Retraction

Retracted: Probability Density Evolution Algorithm for Stochastic Dynamical Systems Based on Fractional Calculus

Journal of Mathematics

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Y. Yan and X. Yu, "Probability Density Evolution Algorithm for Stochastic Dynamical Systems Based on Fractional Calculus," *Journal of Mathematics*, vol. 2021, Article ID 9218857, 9 pages, 2021.

Research Article

Probability Density Evolution Algorithm for Stochastic Dynamical Systems Based on Fractional Calculus

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With the increasing load and speed of trains, the problems caused by various random excitations (such as safety and passenger comfort) have become more prominent and thus arises the necessity to analyze stochastic dynamical systems, which is important in both academic and engineering circles. The existing analysis methods are inadequate in terms of computational accuracy, computational efficiency, and applicability in solving complex problems. For that, a new efficient and accurate method is used in this paper, suitable for linear and nonlinear random vibration analysis of large structures as well as static and dynamic reliability assessment. It is the direct probability integration method, which is extended and applied to the random vibration reliability analysis of dynamical systems. Dynamical models of the dynamic system and coupled system "three-car vehicle-rail-bridge" are established, the time-varying differential equations of motion are derived in detail, and the dynamic response of the system is calculated using the explicit Newmark algorithm. The simulation results show the influence of the number of representative points on the smoothness of the image of the probability density function and the accuracy of the calculation results.

1. Introduction

Railroad transportation is one of the major modes of transportation in the world today, and the mileage of high-speed railroad in China has reached 38,000 km, and it is planned to realize the "eight horizontal and eight vertical" high-speed railroad network by 2030. China is a vast country with many mountains and rivers, and bridge structures occupy a high proportion in the lines. The three subsystems of train, track, and bridge are coupled into a large system through wheel-rail contact forces and bridge-rail interactions, and the dynamic response of the vehicle, track, and bridge structures is triggered by the system excitation [1]. The train operation will have a dynamic impact on the track and bridge structure, causing the track and bridge to vibrate, affecting its working condition and service life; the vibration of the track and bridge structure will in turn affect the train operation smoothness and passenger comfort [2, 3]. This train-track-bridge dynamic interaction problem is a typical large system dynamics problem, which is usually called vehicle-track-bridge coupled vibration [4]. At present, this latter problem has received wide attention from the engineering and academic communities.

With the development of railroad transportation tending to be high speed and lighter, it is necessary to ensure high smoothness, stability, and reliability of the line in order to ensure the safety and stability of high-speed train operation as well as passenger comfort [5]. The stochastic uncertainties are widely present in the vehicle-rail-bridge coupled system (stochastic dynamical system), such as bridge (track) uneven excitation, wind load and seismic action, wheel geometry deviation, and wheel pair serpentine motion, which are directly related to whether the train can run safely and smoothly [6]. The scale of railways is getting bigger and bigger, and there is a strong interest in theory and application. Therefore, studying the dynamic interaction of the "car-iron-bridge" coupling system, as well as stochastic dynamic analysis, correct evaluation of the safety of vehicle operation, etc. have become the focus of research.

The existing methods for the analysis of the dynamics of stochastic dynamic systems such as Monte Carlo simulation, probability density evolution method, and diffusion process method have different degrees of defects, such as low computational accuracy and efficiency, difficult to solve, large computational cost, and insufficient applicability to compute complex problems [8]. Therefore, it is important to find a more accurate and efficient method to evaluate the stochastic vibration and bridge dynamics reliability of the coupled vehicle-rail-bridge system and to study the trend and statistical law of the representative stochastic response of the coupled vehicle-rail-bridge system under the joint action of deterministic load and stochastic excitation, which is important for vehicle dynamics performance evaluation and vehicle bridge maintenance. It is of great significance for the evaluation of vehicle dynamics performance and vehicle bridge maintenance. Guohai Chen and Dixiong Yang proposed a general and efficient method for structural linear and nonlinear stochastic vibration and hydrodynamic reliability analysis: the direct probability integration method (DPIM) [9]. The probability density integral equation for the propagation of randomness in static and dynamic systems is derived uniformly based on the principle of probability conservation [10]. Using probability space dissection and Dirac function smoothing techniques, an efficient solution of the probability density integral equation is achieved, and the probability density functions of the stochastic response of the static and dynamic systems as well as the reliability are obtained by decoupling the calculation from the deterministic dynamics equations. In this paper, the coupled vehicle-bridge and three-car vehicle-rail-bridge systems are modeled, respectively, and the stochastic dynamics of the dynamic system is analyzed by using the direct probability integral method and compared with the Monte Carlo simulation results to analyze the change trend and statistical law of the stochastic dynamics response of the coupled system and to verify the accuracy and efficiency of the direct probability integral method [11]. The results are compared with Monte Carlo simulations to analyze the change trend and statistical law of the coupled system stochastic dynamics response, to verify the accuracy and efficiency of the direct probability integration method, and to provide new ideas and methods for the stochastic dynamics analysis of the power system [12].

2. Related Works

Stochastic dynamic system vibration analysis is mainly used to analyze the impact of system parameters and external environment on the safety and comfort of the train by solving the statistical law of the system response of interest and to provide guidance for safer, smoother, and high-speed operation of the train. The dynamic reliability of a bridge structure represents its probability of not breaking or failing in a given time under the action of internal and external random excitation of the system. The failure criteria commonly used for bridge dynamic reliability analysis are the first beyond failure criterion and the fatigue failure criterion. Among them, fatigue failure refers to the structural response under cyclic random loading, and the structural damage occurs when the structural response is repeated in the threshold range until the damage accumulates to a certain limit value, and its complex fatigue damage mechanism limits the development and application of this method [13]. The first time beyond failure criterion considers that the structural dynamic response (this paper mainly consider the bridge span displacement) first exceeds a given threshold value when the structure is damaged. This criterion is concise and widely used, and the dynamic reliability assessment of the bridge structure in the vehicle-bridge coupled system and the vehicle-rail-bridge coupled system is also based on the first time beyond failure criterion [14].

The methods of analyzing stochastic dynamical system are equally divided into the time domain method and the frequency domain method. The latter method takes the power spectral density function as the core of analysis and is mainly applicable to the analysis of stochastic dynamics of linear vehicle-rail-bridge coupled systems. Zeng Sanyuan and Wu Yingxuan derived the influence of suspension parameters on the vehicle vertical vibration amplitude as well as bridge structural safety by studying the rail excitation function and its application to vehicle stochastic vibration calculation. By solving the dynamic response of the simple vehicle model and the vehicle-track coupled system model, Chen Guo et al. analyzed their power spectra and found that the agreement is better in the low-frequency region, but there will be a large gap in the high-frequency region. Lin Jiahao proposed the virtual excitation method to transform the complex random track upset into simple harmonic virtual track upset, which can improve the calculation accuracy and efficiency, gave a new method to evaluate the train operation comfort based on the elastic vehicle model [15], combined the virtual excitation method and the fine integration method to analyze the nonsmooth stochastic response of a bridge system under the action of random track upset, and concluded that the root mean square of the system response would be doubled for each level of track upset. Zeng et al. used the virtual excitation method to calculate the vehicle-bridge coupled system under the assumption of wheel-rail close connection and the wheel-rail transverse creep rate; considering the vehicle moving load, the random dynamic response of the coupled vehicle-bridge system under the combined effect of vehicle moving load, random uneven excitation of the bridge deck and ground vibration load was calculated using the virtual excitation method and its statistical law was analyzed. The effect of train speed, track irregularity, etc., on the system dynamic response amplitude and driving safety by calculating the amplitude-frequency transfer function between track irregularity and vehicle vertical acceleration is analyzed [16].

The probability density evolution method proposed by Jie Li and Jianbing Chen is a more common method for the dynamics analysis of stochastic dynamical systems, which has higher computational efficiency compared to MCS. Xu and Zhai calculated the three-dimensional probability

density evolution surface of the response of the vehicle body transverse displacement, transverse acceleration, and wheel-rail force by establishing a detailed vehicle-rail (plate rail) coupled system dynamics model. Yu et al. established a vertical vehicle-bridge coupled system and used PDEM to calculate the effect of the randomness of vehicle suspension parameters on the dynamic response of the system. However, Goller et al. pointed out that the probability density evolution method has a complicated solution process, not applicable to cases of large number of random variables [17].

As railroad transportation becomes more and more developed, the number of traffic safety accidents caused by wheel-rail nonlinear contact and various stochastic excitations during vehicle travel is increasing. However, the existing analysis methods have problems such as insufficient computational accuracy, low efficiency, and complex solutions. Therefore, there is a need to develop more accurate, efficient, simple, and universal methods for analyzing the stochastic dynamics of coupled vehicle-rail-bridge systems. The direct probabilistic integration method (DPIM) [9], which is based on the integral formulation of the probability conservation principle, is a new method for linear and nonlinear stochastic vibration analysis of large complex structures, static and dynamic reliability assessment of structures, with significant computational accuracy and efficiency advantages. It is of great academic and theoretical significance and engineering application value.

3. Differential Equations for Vibration of Rail Structures

In practical engineering problems, the infinite-length Euler beam model for steel rails is often simplified to a finite-length simply supported beam model [10]. When a vehicle crosses the bridge, the higher-order modes in the structural system of the track and the bridge are not easily excited, and usually only a few lower modes will be excited, besides to a few modes with lower self-oscillation frequencies that are superimposed to express the vibration state of the structure. This paper uses the vibration superposition method to solve the midpoint displacement of the track and the span displacement of the bridge.

We start our study by deriving a basic form second-order ordinary differential equation for rail vertical vibration so that, at the end of this section, we have all calculations done by steps, and as a result of this, we can follow the same way to derive the differential equation form of the vertical vibration of a bridge.

The vertical, lateral, and torsional vibrations of the track structure under the moving vehicle load are given by the following formula:

$$EI_Y \frac{\partial^4 z_r(x,t)}{\partial x^4} + m_r \frac{\partial^2 z_r(x,t)}{\partial t^2} = -\sum_{i=1}^{N_s} F_{rvi}(t)\delta(x-x_i) + \sum_{j=1}^{N_w} P_{vj}(t)\delta(x-x_{wj}),$$

$$EI_Z \frac{\partial^4 y_r(x,t)}{\partial x^4} + m_r \frac{\partial^2 y_r(x,t)}{\partial t^2} = -\sum_{i=1}^{N_s} F_{rHi}(t)\delta(x-x_i) + \sum_{j=1}^{N_w} P_{Hj}(t)\delta(x-x_{wj}), \quad (1)$$

$$\rho I_0 \frac{\partial^2 \phi_r(x,t)}{\partial t^2} + GI_t \frac{\partial^2 \phi_r(x,t)}{\partial x^2} = -\sum_{i=1}^{N_s} F_{rTi}(t)\delta(x-x_i) + \sum_{j=1}^{N_w} P_{Tj}(t)\delta(x-x_{wj}),$$

where E and G represent Young's modulus of elasticity and shear modulus of the rail, respectively, m_r represents the mass of the rail, which is equal to the product of the cross-sectional area of the rail A_r and the mass density ρ_r, I_y and I_z represent the moments of inertia of the rail section with respect to the horizontal and vertical axes, respectively, I_0 and I_t represent the extreme moments of inertia and torsional moments of inertia of the rail section, respectively, N_s and N_w represent the number of fastener nodes and the number of vehicle axles, respectively, F_{rvi}, F_{rHi}, and F_{rTi} denote the vertical, lateral, and torsional reaction forces at the ith pivot point of the rail, respectively, P_{vj}, P_{Hj}, and P_{Tj} denote the vertical, lateral, and torsional forces on the rail by the jth wheel, respectively, and x_i and x_{wj} denote the x-coordinate of the ith pivot point of the rail and the x-coordinate of the jth wheel pair, respectively.

Equation (1) is a fourth-order partial differential equation, which needs to be transformed into a second-order ordinary differential equation in order to solve it. The regular vibration coordinate $qk(t)$ of the rail is introduced, and the modal order of the interception is M. According to the regular vibration function of the simply supported beam, the vertical vibration of the rail is expressed as

$$Z_k(x) = \sqrt{\frac{2}{m_r l}} \sin \frac{k\pi x}{l}, \quad (2)$$

and the solution of equation (1) can be expressed as

$$Z_r(x,t) = \sum_{k=1}^{M} Z_k(x) q_k(t). \quad (3)$$

Now, substituting (3) in equation (1),

$$\sum_{k=1}^{M} EI_Y \frac{\mathrm{d}^4 Z_k(x)}{\mathrm{d}x^4} q_k(t) + \sum_{k=1}^{M} m_r Z_k(x) \ddot{q}_k(t)$$

$$= -\sum_{i=1}^{N_s} F_{rvi}(t)\delta(x - x_i) + \sum_{j=1}^{N_w} P_{vj}\delta(x - x_{wj}). \tag{4}$$

Then, multiplying both sides of equation (4) by $Z_h(x)$ ($h = 1, 2, \ldots, M$) and integrating x from 0 to l using orthogonality of modes, we obtain

$$\int_0^l Z_h(x)Z_k(x)\mathrm{d}x = 0, \quad (h \neq k). \tag{5}$$

Thus,

$$\int_0^l EI_Y \frac{\mathrm{d}^4 Z_k(x)}{\mathrm{d}x^4} Z_k(x)q_k(t)\mathrm{d}x + \int_0^l m_r Z_k(x)Z_k(x)\ddot{q}_k(t)\mathrm{d}x$$

$$= -\sum_{i=1}^{N_s} \int_0^l F_{rvi}(t)Z_k(x)\delta(x - x_i)\mathrm{d}x$$

$$+ \sum_{j=1}^{Nw} \int_0^l P_{vj}(t)Z_k(x)\delta(x - x_{wj})\mathrm{d}x, \quad (k = 1 \sim M), \tag{6}$$

and by the nature of the Dirac δ function, equation (6) can be organized as

$$m_r \ddot{q}_k(t) \int_0^l Z_k^2(x)\mathrm{d}x + EI_Y q_k(t) \int_0^l Z_k(x) \frac{\mathrm{d}^4 Z_k(x)}{\mathrm{d}x^4} \mathrm{d}x$$

$$= -\sum_{i=1}^{N_s} F_{rvi}(t)Z_k(x_i) + \sum_{j=1}^{Nw} P_{vj}(t)Z_k(x_{wj}), \quad (k = 1 \sim M). \tag{7}$$

Moreover, using

$$\int_0^l Z_k^2(x)\mathrm{d}x = \frac{1}{m_r} \int_0^l Z_k(x) \frac{\mathrm{d}^4 Z_k(x)}{\mathrm{d}x^4} \mathrm{d}x$$

$$= \int_0^l \frac{2}{m_r l} \left(\frac{k\pi}{l}\right)^4 \sin^2 \frac{k\pi x}{l} \mathrm{d}x = \left(\frac{k\pi}{l}\right)^4 \int_0^l Z_k^2(x)\mathrm{d}x \tag{8}$$

$$= \frac{1}{m_r} \left(\frac{k\pi}{l}\right)^4,$$

equation (7) is reduced to

$$\ddot{q}_k(t) + \frac{EI_Y}{m_r} \left(\frac{k\pi}{l}\right)^4 q_k(t) = -\sum_{i=1}^{N_s} F_{rvi}(t)Z_k(x_i)$$

$$+ \sum_{j=1}^{N_w} P_{vj}(t)Z_k(x_{wj}), \quad (k = 1 \sim M), \tag{9}$$

where formula (9) is the basic form of the second-order ordinary differential equation for rail vertical vibration.

4. Dynamics Analysis of Stochastic Dynamical Systems Based on the Direct Probability Integration Method

In this section, a vertical vehicle-bridge coupled system is established, in which a multirigid body model is used for the vehicle and a planar beam unit finite element model is used for the bridge, as shown in Figure 1. Assuming elastic contact between the wheels and the bridge deck, a spring with a stiffness of 2k can be used for the simulation.

The equations of motion of the vehicle and bridge systems can be expressed, respectively, as

$$M_v \mathbf{Y}_v + \mathbf{C}_v \dot{\mathbf{Y}}_v + \mathbf{K}_v \ddot{\mathbf{Y}}_v = L_v F(t),$$
$$M_b \mathbf{Y}_b + \mathbf{C}_b \dot{\mathbf{Y}}_b + \mathbf{K}_b \ddot{\mathbf{Y}}_b = L_b(x)[-F(t) + G], \tag{10}$$

where M_v, $\mathbf{C}_v$, and $\mathbf{K}_v$ and M_b, $\mathbf{C}_b$, and $\mathbf{K}_b$ denote the mass matrix, damping matrix, and stiffness matrix of the vehicle-bridge system, respectively, $\mathbf{Y}_v$, $\dot{\mathbf{Y}}_v$, and $\ddot{\mathbf{Y}}_v$ and $\mathbf{Y}_b$, $\dot{\mathbf{Y}}_b$, and $\ddot{Y}_b$ denote the displacement vector, velocity vector, and acceleration vector of the considered system, respectively, $F(t)$ and G denote the transient contact force between the wheels and the bridge deck and the vehicle self-weight, respectively, and L_v and $L_b(x)$ denote the positioning vectors of $F(t)$ and $[-F(t) + G]$, respectively.

When solving the equations of motion for the coupled vehicle-bridge system, the displacement compatibility condition between the wheels and the bridge deck should also be considered:

$$y_v(t) - y_b(x, t) - r(x) = -\frac{F(t)}{k_2}, \tag{11}$$

where $y_v(t)$ represents the vertical displacement of the wheel, $y_b(x, t)$ represents the vertical displacement of the contact point between the wheel and the bridge deck, and $r(x)$ represents the unevenness of the bridge deck.

4.1. Stochastic Dynamical System Modeling and Dynamic Response Solving. The system parameters are taken from the literature: bridge span $L = 40$ m, linear density $\rho A = 6067$ (kg/m), flexural stiffness $EI = 2658069$ kN/m^2, without considering bridge damping, body mass $m_1 = 12000$ kg, wheel mass $m_2 = 500$ kg, body suspension stiffness factor $k_1 = 280$ (kN/m), damping factor $c_1 = 2\xi\sqrt{m_1 k_1}, \xi = 0.1$, wheel stiffness $k_2 = 156$ (kN/m), and vehicle travel speed is $v = 20$ (m/s).

The differential equations of motion for the bridge, vehicle body, and wheels in the vehicle-bridge coupled system are
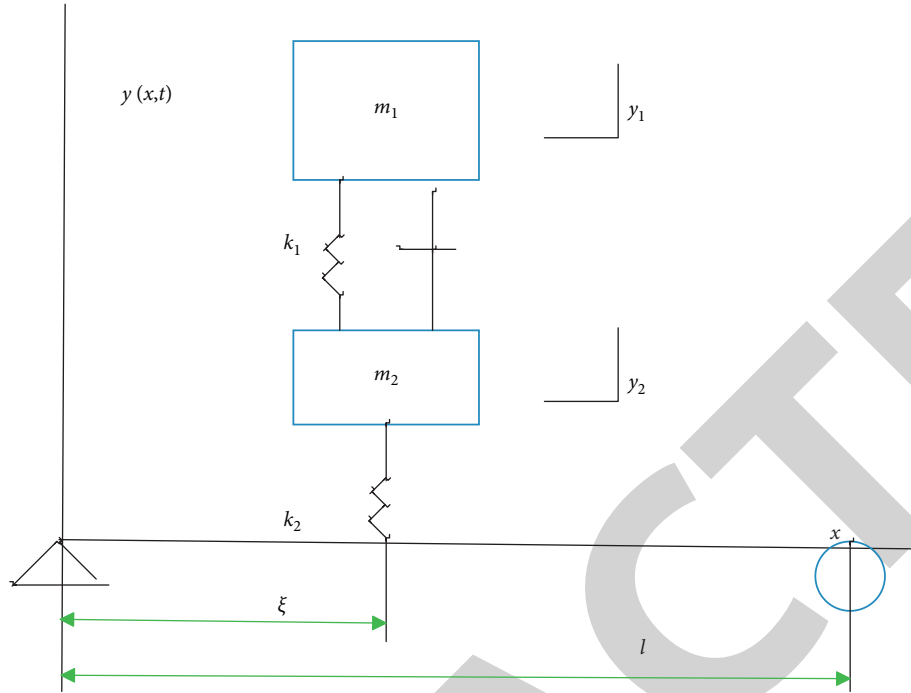
Figure 1: Dynamics model of the train-bridge coupling system.

$$EI y^{(4)}(x,t) + c\dot{y}(x,t) + \rho A\ddot{y}(x,t) = -\{m_1\ddot{y}_1(t) + m_2\ddot{y}_2(t) + (m_1 + m_2 g)\}\delta(x - \xi)$$
$$m_1\ddot{y}_1(t) + c_1\dot{y}_1(t) - c_1\dot{y}_2(t) + k_1 y_1(t) - k_1 y_2(t) = 0, \tag{12}$$
$$m_2\ddot{y}_2(t) - c_1\dot{y}_1(t) + c_1\dot{y}_2(t) - k_1 y_1(t) + (k_1 + k_2)y_2(t) - k_2 y(\xi,t) = k_2\eta(\xi),$$

where $y_1(t)$, $y_2(t)$, and $y(x,t)$ represent the vertical displacement response of the vehicle, wheels, and bridge at position x, respectively, ξ represents the distance of the vehicle through the bridge and ξ is a function of time t, and η represents the unevenness of the bridge deck and η is also a function of ξ. Known bridge vibration function is

$$\varphi_i(x) = \sqrt{2}\sin\frac{i\pi x}{l}, \tag{13}$$

and has the orthogonal property

$$\int_0^l \rho A\varphi_i(x)\varphi_j(x)\mathrm{d}x = m_b\delta_{ij}, \tag{14}$$

where ρAl is the total bridge mass and δ_{ij} is the Kronecker symbol.

The ith order inherent frequency of the bridge is

$$\omega_i = \left(\frac{i\pi}{l}\right)^2\sqrt{\frac{EI}{\rho A}}, \quad i = 1, 2, \ldots. \tag{15}$$

The relationship between the bridge vibrational function and the intrinsic frequency and flexural stiffness is as follows:

$$EI\frac{\mathrm{d}^4\varphi_i(x)}{\mathrm{d}x^4} = \omega_i^2\rho A\varphi_i(x). \tag{16}$$

The bridge displacement response $y(x, t)$ is solved by the vibration superposition method by taking the first n orders of the modal response of the bridge:

$$y(x,t) = \sum_{i=1}^{n}\varphi_i(x)q_i(t), \tag{17}$$

where $q_i(t)$ is the modal response corresponding to the oscillation function $\varphi_i(x)$.

The mass matrix of the coupled system is

$$\mathbf{M}(t) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & \dfrac{m_1}{m_b}\varphi_1(\xi) & \dfrac{m_2}{m_b}\varphi_1(\xi) \\ 0 & 1 & 0 & \cdots & 0 & \dfrac{m_1}{m_b}\varphi_2(\xi) & \dfrac{m_2}{m_b}\varphi_2(\xi) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & \dfrac{m_1}{m_b}\varphi_n(\xi) & \dfrac{m_2}{m_b}\varphi_n(\xi) \\ 0 & 0 & 0 & \cdots & 0 & m_1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & m_2 \end{bmatrix}. \tag{18}$$

Using the explicit Newmark algorithm to solve the differential equations of motion of the coupled vehicle-bridge system, the vector to be solved $\mathbf{q} = \begin{bmatrix} q_1 & q_2 \cdots q_n & y_1 & y_2 \end{bmatrix}^T$ is calculated, and the time analysis step is taken $\Delta T = 0.01$ s, so the total number of time steps $S = (T/\Delta T)$.

5. Stochastic Vibration Simulation Analysis of Stochastic Dynamical Systems

The differential equations of motion of the coupled vehicle-bridge system are derived above, and the deterministic response of the system is calculated under the action of the moving vehicle load. However, in the actual process of vehicle crossing the bridge, it is inevitable that it will be subjected to random uneven excitation of the bridge deck, which has a great impact on the driving safety and passenger comfort. Therefore, it is important to analyze the stochastic dynamics of the coupled vehicle-bridge system under the joint action of vehicle moving load and bridge deck uneven excitation and to study the trend and statistical law of the stochastic response of the system in the process of vehicle crossing the bridge.

5.1. Generate Bridge Deck Unevenness Time Domain Samples.
Assuming that the bridge unevenness field is a uniform random field with zero mean, the power spectral density function is considered as

$$S_r(\omega) = \frac{1}{\pi} \frac{4\gamma\alpha\beta\omega_0^2}{\left(\omega_0^2 - \omega^2\right)^2 + 4\alpha^2\beta^2}, \tag{19}$$

where $\omega_0^2 = \alpha^2 + \beta^2, \alpha = 0.1$ (rad/m), $\beta = 0.1$ (rad/m), and $\gamma = 1$ (cm² · m/rad) and the frequency is in the range $[-10, 10]$ rad/m.

Since the stochastic dynamics analysis of the vehicle-bridge coupled system belongs to the ultra-high-dimensional problem, in order to facilitate the solution, the original D-dimensional space is firstly decomposed into $D/2$-dimensional subspaces using the stratified sampling method, and then, the bridge deck stochastic uneven time domain samples are generated according to the Fourier inverse transform method, and the simulated values of the power spectrum are compared with the analyzed values to verify the correctness of the samples, as shown in Figures 2 and 3, respectively.

5.2. Random Vibration Analysis of Coupled Vehicle-Bridge System Based on the Direct Probability Integration Method.
The random excitation caused by the bridge deck irregularities is obtained by multiplying the time domain samples of the bridge deck irregularities generated in the previous section with the wheel stiffness. Using the direct probability integration method, the probability density function of the response of the coupled vehicle-bridge system under the combined effect of the vehicle moving load and the bridge deck upset excitation can be solved and its mathematical statistical law can be analyzed.
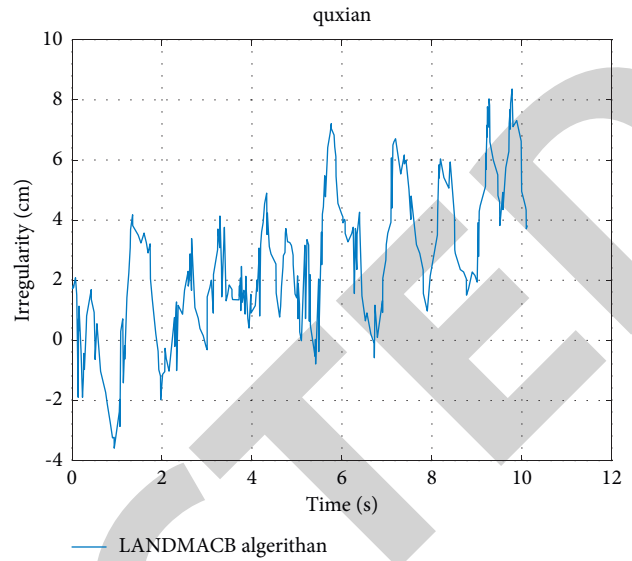

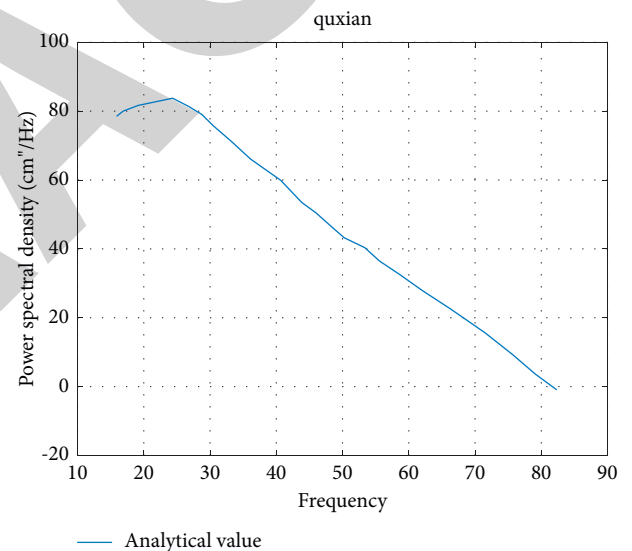FIGURE 2: Time domain sample of bridge deck unevenness.

FIGURE 3: Simulated and resolved power spectrum of bridge deck irregularities.

Taking the number of representative points, $N = 984$, the representative responses of N bridge span displacements and vehicle accelerations are shown in Figures 4 and 5, respectively.

Analysis of Figure 5 shows that the variability of the bridge span displacement is very small until the vehicle reaches the midbridge position, after which the variability increases slightly, but overall the variability is small. Analysis of Figure 6 shows that the variability of the vehicle acceleration response is consistently higher throughout the bridge crossing process, with greater variability near the midspan position and when the vehicle is about to exit the bridge. Comparing the bridge span displacement and vehicle acceleration representative response, it is obvious that the variability of the N-body acceleration response is greater for
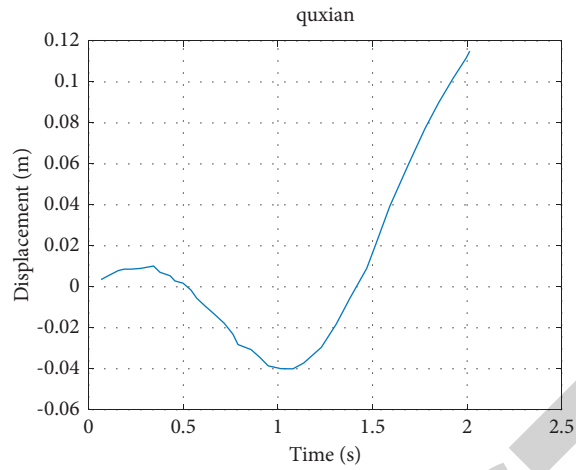
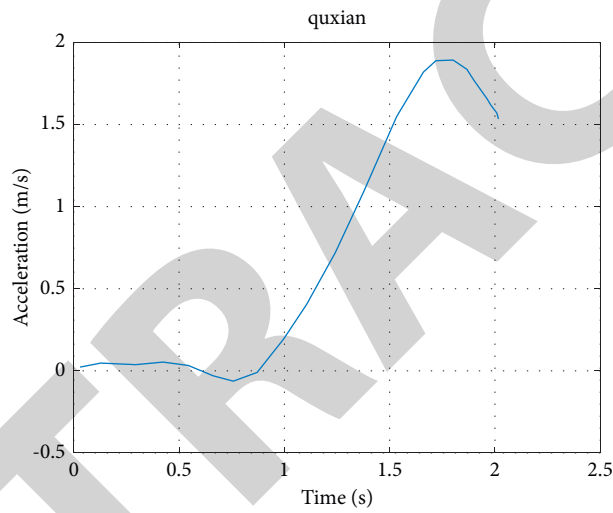Figure 4: Midspan displacement of the bridge ($N = 984$).

Figure 5: Vehicle acceleration of the simply supported girder bridge ($N = 984$).
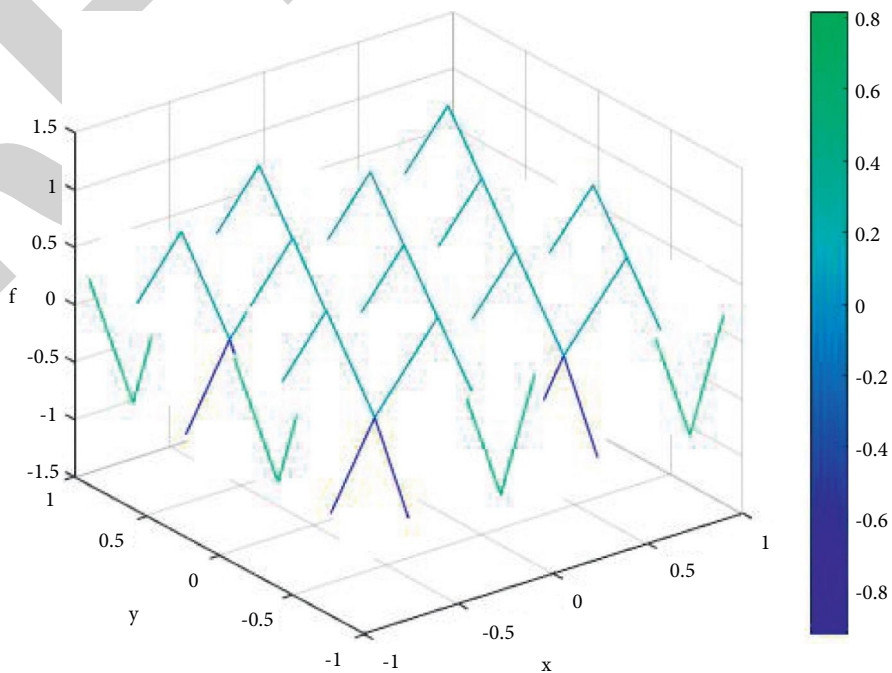

Figure 6: Probability density function surface of bridge midspan displacement.
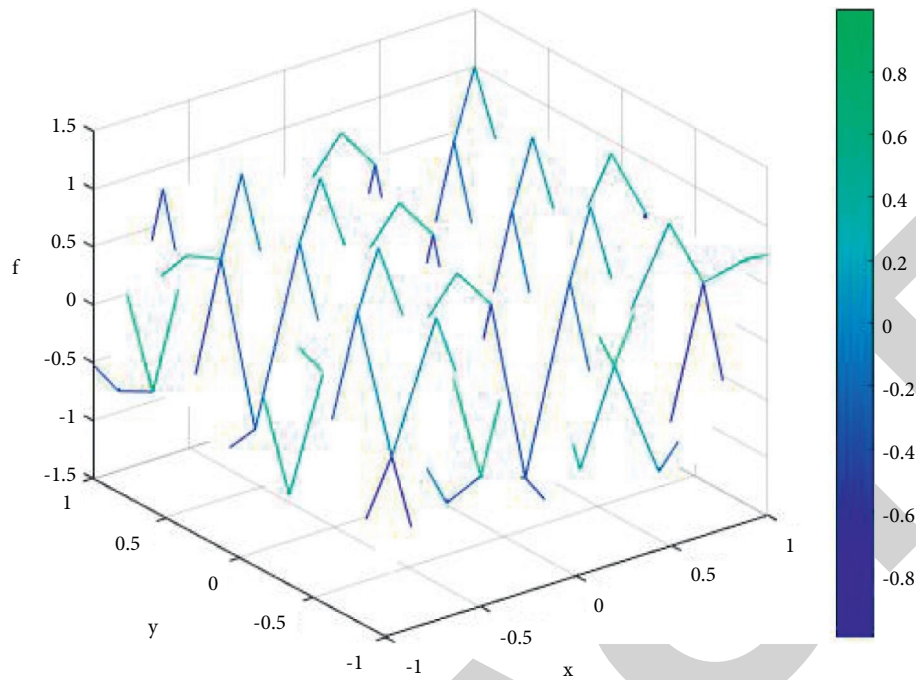
Figure 7: Body acceleration probability density function surface.

the bridge span displacement response, i.e., the vibration of the vehicle is more complex than that of the bridge. This is because the random uneven excitation of the bridge deck acts directly on the vehicle and then acts on the bridge structure through the wheel-bridge contact force. If the number of representative points is reduced by taking $N = 607$ or $N = 374$, the statistical laws of the bridge midspan displacement and vehicle acceleration response are similar to those for $N = 984$ and will not be repeated. Body acceleration probability density function surface is shown in Figure 7.

The 3D probability density function surfaces of the bridge span displacement and vehicle acceleration response can be obtained by substituting the obtained representative responses of the bridge span displacement and vehicle acceleration into equation (19), as shown in Figures 6 and 7, respectively. Based on the probability density function surfaces, the statistical patterns of the bridge span displacement and vehicle acceleration responses can be seen more intuitively. At the initial moment, the variability of the response is very small, and the peak value of PDF image is very high, and then, the peak value of PDF decreases as the variability of response increases gradually.

Intercepting the three-dimensional probability density function surface with the time plane at any moment which can be obtained at that moment of the two-dimensional probability density function curve, we choose $t = 0.7$ s, $t = 1.1$ s, and $t = 1.5$ s three planes to intercept the bridge span displacement three-dimensional PDF image of the results shown in Figure 7; choose $t = 0.8$ s, $t = 1.1$ s, and $t = 1.4$ s three planes to intercept the three-dimensional acceleration of the car body PDF images. It can be seen that the PDF curves of the bridge span displacement and vehicle acceleration response at different moments show different evolutionary patterns. The peak values of the PDF images of the span

displacements of the bridge differed greatly at different moments, and the peak values of the PDF images decreased gradually as the train traveled on the bridge; the PDF images of the vehicle acceleration showed a high peak value except for the initial moment, and the peak values of the PDF images differed less at each moment afterwards. Overall, the peak of the probability density function curve of the bridge span displacement is two orders of magnitude higher than that of the vehicle acceleration, and the variability of the bridge span displacement response is relatively small and the probability density function curve is smoother.

6. Conclusions

In this paper, the direct probability integral method is applied to the stochastic vibration reliability analysis of the dynamical system. A dynamical model of the dynamic system and the coupled system "three-car vehicle-rail-bridge" is established, where its time-varying differential equations of motion are derived in detail, and the dynamic response of the system is calculated using the explicit Newmark algorithm. The simulation results show the influence of the number of representative points on the smoothness of the image of the probability density function and the accuracy of the calculation results.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Xiaohong Yu was responsible for the collection and pre-processing of the entire experimental data.

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