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Research Article

Probabilistic Linguistic Three-Way Multi-Attibute Decision Making for Hidden Property Evaluation of Judgment Debtor

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Most law enforcement cases executed by the courts in China have behaviours of evading, evading, or even violently resisting execution or passively waiting for enforcement, which seriously affects the authority of legal judgments and the judiciary's credibility. Therefore, we develop a hidden property evaluation model based on the probabilistic linguistic three-way multi-attribute decision-making (PL3W-MADM) method. Considering the advantages of probabilistic linguistic term sets (PLTSs) expressing the evaluation information and their probabilities on judgment debtor given by expert judges, we extend the three-way decision method to a probabilistic linguistic environment and develop the strict PL3W-MADM model and flexible PL3W-MADM model. Then, the PL3W-MADM models are used to construct the hidden property evaluation model of judgment debtors. Finally, the developed hidden property evaluation model can quickly and effectively classify the judgment debtors into three categories: hidden behaviour, no hidden behaviour or lack of information, and temporary inability to judge. The results show that the developed model is more suitable for hidden property evaluation than the strict PL3W-MADM model and the flexible PL3W-MADM model.

1. Introduction

Most law enforcement cases executed by the courts in China have behaviours of evading, evading, or even violently resisting execution or passively waiting for enforcement. Many law enforcement cases cannot be executed smoothly, which seriously affects the authority of legal judgments and the judiciary's credibility. Hidden property evaluation of judgment debtors focuses on whether the judgment debtor has concealed property and what manner of concealment is used. In law enforcement cases, assessing how the judgment debtor has concealed property plays a key role in discovering property for enforcement, searching for clues to enforce property, and improving efficiency in the enforcement of cases.

Under the existing model, the judgment of whether the judgment debtor has the behaviour of concealing property mainly depends on the experience of the judges. The main characteristics of the existing judgment methods are as follows. Firstly, there is no unified evaluation standard. The expert judges can only evaluate whether the judgment debtor has concealed property behaviour according to their case handling experience, and there is no unified evaluation standard. Secondly, in the assessment process, the assessment information is based on the results of the expert judge's qualitative analysis, which is usually expressed in the form of fuzzy linguistic and lacks the integration of the opinions of multiple expert judges, let alone the integration of assessment information from multiple expert judges. Thirdly, the judgment debtors are mainly divided into two types: with

and without concealment of property, which lacks consideration for judgment debtors who do not have sufficient information. It is unreasonable to hastily categorize judgment debtors as having or not having concealment when they do not have sufficient information.

Because of the above characteristics, fuzzy linguistic variables provide a useful tool to express the judges' uncertain information evaluating the judgment debtors. Considering judging whether a judgment debtor has concealed property involves multiple attributes, the fuzzy linguistic multiattribute decision-making is suitable for solving such problems [1–5]. Therefore, we propose a probabilistic linguistic three-way multiattribute decision-making (PL3W-MADM) method to evaluate the concealment property, and the flowchart is shown in Figure 1. The main idea of the PL3W-MADM method for assessing a judgment debtor's concealment manner is that probabilistic linguistic term sets (PLTSs) are used to integrate the expert judges' assessment information on various concealment manner of the judgment debtor. The judgment debtors are classified into three categories: those with concealment behaviour, those without concealment behaviour, or those with insufficient information to judge for the time being by setting the upper and lower limits of the uncertainty region.

The main contributions of our work are as follows. Firstly, the concealment patterns are summarized and analyzed by how property is found, and the concealment patterns are used as assessment criteria to determine whether the judgment has concealed property. The expert judge will assess each concealment pattern of the judgment debtor based on the information already available. Secondly, considering that the expert judges' different opinions, the PLTSs are used to integrate the assessment information provided by all expert judges, which considered both the fuzzy linguistic assessment information and the corresponding probabilities, which provides a very practical tool in solving such problems. Thirdly, using the three-way decision (3WD) method, the judgment debtors are classified into three categories based on all expert judges' assessment results on the judgment debtor: with concealment, without concealment, or insufficient information to judge for the time being. In the assessment process, it is temporarily impossible to judge whether the judgment debtor has hidden property according to the available assessment information. We can adapt delayed decision-making and wait until the information is fully grasped before making a judgment. The 3WD method provides a better idea for solving it. Fourthly, for the judgment debtor who has concealed his property, we can analyze which way he has used to conceal his property. For the judgment debtor who cannot be identified due to insufficient information, we can quantify the possibility of concealment property.

The structure of our work is organized as follows. Section 2 provides some related works on the hidden property analysis and 3WD method. Some basic concepts on 3WD and PLTSs are introduced inSection 3. Section 4 extends the 3WD method into a probabilistic linguistic environment and develops a strict PL3W-MADM method and flexible PL3W-MADM method. Considering the advantages of the strict PL3W-MADM method and flexible PL3W-MADM method, Section 5 proposes a comprehensive PL3W-MADM method for the hidden property evaluation of judgment debtors. A numerical example is given to illustrate the effectiveness of the developed hidden property analysis method in Section 6. Section 7 makes some conclusions.

2. Literature Reviews

The judgment debtor's hidden property evaluation model focuses on the judgment debtor's concealment and how the property is concealed. Wu et al. [6, 7] and Zhang et al. [8] developed some hesitant fuzzy linguistic methods for hidden property analysis of judgment debtors. Under the existing assessment model, judgment debtors are divided into those who have concealed property and those who have not. This decision, which can only be made to accept or reject based on the assessment information, is called a two-way decision. One can make a straightforward yes or no decision on something for which one has comprehensive information. However, for issues such as concealment, where there is insufficient information to decide with sufficient certainty and further investigation is required, the concept of three-branch decision-making is introduced.

The 3WD method, as an extension of two decision-making, is a concept put forward by Yao in 2009 based on decision rough set theory [9, 10]. The two decisions can only make a judgment of acceptance or rejection based on information. As the optimization of a two-way decision, a 3WD method is allowed to make a delayed decision based on accepting or rejecting the resolution. A simple summary is that in the actual decision-making process, people usually make three types of opinions: accept the decision, reject the decision, and delay the decision because of the different degree of mastery of things information. People can directly decide to accept or reject the comprehensive report. Simultaneously, they cannot decide and need to further investigate insufficient information, which is called a delayed decision. In real life, three choices are widely used. For example, when the expert judge judges whether the judgment debtor conceals the property involved in the case, the judgment debtor will decide whether the judgment debtor covers the property and give the corresponding judgment according to the situation's seriousness conclusive evidence proving that the judgment debtor conceals the property. The judge can only treat the person who is incapable of execution and has no concealment of the property as no concealment of the property. Suppose there is no visible evidence pointing to the property concealment of the person subjected to execution. In that case, we can only further collect its relevant financial accounts, investigate its surrounding appropriate personnel, and make conclusions.

The construction of the evaluation function is an essential step of the 3WD method. The current research mainly focuses on constructing the evaluation function from conditional probability and loss function.

The most widely used evaluation function of the decision rough set is a conditional probability, which extends and expands the calculation method of conditional probability. Yao

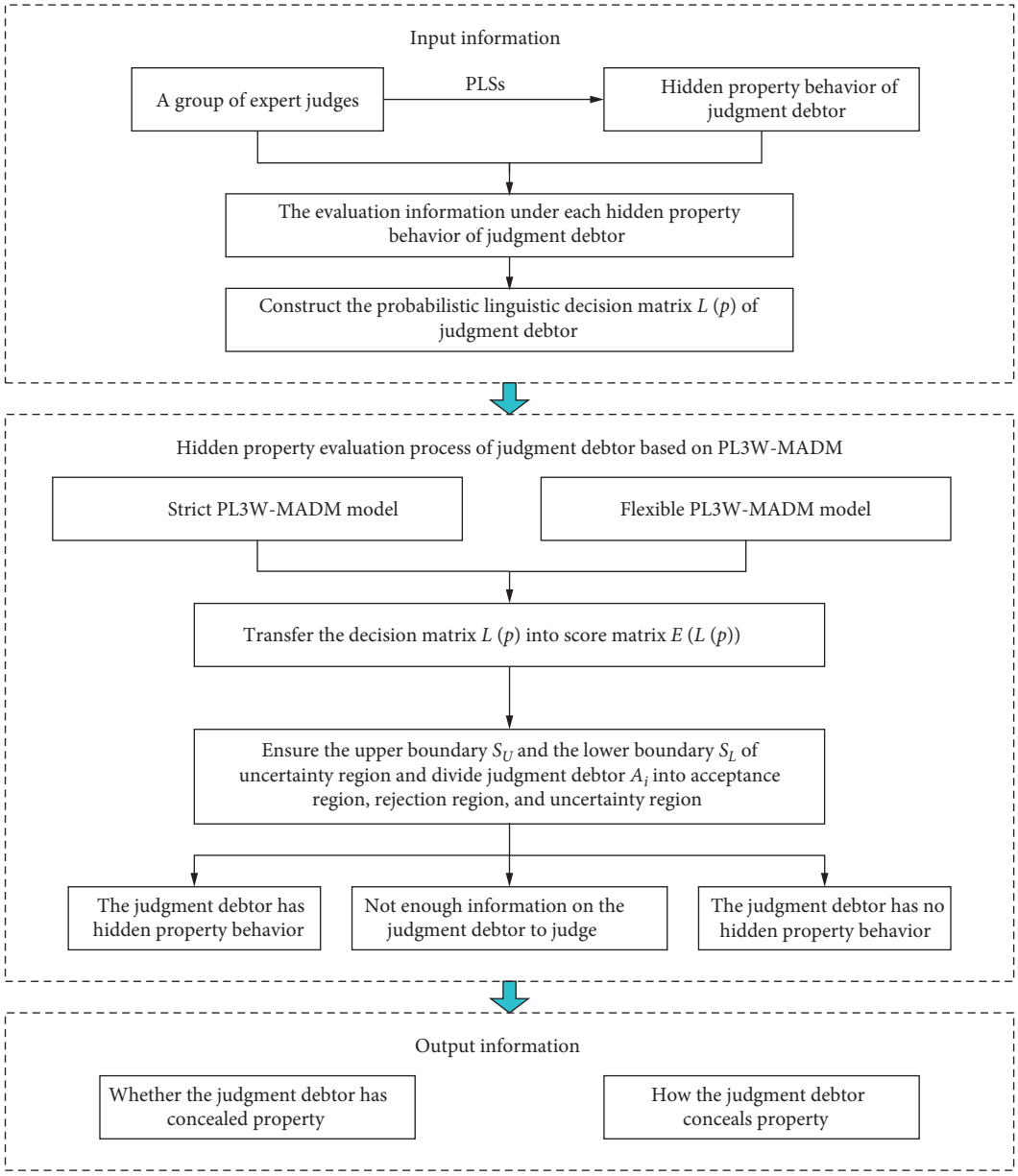

Figure 1: The flowchart of the hidden property evaluation based on the PL3W-MADM model.

et al. [11] proposed a naive Bayesian decision rough set model (NBRS) based on Naive Bayesian classification and rough decision set, which can estimate conditional probability based on Bayesian theory and naive probability independent hypothesis. Liu et al. [12] put forward a new method of discriminant analysis combining the theory of logical regression and decision rough set, using logical regression to estimate the conditional probability of rough decision set, and using decision rough set theory and Bayes decision procedure to calculate the threshold value, which provides a flexible mechanism of threshold interpretation for discriminant analysis. Jia et al. [13] proposed the attribute reduction method of decision risk minimization, which takes the conditional probability of each sample as the search space and takes the risk loss minimization as the goal. The loss function and threshold value obtained can minimize the decision risk.

In the loss function study, the decision rough set model's threshold parameters were mainly determined by a loss function, which plays an important role in three decision-making. Yu et al. [14] proposed a clustering mode cost evaluation method and a clustering effectiveness index, which helps to select the termination point of the clustering algorithm and helps to deal with overlapping cross clustering with different granularity. Herbert et al. [15–17] combined the loss function of rough decision set with the game theory of classification and measurement, played the game in two dimensions of accuracy and generality, and determined the balance point of the game through cooperation or competition to achieve the optimal goal of the size of each decision region. Yao and Deng [10] reduced the number of variables in the threshold by using the loss value with relative value characteristics and applying it to each model's threshold

expression. To find the optimal threshold value of the probabilistic rough set, Deng and Yao [18] introduced information theory, using information entropy as an uncertainty measure, and proposed an information theory method with uncertainty as an objective function and used to explain and determine the threshold value. To determine the probabilistic rough set's optimal threshold value, Azam and Yao [19] introduced the game theory to study the relationship between the probability threshold value and the uncertainty level of their impact on different regions and proposed a configuration mechanism to achieve the optimal threshold value. Liang et al. [20] and Liu et al. [21–23] introduced the loss function's uncertainty evaluation form. Considering that the decision rough set's loss values are in the order of triangular fuzzy number, probability distribution, interval number, and fuzzy number, a new decision rough set model is proposed, which widens the range of loss values. Liu et al. [24] developed a "four-level probabilistic rough set model" by studying the probabilistic rough set model generated under different loss function conditions and the relationship between them. This model can generate ten probabilistic rough set models and two models: two-way and three-way decisions. Liang et al. [25] introduced dual hesitation fuzzy set into decision rough set theory. They proposed a new three-way decision model: considering the loss function of the dual-hesitant fuzzy sets (DHFSs) based on a rough decision set, proposed dual-competency fuzzy decision rough set model. They analyzed the normalization principle of the loss function in dual hesitation fuzzy environment, studied some properties of expected loss, and designed two methods to drive the 3WD method by using a new model, which enriches the comparison of the expected loss. Liang et al. [26] introduced group decision-making into three decision-making of decision rough set theory and proposed three decision-making based on group decision-making. It focused on the measurement and analysis of loss function in-group decision-making environment. The principle of reasonable granularity adopts important and most experts' suggestions to measure each loss function, which provides a new explanation for determining the loss function. Liang and Liu [27] introduced a hesitant fuzzy set into decision rough set theory and proposed a new model of hesitant fuzzy decision rough sets (HFDTRSs). Under the hesitant fuzzy information, the properties of expected loss and their corresponding scores were studied carefully, which provides a solution for determining the loss of a rough decision set. Zhang et al. [28] combined 3WD with uncertain classification, defined two kinds of classification errors and two kinds of uncertain classification of the probabilistic rough set model, considered the cost parameters of two kinds of classification errors and two kinds of uncertain classification, got a pair of thresholds of probabilistic rough set model again, and proposed three decision-making models based on two kinds of uncertain classification. Liu et al. [29] proposed a three-way decision model based on the incomplete information system and defined a new relation to describe the similarity degree of incomplete information. Given the missing value in the incomplete information system, the loss function is obtained using an interval number. A mixed information table containing incomplete information and loss function is used to process the new three decision models. Ma and Lei [30–32] developed some hesitant fuzzy linguistic three-way decision methods to solve the green supplier selection problem.

To sum up, although the existing 3WD methods are relatively abundant, there are still the following places needing to be improved:

(1) With the development of 3WD theory and fuzzy theory, the 3WD method has been applied in the environment of the fuzzy number, interval number, intuitionistic fuzzy number, and hesitant fuzzy number. The fuzzy linguistic has advantages in the expression of uncertainty and fuzziness, especially in qualitative evaluation, such as evaluating whether the judgment debtor conceals the property in the case. It is urgent to extend the 3WD method to fuzzy linguistic, hesitant fuzzy linguistic and probabilistic linguistic environment to solve these problems.

(2) At present, the evaluation functions of the 3WD methods mainly use the loss function, which is tedious in the calculation, low inefficiency in the decision-making process. The score measure is relatively simple and feasible to calculate, and as an evaluation function, it can improve decision-making efficiency.

3. Preliminaries

3.1. 3WD Model. Liang et al. [33] extended the 3WD method to the multiattribute decision-making (MADM) problem. The 3WD models can be divided into single-attribute and multiattribute.

Definition 1 (see [34]). Let A is a fuzzy set in the set U, the membership function is $A(x)$, and $E(A)(x) = A(x)$, then the 3WD method is defined in the universe $(U, \mathrm{Map}(U, [0,1]), [0,1], E)$ as follows:

(1) Acceptance region: $\mathrm{ACP}_{(\alpha,\beta)}(E, A) = \{x \in U | E(A)(x) \geq \beta\}$,

(2) Rejection region: $\mathrm{REJ}_{(\alpha,\beta)}(E, A) = \{x \in U | E(A)(x) \leq \alpha\}$,

(3) Uncertain region: $\mathrm{UNC}_{(\alpha,\beta)}(E, A) = \{x \in U | \alpha < E(A)(x) < \beta\}$,

where $A \in \mathrm{Map}(V, L_C)$, $\alpha, \beta \in L_D$, and $0 \leq \alpha < \beta \leq 1$.

The 3WD method divides the objects into three regions: the acceptance region, the rejection region, and the uncertainty region. The three regions are denoted by $\mathrm{ACP}_{(\alpha,\beta)}(E, A)$, $\mathrm{REJ}_{(\alpha,\beta)}(E, A)$, and $\mathrm{UNC}_{(\alpha,\beta)}(E, A)$, respectively. When the membership value is numerical, the three regions of the three-way decision are shown in Figure 2.

In Figure 1, 0 and 1 represent the worst and best evaluations, respectively. α and β are the lower and upper limits of membership of the uncertainty region are shown, respectively. From the 3WD method, it can be seen that

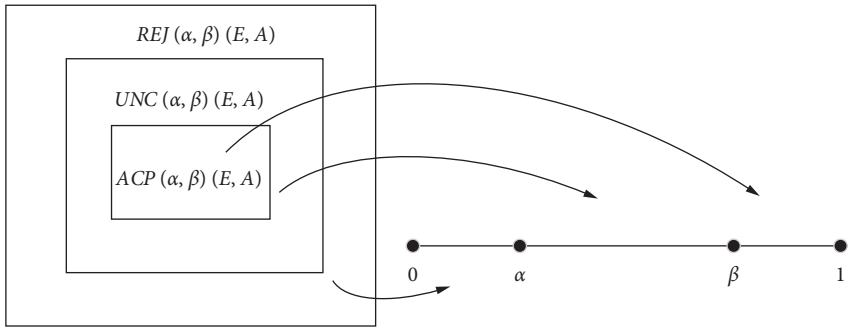

Figure 2: Three regions divided by 3WD.

evaluation targets with membership values less than the lower membership boundary α belong to the rejection region. Evaluation targets with membership values larger than the upper membership boundary β belong to the acceptance region. The rest of the targets have membership values between the lower membership boundary α and the upper membership boundary β and therefore belong to the uncertainty region.

Definition 1 applies only to the case of single-attribute decision-making and is called as three-way single-attribute decision-making (3W-SADM). When the evaluation object is multiattribute, we use the three-way multiattribute decision-making (3W-MADM) method.

Definition 2 (see [35]). Let $X = \{x_1, x_2, \ldots, x_m\}$ be a set of evaluated objects, $u_{ij}(x)$ $(j = 1, 2, \ldots, n)$ is the attribute value of the evaluated object x_i $(i = 1, 2, \ldots, m)$, $u_{0,j}$ is the threshold of the j-th attribute, and then the 3W-MADM method is defined as follows:

(1) Acceptance region: $\mathrm{ACP}_{u_0}(X) = \{x_i \in X | u_{ij}(x) \geq u_{0,j}\}$.
(2) Rejection region: $\mathrm{REJ}_{u_0}(X) = \{x_i \in X | u_{ij}(x) < u_{0,j}\}$.
(3) Uncertainty region: $\mathrm{UNC}_{u_0}(X) = (\mathrm{ACP}_{u_0}(X) \cup \mathrm{REJ}_{u_0}(X))^C$.

The definition of the 3W-MADM model shows that if the evaluated object is rated sufficiently good for all characteristics, it can be assigned directly to the acceptance region. Conversely, if the evaluated object is rated sufficiently poor for all characteristics, it can be assigned directly to the rejection region. However, some objects are evaluated with neither good enough nor bad enough ratings for all attributes, so they are initially assigned to the uncertainty region and need to be analyzed further.

Specifically, for some evaluated objects, the evaluated values of all characteristics are greater than or equal to the threshold, i.e., $u_{ij}(x) \geq u_{0,j}$, which means that the decision-maker accepts the evaluated object as belonging to the acceptance region in terms of all characteristics. For some evaluated objects, the evaluated values of all characteristics are less than the threshold, i.e., $u_{ij}(x) < u_{0,j}$, which means that the decision-maker rejects the evaluated object in terms of all characteristics. For other evaluated objects, some characteristics are greater than the threshold, and some characteristics are less than the threshold, which means that the decision-maker does not fully accept and does not fully reject the evaluated object and therefore classifies it into the uncertainty region.

3.2. PLTSs. Here, we introduce some concepts on the PLTSs.

Definition 3 (see [36, 37]). Let $S = \{s_t | t = 0, 1, \ldots, \tau\}$ be a linguistic term set, s_α and s_β be two linguistic terms, $s_\alpha, s_\beta \in S$, $\lambda_1, \lambda_2 \in [0, 1]$, and the operation laws are defined as

(1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$,
(2) $\lambda s_\alpha = s_{\lambda\alpha}$,
(3) $\lambda(s_\alpha + s_\beta) = \lambda s_\alpha + \lambda s_\beta$,
(4) $\lambda_1 s_\alpha + \lambda_2 s_\beta = s_{\lambda_1\alpha+\lambda_2\beta}$.

To help the decision-maker evaluate the alternative more accurately, Pang et al. [38] proposed the concept of probabilistic linguistic term sets (PLTSs), which requires the decision-maker to give his preference for each linguistic term while using the linguistic term evaluating the alternative. The definitions of PLTSs are as follows:

Definition 4 (see [38]). Let $S = \{s_t | t = 0, 1, \ldots, \tau\}$ is a set of linguistic terms, then the PLTS is defined as follows:

$$L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) | L^{(k)} \in S, p^{(k)} \geq 0, \quad k = 1, 2, \ldots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1 \right\}, \tag{1}$$

where $L^{(k)}$ is a linguistic term in $L^{(k)}(p^{(k)})$, $p^{(k)}$ is the probability of $L^{(k)}$, and $\#L(p)$ is the number of the linguistic terms.

Definition 5 (see [38]). Let $L(p)$ is a PLTS, $\sum_{k=1}^{\#L(p)} p^{(k)} < 1$, and the standardized PLTS is defined as $\overline{L}(p) = \{L^{(k)}(\overline{p}(k)) | k = 1, 2, \ldots, \#L(p)\}$, where $\overline{p}^{(k)} = p^{(k)} / \sum_{k=1}^{\#L(p)} p^{(k)}$.

Definition 6 (see [38–40]). For the three standardized PLTSs $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$, $L_1(p) = \{L_1^{(k_1)}(p_1^{(k_1)}) | k_1 = 1, 2, \ldots, \#L_1(p)\}$, and $L_2(p) = \{L_2^{(k_2)}(p_2^{(k_2)}) | k_2 = 1, 2, \ldots, \#L_2(p)\}$, $\gamma \in [0, 1]$, λ is a positive real number, $\eta^{(k)} \in g(L(p))$, $\eta_1^{(k_1)} \in g(L_1(p))$, $\eta_2^{(k_2)} \in g(L_2(p))$, and the transfer function g is defined as

$$\begin{aligned} &g\colon [0, \tau] \longrightarrow [0, 1], \\ &g(L(p)) = \left\{\frac{r^{(k)}}{\tau}\left(p^{(k)}\right)\right\} = L_\gamma(p), \\ &g^{-1}\colon [0, 1] \longrightarrow [0, \tau], \\ &g^{-1}\left(L_\gamma(p)\right) = \left\{s_{\gamma\tau}\left(p^{(\gamma)}\right) | \gamma \in [0, 1]\right\} = L(p), \end{aligned} \tag{2}$$

where $\eta^{(k)} = r^{(k)}/\tau$, $r^{(k)}$ is the subscript of the linguistic term $L^{(k)}$.

The operation laws of PLTSs are defined as follows:

(1) $\lambda L(p) = g^{-1}(\cup_{\eta^{(k)} \in g(L(p))}\{(1 - (1 - \eta^{(k)})^\lambda)(p^{(k)})\})$,
(2) $L^\lambda(p) = g^{-1}(\cup_{\eta^{(k)} \in g(L(p))}\{(\eta^{(k)})^\lambda (p^{(k)})\})$,
(3) $L_1(p) \oplus L_2(p) = g^{-1}(\cup_{\eta_1^{(k_1)} \in g(L_1(p)), \eta_2^{(k_2)} \in g(L_2(p))}\{(\eta_1^{(k_1)} + \eta_2^{(k_2)} - \eta_1^{(k_1)}\eta_2^{(k_2)})(p_1^{(k_1)} p_2^{(k_2)})\})$,
(4) $L_1(p) \otimes L_2(p) = g^{-1}(\cup_{\eta_1^{(k_1)} \in g(L_1(p)), \eta_2^{(k_2)} \in g(L_2(p))}\{\Pi(p_1^{(k_1)} p_2^{(k_2)})\})$,

where

$$\Pi = \begin{cases} \dfrac{\eta_1^{(k_1)} - \eta_2^{(k_2)}}{1 - \eta_2^{(k_2)}}, & \left(\eta_1^{(k_1)} \geq \eta_2^{(k_2)}, \eta_2^{(k_2)} \neq 1\right), \\ 0. \end{cases} \tag{3}$$

Definition 7 (see [38]). For the PLTS $L(p) = \{L^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#L(p)\}$, $r^{(k)}$ is the subscript of the linguistic term $L^{(k)}$, and the score function of $L(p)$ is defined as

$$E(L(p)) = s_{\overline{\alpha}}, \tag{4}$$

where $\overline{\alpha} = \sum_{k=1}^{\#L(p)} r^{(k)} p^{(k)} / \sum_{k=1}^{\#L(p)} p^{(k)}$.

For any two PLTSs $L_1(p)$ and $L_2(p)$, if $E(L_1(p)) > E(L_1(p))$, then $L_1(p) \succ L_2(p)$, $L_1(p)$ is better than $L_2(p)$; if $E(L_1(p)) < E(L_1(p))$, then $L_1(p) \prec L_2(p)$, $L_2(p)$ is better than $L_1(p)$.

Definition 8 (see [41]). For any two standardized PLTSs $L_1(p)$ and $L_2(p)$, the distance between $L_1(p)$ and $L_2(p)$ is defined as

$$d(L_1(p), L_2(p)) = \sqrt{\frac{\sum_{k=1}^{\#L_1(p)}\left(r_1^{(k)} p_1^{(k)} - r_2^{(k)} p_2^{(k)}\right)}{\#L_1(p)}}, \tag{5}$$

where $r_1^{(k)}$ and $r_2^{(k)}$ are the subscripts of $L_1(p)$ and $L_2(p)$, respectively.

Definition 9 (see [41]). Let $L_j(p) = \{L_j^{(k)}(p_j^{(k)}) | k = 1, 2, \ldots, \#L_j(p)\}$ be a set of PLTSs, $j = 1, 2, \ldots, n$, and the probabilistic linguistic weighted aggregate (PLWA) operator is defined as

$$\mathrm{PLWA}(L_1(p), L_2(p), \ldots, L_n(p)) = \sum_{j=1}^{n} w_j L_j(p) = g^{-1}\left(\bigcup_{\eta_j^{(k)} \in g(L_j(p))}\left\{\left(1 - \prod_{j=1}^{n}\left(1 - \eta_j^{(k)}\right)^{w_j}\right)\left(\prod_{g=1}^{t} p_j^{(k)}\right)\right\}\right), \tag{6}$$

where w_j is the weight of $L_j(p)$.

4. The 3WD Models under Probabilistic Linguistic Environment

4.1. The 3WD Model under Probabilistic Linguistic Environment. In the multiattribute decision-making problem under the probabilistic linguistic environment, if the decision-makers agree that all the attributes of the alternative are excellent, they will choose to accept the alternative directly; if the decision-makers agree that all the attributes of the alternative are evil, they will choose to reject the alternative directly. For the other attributes, they are neither good enough nor bad enough, decision-makers often hesitate to decide whether to accept or reject the alternatives completely. Based on this idea, we put forward 3W-MADM models in the probabilistic linguistic environment, which can be regarded as the fusion of probabilistic linguistic evaluation information and 3W-MADM method, called it the PL3W-MADM method.

Definition 10. Let $L_{ij}(p_{ij})$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ be the probabilistic linguistic evaluation value of alternative A_i $(i = 1, 2, \ldots, m)$ under the attribute C_j $(j = 1, 2, \ldots, n)$, $L_{ij}(p_{ij})$ is taken from the linguistic term set $S = \{s_0, s_1, \ldots, s_k\}$, $E(L_{ij}(p_{ij}))$ is the score measure of $L_{ij}(p_{ij})$ and $s_{0,j} \leq s_{L,j} < s_{U,j} \leq s_{k,j}$, and then the PL3W-MADM is defined as follows:

Acceptance region: $\mathrm{REJ}_{(S_L, S_U)}(E, A_i) = \{x_j \in U_j | E(L_i(p)) \prec S_L\}$,

Rejection region: $\mathrm{ACP}_{(S_L,S_U)}(E, L_i(p)) = \{x_j \in U_j | E(L_i(p))(x_j) \succ S_U\}$,

Uncertainty region: $\mathrm{UNC}_{(S_L,S_U)}(E, L_j(p)) = (\mathrm{ACP}_{(S_L,S_U)}(E, L_j(p)) \cup \mathrm{REJ}_{(S_L,S_U)}(E, L_j(p))^C$,

where S_L, S_U are the lower boundary and upper boundary of the uncertainty region.

The PL3W-MADM model can divide the targets into three regions, which is shown in Figure 3.

From Figure 3, the evaluation function is expressed by a probabilistic linguistic variable. $\mathrm{REJ}(L(p))$, $\mathrm{ACP}(L(p))$, and $\mathrm{UNC}(L(p))$ are rejection region, acceptance region, and uncertainty region. $S_0 = \{s_{0,1}, s_{0,2}, \ldots, s_{0,n}\}$ and $S_k = \{s_{k,1}, s_{k,2}, \ldots, s_{k,n}\}$ are the worst and best evaluation values for the targets. $S_L = \{s_{L,1}, s_{L,2}, \ldots, s_{L,n}\}$ and $S_U = \{s_{U,1}, s_{U,2}, \ldots, s_{U,n}\}$ are the lower and upper boundaries of the uncertainty region. When the evaluation values of the targets are lower than S_L, then the targets are divided into rejection region; when the evaluation values are larger than S_U, then the targets are divided into acceptance region; the other targets are divided into uncertainty region.

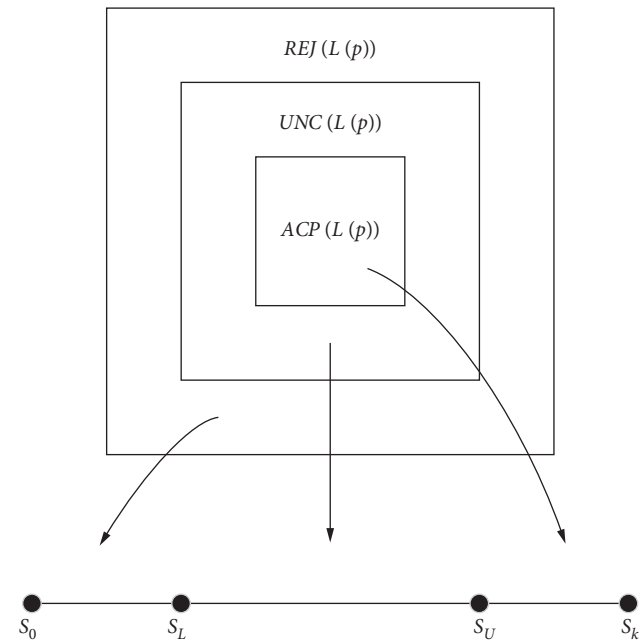

Figure 3: The three regions divided by the PL3W-MADM model.

Example 1. A group of 10 expert judges makes an empirical judgment on whether the 10 judgment debtors have concealed property in terms of their operational behaviour in terms of bank accounts, real estate, etc. It is appropriate to use PLTSs to integrate the group's evaluation of each of the judgment debtor's attributes. Assume that $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ = {very unlikely, unlikely, unlikely, general, likely, very likely, extremely likely} is the linguistic term set to evaluate each characteristic of the judgment debtor, and $s_L = s_2$ and $s_U = s_4$ are the lower and upper bounds of the uncertainty region provided by the expert judges. A judgment debtor A_i has frequent bank account transactions and property transfers to other people during the case. The values of the bank account and real estate operations are $\{s_5(0.6), s_6(0.4)\}$ and $\{s_6(1)\}$, respectively, which means that 60% of the expert judges think that the judgment debtor is likely to conceal or transfer property through the bank account, and 40% believe that the judgment debtor is highly likely to conceal or transfer property through the bank account. At the same time, all expert judges believe that the judgment debtor is very likely to conceal property by transferring property; the scores of these two evaluation values are $s_{5.4}$ and s_6, which are greater than the upper bound s_4. Therefore, the judgment debtor is put into the acceptance region and is considered to have concealed property. Suppose the evaluation values are $\{s_1(0.7), s_2(0.3)\}$ and $\{s_1(1)\}$. That is, 70% of the expert judges think it is impossible for the judgment debtor to hide property through bank accounts, and 30% think it is unlikely, while all expert judges think it is impossible for the judgment debtor to transfer property through real estate. The evaluation values are scored as $s_{1.3}$ and s_1, which are less than the lower bound s_2. Therefore, the judgment debtor is divided into a rejection region and is considered not to have hidden property. In the case of the other evaluation values, it was considered that the expert judges are unable to determine whether the judgment debtor has concealed property, and further confirmation is needed based on other information.

4.2. Strict PL3W-MADM Model. For the MADM problem, the attribute values are evaluated by a group of experts. Therefore, the integrated expert group's evaluation value is reasonable in the form of a probabilistic linguistic variable. We provide the concrete steps of the PL3W-MADM method. According to the expert group members' different attitudes, we divide it into the strict PL3W-MADM method and the flexible PL3W-MADM method.

If the expert group's attitude is conservative, that is to say, the evaluation value of each attribute must meet certain conditions to accept or reject the alternative. In this case, we propose the PL3W-MADM model.

Let $L_{ij}(p_{ij})$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ be the evaluation value of the alternative A_i $(i = 1, 2, \ldots, m)$ under the attribute A_i $(i = 1, 2, \ldots, m)$, and $L_{ij}(p_{ij})$ is taken from the linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ = {very unlikely, unlikely, unlikely, general, likely, very likely, extremely likely}. The decision matrix of the PL3W-MADM model is shown in Table 1.

Next, we provide the steps of the strict PL3W-MADM model as follows:

Step 1: construct the probabilistic linguistic decision matrix $L(p) = (L_{ij}(p_{ij}))_{m\times n}$.

Step 2: transfer the decision matrix into the score matrix $E(L_{ij}(p_{ij}))$.

Step 3: ensure the upper boundary s_U and the lower boundary s_L, and then obtain their corresponding rejection region, acceptance region and uncertainty region.

Table 1: Decision matrix of the PL3W-MADM model.

	C_1	C_2	...	C_n
A_1	$L_{11}(p_{11})$	$L_{12}(p_{12})$	...	$L_{1n}(p_{1n})$
A_2	$L_{21}(p_{21})$	$L_{22}(p_{22})$	...	$L_{2n}(p_{2n})$
...	...	...	...	...
A_m	$L_{m1}(p_{m1})$	$L_{m2}(p_{m2})$	...	$L_{mn}(p_{mn})$

Assume that the expert group members agree that the uncertainty region's lower and upper boundaries are s_L and s_U, respectively. Therefore, for any attribute C_j of an alternative A_i, if all the evaluated values $E(L_{ij}(p_{ij}))\prec s_{L,j}$, then $A_i \in \mathrm{REJ}_{(s_L,s_U)}$; namely, all the evaluated values are less than the lower boundary $s_{L,j}$, then the alternative A_i is rejected and divided into the rejection region. If all the evaluated values $E(L_{ij}(p_{ij}))\succ s_{U,j}$, then $A_i \in \mathrm{ACP}_{(s_L,s_U)}$; namely, all the evaluated values are greater than the upper bound $s_{U,j}$, then the alternative A_i is accepted and divided into the acceptance region. The other alternatives fall into the uncertainty region.

Step 4: transfer the attribute value in the uncertainty region into a fuzzy number and construct the fuzzy decision matrix.

The linguistic term set S in the certainty region is transferred into triangle fuzzy number by the method [42–46] in Table 2.

Step 5: aggregate the fuzzy information of the alternative A_i in the uncertainty region and obtain the weighted evaluation value V_i.

Let w_j be the weight of each attribute C_j $(j = 1, 2, \ldots, n)$ and satisfy $\sum_{j=1}^{n} w_j = 1$. The weighted evaluation value V_i of the alternative A_i is calculated by $V_i = WA(u_{i1}, u_{i2}, \ldots, u_{in}) = \sum_{j=1}^{n} w_j E(L_{ij}(p_{ij}))$.

Step 6: defuzzify the weighted evaluation value of the alternative A_i and obtain the defuzzified value m_i, and sort the alternatives by descending order.

Let $\tilde{a} = (a^L, a^M, a^N)$ be a triangle fuzzy number, and the defuzzied value $m(\tilde{a})$ of $\tilde{a}$ is defined as

$$m(\tilde{a}) = \frac{a^L + 2a^M + a^N}{4}. \tag{7}$$

Let $\tilde{a}_1 = (a_1^L, a_1^M, a_1^N)$ and $\tilde{a}_2 = (a_2^L, a_2^M, a_2^N)$ be the two triangle fuzzy numbers, if $m(\tilde{a}_1) > m(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$.

The flowchart of strict PL3W-MADM model is shown in Figure 4.

4.3. Flexible PL3W-MADM Model. If the expert judge group has a more flexible attitude, i.e., if at least one attribute value reaches a certain condition, the alternative will be accepted or rejected. In this case, we propose the flexible PL3W-MADM model.

Similar to the strict PL3W-MADM model, we can also provide the flexible PL3W-MADM model with the following specific decision steps:

Table 2: Linguistic term set and their corresponding triangle fuzzy number.

Linguistic terms	The corresponding triangle fuzzy number
s_0	(0, 0, 0.17)
s_1	(0, 0.17, 0.33)
s_2	(0.17, 0.33, 0.5)
s_3	(0.33, 0.5, 0.67)
s_4	(0.5, 0.67, 0.83)
s_5	(0.67, 0.83, 1)
s_6	(0.83, 1, 1)

Step 1: construct the probabilistic linguistic decision matrix $L(p) = (L_{ij}(p_{ij}))_{m\times n}$.

Step 2: transfer the decision matrix into the score matrix $E(L_{ij}(p_{ij}))$.

Step 3: ensure the upper boundary s_U and the lower boundary s_L, and then obtain their corresponding rejection region, acceptance region and uncertainty region.

Assume that the expert group members agree that the lower and upper boundaries of the uncertainty region are s_L and s_U, respectively. Therefore, for any attribute C_j of an alternative A_i, if an evaluated value exists $E(L_{ij}(p_{ij}))\prec s_{L,j}$ and all the evaluated values $E(L_{ij}(p_{ij}))\prec s_{U,j}$, then $A_i \in \mathrm{REJ}_{(s_L,s_U)}$; namely, the alternative A_i can be rejected with a high degree of certainty from at least one attribute and not certain accepted considering all the attributes, then the alternative A_i is rejected and divided into the rejection region. If an evaluated value exists $E(L_{ij}(p_{ij}))\succ s_{U,j}$ and all the evaluated values $E(L_{ij}(p_{ij}))\succ s_{L,j}$, then $A_i \in \mathrm{ACP}_{(s_L,s_U)}$; namely, the alternative A_i can be accepted with a high degree of certainty from at least one attribute and not certain rejected considering all the attributes, then the alternative A_i is accepted and divided into the acceptance region. The other alternatives fall into the uncertainty region.

Step 4: transfer the attribute value in the uncertainty region into a fuzzy number and construct the fuzzy decision matrix.

Step 5: aggregate the fuzzy information of the alternative A_i in the uncertainty region and obtain the weighted evaluation value V_i.

Step 6: defuzzify the weighted evaluation value of the alternative A_i and obtain the defuzzified value m_i, and sort the alternatives by descending order.

The flowchart of the flexible PL3W-MADM model is shown in Figure 5.

5. The Developed Hidden Property Evaluation Model of Judgment Debtor Based on PL3W-MADM Model

The developed PL3W-MADM method is used to evaluate the hidden property of judgment debtors. The advantages of the developed method are as follows. Firstly, the fuzzy

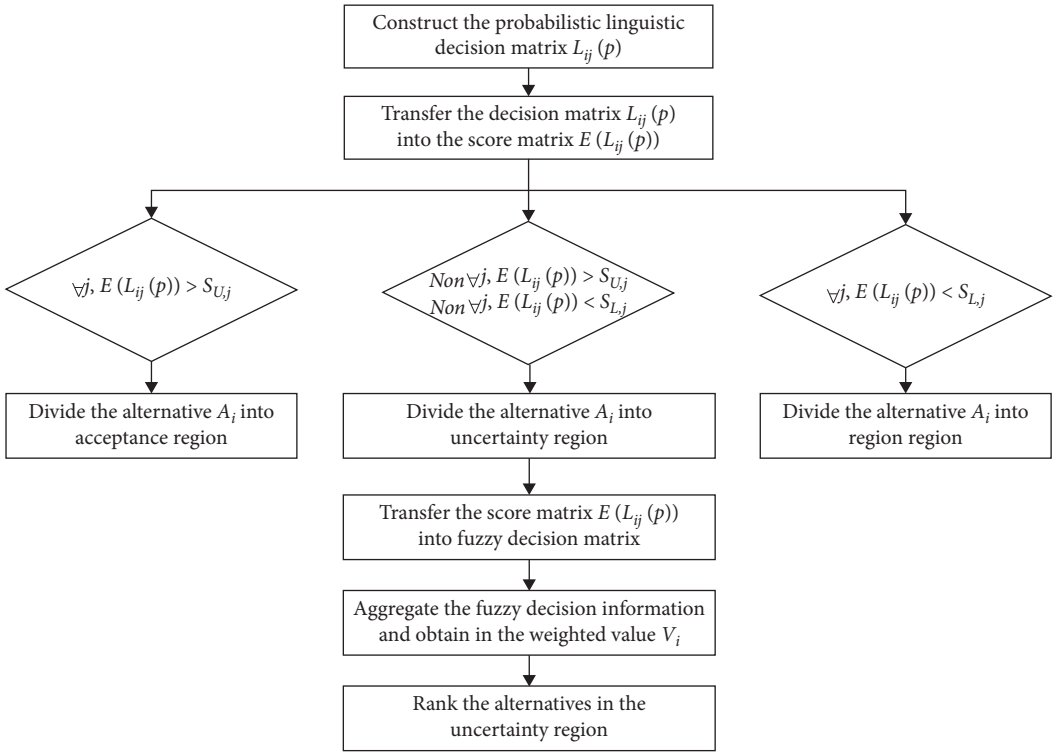

Figure 4: The flowchart of the strict PL3W-MADM model.

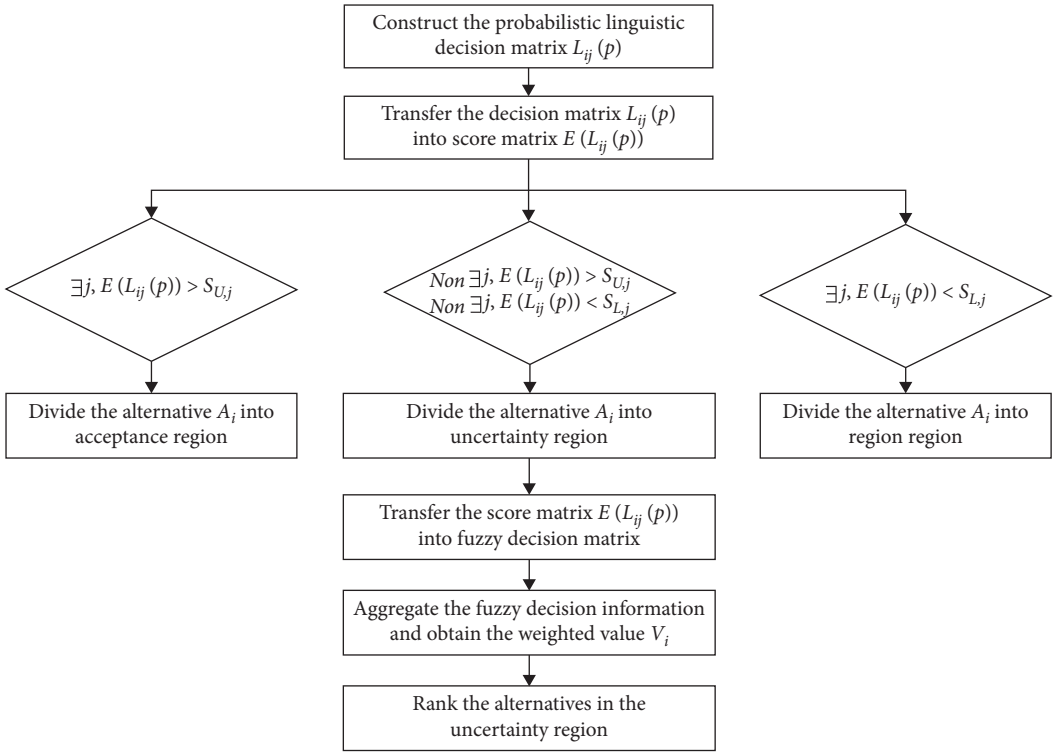

Figure 5: The flowchart of the hidden property evaluation of judgment debtor based on the PL3W-MADM model.

linguistic variables are used to express the uncertainty of the judges' evaluation information on the judgment debtors. Secondly, the PLTSs have some advantages of retaining information from multiple expert judges' assessments and their probabilities. Thirdly, the developed PL3W-MADM method is more suitable and reasonable for evaluating the hidden property of judgment debtors. The concealment property evaluation model's special nature is considered, i.e.,

as long as one concealment property behaviour is particularly suspicious. The judgment debtor can be considered to have concealed property, and in the case that all concealment methods are not suspicious, then the judgment debtor can be considered not to have concealed property. In other words, the rejection region uses the strict PL3W-MADM model, and the acceptance region uses the flexible PL3W-MADM model.

5.1. Analysis of the Behaviour Mode of Concealing Property. To avoid execution, the person subjected to execution often covers and transfers the property in various ways. Through the summary and analysis of the existing sample cases, it is found that the measures taken by the execute to avoid execution are flexible and diverse, from the subject property to the real estate to the right, from divorce to pawn to cancellation of account. According to the way of property discovery, we mainly divide the behaviour of concealed property into seven categories: bank account number, real estate (housing, land, etc.), movable property (motor vehicle, etc.), equity (including stock), insurance account, Alipay account, and other property or way, as shown in Table 3.

Referring to Table 3, the expert team composed of the expert judge evaluates the judgment debtor according to each concealment behaviour's number and degree. The linguistic term set of the expert judge's evaluation linguistic for the various hidden behaviours of the judgment debtor is $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{$extremely unlikely, unlikely, unlikely, general, likely, very likely, very likely$\}$.

5.2. Hidden Property Evaluation Model of the Judgment Debtor. Assuming that there are n types of concealment property behaviours, the expert group evaluates the various concealment property behaviour C_j $(j = 1, 2, \ldots, n)$ of the judgment debtor A_i $(i = 1, 2, \ldots, m)$, with the values expressed in PLTSs. The attribute values $L_{ij}(p_{ij})$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ are taken from the linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{$very unlikely, unlikely, unlikely, general, likely, very likely, highly likely$\}$. The steps of the hidden property evaluation model are as follows:

Step 1: construct the probabilistic linguistic multi-attribute decision matrix $L(p) = (L_{ij}(p_{ij}))_{m\times n}$ of judgment debtors.

Step 2: transfer the decision matrix $L(p) = (L_{ij}(p_{ij}))_{m\times n}$ into the score matrix $(E(L(p)))_{m\times n}$.

Step 3: ensure the upper and lower boundaries of the uncertainty region and divide the universe into rejection region, acceptance region, and uncertainty region.

Assume that the expert group members agree that the lower and upper boundaries of the uncertainty region are s_L and s_U, respectively. Therefore, for any hidden property behaviour C_j of a judgment debtor A_i, if an attribute value exists $E(L_{ij}(p_{ij})) \succ s_{U,j}$, then $A_i \in \mathrm{ACP}_{(s_L, s_U)}$; namely, the expert group members agree that at least one concealment property behaviour is evident and therefore the judgment debtor A_i has concealed property and are placed in the acceptance region. If all the evaluated values $E(L_{ij}(p_{ij})) \prec s_{U,j}$, then $A_i \in \mathrm{REJ}_{(s_L, s_U)}$; namely, no concealment property behaviours of the judgment debtor A_i have been identified so far, and the judgment debtor A_i is divided into the rejection region. Other judgment debtors cannot be judged for the time being on the basis of the information currently available as to whether or not they have concealed property and will gather information for further confirmation, and they are therefore classified in the uncertainty region.

Step 4: transfer the attribute value of the judgment debtors in the uncertainty region into a fuzzy number, and construct the fuzzy decision matrix.

Step 5: aggregate the fuzzy information of the judgment debtors in the uncertainty region and calculate the weighted evaluation value V_i.

Step 6: defuzzify the evaluation values of judgment debtors and obtain the defuzzified value m_i of judgment debtor A_i, and sort by descending order.

The flowchart of the hidden property evaluation model of judgment debtors is shown in Figure 6.

6. Numerical Example

6.1. The Process of the Hidden Property Evaluation. We demonstrate the validity of the hidden property evaluation model by using the example of whether the judgment debtor has concealed property behaviour. There are 10 judgment debtors (A_1–A_{10}), and the ways of concealing property behaviour are classified into seven types, including bank accounts (C_1), real estates such as houses and land (C_2), movable property such as motor vehicles (C_3), equity (including shares) (C_4), insurance accounts (C_5), PayPal accounts (C_6), and other property or ways (C_7). A team of experts formed by the enforcement judge assesses the judgment debtor according to the number and degree of each concealment method behaviour, with the set of assessment language terms $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{$very unlikely, unlikely, unlikely, general, likely, very likely, extremely likely$\}$. The steps of the concealed property behaviour evaluation model are as follows.

Step 1. Construct the probabilistic linguistic multiattribute decision matrix $L(p) = (L_{ij}(p_{ij}))_{m\times n}$ of judgment debtors, and the decision matrix is shown in Table 4. For example, the judgment debtor A_1, the operation of bank accounts $L_{11}(p_{11})$, and real estate $L_{12}(p_{12})$ are evaluated at $\{s_5(0.6), s_6(0.4)\}$ and $\{s_6(1)\}$, respectively. It mainly refers to the frequent transactions of bank accounts and the transfer of property to other people during the litigation of bank account cases. 60% of the expert judges believe that the judgment debtor is likely to conceal or transfer property through bank accounts, and 40% believe that the judgment debtor is highly likely to conceal or transfer property

Table 3: Summary of concealing property behaviours.

Categories	Behaviour of concealing property
Bank account	1. During or after the litigation, the bank account has a large number of capital transactions; 2. Cancel or change grain subsidy account, wage account, and tax refund account; 3. Borrow other people's accounts; 4. Use the frozen account for consumption transfer; 5. Repayment of credit card account after consumption, etc.
Real estate (house, land, etc.)	1. Transfer the home and property to another person's name; 2. House and land acquisition and relocation; 3. Lease and subcontract of homes and land; 4. Hold or disposing of unregistered houses and land; 5. Set up false creditor's rights to make the house mortgage again, etc.
Movable property (motor vehicle, etc.)	1. Transferring the car to another person's name; 2. Concealing, transferring, or refusing to hand over the seized vehicle; 3. The vehicle is mortgaged, pledged, or the debt is paid with the property; 4. Still repaying a large number of car loans; 5. Moving away, selling off, leasing ships, mechanical equipment, and other movable properties.
Shares	Transferring to other people's names intentionally or maliciously.
Insurance account	1. Concealing insurance claims; 2. Concealing investment income insurance or commercial insurance.
Alipay account	Transferring through other Alipay account.
Other property or means	1. Giving up the right of inheritance; 2. Transferring critical assets to others through the countersigning of the debt settlement agreement; 3. Advancing supervision expenses but not used for repayment; 4. Agricultural machinery subsidies; 5. Concealing the winning amount of the lottery.

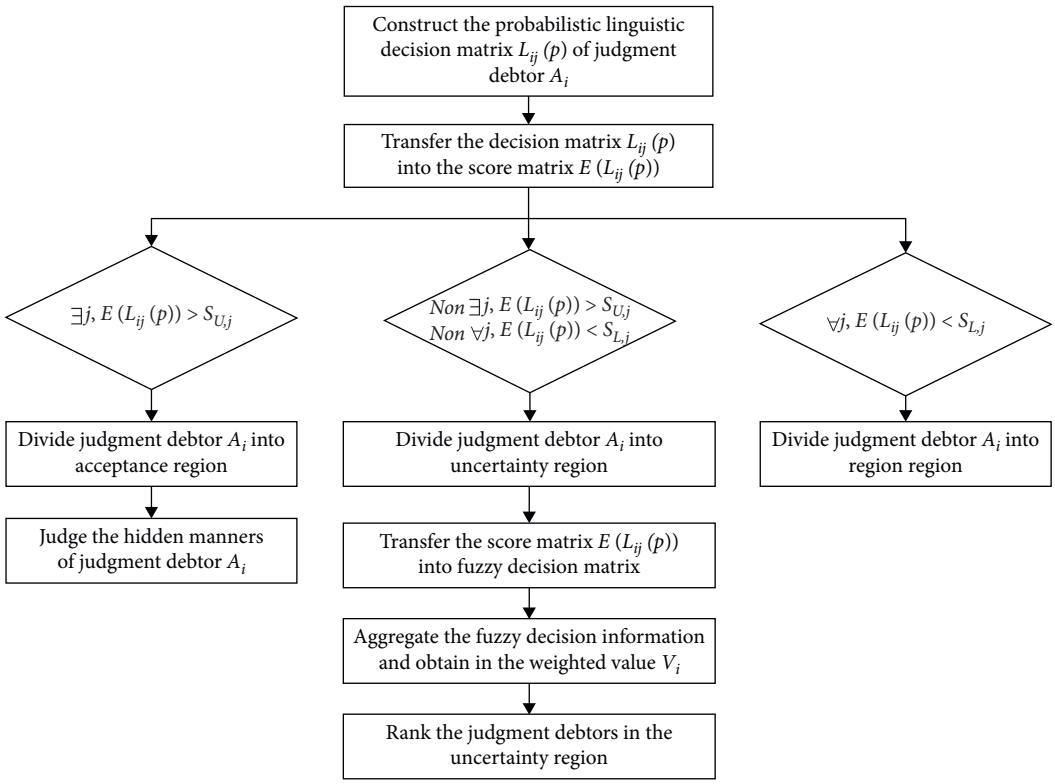

Figure 6: Flowchart of hidden property evaluation of judgment debtors.

Table 4: The probabilistic decision matrix of judgment debtors.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	$\{(s_5, 0.6), (s_6, 0.4)\}$	$\{(s_4, 0.4), (s_5, 0.4), (s_6, 0.2)\}$	$\{(s_4, 0.9), (s_5, 0.1)\}$	$\{(s_5, 0.7), (s_6, 0.3)\}$	$\{(s_5, 0.4), (s_6, 0.6)\}$	$\{(s_3, 0.1), (s_4, 0.7), (s_5, 0.2)\}$	$\{(s_5, 0.6), (s_6, 0.4)\}$
A_2	$\{(s_5, 0.7), (s_6, 0.3)\}$	$\{(s_3, 0.3), (s_4, 0.5), (s_6, 0.2)\}$	$\{(s_3, 0.3), (s_4, 0.6), (s_5, 0.1)\}$	$\{(s_5, 0.5), (s_6, 0.5)\}$	$\{(s_4, 0.3), (s_5, 0.7)\}$	$\{(s_4, 0.3), (s_5, 0.5), (s_6, 0.2)\}$	$\{(s_5, 0.3), (s_6, 0.7)\}$
A_3	$\{(s_4, 0.5), (s_5, 0.5\}$	$\{(s_3, 0.2), (s_4, 0.6), (s_5, 0.2)\}$	$\{(s_3, 0.3), (s_4, 0.5), (s_5, 0.2)\}$	$\{(s_3, 0.2), (s_4, 0.6), (s_5, 0.2)\}$	$\{(s_4, 0.8), (s_5, 0.2)\}$	$\{(s_3, 0.2), (s_4, 0.5), (s_5, 0.3)\}$	$\{(s_3, 0.4), (s_4, 0.6)\}$
A_4	$\{(s_4, 0.7), (s_5, 0.3)\}$	$\{(s_4, 0.7), (s_5, 0.3)\}$	$\{(s_4, 0.6), (s_5, 0.4)\}$	$\{(s_4, 0.7), (s_5, 0.3)\}$	$\{(s_4, 0.5), (s_5, 0.5)\}$	$\{(s_3, 0.2), (s_4, 0.5), (s_5, 0.3)\}$	$\{(s_2, 0.4), (s_3, 0.4), (s_4, 0.2)\}$
A_5	$\{(s_3, 0.6), (s_4, 0.4)\}$	$\{(s_1, 0.4), (s_2, 0.4), (s_3, 0.2)\}$	$\{(s_2, 0.9), (s_3, 0.1)\}$	$\{(s_2, 0.7), (s_3, 0.3)\}$	$\{(s_3, 0.4), (s_4, 0.6)\}$	$\{(s_1, 0.1), (s_2, 0.7), (s_3, 0.2)\}$	$\{(s_3, 0.6), (s_4, 0.4)\}$
A_6	$\{(s_3, 0.7), (s_4, 0.3)\}$	$\{(s_1, 0.3), (s_2, 0.5), (s_3, 0.2)\}$	$\{(s_1, 0.3), (s_2, 0.6), (s_3, 0.1)\}$	$\{(s_3, 0.5), (s_4, 0.5)\}$	$\{(s_2, 0.4), (s_3, 0.6)\}$	$\{(s_2, 0.3), (s_3, 0.5), (s_4, 0.2)\}$	$\{(s_3, 0.3), (s_4, 0.7)\}$
A_7	$\{(s_3, 0.5), (s_4, 0.5\}$	$\{(s_2, 0.2), (s_3, 0.6), (s_4, 0.2)\}$	$\{(s_2, 0.3), (s_3, 0.5), (s_4, 0.2)\}$	$\{(s_2, 0.2), (s_3, 0.6), (s_4, 0.2)\}$	$\{(s_3, 0.8), (s_4, 0.2)\}$	$\{(s_2, 0.2), (s_3, 0.5), (s_4, 0.3)\}$	$\{(s_2, 0.4), (s_3, 0.6)\}$
A_8	$\{(s_3, 0.7), (s_4, 0.3)\}$	$\{(s_3, 0.7), (s_4, 0.3)\}$	$\{(s_3, 0.6), (s_4, 0.4)\}$	$\{(s_1, 0.7), (s_2, 0.3)\}$	$\{(s_2, 0.5), (s_3, 0.5)\}$	$\{(s_2, 0.2), (s_3, 0.5), (s_4, 0.3)\}$	$\{(s_1, 0.4), (s_2, 0.4), (s_3, 0.2)\}$
A_9	$\{(s_1, 0.6), (s_2, 0.4)\}$	$\{(s_0, 0.4), (s_1, 0.4), (s_2, 0.2)\}$	$\{(s_0, 0.9), (s_1, 0.1)\}$	$\{(s_1, 0.7), (s_2, 0.3)\}$	$\{(s_1, 0.3), (s_2, 0.7)\}$	$\{(s_0, 0.1), (s_1, 0.7), (s_2, 0.2)\}$	$\{(s_1, 0.6), (s_2, 0.4)\}$
A_{10}	$\{(s_2, 0.7), (s_3, 0.3)\}$	$\{(s_0, 0.3), (s_1, 0.5), (s_2, 0.2)\}$	$\{(s_0, 0.3), (s_1, 0.6), (s_2, 0.1)\}$	$\{(s_1, 0.5), (s_2, 0.5)\}$	$\{(s_1, 0.4), (s_2, 0.6)\}$	$\{(s_1, 0.3), (s_2, 0.5), (s_3, 0.2)\}$	$\{(s_1, 0.3), (s_2, 0.7)\}$

through bank accounts, while all expert judges believe that the judgment debtor is highly likely to conceal property through the transfer of property.

Step 2. Transfer the decision matrix $L(p) = (L_{ij}(p_{ij}))_{m\times n}$ into the score matrix $(E(L(p)))_{m\times n}$, and the score matrix is shown Table 5.

Step 3. Ensure the upper and lower boundaries of the uncertainty region and divide the universe into rejection region, acceptance region, and uncertainty region.

Assume that the expert group members agree that the lower and upper boundaries of the uncertainty region are $s_L = \{s_2, s_2, s_2, s_2, s_2, s_2, s_2\}$ and $s_U = \{s_4, s_4, s_4, s_4, s_4, s_4, s_4\}$, respectively. Therefore, for any judgment debtor $A_i (i = 1, \ldots, 10)$, if an attribute value exists $E(L_{ij}(p_{ij})) \succ s_{U,j}$, then $A_i \in \mathrm{ACP}_{(s_L, s_U)}$; namely, the expert group members agree that at least one concealment property behaviour of the judgment debtors A_1, A_2, A_3, and A_4 is evident and therefore the judgment debtors A_1, A_2, A_3, and A_4 have concealed property and are placed in the acceptance region. If all the evaluated values $E(L_{ij}(p_{ij})) \prec s_{U,j}$, then $A_i \in \mathrm{REJ}_{(s_L, s_U)}$; namely, no concealment property behaviours of the judgment debtor A_9 have been identified so far, and the judgment debtor A_9 is divided into the rejection region. The judgment debtors A_5, A_6, A_7, A_8, and A_{10} cannot be judged for the time being on the basis of the information currently available as to whether or not they have concealed property and will gather information for further confirmation, and are therefore classified in the uncertainty region. The evaluation results by the developed model are shown in Table 6.

Step 4. Transfer the attribute values in the uncertainty region into fuzzy numbers and construct the fuzzy matrix.

Table 5: The probabilistic linguistic score matrix of judgment debtors.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	$s_{5.4}$	$s_{4.8}$	$s_{4.1}$	$s_{5.3}$	$s_{5.6}$	$s_{4.1}$	$s_{5.4}$
A_2	$s_{5.3}$	$s_{4.1}$	$s_{3.8}$	$s_{5.5}$	$s_{4.7}$	$s_{4.9}$	$s_{5.7}$
A_3	$s_{4.5}$	s_4	$s_{3.9}$	s_4	$s_{4.2}$	$s_{4.1}$	$s_{3.6}$
A_4	$s_{4.3}$	$s_{4.3}$	$s_{4.4}$	$s_{4.3}$	$s_{4.5}$	$s_{4.1}$	$s_{2.8}$
A_5	$s_{3.4}$	$s_{1.8}$	$s_{2.1}$	$s_{2.3}$	$s_{3.6}$	$s_{2.1}$	$s_{3.4}$
A_6	$s_{3.3}$	$s_{1.9}$	$s_{1.8}$	$s_{3.5}$	$s_{2.6}$	$s_{2.9}$	$s_{3.7}$
A_7	$s_{3.5}$	s_3	$s_{2.9}$	s_3	$s_{3.2}$	$s_{3.1}$	$s_{2.6}$
A_8	$s_{3.3}$	$s_{3.3}$	$s_{3.4}$	$s_{1.3}$	$s_{2.5}$	$s_{3.1}$	$s_{1.8}$
A_9	$s_{1.4}$	$s_{0.8}$	$s_{0.1}$	$s_{1.3}$	$s_{1.7}$	$s_{1.1}$	$s_{1.4}$
A_{10}	$s_{2.3}$	$s_{0.9}$	$s_{0.8}$	$s_{1.5}$	$s_{1.6}$	$s_{1.9}$	$s_{1.7}$

Table 6: The evaluation results by the developed hidden property evaluation model.

	Evaluation results
Acceptance region	A_1, A_2, A_3, A_4
Uncertainty region	A_5, A_6, A_7, A_8, A_{10}
Rejection region	A_9

The possibility of the concealed behaviours of the judgment debtors A5, A6, A7, A8, and A10 in the uncertain region should be analyzed. The attribute values of the judgment debtors A_5, A_6, A_7, A_8, and A_{10} are converted into fuzzy numbers according to the correspondence between the linguistic term set S and the triangular fuzzy numbers in Table 1, and the fuzzy matrix is shown in Table 7.

Step 5. Aggregate the fuzzy information for each judgment debtor in the uncertainty region.

In this problem, assuming the same weight for each concealment, all the evaluated values of the judgment debtors A_5, A_6, A_7, A_8, and A_{10} in the uncertainty region are weighted together using the formula: $V_i = WA\ (u_{i1}, u_{i2},$

Table 7: The fuzzy decision matrix in the uncertainty region.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_5	[0.4, 0.57, 0.73]	[0.13, 0.3, 0.47]	[0.18, 0.35, 0.52]	[0.22, 0.38, 0.55]	[0.43, 0.6, 0.77]	[0.18, 0.35, 0.52]	[0.4, 0.57, 0.73]
A_6	[0.38, 0.55, 0.72]	[0.15, 0.32, 0.48]	[0.13, 0.3, 0.47]	[0.42, 0.58, 0.75]	[0.27, 0.43, 0.6]	[0.32, 0.48, 0.65]	[0.45, 0.62, 0.78]
A_7	[0.42, 0.58, 0.75]	[0.33, 0.5, 0.67]	[0.32, 0.48, 0.65]	[0.33, 0.5, 0.67]	[0.37, 0.53, 0.7]	[0.35, 0.52, 0.68]	[0.27, 0.43, 0.6]
A_8	[0.38, 0.55, 0.72]	[0.38, 0.55, 0.72]	[0.4, 0.57, 0.73]	[0.4, 0.22, 0.38]	[0.25, 0.42, 0.58]	[0.35, 0.52, 0.68]	[0.13, 0.3, 0.47]
A_{10}	[0.22, 0.38, 0.55]	[0, 0.15, 0.32]	[0, 0.13, 0.3]	[0.08, 0.25, 0.42]	[0.1, 0.27, 0.43]	[0.15, 0.32, 0.48]	[0.12, 0.28, 0.45]

$\ldots, u_{in}) = \sum_{j=1}^{n} w_j E(L_{ij}(p_{ij}))$, to obtain the weighted evaluated values, and the results of which are shown in Table 6.

Step 6. Defuzzify and rank the weighted evaluation values of the judgment debtors.

The weighted aggregated values of judgment debtors A_5, A_6, A_7, A_8, and A_{10} are defuzzified as shown in Table 8, and the ranking result is $A_7 > A_6 > A_5 = A_8 > A_{10}$, which means that among the judgment debtors in the uncertainty region. The judgment debtor A_7 is the most likely to have concealed property, and the judgment debtor A_{10} is the least likely to have concealed property.

The results of the developed concealment property evaluation model show that at least one of the concealment behaviours of the judgment debtors A_1, A_2, A_3, and A_4 is more obvious. So, the judgment debtors A_1, A_2, A_3, and A_4 are deemed to have concealment behaviours, and the judgment debtors A_1, A_2, A_3, and A_4 are assigned to the acceptance region; the judgment debtor A_9 is not found to have any concealment behaviours for the time being. So, the judgment debtor A_9 is assigned to the rejection region; the judgment debtors A_5, A_6, A_7, A_8, and A_{10} are deemed to have basically no concealment behaviours based on the information currently available to them. The information currently available to the judgment debtors A_5, A_6, A_7, A_8, and A_{10} is not enough to determine whether or not they have concealed property, and information will be collected for further confirmation. Therefore, they are placed in the uncertainty region. Through the analysis, it is found that the judgment debtor A_7 is most likely to have concealed property, and the judgment debtor A_{10} is least likely to conceal property.

To further demonstrate the effectiveness of the proposed concealed property evaluation model, it is compared with the evaluation results of the strict PL3W-MADM model and flexible PL3W-MADM model, as shown in Table 9. Among the three methods, the judgment debtor A_1 is always considered to have concealed behaviour, A_9 is always considered to have no concealed behaviour, and A_8 is always temporarily unable to be judged due to insufficient information. However, the judgment debtors A_2, A_3, and A_4 fall into the uncertainty region in the strict PL3W-MADM model, while they fall into the acceptance region in the other two methods. The judgment debtors A_5, A_6, A_7, and A_{10} fall into the rejection region in the flexible PL3W-MADM model, while they fall into the uncertainty region in the other two methods, which is obviously unreasonable. The hidden property evaluation model's special feature is that as long as there is one particularly suspicious concealment behaviour, the judgment debtor can be considered concealed property. Therefore, the flexible PL3W-MADM model is used for the acceptance region. Suppose all the hidden property behaviours are not suspicious. In that case, the judgment debtor can be considered not to have concealed property, so the strict PL3W-MADM model is used for the rejection region.

Table 8: The defuzzied value of each judgment debtor in the uncertainty region.

	The weighted value of each judgment debtor	The defuzzied value
A_5	[0.28, 0.45, 0.62]	0.45
A_6	[0.30, 0.47, 0.64]	0.47
A_7	[0.36, 0.51, 0.67]	0.51
A_8	[0.28, 0.45, 0.62]	0.45
A_{10}	[0.08, 0.25, 0.42]	0.25

6.2. Comparative Analysis. To illustrate the effectiveness and reasonableness of the developed method, we compare it with the distance-based method [6].

The main idea of the distance-based method is to calculate the distance between the alternative judgment debtor and the ideal concealed judgment debtor. The closer the distance, the more likely the alternative judgment debtor is to have concealed property. Suppose that the ideal concealed judgment debtor is $s^+ = \{s_6, s_6, s_6, s_6, s_6, s_6, s_6\}$, and the distances between the alternative judgment debtors and the ideal concealed judgment debtor are shown in Table 10. The ranking result is $A_1 > A_2 > A_4 > A_3 > A_7 > A_6 > A_5 = A_8 > A_{10} > A_9$. Therefore, the most likely to conceal property is A_1, and the most unlikely to hide is A_9.

The distance-based method could obtain the ranking result by the comprehensive evaluation values. However, it cannot judge whether the alternative judgment debtor has concealed property and the comprehensive evaluation values cannot reflect exactly which method the judgment debtor used to conceal his property. Obviously, it is not suitable and reasonable for solving the problem. Therefore, the developed hidden property evaluation model is more reasonable in determining whether the judgment debtor is concealing property and what concealment behaviour there is.

Table 9: The comparative analysis of the developed hidden property evaluation method and other methods.

	The developed method	Strict PL3W-MADM model	Flexible PL3W-MADM model
Acceptance region	A_1, A_2, A_3, A_4	A_1	A_1, A_2, A_3, A_4
Uncertainty region	A_5, A_6, A_7, A_8, A_{10}	A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{10}	A_8
Rejection region	A_9	A_9	A_5, A_6, A_7, A_9, A_{10}

Table 10: The distances between the alternative judgment debtors and the ideal concealed judgment debtor and their corresponding rankings.

	The distances	The rankings
A_1	1.04	1
A_2	1.14	2
A_3	1.96	4
A_4	1.90	3
A_5	3.33	7
A_6	3.19	6
A_7	2.96	5
A_8	3.33	7
A_9	4.89	10
A_{10}	4.47	9

7. Conclusions

We present a new hidden property evaluation method based on PL3W-MADM method to judge whether the judgment debtor has concealed property and which hidden property behaviour they have. The key findings of our work are as follows. Firstly, the behaviours of concealing property according to the existing sample cases are summarized and divided into seven categories: bank account, real estate (housing, land, etc.), movable property (vehicle, etc.), equity (including stock), insurance account, Alipay account, and other property or way, and the evaluation criteria are provided. Secondly, the PLTS is used to express nonuniform evaluation information given by the expert judges, and the 3WD model is extended to the probabilistic linguistic environment. The strict PL3W-MADM model and the flexible PL3W-MADM model are developed. Thirdly, considering the advantages of the strict PL3W-MADM model and the flexible PL3W-MADM model, the hidden property evaluation model is used to establish to evaluate the hidden property behaviour of the judgment debtor. Finally, through the evaluation of various hidden property behaviours by the expert judges, the developed hidden property evaluation model can quickly and effectively classify the judgment debtors into three categories: hidden behaviour, no hidden behaviour or lack of information, and temporary inability to judge. The results show that the developed model is more suitable for hidden property evaluation than the strict PL3W-MADM model and the flexible PL3W-MADM model.

However, there are some limitations on the developed method. For example, the attribute values cannot be quantified, and the parameter selection of the developed method needs further improvement and lack of multi-attribute consideration in assessing the likelihood of concealment property. In future work, fuzzy MADM methods have some advantages for expressing the uncertain information of concealed property [47–49], and we can combine the advantages of fuzzy MADM methods to further research on the possibility of concealed property.

Data Availability

No data were used to support this study.

Disclosure

Jinhui He and Huirong Zhang are the co-first authors.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

Jinhui He and Huirong Zhang contributed equally to this work.

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