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Research Article

Computation of Edge Resolvability of Benzenoid Tripod Structure

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In chemistry, graphs are commonly used to show the structure of chemical compounds, with nodes and edges representing the atom and bond types, respectively. Edge resolving set λ_e is an ordered subset of nodes of a graph C, in which each edge of C is distinctively determined by its distance vector to the nodes in λ. The cardinality of a minimum edge resolving set is called the edge metric dimension of C. An edge resolving set $L_{e,f}$ of C is fault-tolerant if $\lambda_{e,f} \setminus b$ is also an edge resolving set, for every b in $\lambda_{e,f}$. Resolving set allows obtaining a unique representation for chemical structures. In particular, they were used in pharmaceutical research for discovering patterns common to a variety of drugs. In this paper, we determine the exact edge metric and fault-tolerant edge metric dimension of benzenoid tripod structure and proved that both parameters are constant.

1. Introduction

Mathematical chemistry has recently presented a wide range of approaches to understanding the chemical structures that underpin existing chemical theories and developing and exploring new mathematical models of chemical phenomena and applying mathematical concepts and processes to chemistry. Only a few scientists have been convinced to exploit linkages between mathematics and chemistry and the possibility of using arithmetic to deduce known and anticipate new chemical characteristics, throughout the history of science. In many areas of physical chemistry, such as thermodynamics and compound energy, extensive use of mathematical approaches is commonplace. After physicists revealed in the first few years of the twentieth century that the key features of chemical compounds can be predicted using quantum theory approaches, a significant need for mathematics in chemistry arose. The main driving force that drove the mathematics and its concepts into chemistry laboratories was the realization that chemistry cannot be comprehended without knowledge of quantum physics, including its complicated mathematical instruments. For the different study of mathematical chemistry in terms of graph theory, we suggest some literature here [1].

Chemical graph theory is a branch of mathematical science that is used to characterise the structural properties of molecules, processes, crystals, polymers, clusters, and other objects. The vertex of a chemical graph theory might be an electron, an atom, a molecule, a collection of atoms, intermediates, orbitals, and many other things. Intermolecular bonding, bonded and nonbonded connections, basic reactions, and other forces such as van der Waals forces, Keesom forces, and Debye forces can all be used to illustrate the relationships between vertices of a structure.

The general convex polytope structures are discussed in [2, 3], in which authors consider the problem of edge metric resolvability. In the reply to aroused question from seminal work of edge resolvability, Raza and Bataineh [4] answered some questions and provided interesting results as well as analysis between vertex and edge resolvability. The detailed discussion of identifying edges and vertices of general graph is studied in [5]. Necklace graph's edge resolvability is discussed in [6]. Polycyclic aromatic hydrocarbons in terms of edge and fault-tolerant edge resolvability are deeply

investigated in [7]. The generalized version of edge resolvability is introduced in [8]. Few efficient techniques of finding edge resolving set are found in [9]. The graph having larger edge resolvability rather than vertex resolvability is generally studied in [10]. The generalized Peterson graph's edge resolvability is found in [11]. The k-multiwheel graph point of discussion with its edge resolvability is found in [12].

Slater [13] presented the idea of resolving sets, which was later discussed by Harary and Melter [14]. Metric generators, as detailed in [15, 16], allow for alternative representations of chemical substances. Precisely, they were used in pharmaceutical research for determining patterns similar to a variety of drugs [17]. Metric dimension has various other applications, such as robot navigation [18], weighing problems [19], computer networks [20], combinatorial optimization [21], image processing, facility location problems, and sonar and coastguard loran [13]; for further detail, see [22, 23]. Due to its variety of applications, the concept of metric dimension is widely used to solve many difficult problems. Hussain et al. [24], Krishnan et al. [25], and Siddiqui et al. [26] have computed the resolvability parameter for alpha-boron nanotube, certain crystal structures, and certain nanotube lattices, respectively. For the NP-harness of these topics, for example, and for the metric dimension [27, 28] and application along with its NP-hardness, see [29]. Rather than separating two unique vertices of a graph based on a subset of vertices, two edges might be distinguished based on the same subset of vertices. Kelenc et al. [30] created a new parameter called the edge metric dimension to represent this idea. They employed graph metric to identify each pair of edges based on the graph's distance from a selection of vertices.

The following are basic preliminaries for the concepts studied here.

Definition 1 (see [31]). Assume that C is an associated graph of chemical structure/network, whose vertex/node set are denoted by symbol $N(C)$ or simply N, while $B(C)$ or B is the edge/bond set, and the shortest distance between two bonds $b_1, b_2 \in N(C)$ is denoted by S_{b_1,b_2} and calculated by counting the number of bonds while moving through the $b_1 - b_2$ path.

Definition 2 (see [32]). The distance between an edge $e = b_1b_2 \in B(C)$ and a node $b \in N(C)$ is counted by the relation $Se, b = \min\{Sb_1, b, Sb_2, b\}$. Assuming a subset of selected nodes λ_e, if the position $\mathfrak{p}(e|\lambda_e)$ of each edge e is unique of a graph, then λ_e is called as edge metric resolving set and $\dim_e(C)$ is the minimum count of members of λ_e, called as edge metric dimension.

Definition 3. Assuming that any of the member of edge metric resolving set λ_e is not working or any of the node from κ members is spoiled, then one cannot get the unique position of the entire edge set. To tackle this issue, the definition is known as fault-tolerant edge resolving set which is dealt with by eliminating any of the member from λ_e and still obtains the unique position of the entire edge set of a graph, symbolized as $\lambda_{e,f}$, and the minimum members in the set denoted as $\dim_{e,f}(C)$ and named as fault-tolerant edge metric dimension.

Theorem 1 (see [30]). *If $dim_e(C)$ is the edge metric dimension, then $dim_e(C) = 1$, iff C is a path P_n.*

2. Construction of Tripod Structure

Because it is significant in theoretical chemistry, benzenoid systems are natural graph representations of benzenoid hydrocarbons. It is a well-known fact that hydrocarbons generated from benzenoids are important and beneficial in the chemical, food, and environmental industries, according to [33]. The authors in [34] describe the benzenoid system we mentioned above in our work. Polynomial types were discussed in relation to various catacondensed and pericondensed benzenoid structures. This is a pericondensed benzenoid tripod construction. It has $4(\delta_1 + \delta_2 + \delta_3) - 8$ nodes and $5(\delta_1 + \delta_2 + \delta_3) - 11$ bonds, with all the running parameters $\delta_1, \delta_2, \delta_3 \geq 2$. Furthermore, Jamil et al. [35] provide a comprehensive topological investigation of benzenoid structures, and metric-based study of benzenoid networks is available in [36]. The node and bond or vertex and edge set for the benzenoid structure $T(\delta_1, \delta_2, \delta_3)$ is shown as follows. In our primary results, we apply the labelling of nodes and edges specified in Figure 1.

$$
\begin{aligned}
N(T(\delta_1,\delta_2,\delta_3)) &= \{a_\kappa\colon 1 \le \kappa \le 2\delta_3\} \cup \{b_\kappa\colon 1 \le \kappa \le 2\delta_1\} \cup \{c_\kappa, c'_\kappa\colon 1 \le \kappa \le 2\delta_2 - 1\} \\
&\cup \{a'_\kappa\colon 1 \le \kappa \le 2\delta_3 - 3\} \cup \{b'_\kappa\colon 1 \le \kappa \le 2\delta_1 - 3\}, \\
B(T(\delta_1,\delta_2,\delta_3)) &= \{a_\kappa a_{\kappa+1}\colon 1 \le \kappa \le 2\delta_3 - 1\} \cup \{b_\kappa b_{\kappa+1}\colon 1 \le \kappa \le 2\delta_1 - 1\} \\
&\cup \{c_\kappa c_{\kappa+1}, c'_\kappa c'_{\kappa+1}\colon 1 \le \kappa \le 2\delta_2 - 2\} \cup \{a'_\kappa a'_{\kappa+1}\colon 1 \le \kappa \le 2\delta_3 - 4\} \\
&\cup \{b'_\kappa b'_{\kappa+1}\colon 1 \le \kappa \le 2\delta_1 - 4\} \cup \{a_\kappa a'_\kappa\colon 1 \le \kappa(\text{odd}) \le 2\delta_3 - 3\} \\
&\cup \{b_{\kappa+3} b'_\kappa\colon 1 \le \kappa(\text{odd}) \le 2\delta_1 - 3\} \cup \{c_\kappa c'_\kappa\colon 1 \le \kappa(\text{odd}) \le 2\delta_2 - 1\} \\
&\cup \{a_{2\delta_3} b_1, a_{2\delta_3-1} c'_1, b_2 c_1, a_{2\delta_3-3} c'_2, b'_1 c_2,\}.
\end{aligned}
\tag{1}
$$

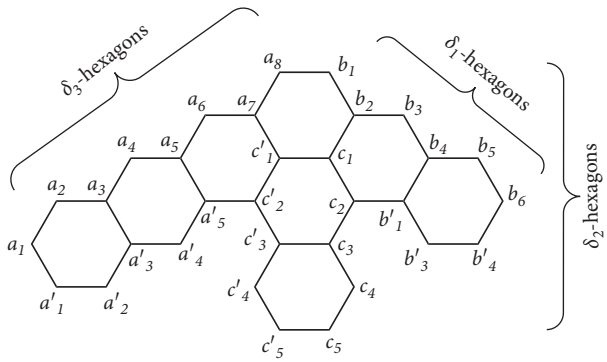

Figure 1: Benzenoid tripod with $\{\delta_1, \delta_2, \delta_3\} = \{3, 3, 4\}$.

Lemma 1. *If $T(\delta_1, \delta_2, \delta_3)$ is a graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \geq 2$, then the minimum members in its edge resolving set are two.*

Proof. The total number of nodes in the corresponding graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \geq 2$ is $4(\delta_1 + \delta_2 + \delta_3) - 8$ and to check the possibilities of edge resolving set with cardinality two is $C(4(\delta_1 + \delta_2 + \delta_3) - 8, 2) = ((4(\delta_1 + \delta_2 + \delta_3) - 8)!/(2 \times (4(\delta_1 + \delta_2 + \delta_3) - 10)!))$. Here, we are checking with cardinality two because by Theorem 1, the edge resolving set with cardinality one is reserved for path graph only. Now due to the NP-hardness of choosing edge resolving set, we cannot find the exact number of edge resolving sets for a graph, but from $((4(\delta_1 + \delta_2 + \delta_3) - 8)!/(2 \times (4(\delta_1 + \delta_2 + \delta_3) - 10)!))$-possibilities, we choose a subset λ_e and defined as $\lambda_e = \{a_1, b_1\}$. Now to prove this claim that λ_e is actually one of the candidates for the edge resolving set of benzenoid tripod graph or $T(\delta_1, \delta_2, \delta_3)$, we will follow Definition 2. To fulfill the requirements of definition, we will check the unique positions or locations of each edge and the methodology is defined above in Definition 2.

Positions $\mathfrak{p}(a_\kappa a_{\kappa+1}|\lambda_e)$ with respect to λ_e, for the edges $a_\kappa a_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_3 - 1$, are given as

$$\mathfrak{p}(a_\kappa a_{\kappa+1}|\lambda_e) = (\kappa - 1, 2\delta_3 - \kappa). \tag{2}$$

Positions $\mathfrak{p}(b_\kappa b_{\kappa+1}|\lambda_e)$ with respect to λ_e, for the edges $b_\kappa b_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_1 - 1$, are given as

$$\mathfrak{p}(b_\kappa b_{\kappa+1}|\lambda_e) = (2\delta_3 + \kappa - 1, \kappa - 1). \tag{3}$$

Positions $\mathfrak{p}(c_\kappa c_{\kappa+1}|\lambda_e)$ with respect to λ_e, for the edges $c_\kappa c_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_2 - 2$, are given as

$$\mathfrak{p}(c_\kappa c_{\kappa+1}|\lambda_e) = \begin{cases} (2\delta_3, \kappa + 1), & \text{if } \kappa = 1, 2, \\ (2\delta_3 + \kappa - 3, \kappa + 1), & \text{if } \kappa = 3, 4, \ldots, 2\delta_2 - 1. \end{cases} \tag{4}$$

Positions $\mathfrak{p}(a'_\kappa a'_{\kappa+1}|\lambda_e)$ with respect to λ_e, for the edges $a'_\kappa a'_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_3 - 4$, are given as

$$\mathfrak{p}(a'_\kappa a'_{\kappa+1}|\lambda_e) = (\kappa, 2\delta_3 + 1 - \kappa). \tag{5}$$

Positions $\mathfrak{p}(b'_\kappa b'_{\kappa+1}|\lambda_e)$ with respect to λ_e, for the edges $b'_\kappa b_{\kappa+1}{}'$ with $\kappa = 1, 2, \ldots, 2\delta_1 - 4$, are given as

$$\mathfrak{p}(b'_\kappa b'_{\kappa+1}|\lambda_e) = (2\delta_3 + \kappa + 1, \kappa + 3). \tag{6}$$

Positions $\mathfrak{p}(c'_\kappa c'_{\kappa+1}|\lambda_e)$ with respect to λ_e, for the edges $c'_\kappa c'_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_2 - 2$, are given as

$$\mathfrak{p}(c'_\kappa c_{\kappa+1}{}'|\lambda_e) = \begin{cases} (2(\delta_3 - 1), \kappa + 2), & \text{if } \kappa = 1, \\ (2(\delta_3 - 2) + \kappa, \kappa + 2), & \text{if } \kappa = 2, 3, \ldots, 2\delta_2 - 1. \end{cases} \tag{7}$$

Positions $\mathfrak{p}(a_\kappa a'_\kappa|\lambda_e)$ with respect to λ_e, for the edges $a_\kappa a'_\kappa$ with $\kappa = 1, 3, \ldots, 2\delta_3 - 3$, are given as

$$\mathfrak{p}(a_\kappa a'_\kappa|\lambda_e) = (\kappa - 1, 2\delta_3 - \kappa + 1). \tag{8}$$

Positions $\mathfrak{p}(b_{\kappa+3} b'_\kappa|\lambda_e)$ with respect to λ_e, for the edges $b_{\kappa+3} b'_\kappa$ with $\kappa = 1, 3, \ldots, 2\delta_1 - 3$, are given as

$$\mathfrak{p}(b_{\kappa+3} b'_\kappa|\lambda_e) = (2\delta_3 + \kappa + 1, \kappa + 2). \tag{9}$$

Positions $\mathfrak{p}(c_\kappa a'_\kappa|\lambda_e)$ with respect to λ_e, for the edges $c_\kappa c'_\kappa$ with $\kappa = 1, 3, \ldots, 2ml - 3$, are given as

$$\mathfrak{p}(c_\kappa c'_\kappa|\lambda_e) = \begin{cases} (2\delta_3 - 1, \kappa + 1), & \text{if } \kappa = 1, \\ (2(\delta_3 - 2) + \kappa, \kappa + 1), & \text{if } \kappa = 2, 3, \ldots, 2\delta_2 - 1. \end{cases} \tag{10}$$

Positions of the joint edges are given as

$$\begin{aligned} \mathfrak{p}\left(a_{2\delta_3} b_1|\lambda_e\right) &= (2\delta_3 - 1, 0), \\ \mathfrak{p}\left(a_{2\delta_3} c'_1|\lambda_e\right) &= (2\delta_3 - 2, 2), \\ \mathfrak{p}(b_2 c_1|\lambda_e) &= (2\delta_3, 1), \\ \mathfrak{p}\left(a_{2\delta_3 - 3}{}' c'_2|\lambda_e\right) &= (2\delta_3 - 3, 4), \\ \mathfrak{p}(b'_1 c_2|\lambda_e) &= (2\delta_3 + 1, 3). \end{aligned} \tag{11}$$

The given positions $\mathfrak{p}(.|\lambda_e)$ of all $5(\delta_1 + \delta_2 + \delta_3) - 11$-bonds of $T(\delta_1, \delta_2, \delta_3)$ graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \geq 2$, with respect to λ_e, are unique and no two bonds have the same position $\mathfrak{p}$. So we can conclude that we resolve the bonds of $T(\delta_1, \delta_2, \delta_3)$ with two nodes. It is implied that the minimum members in the edge resolving set of $T(\delta_1, \delta_2, \delta_3)$ are two. □

Remark 1. If $T(\delta_1, \delta_2, \delta_3)$ is a graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \geq 2$, then

$$\dim_e(T(\delta_1, \delta_2, \delta_3)) = 2. \tag{12}$$

Proof. From the definition of edge metric dimension, the concept is solemnly based on the selected subset (λ_e) chosen in such a way that the entire edge set has unique position with respect to the selected nodes or subset. In Lemma 1, we already discussed the possibility of selected subset (edge resolving set) and, according to the definition, its minimum possible cardinality. In that lemma, we choose $\lambda_e = \{a_1, b_1\}$ as an edge resolving set for the graph of benzenoid tripod or $T(\delta_1, \delta_2, \delta_3)$ for all the possible combinatorial values of $\delta_1, \delta_2, \delta_3 \geq 2$. We also proved in such lemma that $|\lambda_e| = 2$ is the least possible cardinality of edge resolving set for the

benzenoid tripod $T(\delta_1, \delta_2, \delta_3)$. It is enough for the proof of what we claim in the statement that edge metric dimension of benzenoid tripod is two, which completes the proof. □

Lemma 2. *If $T(\delta_1, \delta_2, \delta_3)$ is a graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \geq 2$, then the minimum members in its fault-tolerant edge resolving set are four.*

Proof. The total number of nodes in the corresponding graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \geq 2$, are $4(\delta_1 + \delta_2 + \delta_3) - 8$ and to check the possibilities of fault-tolerant edge resolving set with cardinality four are $C(4(\delta_1 + \delta_2 + \delta_3) - 8, 4) = ((4(\delta_1 + \delta_2 + \delta_3) - 8)!/(2 \times (4(\delta_1 + \delta_2 + \delta_3) - 12)!))$. Here we are checking with cardinality four, and later, we will also check cardinality three. Now due to the NP-hardness of choosing fault-tolerant edge resolving set, we cannot find the exact number of fault-tolerant edge resolving sets for a graph, but from $((4(\delta_1 + \delta_2 + \delta_3) - 8)!/(2 \times (4(\delta_1 + \delta_2 + \delta_3) - 12)!))$-possibilities, we choose a subset $\lambda_{e,f}$ and defined as $\lambda_{e,f} = \{a_1, b_1, a_{2\delta_3}, b_{2\delta_1}\}$. Now to prove this claim that $\lambda_{e,f}$ is actually one of the candidates for the fault-tolerant edge resolving set of benzenoid tripod graph or $T(\delta_1, \delta_2, \delta_3)$, we will follow Definition 3. To fulfill the requirements of definition, we will check the unique positions or locations of each node and the methodology is defined above in Definition 3.

Positions $\mathfrak{p}(a_\kappa a_{\kappa+1}|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $a_\kappa a_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_3 - 1$, are given as

$$\mathfrak{p}\left(a_\kappa a_{\kappa+1}|\lambda_{e,f}\right) = \begin{cases} (\kappa - 1, 2\delta_3 - \kappa, 2\delta_3 - 1 - \kappa, 2(\delta_3 + \delta_1) - 1 - \kappa), & \text{if } \kappa = 1, 2, \ldots, 2\delta_3 - 2, \\ (\kappa - 1, 2\delta_3 - \kappa, 2\delta_3 - 1 - \kappa, 2\delta_3), & \text{if } \kappa = 2\delta_3 - 1. \end{cases} \tag{13}$$

Positions $\mathfrak{p}(b_\kappa b_{\kappa+1}|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $b_\kappa b_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_1 - 1$, are given as

$$\mathfrak{p}\left(b_\kappa b_{\kappa+1}|\lambda_{e,f}\right) = (2\delta_3 + \kappa - 1, \kappa - 1, \kappa, 2\delta_1 - 1 - \kappa). \tag{14}$$

Positions $\mathfrak{p}(c_\kappa c_{\kappa+1}|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $c_\kappa c_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_2 - 2$, are given as

$$\mathfrak{p}\left(c_\kappa c_{\kappa+1}|\lambda_{e,f}\right) = \begin{cases} (2\delta_3, \kappa + 1, \kappa + 2, 2\delta_1 - 2), & \text{if } \kappa = 1, \\ (2\delta_3, \kappa + 1, \kappa + 2, 2(\delta_1 - 2) + \kappa), & \text{if } \kappa = 2, \\ (2\delta_3 + \kappa - 3, \kappa + 1, \kappa + 2, 2(\delta_1 - 2) + \kappa), & \text{if } \kappa = 3, 4, \ldots, 2\delta_2 - 1. \end{cases} \tag{15}$$

Positions $\mathfrak{p}(a'_\kappa a'_{\kappa+1}|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $a'_\kappa a'_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_3 - 4$, are given as

$$\mathfrak{p}\left(a'_\kappa a'_{\kappa+1}|\lambda_{e,f}\right) = (\kappa, 2\delta_3 + 1 - \kappa, 2\delta_3 - \kappa, 2(\delta_3 + \delta_1 - 1) - \kappa). \tag{16}$$

Positions $\mathfrak{p}(b'_\kappa b'_{\kappa+1}|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $b'_\kappa b'_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_1 - 4$, are given as

$$\mathfrak{p}\left(b'_\kappa b'_{\kappa+1}|\lambda_{e,f}\right) = (2\delta_3 + \kappa + 1, \kappa + 3, \kappa + 4, 2\delta_1 - 3 - \kappa). \tag{17}$$

Positions $\mathfrak{p}(c'_\kappa c'_{\kappa+1}|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $c'_\kappa c'_{\kappa+1}$ with $\kappa = 1, 2, \ldots, 2\delta_2 - 2$, are given as

$$\mathfrak{p}\left(c'_\kappa c'_{\kappa+1}|\lambda_{e,f}\right) = \begin{cases} (2(\delta_3 - 1), \kappa + 2, \kappa + 1, 2\delta_1), & \text{if } \kappa = 1, \\ (2(\delta_3 - 1), \kappa + 2, \kappa + 1, 2\delta_1), & \text{if } \kappa = 2, \\ (2(\delta_3 - 2) + \kappa, \kappa + 2, \kappa + 1, 2\delta_1 - 3 + \kappa), & \text{if } \kappa = 3, 4, \ldots, 2\delta_2 - 1. \end{cases} \tag{18}$$

Positions $\mathfrak{p}(a_\kappa a'_\kappa|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $a_\kappa a'_\kappa$ with $\kappa = 1, 3, \ldots, 2\delta_3 - 3$, are given as:

$$\mathfrak{p}\left(a_\kappa a'_\kappa|\lambda_{e,f}\right) = (\kappa - 1, 2\delta_3 - \kappa + 1, 2\delta_3 - \kappa, 2(\delta_3 + \delta_1) - 1 - \kappa). \tag{19}$$

Positions $\mathfrak{p}(b_{\kappa+3} b'_\kappa|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $b_{\kappa+3} b'_\kappa$ with $\kappa = 1, 3, \ldots, 2\delta_1 - 3$, are given as

$$\mathfrak{p}\left(b_{\kappa+3} b'_\kappa|\lambda_{e,f}\right) = (2\delta_3 + \kappa + 1, \kappa + 2, \kappa + 3, 2\delta_1 - 3 - \kappa). \tag{20}$$

Positions $\mathfrak{p}(c_\kappa a_\kappa'|\lambda_{e,f})$ with respect to $\lambda_{e,f}$, for the edges $c_\kappa c_\kappa'$ with $\kappa = 1, 3, \ldots, 2ml - 3$, are given as

$$\mathfrak{p}\left(c_\kappa c_\kappa'|\lambda_{e,f}\right) = \begin{cases} \left(2\delta_3 - 1, \kappa + 1, \kappa + 1, 2\delta_1 - 1\right), & \text{if } \kappa = 1, \\ \left(2\left(\delta_3 - 2\right) + \kappa, \kappa + 1, \kappa + 1, 2\left(\delta_1 - 2\right) + \kappa\right), & \text{if } \kappa = 2, 3, \ldots, 2\delta_2 - 1. \end{cases} \tag{21}$$

Positions of the joint edges with respect to $\lambda_{e,f}$ are given as

$$\begin{aligned} \mathfrak{p}\left(a_{2\delta_3}b_1|\lambda_{e,f}\right) &= \left(2\delta_3 - 1, 0, 0, 2\delta_1 - 1\right), \\ \mathfrak{p}\left(a_{2\delta_3}c_1'|\lambda_{e,f}\right) &= \left(2\delta_3 - 2, 2, 1, 2\delta_1\right), \\ \mathfrak{p}\left(b_2c_1|\lambda_{e,f}\right) &= \left(2\delta_3, 1, 2, 2\delta_1 - 2\right), \\ \mathfrak{p}\left(a_{2\delta_3'-3}c_2'|\lambda_{e,f}\right) &= \left(2\delta_3 - 3, 4, 3, 2\delta_1 + 1\right), \\ \mathfrak{p}\left(b_1'c_2|\lambda_{e,f}\right) &= \left(2\delta_3 + 1, 3, 4, 2\delta_1 - 3\right). \end{aligned} \tag{22}$$

On the behalf of given fact for the fulfillment of definition of fault-tolerant edge resolving set, we can say that $\lambda_{e,f}$ with cardinality four is possible, but when it comes to optimized value of $|\lambda_{e,f}|$, we still need to investigate about the minimum value of $|\lambda_{e,f}|$. Following are some possible cases to check that whether $|\lambda_{e,f}| = 3$ is possible or not. However, we find the fault-tolerant edge resolving set with the help of algorithm satisfying that $|\lambda_{e,f}| \neq 3$, but for the proving purpose we build some general cases and try to conclude that only $|\lambda_{e,f}| > 3$ is possible.

Case 1: assume that $\lambda_{e,f}' \subset \{a_\kappa : \kappa = 1, 2, \ldots, 2\delta_3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s a_s'|\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_3 - 1$ and $1 \leq s\,(\text{odd}) \leq 2\delta_3 - 3$.

Case 2: assume that $\lambda_{e,f}' \subset \{b_\kappa : \kappa = 1, 2, \ldots, 2\delta_1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s' a_{s+1}'|\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_3 - 1$ and $1 \leq s \leq 2\delta_3 - 4$.

Case 3: assume that $\lambda_{e,f}' \subset \{c_\kappa : \kappa = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_{r+3}b_r'|\lambda_{e,f}') = p(a_s a_{s+1}'|\lambda_{e,f}')$, where $1 \leq r\,(\text{odd}) \leq 2\delta_1 - 3$ and $1 \leq s \leq 2\delta_3 - 1$.

Case 4: assume that $\lambda_{e,f}' \subset \{a_{\kappa'} : \kappa = 1, 2, \ldots, 2\delta_3 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(c_s c_s'|\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_3 - 1$ and $1 \leq s\,(\text{odd}) \leq 2\delta_2 - 1$.

Case 5: assume that $\lambda_{e,f}' \subset \{b_{\kappa'} : \kappa = 1, 2, \ldots, 2\delta_1 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s' a_{s+1}'|\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_3 - 1$ and $1 \leq s \leq 2\delta_3 - 4$.

Case 6: assume that $\lambda_{e,f}' \subset \{c_{\kappa'} : \kappa = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s a_{s+1}|\lambda_{e,f}')$, where $1 \leq r, s \leq 2\delta_3 - 1$.

Case 7: assume that $\lambda_{e,f}' \subset \{a_\kappa, b_j : \kappa = 1, 2, \ldots, 2\delta_3, j = 1, 2, \ldots, 2\delta_1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_s' b_{s+1}'|\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_1 - 1$ and $1 \leq s \leq 2\delta_1 - 4$.

Case 8: assume that $\lambda_{e,f}' \subset \{a_\kappa, c_j : \kappa = 1, 2, \ldots, 2\delta_3, j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_{r+3}b_r'|\lambda_{e,f}') = p(b_s' b_{s+1}'|\lambda_{e,f}')$, where $1 \leq r\,(\text{odd}) \leq 2\delta_1 - 3$ and $1 \leq s \leq 2\delta_1 - 4$.

Case 9: assume that $\lambda_{e,f}' \subset \{a_\kappa, a_j' : \kappa = 1, 2, \ldots, 2\delta_3, j = 1, 2, \ldots, 2\delta_3 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_{s+3}|b_r'\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_1 - 1$ and $1 \leq s\,(\text{odd}) \leq 2\delta_1 - 3$.

Case 10: assume that $\lambda_{e,f}' \subset \{a_\kappa, b_j' : \kappa = 1, 2, \ldots, 2\delta_3, j = 1, 2, \ldots, 2\delta_1 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s a_s'|\lambda_{e,f}')$, where $1 \leq r \leq 2\delta_3 - 1$ and $1 \leq s\,(\text{odd}) \leq 2\delta_3 - 3$.

Case 11: assume that $\lambda_{e,f}' \subset \{a_\kappa, c_j' : \kappa = 1, 2, \ldots, 2\delta_3, j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition

according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_s' b_{s+1}'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_1 - 1$ and $1 \le s \le 2\delta_1 - 4$.

Case 12: assume that $\lambda_{e,f}' \subset \{b_\kappa, c_j : \kappa = 1, 2, \ldots, 2\delta_1, j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s a_s'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_3 - 1$ and $1 \le s\,(\text{odd}) \le 2\delta_3 - 3$.

Case 13: assume that $\lambda_{e,f}' \subset \{b_\kappa, a_j' : \kappa = 1, 2, \ldots, 2\delta_1, j = 1, 2, \ldots, 2\delta_3 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s a_s'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_3 - 1$ and $1 \le s\,(\text{odd}) \le 2\delta_3 - 3$.

Case 14: assume that $\lambda_{e,f}' \subset \{b_\kappa, b_j' : \kappa = 1, 2, \ldots, 2\delta_1, j = 1, 2, \ldots, 2\delta_3 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s a_s'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_3 - 1$ and $1 \le s\,(\text{odd}) \le 2\delta_3 - 3$.

Case 15: assume that $\lambda_{e,f}' \subset \{b_\kappa, c_j' : \kappa = 1, 2, \ldots, 2\delta_1, j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s' a_{s+1}'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_3 - 1$ and $1 \le s \le 2\delta_3 - 4$.

Case 16: assume that $\lambda_{e,f}' \subset \{c_\kappa, a_j' : \kappa = 1, 2, \ldots, 2\delta_2 - 1, j = 1, 2, \ldots, 2\delta_3 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_s' b_{s+1}'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_1 - 1$ and $1 \le s \le 2\delta_1 - 4$.

Case 17: assume that $\lambda_{e,f}' \subset \{c_\kappa, b_j' : \kappa = 1, 2, \ldots, 2\delta_2 - 1, j = 1, 2, \ldots, 2\delta_1 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s' a_{s+1}'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_3 - 1$ and $1 \le s \le 2\delta_3 - 4$.

Case 18: assume that $\lambda_{e,f}' \subset \{c_\kappa, c_j' : \kappa, j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_s b_{s+1}|\lambda_{e,f}')$, where $1 \le r, s \le 2\delta_1 - 1$.

Case 19: assume that $\lambda_{e,f}' \subset \{a_\kappa', b_j' : \kappa = 1, 2, \ldots, 2\delta_3 - 3, j = 1, 2, \ldots, 2\delta_1 - 3\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_s' b_{s+1}'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_1 - 1$, $1 \le s \le 2\delta_1 - 4$.

Case 20: assume that $\lambda_{e,f}' \subset \{a_\kappa', c_j' : \kappa = 1, 2, \ldots, 2\delta_3 - 3, \quad j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(b_r b_{r+1}|\lambda_{e,f}') = p(b_s' b_{s+1}'|\lambda_{e,f}')$, where $1 \le r \le 2\delta_1 - 1$, $1 \le s \le 2\delta_1 - 4$.

Case 21: assume that $\lambda_{e,f}' \subset \{b_\kappa', c_j' : \kappa = 1, 2, \ldots, 2\delta_1 - 3, j = 1, 2, \ldots, 2\delta_2 - 1\}$, with a condition according to our requirement of theorem that $|\lambda_{e,f}'| = 3$, and removal of any vertex from $\lambda_{e,f}'$ to fulfill the definition. The result is implied in the same edge's position and contradicts our assumption with the fact that $p(a_r a_{r+1}|\lambda_{e,f}') = p(a_s' a_{s+1}'|\lambda_{e,f}')$, where $1 \le r, s \le 2\delta_1$.

The given positions $\mathfrak{p}(.|\lambda_{e,f})$ of all $5(\delta_1 + \delta_2 + \delta_3) - 11$-bonds of $T(\delta_1, \delta_2, \delta_3)$ graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \ge 2$, with respect to $\lambda_{e,f}$, having $|\lambda_{e,f}| = 4$, are unique and no two bonds have the same position $\mathfrak{p}$. It can also be accessed that eliminating any of arbitrary nodes from $\lambda_{e,f}$ will not affect the definition of edge resolving set. We also checked that the fault-tolerant edge resolving set $\lambda_{e,f}$ with $|\lambda_{e,f}| = 3$ resulted in two edges having the same position $\mathfrak{p}$. So we can conclude that we resolve the bonds of $T(\delta_1, \delta_2, \delta_3)$ with four nodes. It is implied that the minimum members in the fault-tolerant edge resolving set of $T(\delta_1, \delta_2, \delta_3)$ are four. □

Remark 2. If $T(\delta_1, \delta_2, \delta_3)$ is a graph of benzenoid tripod with $\delta_1, \delta_2, \delta_3 \ge 2$, then

$$\dim_{e,f}(T(\delta_1, \delta_2, \delta_3)) = 4. \tag{23}$$

Proof. From the definition of fault-tolerant edge metric dimension (same as in parent concept), the concept is solemnly based on the selected subset ($\lambda_{e,f}$) chosen in such a way that the entire edge set has unique position with respect to the selected nodes or subset. Addition or removal of any arbitrary single member of $\lambda_{e,f}$ does not affect the resolvability of edges or position of the entire edge set of graph remains unique. In Lemma 2, we already discussed the possibility of selected subset (fault-tolerant edge resolving set) and, according to the definition, its minimum possible

cardinality. In that lemma, we choose $\lambda_{e,f} = \{a_1, b_1, a_{2\delta_3}, b_{2\delta_1}\}$ as a fault-tolerant edge resolving set for the graph of benzenoid tripod or $T(\delta_1, \delta_2, \delta_3)$ for all the possible combinatorial values of $\delta_1, \delta_2, \delta_3 \geq 2$. We also proved in such lemma that $|\lambda_{e,f}| = 4$ is the least possible cardinality of fault-tolerant edge resolving set for the benzenoid tripod $T(\delta_1, \delta_2, \delta_3)$. It is enough for the proof of what we claim in the statement that fault-tolerant edge metric dimension of benzenoid tripod is four, which completes the proof. □

3. Conclusion

Mathematical chemistry, particularly graphical chemistry, has made it simpler to examine complicated networks and chemical structures in their simplest forms. Similarly, resolvability is a parameter in which the complete node or edge set, and occasionally both, reconfigure themselves into unique outfits in order to call or access them. Edge metric dimension is also a parameter with this property to gain each edge into a unique form. In this work, we consider benzenoid tripod structure to achieve its resolvability and found its minimum edge-resolving set. We concluded that edge metric and fault-tolerant edge metric resolving set are with constant and exact number of members for this structure.

Data Availability

There are no data associated with this manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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