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*Research Article*

# Several Characterizations on Degree-Based Topological Indices for Star of David Network

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In order to make quantitative structure-movement/property/danger relations, topological indices (TIs) are the numbers that are related to subatomic graphs. Some fundamental physicochemical properties of chemical compounds, such as breaking point, protection, and strain vitality, correspond to these TIs. In the compound graph hypothesis, the concept of TIs was developed in view of the degree of vertices. In investigating minimizing exercises of Star of David, these indices are useful. In this study, we explore the different types of Zagreb indices, Randić indices, atom-bond connectivity indices, redefined Zagreb indices, and geometric-arithmetic index for the Star of David. The edge partitions of this network are tabled based on the sum of degrees-of-end vertices and the sum of degree-based edges. To produce closed formulas for some degree-based network TIs, these edge partitions are employed.

## 1. Introduction

Graph theory is a branch of mathematics in which we use graph parameter methods to precisely expose the compound phenomenon. For example, the graph theory characterizes an area between different disciplines of science when applied to the investigation of molecular structures, which is known as molecular topology or the theory of chemical graphs. A significant part of the analysis was supported by chemical graph theory [1]. Chemist can be performed for the statistical demonstration of chemical marvel by means of graph theory. In quantitative structure activity, researchers tried to figure out what structural characteristics will be developed. Physicochemical features and topological measures are discussed by Wiener. Different types of graph descriptors, such as distance-based, degree-based, spectral, and polynomial-related descriptors, have been well defined and explored extensively in the literature. Vertex degree-based descriptors are the most important of these classes, and they play a crucial role in chemical graph theory. These descriptors are combined to infer physicochemical, biological, and pharmacological qualities such stability, chirality, melting point, boiling point, similarity, connectedness, entropy, enthalpy of formation, surface tension, density, critical temperature, and others. Mathematicians and chemists use a variety of topological indices in these types of studies. The quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) research use the index, the Randic index, the Zagreb indices, and the ABC index to measure by Yang et al. [2] in the bioactivity of chemical compounds [3]. Topological indices provide numerical representations, molecular size, shape, branching, and other properties that are used to compare chemical compounds' topological similarities and in QSPR/QSAR research [4, 5]. There are several properties related to new families of graphs that are discussed in [6–8] such as metric dimensions and indices. The spectral properties, metric dimensions, and indices of different families of graphs are discussed in [9–14]. These include distance-based and degree-based TIs, as well as related polynomials [15] and classified graph indices, among other forms of topological indices. In 2017, Maji and Ghorai introduced new distance-degree-based topological indices, see [16]. In chemical graph theory and notably in chemistry, degree-based TIs are extremely important and serve a critical function. Furthermore, algebraic graph theory results are discussed in [17–21] by using the notion of totient number which was introduced by Shahbaz and Khalid in 2017.

In this paper, we study some degree-based analysis of TIs of the Star of David network. In Hebrew, the Star of David, or "Magen David" ("Shield of David"), consists of two overlaid equilateral triangles that form a six-pointed star. It cannot be traced back to the Bible or the Talmud, but it is said that it comes from the (presumed) similarity to the shape of the shield of King David. It was neither used as a sign of Jewish identity, although it originated in Antiquity, nor was even restricted to Judaism. The seven-branched candelabrum, still one of Israel's emblems today, was the most famous symbol of Judaism at that time.

We arranged our paper as follows. In Section 2, we give some preliminary concepts related to topological indices of different kinds. In Section 3, we construct the Star of David network and proposed their algorithm. In Section 4, we compute several results on topological indices for the proposed network. In Section 5, we give a comparison of topological indices for proposed networks. In Section 6, we give the concluding remarks about our proposed work. In future work, one can compute more indices on proposed Star of David networks.

## 2. Preliminaries

According to this study, a simple connected graph $G$ is made up of vertices $V(G)$ and edges $E(G)$, with $\aleph(\mu)$ being the degree of each vertex and the number of edges intersecting $\mu$."

*Definition 1.* Topological index (TI) is derived by Wiener, which was created in 1945 after investigating alkane's boiling point [22]. According to Randić [23] characterization, the earliest degree-based index is the Randić index, which is defined as

$$R(G) = \sum_{\mu\nu\in E(G)} \frac{1}{\left(\sqrt{\aleph(\mu)\aleph(\nu)}\right)}. \tag{1}$$

As a traditional graph-based molecular structure descriptor, the Randić index has been widely used in chemical and pharmaceutical research. Even the mathematical sense of this index is clear for detail of its QSPR/QSAR application, see [24, 25]. Bollobác and Paul [26] presented the general Randić index, which is defined as

$$R_\alpha(G) = \sum_{\mu\nu\in E(G)} (\aleph(\mu)\aleph(\nu))^\alpha \text{ for } \alpha = 1, \frac{1}{2}, -\frac{1}{2}, -1. \tag{2}$$

*Definition 2.* Gutman and Trinajstić [27] defined the first and second Zagreb indices as follows; also, see [28–30]:

$$\begin{aligned} M_1(G) &= \sum_{\mu\nu\in E(G)} (\aleph(\mu) + \aleph(\nu)), \\ M_2(G) &= \sum_{\mu\nu\in E(G)} (\aleph(\mu)\aleph(\mu)). \end{aligned} \tag{3}$$

*Definition 3.* Ranjini et al. [31] proposed the redefined version of Zagreb indices

The redefined first Zagreb index of graph $G$ is defined as

$$R_eZG_1(\mathrm{G}) = \sum_{\mu\nu\in E(G)} \frac{(\aleph(\mu) + \aleph(\nu))}{(\aleph(\mu)\cdot\aleph(\nu))}. \tag{4}$$

The redefined second Zagreb index of graph $G$ is defined as

$$R_eZG_2(G) = \sum_{\mu\nu\in E(G)} \frac{(\aleph(\mu)\cdot\aleph(\nu))}{(\aleph(\mu) + \aleph(\nu))}. \tag{5}$$

The redefined third Zagreb index of graph $G$ is defined as

$$R_eZG_3(G) = \sum_{\mu\nu\in E(G)} (\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu) + \aleph(\nu)). \tag{6}$$

*Definition 4.* Estrada et al., in [32], proposed degree-based TI $ABC$ and defined it as

$$ABC(G) = \sum_{\mu\nu\in E(G)} \sqrt{\frac{\aleph(\mu) + \aleph(\nu) - 2}{\aleph(\mu)\cdot\aleph(\nu)}}. \tag{7}$$

The atom-bond connectivity index ($ABC$) is a molecular structural descriptor that has lately found surprising applicability in explaining linear and branched alkane stability, as well as cycloalkane strain energy. It is used for modeling the thermodynamic characteristics in organic chemical molecules.

*Definition 5.* The $GA$ index is proposed by Vukicevic and Furtula in [33] and defined as

$$GA(G) = \sum_{\mu\nu\in E(G)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu) + \aleph(\nu)}. \tag{8}$$

The prediction value of the $GA$ index is slightly greater than the Randić connection index for physicochemical characteristics such as entropy and acentric factor, according to [33].

*Definition 6.* Furtula and Gutman, in [34], proposed the forgotten TI and stated it as

$$F(G) = \sum_{\mu\nu\in E(G)} (\aleph(\mu))^2 + (\aleph(\nu))^2. \tag{9}$$

For further study, see [35]. The Star of David networks are shown in Figures 1 and 2.

## 3. Higher Dimension $SD_{(n)}$ Drawing Algorithm for Star of David Network

Step 1 : the Star of David, consists of two equilateral overlapping triangles forming a six-pointed star. Draw David's Star graph $G$, which is one and two dimensional, as seen in figure ??. Algorithm for Star of David is as given below:

(i)
```
≠inclu de⟨iostream⟩
  using namespace std;, ≠ define n ⟍'n'
    int i, j;, int main()
{
    for (i = 0; i ≤ 1; i + +)
    {
      for (j = 1; j ≤ 5 − i; j + +)
  cout ⟨⟨" ";
⟨⟨" ∗ ";
if (i == 1)
  cout ⟨⟨" ∗ ";
  cout ⟨⟨n;
    }
    for (i = 1; i ≤ ; i + +)
      {
    for (j = 1; j ≤ 11; j + +)
        {
if (i == 2‖i == 3)
        {
if (j == 2‖j == 3‖j == 9‖j == 10)
          {
  cout ⟨⟨" ∗ ";
  }
  else
        cout ⟨⟨" ";
      }
  else
          cout ⟨⟨" ∗ ";
      }
            cout ⟨⟨n;
        }
for (i = 1; i ≥ 0; i − −)
          {
for (j = (5 − i); j ≥ 1; j − −)
              cout ⟨⟨" ";
                cout ⟨⟨" ∗ ";
if (i == 1)
                  cout ⟨⟨" ∗ ";
                    cout ⟨⟨n;
          }
            }
```

Step 2 : add two David's stars on the upper and lower sides.

Step 3 : adding one more star of David in each step similarly, we proceed up to $n = 5$. We get the sequence at $n = 5$.

## 4. Results on Indices for Star of David Network

**Theorem 1.** *The atom-bond connectivity index of Star of David network is*

$$ABC(G) = 3.2n^2 - 10.6n - 10.2. \quad (10)$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the atom-bond connectivity index for $G$:

$$\begin{aligned} ABC(G) &= \sum_{\mu\nu\in E(G)} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} \\ &= \sum_{\mu\nu\in E\{2,2\}} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} + \sum_{\mu\nu\in E\{2,4\}} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} + \sum_{\mu\nu\in E\{2,6\}} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} \\ &\quad + \sum_{\mu\nu\in E\{4,4\}} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} + \sum_{\mu\nu\in E\{4,6\}} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} + \sum_{\mu\nu\in E\{6,6\}} \sqrt{\frac{\aleph(\mu)+\aleph(\nu)-2}{\aleph(\mu)\cdot\aleph(\nu)}} \\ &= 4\sqrt{\frac{2+2-2}{2\cdot 2}} + 8\sqrt{\frac{2+4-2}{2\cdot 4}} + (12n-16)\sqrt{\frac{2+6-2}{2\cdot 6}} \\ &\quad + (6n-6)\sqrt{\frac{4+4-2}{4\cdot 4}} + (12n-20)\sqrt{\frac{4+6-2}{4\cdot 6}} + 6(n-2)^2\sqrt{\frac{6+6-2}{6\cdot 6}}. \end{aligned} \quad (11)$$

Figure 1: Star of David.

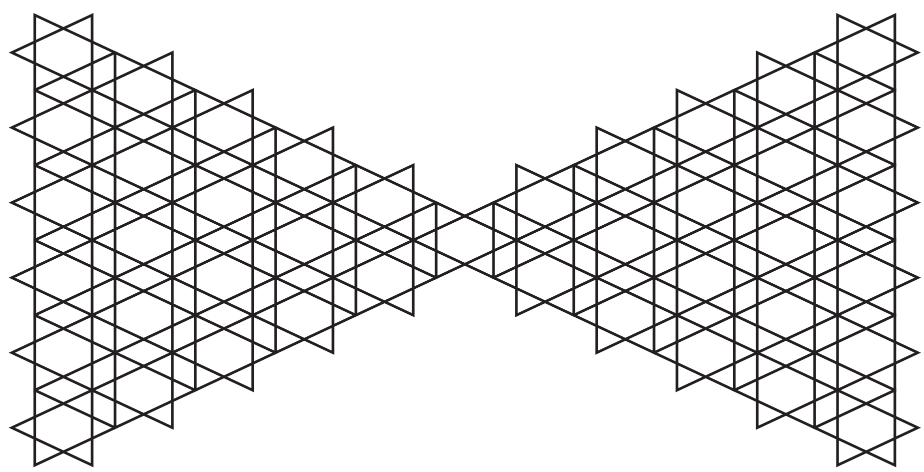

Figure 2: Star of David network.

Table 1: Star of David network edge partition.

| Types of edges | $E_{\{2,2\}}$ | $E_{\{2,4\}}$ | $E_{\{2,6\}}$ | $E_{\{4,4\}}$ | $E_{\{4,6\}}$ | $E_{\{6,6\}}$ |
|---|---|---|---|---|---|---|
| Number of edges | $(2,2)$ | $(2,4)$ | $(2,6)$ | $(4,4)$ | $(4,6)$ | $(6,6)$ |
| Frequency | 4 | 8 | $12n-16$ | $6n-6$ | $12n-20$ | $6(n-2)^2$ |

We get the outcomes after estimates:

$$ABC(G) = 3.2n^2 - 10.6n - 10.2. \quad (12)$$

**Theorem 2.** *The geometric-arithmetic index of Star of David network is*

$$GA(G) = 6n^2 + 4.2n - 6.46. \quad (13)$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of geometric-arithmetic index for $G$:

$$\begin{aligned} GA(G) &= \sum_{\mu\nu\in E(G)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)} \\ &= \sum_{\mu\nu\in E(2,2)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)} + \sum_{\mu\nu\in E(2,4)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)} + \sum_{\mu\nu\in E(2,6)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)} \end{aligned}$$

$$+\sum_{\mu\nu\in E(4,4)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)}+\sum_{\mu\nu\in E(4,6)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)}+\sum_{\mu\nu\in E(6,6)} 2\frac{\sqrt{\aleph(\mu)\cdot\aleph(\nu)}}{\aleph(\mu)+\aleph(\nu)}$$
$$= 4\cdot 2\frac{\sqrt{2\cdot 2}}{2+2}+8\cdot 2\frac{\sqrt{2\cdot 4}}{2+4}+(12n-16)\cdot 2\frac{\sqrt{2\cdot 6}}{2+6} \quad (14)$$
$$+(6n-6)\cdot 2\frac{\sqrt{4\cdot 4}}{4+4}+(12n-20)\cdot 2\frac{\sqrt{4\cdot 6}}{4+6}+6(n-2)^2\cdot 2\frac{\sqrt{6\cdot 6}}{6+6}$$
$$= 6n^2+4.2n-6.46.$$

**Theorem 3.** *The first Zagreb index of Star of David network is*

$$M_1(G) = 72n^2-24n-24. \quad (15)$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the first Zagreb index for $G$:

$$M_1(G) = \sum_{\mu\nu\in E(G)} (\aleph(\mu)+\aleph(\nu))$$
$$= \sum_{\mu\nu\in E(2,2)} (\aleph(\mu)+\aleph(\nu))+\sum_{\mu\nu\in E(2,4)} (\aleph(\mu)+\aleph(\nu))+\sum_{\mu\nu\in E(2,6)} (\aleph(\mu)+\aleph(\nu))$$
$$+\sum_{\mu\nu\in E(4,4)} (\aleph(\mu)+\aleph(\nu))+\sum_{\mu\nu\in E(4,6)} (\aleph(\mu)+\aleph(\nu))+\sum_{\mu\nu\in E(6,6)} (\aleph(\mu)+\aleph(\nu)) \quad (16)$$
$$= 4(2+2)+8(2+4)+(12n-16)(2+6)+(6n-6)(4+4)$$
$$+(12n-20)(4+6)+6(n-2)^2(6+6).$$

We get the outcomes after estimates:

$$M_1(G) = 72n^2-24n-24. \quad (17)$$

**Theorem 4.** *The second Zagreb index of Star of David network is*

$$M_2(G) = 216n^2-336n+176. \quad (18)$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the second Zagreb index for $G$:

$$M_2(G) = \sum_{\mu\nu\in E(G)} (\aleph(\mu)\cdot\aleph(\nu))$$
$$= \sum_{\mu\nu\in E(2,2)} (\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(2,4)} (\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(2,6)} (\aleph(\mu)\cdot\aleph(\nu))$$
$$+\sum_{\mu\nu\in E(4,4)} (\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(4,6)} (\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(6,6)} (\aleph(\mu)\cdot\aleph(\nu)) \quad (19)$$
$$= 4(2\cdot 2)+8(2\cdot 4)+(12n-16)(2\cdot 6)+(6n-6)(4\cdot 4)+(12n-20)(4\cdot 6)$$
$$+6(n-2)^2(6\cdot 6)$$
$$= 216n^2-336n+176.$$

**Theorem 5.** *The redefined first Zagreb index of Star of David network is*

$$R_eZG_1(G) = 2n^2+8n-3.97. \quad (20)$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the redefined first Zagreb index for $G$:

$$
\begin{aligned}
R_eZG_1(G) &= \sum_{\mu\nu\in E(G)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} \\
&= \sum_{\mu\nu\in E(2,2)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} + \sum_{\mu\nu\in E(2,4)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} + \sum_{\mu\nu\in E(2,6)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} \\
&\quad + \sum_{\mu\nu\in E(4,4)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} + \sum_{\mu\nu\in E(4,6)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} + \sum_{\mu\nu\in E(6,6)} \frac{\aleph(\mu)+\aleph(\nu)}{\aleph(\mu)\cdot\aleph(\nu)} \\
&= 4\cdot\frac{2+2}{2\cdot 2} + 8\cdot\frac{2+4}{2\cdot 4} + (12n-16)\cdot\frac{2+6}{2\cdot 6} + (6n-6)\frac{4+4}{4\cdot 4} \\
&\quad + (12n-20)\frac{4+6}{4\cdot 6} + 6(n-2)^2\frac{6+6}{6\cdot 6}.
\end{aligned} \tag{21}
$$

We get the outcomes after estimates:

$$R_eZG_1(G) = 2n^2 + 8n - 3.97. \tag{22}$$

**Theorem 6.** *The redefined second Zagreb index of Star of David network is*

$$R_eZG_2(G) = 18n^2 - 13.2n + 2.67. \tag{23}$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the redefined second Zagreb index for $G$:

$$
\begin{aligned}
R_eZG_2(G) &= \sum_{\mu\nu\in E(G)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} \\
&= \sum_{\mu\nu\in E(2,2)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} + \sum_{\mu\nu\in E(2,4)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} + \sum_{\mu\nu\in E(2,6)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} \\
&\quad + \sum_{\mu\nu\in E(4,4)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} + \sum_{\mu\nu\in E(4,6)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} + \sum_{\mu\nu\in E(6,6)} \frac{\aleph(\mu)\cdot\aleph(\nu)}{\aleph(\mu)+\aleph(\nu)} \\
&= 4\cdot\frac{2\cdot 2}{2+2} + 8\cdot\frac{2\cdot 4}{2+4} + (12n-16)\cdot\frac{2\cdot 6}{2+6} + (6n-6)\frac{4\cdot 4}{4+4} \\
&\quad + (12n-20)\frac{4\cdot 6}{4+6} + 6(n-2)^2\frac{6\cdot 6}{6+6}.
\end{aligned} \tag{24}
$$

We get the outcomes after estimates:

$$R_eZG_2(G) = 18n^2 - 13.2n + 2.67. \tag{25}$$

**Theorem 7.** *The redefined third Zagreb index of Star of David network is*

$$R_eZG_3(G) = 2592n^2 - 5568n + 3712. \tag{26}$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the redefined third Zagreb index for $G$:

$$
\begin{aligned}
R_eZG_3(G) &= \sum_{\mu\nu\in E(G)} (\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu)) \\
&= \sum_{\mu\nu\in E(2,2)} (\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu)) + \sum_{\mu\nu\in E(2,4)} (\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu))
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{\mu\nu\in E(2,6)}(\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu))+\sum_{\mu\nu\in E(4,4)}(\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu))\\
&+\sum_{\mu\nu\in E(4,6)}(\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu))+\sum_{\mu\nu\in E(6,6)}(\aleph(\mu)\cdot\aleph(\nu))(\aleph(\mu)+\aleph(\nu))\\
&=4(2\cdot 2)(2+2)+8(2\cdot 4)(2+4)+(12n-16)(2\cdot 6)(2+6)\\
&+(6n-6)(4\cdot 4)(4+4)+(12n-20)(4\cdot 6)(4+6)+6(n-2)^2(6\cdot 6)(6+6).
\end{aligned}
\tag{27}
$$

We get the outcomes after estimates:

$$R_eZG_3(G)=2592n^2-5568n+3712. \tag{28}$$

**Theorem 8.** *The forgotten TI of Star of David network is*

$$F(G)=432n^2-432n+48. \tag{29}$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of forgotten TI for $G$:

$$
\begin{aligned}
F(G)&=\sum_{\mu\nu\in E(G)}(\aleph(\mu))^2+(\aleph(\nu))^2\\
&=\sum_{\mu\nu\in E(2,2)}(\aleph(\mu))^2+(\aleph(\nu))^2+\sum_{\mu\nu\in E(2,4)}(\aleph(\mu))^2+(\aleph(\nu))^2+\sum_{\mu\nu\in E(2,6)}(\aleph(\mu))^2+(\aleph(\nu))^2\\
&+\sum_{\mu\nu\in E(4,4)}(\aleph(\mu))^2+(\aleph(\nu))^2+\sum_{\mu\nu\in E(4,6)}(\aleph(\mu))^2+(\aleph(\nu))^2+\sum_{\mu\nu\in E(6,6)}(\aleph(\mu))^2+(\aleph(\nu))^2\\
&=4\left(2^2+2^2\right)+8\left(2^2+4^2\right)+(12n-16)\left(2^2+6^2\right)+(6n-6)\left(4^2+4^2\right)\\
&+(12n-20)\left(4^2+6^2\right)+6(n-2)^2\left(6^2+6^2\right).
\end{aligned}
\tag{30}
$$

We get the outcomes after estimates:

$$F(G)=432n^2-432n+48. \tag{31}$$

**Theorem 9.** *The Randić indices of Star of David network are*

$$
\begin{aligned}
R_1(G)&=216n^2-336n+176,\\
R_{1/2}(G)&=36n^2+220.4n+85.4,\\
R_{-1/2}(G)&=n^2+3.4n-1.4,\\
R_{-1}(G)&=0.17n^2+1.2n-0.54.
\end{aligned}
\tag{32}
$$

*Proof.* Let $G$ be the graph of Star of David network. By using Table 1, we apply the formula of the general Randić index for $G$:

$$R_\alpha(G)=\sum_{\mu\nu\in E(G)}(\aleph(\mu)\cdot\aleph(\nu))^\alpha. \tag{33}$$

For $\alpha=1$,

$$
\begin{aligned}
R_1(G)&=\sum_{\mu\nu\in E(G)}(\aleph(\mu)\cdot\aleph(\nu))\\
&=\sum_{\mu\nu\in E(2,2)}(\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(2,4)}(\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(2,6)}(\aleph(\mu)\cdot\aleph(\nu))\\
&+\sum_{\mu\nu\in E(4,4)}(\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(4,6)}(\aleph(\mu)\cdot\aleph(\nu))+\sum_{\mu\nu\in E(6,6)}(\aleph(\mu)\cdot\aleph(\nu))\\
&=4(2\cdot 2)+8(2\cdot 4)+(12n-16)(2\cdot 6)+(6n-6)(4\cdot 4)\\
&+(12n-20)(4\cdot 6)+6(n-2)^2(6\cdot 6).
\end{aligned}
\tag{34}
$$

We get the outcomes after estimates:

$$R_1(G) = 216n^2 - 336n + 176. \quad (35)$$

For $\alpha = 1/2$,

$$\begin{aligned} R_{1/2}(G) &= \sum_{\mu\nu\in E(G)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} \\ &= \sum_{\mu\nu\in E(2,2)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} + \sum_{\mu\nu\in E(2,4)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} + \sum_{\mu\nu\in E(2,6)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} \\ &\quad \sum_{\mu\nu\in E(4,4)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} + \sum_{\mu\nu\in E(4,6)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} + \sum_{\mu\nu\in E(6,6)} (\aleph(\mu)\cdot\aleph(\nu))^{1/2} \\ &= 4(2\cdot 2)^{1/2} + 8(2\cdot 4)^{1/2} + (12n-16)(2\cdot 6)^{1/2} + (6n-6)(4\cdot 4)^{1/2} \\ &\quad + (12n-20)(4\cdot 6)^{1/2} + 6(n-2)^2(6\cdot 6)^{1/2}. \end{aligned} \quad (36)$$

We get the outcomes after estimates:

$$R_{1/2}(G) = 36n^2 + 220.4n + 85.4. \quad (37)$$

For $\alpha = -1/2$,

$$\begin{aligned} R_{-1/2}(G) &= \sum_{\mu\nu\in E(G)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} \\ &= \sum_{\mu\nu\in E(2,2)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} + \sum_{\mu\nu\in E(2,4)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} + \sum_{\mu\nu\in E(2,6)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} \\ &\quad \sum_{\mu\nu\in E(4,4)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} + \sum_{\mu\nu\in E(4,6)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} + \sum_{\mu\nu\in E(6,6)} (\aleph(\mu)\cdot\aleph(\nu))^{-1/2} \\ &= 4(2\cdot 2)^{-1/2} + 8(2\cdot 4)^{-1/2} + (12n-16)(2\cdot 6)^{-1/2} + (6n-6)(4\cdot 4)^{-1/2} \\ &\quad + (12n-20)(4\cdot 6)^{-1/2} + 6(n-2)^2(6\cdot 6)^{-1/2}. \end{aligned} \quad (38)$$

We get the outcomes after estimates:

$$R_{-1/2}(\mathrm{G}) = n^2 + 3.4n - 1.4. \quad (39)$$

For $\alpha = -1$,

$$\begin{aligned} R_{-1}(G) &= \sum_{\mu\nu\in E(G)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} \\ &= \sum_{\mu\nu\in E(2,2)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} + \sum_{\mu\nu\in E(2,4)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} + \sum_{\mu\nu\in E(2,6)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} \\ &\quad + \sum_{\mu\nu\in E(4,4)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} + \sum_{\mu\nu\in E(4,6)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} + \sum_{\mu\nu\in E(6,6)} (\aleph(\mu)\cdot\aleph(\nu))^{-1} \\ &= 4(2\cdot 2)^{-1} + 8(2\cdot 4)^{-1} + (12n-16)(2\cdot 6)^{-1} + (6n-6)(4\cdot 4)^{-1} \\ &\quad + (12n-20)(4\cdot 6)^{-1} + 6(n-2)^2(6\cdot 6)^{-1}. \end{aligned} \quad (40)$$

We get the outcomes after estimates:

$$R_{-1}(G) = 0.17n^2 + 1.2n - 0.54. \quad (41)$$

*4.1. 3D Graphical Representation of Topological Indices for Star of David Networks.* The TIs of the Star of David Network are illustrated graphically in Figure 3. The evolution of TIs along various parameters is portrayed in graphs. Despite the fact

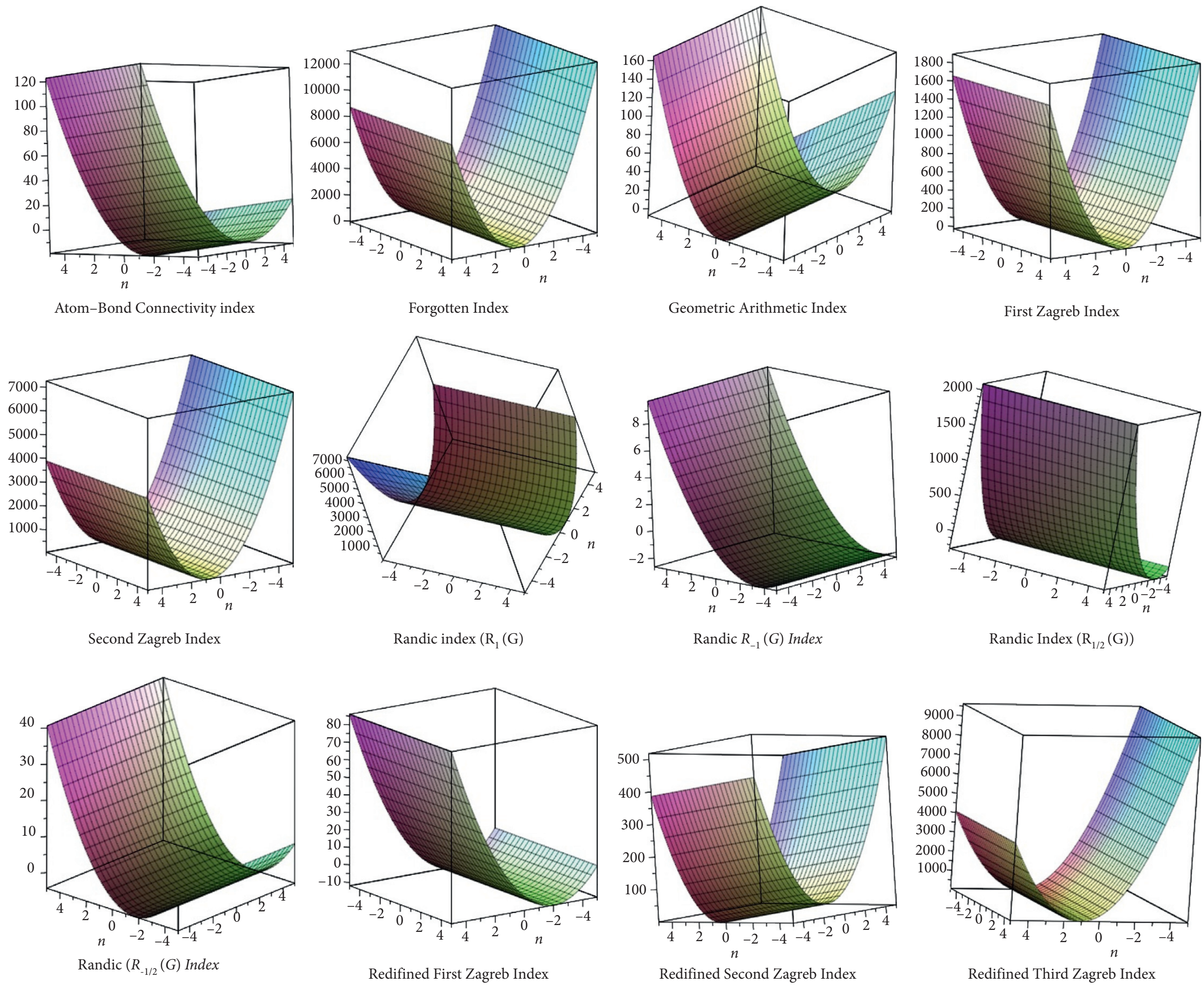


Figure 3: Three-dimensional graphical representation of degree-based topological indices.

Table 2: Numerical comparison of Star of David network.

| $n$ | $ABC(G)$ | $GA(G)$ | $M_1(G)$ | $M_2(G)$ | $ReZG_1(G)$ | $ReZG_2(G)$ | $ReZG_3(G)$ | $F(G)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | −17.6 | −21.8 | 24 | 56 | 6.03 | 7.47 | 736 | 48 |
| 2 | −18.6 | −149.6 | 216 | 368 | 20.03 | 48.27 | 2944 | 912 |
| 3 | −13.2 | −553.4 | 552 | 1112 | 38.03 | 125.07 | 10 336 | 2640 |
| 4 | −1.4 | −1377.2 | 1032 | 2288 | 60.03 | 237.87 | 22 912 | 5232 |
| 5 | 16.8 | −2765 | 1656 | 3896 | 86.03 | 386.67 | 40 672 | 8688 |
| 6 | 41.4 | −4860.8 | 2424 | 5936 | 116.03 | 571.47 | 63 616 | 13 008 |
| 7 | 72.4 | −7808.6 | 3336 | 8408 | 150.03 | 792.27 | 91 744 | 18 192 |
| 8 | 109.8 | −11 752.4 | 4392 | 11 312 | 188.03 | 1049.07 | 125 056 | 24 240 |
| 9 | 153.6 | −16 836.2 | 5592 | 14 648 | 230.03 | 1341.87 | 163 552 | 31 152 |
| 10 | 203.8 | −23 204 | 6936 | 18 416 | 276.03 | 1670.67 | 207 232 | 38 928 |

Table 3: Numerical comparison of Star of David networks.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_1(G)$ | 56 | 368 | 1112 | 2288 | 3896 | 5936 | 8408 | 11 312 | 14 648 | 18 416 |
| $R_{1/2}(G)$ | 341.8 | 670.2 | 1070.6 | 1543 | 2087.4 | 2703.8 | 3392.2 | 4152.6 | 4985 | 5889.4 |
| $R_{-1/2}(G)$ | 3 | 9.4 | 17.8 | 28.2 | 40.6 | 55 | 71.4 | 89.8 | 110.2 | 132.6 |
| $R_{-1}(G)$ | 0.83 | 2.54 | 4.59 | 6.98 | 9.71 | 12.78 | 16.19 | 19.94 | 24.03 | 28.46 |

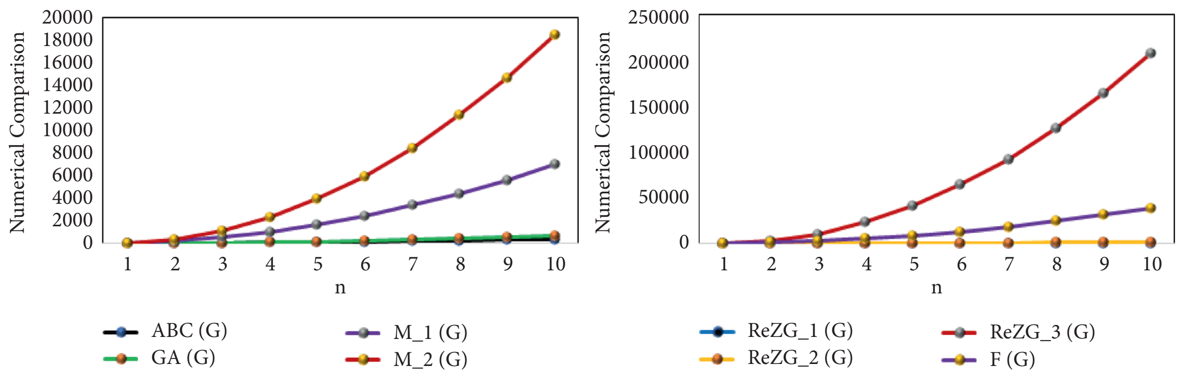


Figure 4: Two-dimensional numerical comparison of Star of David network.

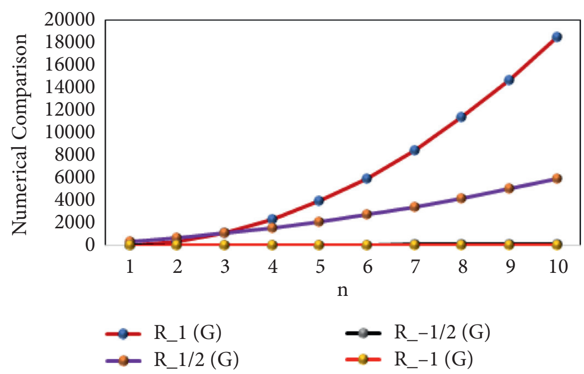


Figure 5: Two-dimensional numerical comparison of Star of David networks.

that the graphs appear to be similar, their gradients differ. In Figures 4 and 5, we give a two-dimensional numerical comparison of Star of David networks. We discuss the numerical comparison of Star of David networks with different degree-based topological indices in Tables 2 and 3.

## 5. Comparison of Results for Topological Indices of Star of David Networks

## 6. Conclusion

TIs for Star of David networks are computed in this paper; as well as, the analytic closed algorithms are reviewed and specified for these networks, namely, the general Randić index, the atomic-bond connectivity index, and the geometric-arithmetic index, as well as the first and second Zagreb index and closed formulas of this network were determined that will help network scientists in understanding and exploring the fundamental topologies of such networks. Computer scientists and chemists who work with Hex-derived networks may find these discoveries valuable.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare no conflicts of interest.

## Authors' Contributions

All authors contributed equally to this manuscript. All authors have read and agreed to the published version of the manuscript.

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