(x_a)$ holds since μ is arbitrary. To show the other inequality $\mathrm{Conv}_{\mathrm{N}_{\mathrm{Conv}}}(\mathrm{F})(x_a)\leq\mathrm{Conv}(\mathrm{F})(x_a)$, put $I=\{\mu\in L^X\mid\mu\notin\mathrm{F}\}$ and $J_\mu=\{\mathrm{G}\in\mathbb{F}(X,L,2)\mid\mu\notin\mathrm{G}\}$. As M is a completely distributive complete lattice,

$$\mathrm{Conv}_{\mathrm{N}_{\mathrm{Conv}}}(\mathrm{F})(x_a)=\bigvee_{g\in\prod_{\mu\in I}J_\mu}\bigwedge_{\mu\in I}\mathrm{Conv}(g(\mu))(x_a). \tag{11}$$

It suffices to prove

$$\bigwedge_{\mu\in I}\mathrm{Conv}(g(\mu))(x_a)\leq\mathrm{Conv}(\mathrm{F})(x_a),\quad\left(\forall g\in\prod_{\mu\in I}J_\mu\right). \tag{12}$$

For each $g\in\prod_{\mu\in I}J_\mu$, by (LM3), we have $\wedge_{\mu\in I}\mathrm{Conv}(g(\mu))(x_a)=\wedge_{\mu\in I}\wedge_{\nu\notin g(\mu)}(\mathrm{N}_{Conv}(x_a)(\nu))^*=\wedge_{\nu\notin\cap_{\mu\in I}g(\mu)}(\mathrm{N}_{Conv}(x_a)(\nu))^*=\mathrm{Conv}(\cap_{\mu\in I}g(\mu))(x_a)$. As $\cap_{\mu\in I}g(\mu)\subseteq\mathscr{F}$, from (LM2), we have $\mathrm{Conv}(\cap_{\mu\in I}g(\mu))(x_a)\leq\mathrm{Conv}(\mathrm{F})(x_a)$, which means

$$\bigwedge_{\mu\in I}\mathrm{Conv}(g(\mu))(x_a)\leq\mathrm{Conv}(\mathrm{F})(x_a),\quad\left(\forall g\in\prod_{\mu\in I}J_\mu\right). \tag{13}$$

By Propositions 1–5, we have the following. □

Theorem 1

(1) (L, M)-FNS is isomorphic to (L, M)-FTCS.

Similar to [25], it is easy to check that (L, M)-FNS is isomorphic to (L, M)-FTop

(2) Thus (L, M)-FTop is isomorphic to (L, M)-FTCS.

Remark 2

(1) If Conv is an (L, M)-fuzzy topological convergence structure on X, then it satisfies the following: if $x_a \leq x_b$, then $\mathrm{Conv}(\mathrm{F})(x_a) \leq \mathrm{Conv}(\mathrm{F})(x_b)$ $(\forall x_a, x_b \in J(L^X), \mathrm{F} \in \mathbb{F}(X, L, 2))$.

(2) From Proposition 5, the following is true due to (LM3):

$$\bigwedge_{i\in I} \mathrm{Conv}(\mathrm{F}_i)(x_a) = \mathrm{Conv}\left(\bigcap_{i\in I} \mathrm{F}_i\right)(x_a). \tag{14}$$

(3) Now we turn our attention to the Moore–Smith convergence theory; for basic notions, refer to [19, 21, 23].

For a molecule net $S = (S(m) \mid m \in D)$ $(S(m) \in J(L^X)$, and D is a directed set), we define an L-filter F_S associated with the net S as follows: $\mathrm{F}_S = \{\mu \mid S \text{ is eventually in } \mu\}$. Conversely, for an L-filter F, define a molecule net $S_{\mathrm{F}} = (S(m) \mid m \in D)$, where $D = \{(x_a, A) \mid x_a \in J(L^X), A \in \mathrm{F}, x_a \leq A\}$ is a directed set, on which the order is equipped with the relation $\leq$: $(x_a, A) \leq (y_b, B)$ if and only if $A \geq B$, and the mapping is $S(x_a, A) = x_a$.

Now define convergence of S as

$$\mathrm{Conv}(S)(x_a) = \wedge\{(\mathrm{N}(x_a)(\mu))^* \mid S \text{ is not eventually in } \mu\}. \tag{15}$$

In an (L, M)-fuzzy topological space (X, T), the following hold. The proof is simple and is missing:

(i) (1) $\mathrm{Conv}(S)(x_a) = \mathrm{Conv}(\mathrm{F}_S)(x_a)$ for each molecule S and $x_a \in J(L^X)$.

(ii) (2) $\mathrm{Conv}(\mathrm{F})(x_a) = \mathrm{Conv}(S_{\mathrm{F}})(x_a)$ for each $\mathrm{F} \in \mathbb{F}(X, L, 2)$ and $x_a \in J(L^X)$.

(iii) (4) In [16], Pang and Fang proposed the concept of topological L-fuzzy Q-convergence spaces (the corresponding category is denoted by L-QFTCS). In these convergence spaces, the value of an L-fuzzy filter converging to a fuzzy point is from $\{0, 1\}$. Thus, Pang and Fang's spaces are different from ours. But L-QFTCS is isomorphic to (L, L)-FTCS when L is a completely distributive lattice with an order-reversing involution.

(iv) (5) For a given set X, let $\mathrm{TopConv}(X, L, M)$ be a set of all (L, M)-fuzzy topological convergence structures on X, and the relation $\leq$ on $\mathrm{TopConv}(X, L, M)$ is defined by $\mathrm{Conv}_1 \leq \mathrm{Conv}_2$ if and only if $\mathrm{Conv}_1(\mathrm{F})(x_a) \leq \mathrm{Conv}_2(\mathrm{F})(x_a)$ $(\forall x_a \in J(L^X) \text{ and } \mathrm{F} \in \mathbb{F}(X, L, 2))$.

The set of all (L, M)-fuzzy neighborhood operators on X is written as $\mathrm{NO}(X, L, M)$, and the relation $\leq$ on $\mathrm{NO}(X, L, M)$ is defined by $\mathrm{N}_1 \leq \mathrm{N}_2$ if and only if $\mathrm{N}_1(x_a)(\mu) \geq \mathrm{N}_2(x_a)(\mu)$ $(\forall x_a \in J(L^X) \text{ and } \mu \in L^X)$.

From Propositions 2 and 5, we have $\mathrm{N}_{\mathrm{Conv}_{\mathrm{N}}} = \mathrm{N}$ and for each $\mathrm{N} \in \mathrm{OP}(X, L, M)$, $\mathrm{Conv}_{\mathrm{N}_{\mathrm{Conv}}} = \mathrm{Conv}$ for each $\mathrm{Conv} \in \mathrm{TopConv}(X, L, M)$. This implies that there exists a bijection between $(\mathrm{TopConv}(X, L, M), \leq)$ and $(\mathrm{OP}(X, L, M), \leq)$. Similar to paper [26], it is easily checked that $(\mathrm{TopConv}(X, L, M), \leq)$ and $(\mathrm{OP}(X, L, M), \leq)$ are complete lattices and they are isomorphic.

By Theorem 1 (1), there exists an isomorphic functor between (L, M)-**FNS** and (L, M)-**FTCS**. Furthermore, the restriction of this functor to $(\mathrm{OP}(X, L, M), \leq)$ is an order isomorphism.

Now we prove that (L, M)-**FPIS** is isomorphic to (L, M)-**FPCS**. Since its proof is similar to Theorem 1, we only give some propositions as follows.

Proposition 6. *For each (L, M)-fuzzy preinterior operator* I *on X, the mapping $Conv_{\mathrm{I}}$: $\mathbb{F}(X, L, 2) \longrightarrow M^{J(L^X)}$, defined by*

$$\begin{gathered} \mathrm{Conv}_{\mathrm{I}}(\mathrm{F})(x_a) = \bigwedge_{\mu \notin \mathrm{F}} (\mathrm{I}(\mu)(x_a))^*, \\ \left(\forall \mathrm{F} \in \mathbb{F}(X, L, 2), x_a \in J\left(L^X\right)\right), \end{gathered} \tag{16}$$

is an (L, M)-fuzzy principal convergence structure on X.

Proposition 7. *For each (L, M)-fuzzy principal convergence structure Conv on X, the mapping I_{Conv}: $L^X \longrightarrow M^{J(L^X)}$, defined by*

$$\begin{gathered} \mathrm{I}_{\mathrm{Conv}}(\mu)(x_a) = \bigwedge_{\mu \notin \mathrm{F}} (\mathrm{Conv}(\mathrm{F})(x_a))^*, \\ \left(\forall \mu \in L^X, x_a \in J\left(L^X\right)\right), \end{gathered} \tag{17}$$

is an (L, M)-fuzzy preinterior operator on X.

Proposition 8. *For any (L, M)-fuzzy principal convergence structure Conv on X, one has $Conv_{\mathrm{I}_{Conv}} = Conv$.*

Proposition 9. *For any (L, M)-fuzzy preinterior operator* I *on X, one has $\mathrm{I}_{Conv_{\mathrm{I}}} = \mathrm{I}$.*

Proposition 10. *If f: $(X, \mathrm{I}_X) \longrightarrow (Y, \mathrm{I}_Y)$ is an (L, M)-FPIS morphism, then f: $(X, Conv_{\mathrm{I}_X}) \longrightarrow (Y, Conv_{\mathrm{I}_Y})$ is an (L, M)-FPCS morphism.*

Proposition 11. *If f: $(X, Conv_X) \longrightarrow (Y, Conv_Y)$ is an (L, M)-FPCS morphism, then f: $(X, \mathrm{I}_{Conv_X}) \longrightarrow (Y, \mathrm{I}_{Conv_Y})$ is an (L, M)-FPIS morphism.*

By Propositions 6–11, we have the following.

Theorem 2. *(L, M)-FPIS is isomorphic to (L, M)-FPCS.*

Based on the above facts, we give the following example.

Example 1. Let $X = \{x\}$ be a single set, $L = \{1, 0.7, 0.5, 0.3, 0\}$ and $M = \{1, 0.5, 0\}$, where unary operation $*$ on M is defined as $1^* = 0$, $0.5^* = 0.5$, and $0^* = 1$. Define $T(0_X) = T(1_X) = 1$; otherwise, $\mathrm{T}(A) = 0.5$, $A \in L^X$. Then

(1) (X, T) is an (L, M)-fuzzy topological space.

(2) Define $\mathrm{N}(x_a)(A) = \vee\{\mathrm{T}(B) \mid x_a \leq B \leq A\}$. It follows from Theorem 3.2 in [25] that (X, N) is an (L, M)-fuzzy neighborhood space. In this way, $\mathrm{N}(x_{0.5})$, which is defined by $\mathrm{N}(x_{0.5})(x_1) = 1$, $\mathrm{N}(x_{0.5})(x_{0.7}) = \mathrm{N}(x_{0.5})(x_{0.5}) = 0.5$, and $\mathrm{N}(x_{0.5})(x_{0.3}) = \mathrm{N}(x_{0.5})(x_0) = 0$, is the (L, M)-fuzzy neighborhood of $x_{0.5}$.

(3) Let F be an L-filter, $\mathrm{Conv}(\mathrm{F})(x_a) = \wedge \{\mathrm{N}(x_a)(A)^* \mid A \notin \mathrm{F}\}$. In this way, supposing $\mathrm{F} = \{x_1, x_{0.7}\}$, we can see that $\mathrm{Conv}(\mathrm{F})(x_{0.5}) = \mathrm{N}(x_{0.5})\ (x_{0.5})^* \wedge \mathrm{N}(x_{0.5})(x_{0.3})^* \wedge \mathrm{N}(x_{0.5})\ (x_0)^* = 0.5$, which means that the degree of F convergence to $x_{0.5}$ is 0.5.

4. Two Approaches to L-Topological Convergence Structures

In this section, we restrict our attention to the case $M = 2$, and thus, we identify a set Y with its characteristic function χ_Y. We write $\mathrm{F} \longrightarrow x_a$ whenever $\mathrm{Conv}(\mathrm{F})(x_a) = 1$ and replace Conv by $\longrightarrow$ in order to emphasize the convergence. In this manner, the notion of an $(L, 2)$-fuzzy topological convergence space and the notion of an $(L, 2)$-fuzzy neighborhood space can be easily given as follows.

Definition 8

(1) A pair $(X, \longrightarrow)$ is an L-convergence space ($\longrightarrow$ is called an L-convergence structure on X) if it satisfies the following conditions:

 (i) (L1) $\uparrow x_a \longrightarrow x_a\, (\forall x_a \in J(L^X))$

 (ii) (L2) If $\mathrm{F} \subseteq \mathrm{G}$ and $\mathrm{F} \longrightarrow x_a$, then $\mathrm{G} \longrightarrow x_a\, (\forall x_a \in J(L^X), \forall \mathrm{F}, \mathrm{G} \in \mathbb{F}(X, L, 2))$

 (Order) If $\mathrm{F} \longrightarrow x_a$ and $x_a \leq x_b$, then $\mathrm{F} \longrightarrow x_b$ $(\forall x_a, x_b \in J(L^X), \forall \mathrm{F} \in \mathbb{F}(X, L, 2))$.

(2) A pair $(X, \longrightarrow)$ is an L-topological convergence space($\longrightarrow$ is called an L-topological convergence structure on X) if it satisfies the following conditions:

 (i) (L1) $\uparrow x_a \longrightarrow x_a\, (\forall x_a \in J(L^X))$

 (ii) (L2) If $\mathrm{F} \subseteq \mathrm{G}$ and $\mathrm{F} \longrightarrow x_a$, then $\mathrm{G} \longrightarrow x_a$ $(\forall x_a \in J(L^X), \forall \mathrm{F}, \mathrm{G} \in \mathbb{F}(X, L, 2))$

 (iii) (L3) $\mathrm{N}(x_a) \longrightarrow x_a$, where $\mathrm{N}(x_a) = \cap_{\mathrm{F} \longrightarrow x_a} \mathrm{F}$ $(\forall x_a \in J(L^X))$

 (iv) (L4) If $\mu \in \mathrm{N}(x_a)$, then there exists $\nu \in L^X$ such that $\nu \in \uparrow x_a \cap \downarrow \mu$ and $\nu \in \mathrm{N}(y_b)$ for each $y_b \in J(L^X) \cap \Downarrow \nu\, (\forall \mu \in L^X, \forall x_a \in J(L^X))$

 Obviously, an L-topological convergence space is an L-convergence space.

(3) (See [25]) A pair (X, N) is an L-neighborhood space (N is called an L-neighborhood system on X) if it satisfies the following conditions:

 (i) (LN0) $1_X \in \mathrm{N}(x_a)$ and $0_X \notin \mathrm{N}(x_a)$ $(\forall x_a \in J(L^X))$

 (ii) (LN1) If $x_a \nleq \mu$, then $\mu \notin \mathrm{N}(x_a)$ $(\forall x_a \in J(L^X), \forall \mu \in L^X)$

 (iii) (LN2) If $\mu \leq \nu$ and $\mu \in \mathrm{N}(x_a)$, then $\nu \in \mathrm{N}(x_a)\, (\forall x_a \in J(L^X), \forall \mu, \nu \in L^X)$

 (iv) (LN3) If $\mu, \nu \in \mathrm{N}(x_a)$, then $\mu \wedge \nu \in \mathrm{N}(x_a)$ $(\forall x_a \in J\ (L^X), \forall \mu, \nu \in L^X)$

 (v) (LN4) If $\mu \in \mathrm{N}(x_a)$, then there exists $\nu \in L^X$ such that$\nu \in \uparrow x_a \cap \downarrow \mu$ and $\nu \in \mathrm{N}(y_b)$ for each $y_b \in J(L^X) \cap \Downarrow \nu (\forall x_a \in J(L^X), \forall \mu \in L^X)$

Remark 3 (see [25]). For a given set X, let $\mathbb{T}(X, L)$ be the set of all L-topologies on X and $\mathbb{N}(X, L)$ the set of all L-topological neighborhood systems on X. Then, the mapping $\varphi_{12}\colon \mathbb{T}(X, L) \longrightarrow \mathbb{N}(X, L)$ defined by $N_T(x_a) = \{\nu \in L^X \mid \exists \mu \in \mathrm{T}, \mu \in \uparrow x_a \cap \downarrow \nu\} (\forall \mathrm{T} \in \mathbb{T}(X, L), \forall x_a \in\ J(L^X))$ is a one-to-one correspondence, whose inverse $\varphi_{21}\colon \mathbb{N}(X, L) \longrightarrow \mathbb{T}(X, L)$ is given by $T_N = \{\nu \in L^X \mid \nu \in \mathrm{N}(y_b)\ \ (\forall y_b \in J(L^X) \cap \Downarrow \nu)\}, (\forall \mathrm{N} \in \mathbb{N}(X, L))$. Thus, by Theorem 1, there exists a one-to-one correspondence between the set of all L-topological convergence structures on X and the set of L-topologies on X.

In [9], Jäger gave a generalization of Kowalsky's diagonal condition. Based on different tools, we give another form of Kowalsky's diagonal condition [2] in fuzzy setting. In the presence of this form, we give a characterization of an L-topological convergence structure.

Theorem 3. *$(X, \longrightarrow)$ is an L-topological convergence space if and only if it satisfies (L1)–(L3), (order) in Definition 8, and the following:*

 (i) (LK4) For any $\mathrm{G} \in \mathbb{F}(X, L, 2)$ and any subfamily $\{\mathrm{F}(y_b)\}_{y_b \in J(L^X)} \subseteq \mathbb{F}(X, L, 2)$ which satisfies

$$\text{(LFD) } \mathrm{F}(y_a), \supseteq \mathscr{F}(y_b), \quad \text{whenever } y_a \leq y_b, \tag{18}$$

 if $\mathrm{G} \longrightarrow x_a$ and $\mathrm{F}(y_b) \longrightarrow y_b$, then $\mathrm{G}(\mathrm{F}_-) \longrightarrow x_a$ $(x_a, y_b \in J(L^X))$, where the mapping $\mathrm{F}_-\colon L^X \longrightarrow L^X$ and the subfamily $\mathrm{G}(\mathrm{F}_-) \subseteq L^X$ are defined as follows:

$$\begin{aligned} &\mathrm{F}_-(\mu) = \vee\{x_a \mid \mu \in \mathrm{F}(x_a)\}, \quad \left(\forall \mu \in L^X\right) \\ &\mathrm{G}(\mathrm{F}_-) = \left\{\mu \in L^X \mid \mathrm{F}_-(\mu) \in \mathrm{G}\right\}. \end{aligned} \tag{19}$$

To prove Theorem 3, we need several lemmas.

Lemma 1. *Let $\mathrm{G} \in \mathbb{F}(X, L, 2)$ and $\{\mathrm{G}(x_a)\}_{x_a \in J(L^X)} \subseteq \mathbb{F}(X, L, 2)$ satisfy (LFD). Then $\mathrm{G}(\mathrm{F}_-) = \{\mu \in L^X \mid \mathrm{F}_-(\mu) \in \mathrm{G}\}$ is an L-filter on X.*

Proof. As $1_X \in \mathrm{F}(x_a) (\forall x_a \in J(L^X))$, $\mathrm{F}_-(1_X) = \vee J\ \ (L^X) = 1_X \in \mathrm{G}$, which means $1_X \in \mathrm{G}(\mathrm{F}_-)$. Since $\{x_a \in J(L^X) \mid 0_X \in \mathrm{F}(x_a)\} = \varnothing$, we have $\mathrm{F}_-(0_X) = 0_X \notin \mathrm{G}$; that is, $0_X \in \mathrm{G}(\mathrm{F}_-)$. Thus, $\mathrm{G}(\mathrm{F}_-)$ satisfies (F1). Obviously, $\mathrm{G}(\mathrm{F}_-)$ satisfies (F2).

For any $\mu, \nu \in \mathrm{G}(\mathrm{F}_-)$, put $J_\mu = \{x_a \in J(L^X) \mid \mu \in \mathrm{F}(x_a)\}$ and $J_\nu = \{x_a \in J(L^X) \mid \nu \in \mathrm{F}(x_a)\}$. Then $\mathrm{F}_-(\mu) = \vee J_\mu$, $\mathrm{F}_-(\nu) = \vee J_\nu$ and $\mathrm{F}_-(\mu) \wedge \mathrm{F}_-(\nu) \in G$ by definition of $\mathrm{G}(\mathrm{F}_-)$. We first show the inequality $\mathrm{F}_-(\mu) \wedge \mathrm{F}_-(\nu) \leq \vee(J_\mu \cap J_\nu)$.

Suppose $x_a \in J(L^X)$ and $x_a \lhd \mathrm{F}_-(\mu) \wedge \mathrm{F}_-(\nu)$, and then $x_a \lhd \mathrm{F}_-(\mu) = \vee J_\mu$. There exists an $x_b \in J_\mu$ (which implies $\mu \in \mathrm{F}(x_b)$) such that $x_a \le x_b$, and thus, $\mathrm{F}(x_b) \subseteq \mathrm{F}(x_a)$ since $\{\mathrm{F}(x_a)\}_{x_a \in J(L^X)}$ satisfies (LFD). Therefore, $\mu \in \mathrm{F}(x_a)$ (i.e., $x_a \in J_\mu$). Similarly, $x_a \in J_\nu$. Consequently $x_a \le \vee(J_\mu \cap J_\nu)$, which implies $\mathrm{F}_-(\mu) \wedge \mathrm{F}_-(\nu) \le \vee(J_\mu \cap J_\nu)$ since L^X is a completely distributive lattice. Notice that $(J_\mu \cap J_\nu) \subseteq \{x_a \in J(L^X) \mid \mu \wedge \nu \in \mathrm{F}(x_a)\}$, we have $\mathrm{F}_-(\mu) \wedge \mathrm{F}_-(\nu) \le \mathrm{F}_-(\mu \wedge \nu)$, and thus, $\mathrm{F}_-(\mu \wedge \nu) \in \mathrm{G}$, which means $\mu \wedge \nu \in \mathrm{G}(\mathrm{F}_-)$. Hence, $\mathrm{G}(\mathrm{F}_-)$ also satisfies (F3). □

Lemma 2. *For a pair (X, N) satisfying conditions (LN0)-(LN3) in Definition 8 (3), the following are equivalent:*

(1) (LN4)

(2) $\{\mathrm{N}(x_a)\}_{x_a \in J(L^X)}$ satisfies (LFD) and $\mathrm{N}(x_a) \subseteq \mathrm{N}(x_a)(\mathrm{N}_-)$ $(\forall x_a \in J(L^X))$

Proof

(1) $\Longrightarrow$ (2). Since (X, N) satisfies conditions (LN0), (LN2), and (LN3), $\{\mathrm{N}(x_a)\}_{x_a \in J(L^X)} \subseteq \mathbb{F}(X, L, 2)$. For any $x_a, x_b \in J(L^X)$ satisfying $x_a \le x_b$ and any $\mu \in \mathrm{N}(x_b) = \mathrm{N}_{\mathrm{T_N}}(x_b)$, there exists a $\nu \in T_N$ such that $x_b \le \nu \le \mu$ since (X, N) satisfies conditions (LN0)–(LN4) (and thus, $N = N_{TN}$; see [25] or Remark 3). It follows from $x_a \le x_b$ that $\mu \in \mathrm{N}_{\mathrm{T_N}}(x_a) = \mathrm{N}(x_a)$. Therefore, $\{\mathrm{N}(x_a)\}_{x_a \in J(L^X)}$ satisfies (LFD), and thus, $\mathrm{N}(x_a)(\mathrm{N}_-)$ is well-defined from Lemma $1\,(\forall x_a \in J(L^X))$.

Suppose that $\mu \in \mathrm{N}(x_a)$, then $x_a \le \nu \le \mu$ and $\nu \in \mathrm{N}(y_c)\,(\forall y_c \in J(L^X) \cap \Downarrow \nu)$ hold for some $\nu \in L^X$ by (LN4). It follows that $\mu \in \mathrm{N}(y_c)\,(\forall y_c \in J(L^X) \cap \Downarrow \nu)$ and $\nu \le \mathrm{N}_-(\mu)$. As $\nu \in \mathrm{T_N}$ (see Remark 3), we have $\nu \in \mathrm{N}_{\mathrm{T_N}}(x_a) = \mathrm{N}(x_a)$, and thus, $\mathrm{N}_-(\mu) \in \mathrm{N}(x_a)$ (i.e., $\mu \in \mathrm{N}(x_a)(\mathrm{N}_-)$). Therefore, $\mathrm{N}(x_a) \subseteq \mathrm{N}(x_a)(\mathrm{N}_-)\,(\forall x_a \in J(L^X))$.

(2) $\Longrightarrow$ (1). Assume $\mu \in \mathrm{N}(x_a)$; then $\mu \in \mathrm{N}(x_a)(\mathrm{N}_-)$ by (2), and then $\mathrm{N}_-(\mu) \in \mathrm{N}(x_a)$ by definition of $\mathrm{N}(x_a)(\mathrm{N}_-)$. We will show that ν (put $\nu = \mathrm{N}_-(\mu)$) is the required one in (LN4). Firstly, for all $y_b \in J(L^X)$ satisfying $\mu \in \mathrm{N}(y_b)$, we have $y_b \le \mu$ by (LN1) and thus, $\mathrm{N}_-(\mu) \le \mu$ by definition of $\mathrm{N}_-(\mu)$. Further, it follows from $\mathrm{N}_-(\mu) \in \mathrm{N}(x_a)$ that $x_a \le \mathrm{N}_-(\mu)$. Therefore, $\nu \in \uparrow x_a \cap \downarrow \mu$. Secondly, we may show $\mathrm{N}_-(\nu) = \nu$. On the one hand, as $\nu \le \mu$, $\mathrm{N}_-(\nu) \le \mathrm{N}_-(\mu) = \nu$. On the other hand, for each $y_b \in J(L^X)$ satisfying $\mu \in \mathrm{N}(y_b)$, we have $\mu \in \mathrm{N}(y_b)(\mathrm{N}_-)$ by (2), and thus, $\mathrm{N}_-(\mu) \in \mathrm{N}(y_b)$ by definition of $\mathrm{N}(y_b)(\mathrm{N}_-)$, which implies $\mathrm{N}_-(\mu) \le \mathrm{N}_-(\mathrm{N}_-(\mu))$. Therefore, $\nu \le \mathrm{N}_-(\nu)$. Finally, for each $z_c \lhd \nu = \mathrm{N}_-(\nu)$, there exists a $z_b \in J(L^X)$ such that $z_c \le z_b$ and $\nu \in \mathrm{N}(z_b)$. $\nu \in \mathrm{N}(z_c)$ since $\{\mathrm{N}(x_a)\}_{x_a \in J(L^X)}$ satisfies (LFD). That is, for each $z_c \lhd \nu$, $\nu \in \mathrm{N}(z_c)$ holds. □

Lemma 3. *For a pair $(X, \longrightarrow)$ satisfying conditions (L1)-(L3) and (order) in Definition 8, the following are equivalent:*

(1) (L4)

(2) (LK4)

Proof

(i) (1) $\Longrightarrow$ (2). It suffices to prove $\mathrm{N}(x_a) \subseteq \mathrm{G}(\mathrm{F}_-)$ by (L3). First, we show $\mathrm{G}(\mathrm{N}_-) \subseteq \mathrm{G}(\mathrm{F}_-)$. Suppose that $\mu \in \mathrm{G}(\mathrm{N}_-)$ (i.e., $\mathrm{N}_-(\mu) \in \mathrm{G}$). For each $y_b \in J(L^X)$ satisfying $\mu \in \mathrm{N}(y_b)$, we have $\mathrm{N}(y_b) \subseteq \mathrm{F}(y_b)$ (and thus, $\mu \in \mathrm{F}(y_b)$) for $\mathrm{F}(y_b) \longrightarrow y_b$ and (L3), which implies $\mathrm{N}_-(\mu) \le \mathrm{F}_-(\mu)$, and thus, $\mathrm{F}_-(\mu) \in \mathrm{G}$; that is $\mu \in \mathrm{G}(\mathrm{F}_-)$. Next, we show $\mathrm{N}(x_a)(\mathrm{N}_-) \subseteq \mathrm{G}(\mathrm{N}_-)$. Since $\mathrm{G} \longrightarrow x_a$, we obtain $\mathrm{N}(x_a) \subseteq \mathrm{G}$. Suppose $\mu \in \mathrm{N}(x_a)(\mathrm{N}_-)$, then $\mathrm{N}_-(\mu) \in \mathrm{N}(x_a)$, and thus, $\mathrm{N}_-(\mu) \in \mathrm{G}$; that is, $\mu \in \mathrm{G}(\mathrm{N}_-)$. Finally, as $\mathrm{N}(x_a) \subseteq \mathrm{N}(x_a)(\mathrm{N}_-)$ (see Lemma 2), we have $\mathrm{N}(x_a) \subseteq \mathrm{G}(\mathrm{F}_-)$ by the preceding two conclusions.

(ii) (2) $\Longrightarrow$ (1). Take $\mathrm{G} = \mathrm{N}(x_a)$ and $\mathrm{F}(y_b) = \mathrm{N}(x_b)\,(\forall y_b \in J(L^X))$. Since (order) is satisfied, $\{\mathrm{F}(y_b)\}_{y_b \in J(L^X)}$ is a subfamily of $\mathbb{F}(X, L, 2)$ satisfying (LFD). $\mathrm{G} \longrightarrow x_a$, and $\mathrm{F}(y_b) \longrightarrow y_b$ $(\forall y_b \in J(L^X))$. Thus, $\mathrm{G}(\mathrm{F}_-) \longrightarrow x_a$ by (2); that is, $\mathrm{N}(x_a) \subseteq \mathrm{G}(\mathrm{F}_-) = \mathrm{N}(x_a)(\mathrm{N}_-)$. It follows from Lemma 2 that (L4) holds. □

Proof of Theorem 3. It follows from Lemma 3.

In paper [11], based on stratified L-fuzzy filter, Gähler's neighborhood condition is studied in stratified L-convergence space. Based on L-filter, Gähler's neighborhood condition is also studied in L-convergence space. □

Theorem 4. *$(X, \longrightarrow)$ is an L-topological convergence space if and only if it satisfies (L1)–(L3), (order) in Definition 8, and the following:*

(i) (LG4) If $\mathrm{F} \longrightarrow x_a$, then $\mathrm{F}(\mathcal{N}_-) \longrightarrow x_a$ $(\forall x_a \in J(L^X), \mathrm{F} \in \mathbb{F}(X, L, 2))$

Proof. Suppose $(X, \longrightarrow)$ is an L-topological convergence space; we show that $(X, \longrightarrow)$ satisfies (LG4). If $\mathrm{F} \longrightarrow x_a$, then, by (L3), $\mathrm{N}(x_a) \subseteq \mathrm{F}$. It can be easily checked that $\mathrm{N}(x_a)(\mathrm{N}_-) \subseteq \mathrm{F}(\mathrm{N}_-)$. It follows from Lemma 2 that $\mathrm{N}(x_a) \subseteq \mathrm{F}(\mathrm{N}_-)$; that is, $\mathrm{F}(\mathrm{N}_-) \longrightarrow x_a$. Conversely, it follows from (L3) that $\mathrm{N}(x_a) \longrightarrow x_a$, and thus, $\mathrm{N}(x_a)(\mathrm{N}_-) \longrightarrow x_a$ by (LG4). This implies $\mathrm{N}(x_a) \subseteq \mathrm{N}(x_a)(\mathrm{N}_-)$. Following Lemma 2, (L4) holds. □

5. The Relation between $(X, \longrightarrow)$ and Li's $(X, \lim)$

An L-topological stratified L-fuzzy convergence space [6] (where L is a complete Heyting algebra) is a pair $(X, \lim)$, where X is a set, and lim: $\mathbb{F}^S(X, L, L) \longrightarrow L^X$ (called an

L-topological stratified L-fuzzy convergence structure on X) is a mapping which satisfies the following conditions:

(i) $(L1)^{*}\lim[x](x)=1\,(\forall x\in X)$.

(ii) $(\mathrm{L2})^{*}\,\mathrm{F}\leq\mathrm{G}$ implies $\lim\mathrm{F}\leq\lim\mathrm{G}\,(\forall\mathrm{F},\mathrm{G}\in\mathbb{F}^{S}(X,L,L))$.

(iii) (Lp) $\lim\mathrm{F}(x)=\wedge_{\mu\in L^{X}}(\mathrm{N}^{x}(\mu)\longrightarrow\mathrm{N}(\mu))$,

(iv) where

$$\mathrm{N}^{x}(\mu)=\bigwedge_{\mathrm{G}\in SF(X,L,L)}(\lim\mathrm{G}(x)\longrightarrow\mathrm{G}(\mu)),\quad(\forall x\in X).\tag{20}$$

(v) (Lt) $\mathrm{N}^{x}(\mu)\leq\vee\{\mathrm{N}^{x}(\nu)\,|\,\nu(y)\leq\mathrm{N}^{y}\quad(\mu)(\forall y\in X)\}$, $(\forall x\in X,\forall\mu\in L^{X})$.

From Proposition 6.4 in [6], there exists a one-to-one correspondence between the set of all L-topological stratified L-fuzzy convergence structures on X and the set of stratified L-topologies on X. Following Jäger's work, Li [12] proved that there exists a one-to-one correspondence between the set of all L-topological L-fuzzy convergence structures on X and the set of L-topologies on X, where an L-topological L-fuzzy convergence structure is a mapping $\lim\colon\mathbb{F}(X,L,L)\longrightarrow L^{X}$ which satisfies $(L1)^{*}$, $(L2)^{*}$, (Lp), and (Lt).

The spaces $(X,\longrightarrow)$ and $(X,\lim)$ are absolutely different in their forms. For the former, a filter is a family of subsets of L^{X}; the point is fuzzy but the convergence is crisp. However, for the latter, a filter is a mapping from L^{X} to L; the point is crisp but the convergence is fuzzy. From Remark 3, there exists a one-to-one correspondence between the set of all L-topological convergence structures on X and the set of L-topologies on X. Thus, these two notions (L-topological L-fuzzy convergence structures [12] and L-topological convergence structures) can be determined reciprocally for L being a completely distributive lattice. To say it more precisely, there exists a one-to-one correspondence φ_{13} from $\mathbb{A}(X,L)$ (the set of all L-topological convergence structures on X) to $\mathbb{B}(X,L)$ (the set of all L-topological L-fuzzy convergence structures on X), which is defined by

$$\begin{gathered}\varphi_{13}(\longrightarrow)\mathrm{F}(x)=\bigwedge_{\mu\in L^{X}}(\mathrm{N}^{x}(\mu)\longrightarrow\mathrm{F}(\mu)),\\(\forall\mathrm{F}\in\mathbb{F}(X,L,L),\forall x\in X),\end{gathered}\tag{21}$$

for each $\longrightarrow$ in $\mathbb{A}(X,L)$, where $\mathrm{N}^{x}(\mu)=\vee\ \{a\in J(L)\,|\,\mu\in\mathrm{N}\ (x_{a})\}$, $(\forall\mu\in L^{X})$; the inverse mapping φ_{31} of φ_{13} is given by $\varphi_{31}(\lim)=\longrightarrow_{\lim}$ for each $\lim\in\in\mathbb{B}(X,L)$, which satisfies

$$\mathrm{F}\longrightarrow_{\lim}x_{a}\Longleftrightarrow\mathrm{N}(x_{a})\subseteq\mathrm{F},\tag{22}$$

where $\mathrm{N}(x_{a})=\{\mu\in L^{X}\,|\,\mathrm{N}^{x}(\mu)\geq a\}\,(\forall x_{a}\in J(L^{X}),\mathrm{F}\in\mathbb{F}(X,L,2))$.

6. Conclusions

In the present paper, we propose the notion of (L,M)-fuzzy topological convergence structure (given by L-filters) and show that such a structure can be used to characterize an (L,M)-fuzzy topology. With convergence theory of molecular nets [19], we give a Moore–Smith convergence theory in (L,M)-fuzzy topological space. For further issues, we are devoted to L-topological convergence structure and give two conditions, Kowalsky's diagonal condition and Gähler's neighborhood condition, in fuzzing setting. From these results, we consider that L-filter is an important and appropriate tool to (L,M)-fuzzy topological space and also to L-topological space.

It is well known that (L,M)-fuzzy topological space has bifuzziness: fuzziness of open sets and fuzziness of openness. So we are sure that there exist other kinds of convergence structures such that all these structures can be categorically isomorphic to (L,M)-fuzzy topologies. For example, with the help of L-filter and idea of Q-neighborhood operator (or L-fuzzy filter and idea of neighborhood operator), we could consider the corresponding convergence structures. This will be our future work. After these efforts, maybe, due to one kind of these convergence structures, we can study other properties of (L,M)-fuzzy topological spaces and (L,M)-fuzzy topological group [27] for more convenience.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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