

Retraction

Retracted: Topological Structures of Lower and Upper Rough Subsets in a Hyperring

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] N. Abughazalah, N. Yaqoob, and K. Shahzadi, "Topological Structures of Lower and Upper Rough Subsets in a Hyperring," *Journal of Mathematics*, vol. 2021, Article ID 9963623, 6 pages, 2021.

Research Article

Topological Structures of Lower and Upper Rough Subsets in a Hyperring

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In this paper, we study the connection between topological spaces, hyperrings (semi-hypergroups), and rough sets. We concentrate here on the topological parts of the lower and upper approximations of hyperideals in hyperrings and semi-hypergroups. We provide the conditions for the boundary of hyp-ideals of a hyp-ring to become the hyp-ideals of hyp-ring.

1. Introduction

Algebraic hyp-structure (hyperstructure) represents a real extension of classical algebraic structure. Algebraic hyp-structures depend on hyperoperations and their properties. Sm-hyp-group (semi-hypergroup) was first introduced by French Mathematician Marty [1] in 1934. The sm-hyp-group concept is the generalization of sm-group (semigroup) concept, likewise the hyp-ring (hyperring) concept is the generalization of ring concept. In [2, 3], authors provided many applications of hyp-structures. There are several creators who added numerous outcomes to the hypothesis of algebraic hyp-structures, for instance, Hila and Dine [4] studied the hyperideals of left almost semi-hypergroups. Tang et al. [5] introduced the idea of hyperfilters in ordered semi-hypergroups, also see [6, 7].

In 1982, Pawlak [8] introduced R-sets (rough sets) for the very first time. R-set theory has been a knowledge discovery in rational databases. Set approximation is divided into two parts, i.e., lower approximation and upper approximation. The applications of R-sets are considered in finance, pattern recognition, industries, information processing, and business. It provides a mathematical tool to find out pattern hidden in data. The major advantages of

R-set approach is that it does not need any primary/secondary information about the data like the theory of probability in statistics and the grade of membership in the theory of fuzzy set. It gives systematic procedures, tools, and algorithms to find out hidden patterns in data, and it permits generating in mechanized way the sets of decision rules from data. Thivagar and Devi [9] introduced the concept of nanotopology via ring structure. R-set theory has been studied by several authors in algebraic structures and also in algebraic hyperstructures. Ahn and Kim applied R-set theory to BE-algebras [10]. Ali et al. [11] studied generalized roughness in $(\varepsilon, \varepsilon \vee q_k)$ -fuzzy filters of ordered semigroups. Biswas and S. Nanda [12] applied R-set theory to groups. Shabir and Irshad [13] applied roughness in ordered semigroups. In [14–22], authors studied roughness in different hyperstructures. Fuzzy sets were also considered by many authors, for instance, Fotea and Davvaz [23] studied fuzzy hyperrings. Ameri and Motameni [24] applied fuzzy set theory to the hyperideals of fuzzy hyperrings. Bayrak and Yamak [25] introduced some results on the lattice of fuzzy hyperideals of a hyperring. Davvaz [26] studied fuzzy Krasner (m, n) -hyperrings. Connections between fuzzy sets and topology are considered in [27–29].

2. Preliminaries and Notations

Definition 1. A topological space refers to a pair (\mathfrak{F}, τ) , where \mathfrak{F} is a nonempty set and τ is a topology on \mathfrak{F} .

Definition 2. A hyp-groupoid (hypergroupoid) $(\mathcal{F}, \hat{\circ})$ is called a sm-hyp-group if, for all a, b, c of \mathcal{F} , we have $(a \hat{\circ} b) \hat{\circ} c = a \hat{\circ} (b \hat{\circ} c)$, which means that

$$\bigcup_{d \in a \hat{\circ} b} d \hat{\circ} c = \bigcup_{e \in b \hat{\circ} c} a \hat{\circ} e. \quad (1)$$

Definition 3. A subset I of a sm-hyp-group \mathcal{F} is called right hyp-ideal (resp., left hyp-ideal) if

- (i) $I \hat{\circ} I \subseteq I$
- (ii) $I \hat{\circ} \mathcal{F} \subseteq I$ (resp., $\mathcal{F} \hat{\circ} I \subseteq I$)

A left and right hyp-ideal I of \mathcal{F} is known as hyp-ideal of \mathcal{F} .

Definition 4. (lower approximation of a subset, see [8]). The l -approximation (lower approximation) of $Y \subseteq U$ w.r.t E (E is an equivalence relation) is a set of all those objects, which are contained in Y . From the diverse representations of an E -relation, we attain three productive definitions of l -approximation:

- (i) $\underline{E}_{\text{Lower}}(Y) = \{a \in U : [a]_E \subseteq Y\}$
- (ii) $\underline{E}_{\text{Lower}}(Y) = \bigcup_{[a]_E \subseteq Y} [a]_E$
- (iii) $\underline{E}_{\text{Lower}}(Y) = \bigcup \{A \in U : A \subseteq Y\}$, where $[a]_E = \{q : qEa\}$

(i) is element-based definition, (ii) is granule-based definition, and (iii) is subsystem-based definition.

Definition 5. (upper approximation of a subset, see [8]). The u -approximation (upper approximation) of a set Y w.r.t E is a set of all those objects which have nonempty intersection with Y . From the unlike representations of an E -relation, we obtain three constructive definitions of u -approximation:

- (i) $\overline{E}_{\text{Upper}}(Y) = \{a \in U : [a]_E \cap Y \neq \emptyset\}$
- (ii) $\overline{E}_{\text{Upper}}(Y) = \bigcup_{[a]_E \cap Y \neq \emptyset} [a]_E$
- (iii) $\overline{E}_{\text{Upper}}(Y) = \bigcap \{A \in U : A \cap Y \neq \emptyset\}$, where $[a]_E = \{q : qEa\}$

The following properties hold in approximation space [8]:

- (1) $\underline{E}_{\text{Lower}}(Y) \subseteq Y \subseteq \overline{E}_{\text{Upper}}(Y)$
- (2) $\underline{E}_{\text{Lower}}(\emptyset) = \emptyset = \overline{E}_{\text{Upper}}(\emptyset)$;
 $\underline{E}_{\text{Lower}}(U) = U = \overline{E}_{\text{Upper}}(U)$
- (3) $\overline{E}_{\text{Upper}}(Y_1 \cup Y_2) = \overline{E}_{\text{Upper}}(Y_1) \cup \overline{E}_{\text{Upper}}(Y_2)$
- (4) $\underline{E}_{\text{Lower}}(Y_1 \cup Y_2) \supseteq \underline{E}_{\text{Lower}}(Y_1) \cup \underline{E}_{\text{Lower}}(Y_2)$
- (5) $\overline{E}_{\text{Upper}}(Y_1 \cap Y_2) \subseteq \overline{E}_{\text{Upper}}(Y_1) \cap \overline{E}_{\text{Upper}}(Y_2)$
- (6) $\underline{E}_{\text{Lower}}(Y_1 \cap Y_2) = \underline{E}_{\text{Lower}}(Y_1) \cap \underline{E}_{\text{Lower}}(Y_2)$
- (7) $Y_1 \subseteq Y_2$ implies $\underline{E}_{\text{Lower}}(Y_1) \subseteq \underline{E}_{\text{Lower}}(Y_2)$,
 $\overline{E}_{\text{Upper}}(Y_1) \subseteq \overline{E}_{\text{Upper}}(Y_2)$
- (8) $\underline{E}_{\text{Lower}}(Y) = \overline{E}_{\text{Upper}}(Y)$

- (9) $\overline{E}_{\text{Upper}}(Y) = \underline{E}_{\text{Lower}}(Y)$
- (10) $\underline{E}_{\text{Lower}} \underline{E}_{\text{Lower}}(Y) = \overline{E}_{\text{Upper}} \underline{E}_{\text{Lower}}(Y) = \underline{E}_{\text{Lower}}(Y)$
- (11) $\overline{E}_{\text{Upper}} \overline{E}_{\text{Upper}}(Y) = \underline{E}_{\text{Lower}} \overline{E}_{\text{Upper}}(Y) = \overline{E}_{\text{Upper}}(Y)$

3. T-Structures of R-Sets Based on Sm-Hyp-Groups

In this section, we develop some concepts related to topology of R-sets based on sm-hyp-groups.

Definition 6. Let \mathcal{F} be a sm-hyp-group, $Y \subseteq \mathcal{F}$, and ξ be a REG-relation (regular relation) on \mathcal{F} . Then, the (l -) u -approximations and boundary of Y with respect to the REG-relation ξ are given as follows:

- (i) $\xi_{\text{Lower}}(Y) = \{x \in \mathcal{F} : \xi(x) \subseteq Y\}$
- (ii) $\xi_{\text{Upper}}(Y) = \{x \in \mathcal{F} : \xi(x) \cap Y \neq \emptyset\}$
- (iii) $\xi^B(Y) = \overline{\xi_{\text{Upper}}}(Y) - \xi_{\text{Lower}}(Y)$

The family of sets

$$\xi^T(Y) = \{\mathcal{F}, \emptyset, \xi_{\text{Lower}}(Y), \overline{\xi_{\text{Upper}}}(Y), \xi^B(Y)\} \quad (2)$$

forms a topology on \mathcal{F} .

Example 1. Let $\mathcal{F} = \{a_{\mathcal{F}}, b_{\mathcal{F}}, c_{\mathcal{F}}, d_{\mathcal{F}}\}$ be a sm-hyp-group under the binary hyperoperation " $\hat{\circ}$ " defined in Cayley (Table 1).

Let

$$\xi = \{(a_{\mathcal{F}}, a_{\mathcal{F}}), (a_{\mathcal{F}}, b_{\mathcal{F}}), (a_{\mathcal{F}}, c_{\mathcal{F}}), (b_{\mathcal{F}}, a_{\mathcal{F}}), (b_{\mathcal{F}}, b_{\mathcal{F}}), (b_{\mathcal{F}}, c_{\mathcal{F}}), (c_{\mathcal{F}}, a_{\mathcal{F}}), (c_{\mathcal{F}}, b_{\mathcal{F}}), (c_{\mathcal{F}}, c_{\mathcal{F}}), (d_{\mathcal{F}}, d_{\mathcal{F}})\} \quad (3)$$

be a REG-relation on the sm-hyp-group \mathcal{F} with the following regular classes:

$$\xi(a_{\mathcal{F}}) = \xi(b_{\mathcal{F}}) = \xi(c_{\mathcal{F}}) = \{a_{\mathcal{F}}, b_{\mathcal{F}}, c_{\mathcal{F}}\} \text{ and } \xi(d_{\mathcal{F}}) = \{d_{\mathcal{F}}\}. \quad (4)$$

Now, let $Y = \{a_{\mathcal{F}}, b_{\mathcal{F}}, d_{\mathcal{F}}\} \subseteq \mathcal{F}$. Then, $\xi_{\text{Lower}}(Y) = \{d_{\mathcal{F}}\}$, $\xi_{\text{Upper}}(Y) = \mathcal{F}$, and $\xi^B(Y) = \{a_{\mathcal{F}}, b_{\mathcal{F}}, c_{\mathcal{F}}\}$. Hence, $\xi^T(Y) = \{\mathcal{F}, \emptyset, \{d_{\mathcal{F}}\}, \{a_{\mathcal{F}}, b_{\mathcal{F}}, c_{\mathcal{F}}\}\}$, which is clearly a topology on \mathcal{F} .

Remark 1. Let \mathcal{F} be a sm-hyp-group, ξ be a REG-relation on \mathcal{F} , and $Y \subseteq \mathcal{F}$.

- (i) If $\xi_{\text{Lower}}(Y) = \emptyset$ and $\overline{\xi_{\text{Upper}}}(Y) = \mathcal{F}$, then $\xi^T(Y) = \{\mathcal{F}, \emptyset\}$ is called the indiscrete topology on \mathcal{F} .
- (ii) If $\xi_{\text{Lower}}(Y) = \overline{\xi_{\text{Upper}}}(Y) = Y$, then the topology $\xi^T(Y) = \{\mathcal{F}, \emptyset, \xi_{\text{Lower}}(Y)\} = \{\mathcal{F}, \emptyset, \overline{\xi_{\text{Upper}}}(Y)\} = \{\mathcal{F}, \emptyset, Y\}$. (5)
- (iii) If $\xi_{\text{Lower}}(Y) = \emptyset$ and $\overline{\xi_{\text{Upper}}}(Y) \neq \mathcal{F}$, then $\xi^T(Y) = \{\mathcal{F}, \emptyset, \overline{\xi_{\text{Upper}}}(Y)\}$.

TABLE 1: Tabular form of the hyperoperation “ \circ ” defined in Example 1.

\circ	$a_{\mathcal{F}}$	$b_{\mathcal{F}}$	$c_{\mathcal{F}}$	$d_{\mathcal{F}}$
$a_{\mathcal{F}}$	$a_{\mathcal{F}}$	$b_{\mathcal{F}}$	$\{a_{\mathcal{F}}, c_{\mathcal{F}}\}$	$d_{\mathcal{F}}$
$b_{\mathcal{F}}$	$b_{\mathcal{F}}$	$b_{\mathcal{F}}$	$b_{\mathcal{F}}$	$d_{\mathcal{F}}$
$c_{\mathcal{F}}$	$\{a_{\mathcal{F}}, c_{\mathcal{F}}\}$	$b_{\mathcal{F}}$	$c_{\mathcal{F}}$	$d_{\mathcal{F}}$
$d_{\mathcal{F}}$	$d_{\mathcal{F}}$	$d_{\mathcal{F}}$	$d_{\mathcal{F}}$	$d_{\mathcal{F}}$

- (iv) If $\xi_{\text{Lower}}(Y) \neq \emptyset$ and $\overline{\xi_{\text{Upper}}}(Y) = \mathcal{F}$, then $\xi^{\tau}(Y) = \{\mathcal{F}, \emptyset, \xi^B(Y)\}$.
- (v) If $\xi_{\text{Lower}}(Y) \neq \xi_{\text{Upper}}(Y)$, where $\xi_{\text{Lower}}(Y) \neq \emptyset$, then $\xi^{\tau}(Y) = \{\mathcal{F}, \emptyset, \xi_{\text{Lower}}(Y), \overline{\xi_{\text{Upper}}}(Y), \xi^B(Y)\}$ is the discrete topology on \mathcal{F} .

Theorem 1. Let \mathcal{F} be a sm-hyp-group, ξ be a REG-relation on \mathcal{F} , and $Y \subseteq \mathcal{F}$. Then,

- (i) $\xi_{\text{Lower}}(Y) \subseteq Y \subseteq \overline{\xi_{\text{Upper}}}(Y)$
(ii) $\xi_{\text{Lower}}(\emptyset) = \emptyset = \overline{\xi_{\text{Upper}}}(\emptyset)$
(iii) $\xi_{\text{Lower}}(\mathcal{F}) = \mathcal{F} = \overline{\xi_{\text{Upper}}}(\mathcal{F})$

Proof

- (i) We have to prove that $\xi_{\text{Lower}}(Y) \subseteq Y \subseteq \overline{\xi_{\text{Upper}}}(Y)$.

First, we prove that $\xi_{\text{Lower}}(Y) \subseteq Y$.

Let

$$x \in \xi_{\text{Lower}}(Y) \Rightarrow \xi(x) \subseteq Y. \quad (6)$$

As $\xi(x)$ is a regular class of x , so $x \in \xi(x)$. However, as $\xi(x) \subseteq Y$, thus $x \in Y$. Now, we prove that $Y \subseteq \overline{\xi_{\text{Upper}}}(Y)$. Let $y \in Y$. As $\xi(y)$ is a regular class of y , so $y \in \xi(y)$. Thus,

$$y \in \xi(y) \cap Y \Rightarrow \xi(y) \cap Y \neq \emptyset. \quad (7)$$

Thus, $y \in \overline{\xi_{\text{Upper}}}(Y)$.

- (ii) The proof of this part is straightforward.
(iii) The proof of this part is straightforward.

It is easy to see from Example 1 that $\overline{\xi_{\text{Upper}}}(Y) \not\subseteq Y \not\subseteq \xi_{\text{Lower}}(Y)$. \square

Proposition 1. Let \mathcal{F} be a sm-hyp-group, ξ be a REG-relation on \mathcal{F} , and Y_1 and Y_2 two subsets of \mathcal{F} such that $Y_1 \subseteq Y_2$. Then,

- (i) $\xi_{\text{Lower}}(Y_1) \subseteq \xi_{\text{Lower}}(Y_2)$
(ii) $\overline{\xi_{\text{Upper}}}(Y_1) \subseteq \overline{\xi_{\text{Upper}}}(Y_2)$
(iii) $\xi^B(Y_1) \subseteq \xi^B(Y_2)$

Proof

- (i) Given $Y_1 \subseteq Y_2$ and $x \in \xi_{\text{Lower}}(Y_1)$, by definition

$$\begin{aligned} &\Rightarrow \xi(x) \subseteq Y_1 \quad \text{for all } x \in \mathcal{F} \\ &\Rightarrow \xi(x) \subseteq Y_1 \subseteq Y_2 \Rightarrow \xi(x) \subseteq Y_2 \quad \text{for all } x \in \mathcal{F}. \end{aligned} \quad (8)$$

Thus, $\xi_{\text{Lower}}(Y_1) \subseteq \xi_{\text{Lower}}(Y_2)$.

- (ii) Let $x \in \overline{\xi_{\text{Upper}}}(Y_1) \Rightarrow \xi(x) \cap Y_1 \neq \emptyset$. Let

$$\begin{aligned} &y \in \xi(x) \cap Y_1 \\ &\Rightarrow y \in \xi(x) \text{ and } y \in Y_1 \\ &\Rightarrow y \in \xi(x) \text{ and } y \in Y_1 \subseteq Y_2 \\ &\Rightarrow y \in \xi(x) \cap Y_2 \text{ and } y \in Y_2 \\ &\Rightarrow \xi(x) \cap Y_2 \neq \emptyset \\ &\Rightarrow x \in \overline{\xi_{\text{Upper}}}(Y_2). \end{aligned} \quad (9)$$

Hence, we get $\overline{\xi_{\text{Upper}}}(Y_1) \subseteq \overline{\xi_{\text{Upper}}}(Y_2)$.

- (iii) From (i) and (ii),

$$\overline{\xi_{\text{Upper}}}(Y_1) - \xi_{\text{Lower}}(Y_1) \subseteq \overline{\xi_{\text{Upper}}}(Y_2) - \xi_{\text{Lower}}(Y_2). \quad (10)$$

Thus, we have $\xi^B(Y_1) \subseteq \xi^B(Y_2)$. \square

Theorem 2. Let \mathcal{F} be a sm-hyp-group and ξ be a REG-relation on \mathcal{F} , $Y_1, Y_2 \subseteq \mathcal{F}$ such that $Y_1 \subseteq Y_2$. Then, $\xi^{\tau}(Y_1) \subseteq \xi^{\tau}(Y_2)$.

Proof. Since $Y_1 \subseteq Y_2 \subseteq \mathcal{F}$, the approximations with respect to the sm-hyp-group satisfy

$$\begin{aligned} &\xi_{\text{Lower}}(Y_1) \subseteq \xi_{\text{Lower}}(Y_2), \\ &\overline{\xi_{\text{Upper}}}(Y_1) \subseteq \overline{\xi_{\text{Upper}}}(Y_2) \text{ and} \\ &\xi^B(Y_1) \subseteq \xi^B(Y_2), \end{aligned} \quad (11)$$

which implies that $\xi^{\tau}(Y_1) \subseteq \xi^{\tau}(Y_2)$. \square

Proposition 2. Suppose ξ and γ are two REG-relations on \mathcal{F} such that $\xi \subseteq \gamma$, and let Y_1 be the nonempty subset of \mathcal{F} . Then,

- (i) $\gamma_{\text{Lower}}(Y_1) \subseteq \xi_{\text{Lower}}(Y_1)$
(ii) $\overline{\xi_{\text{Upper}}}(Y_1) \subseteq \overline{\gamma_{\text{Upper}}}(Y_1)$
(iii) $\xi^B(Y_1) \subseteq \gamma^B(Y_1)$

Proof. Suppose ξ and γ are two REG-relations on \mathcal{F} such that $\xi \subseteq \gamma$, and let Y_1 be the nonempty subset of \mathcal{F} .

- (i) Let $x \in \gamma_{\text{Lower}}(Y_1)$. Then, $\gamma(x) \subseteq Y_1$. Now, as $\xi \subseteq \gamma$, so $\xi(x) \subseteq \gamma(x)$ for any $x \in \mathcal{F}$. Then, we get $\xi(x) \subseteq Y_1$. Hence, $x \in \xi_{\text{Lower}}(Y_1)$.
(ii) Let $x \in \overline{\xi_{\text{Upper}}}(Y_1)$. Then, $\xi(x) \cap Y_1 \neq \emptyset$. Now, as $\xi \subseteq \gamma$, so

$$\begin{aligned} &\xi(x) \subseteq \gamma(x) \quad \text{for any } x \in \mathcal{F} \\ &\Rightarrow \xi(x) \cap Y_1 \subseteq \gamma(x) \cap Y_1 \quad \text{for any } x \in \mathcal{F}. \end{aligned} \quad (12)$$

As $\emptyset \neq \xi(x) \cap Y_1 \subseteq \gamma(x) \cap Y_1$. Thus, $\gamma(x) \cap Y_1 \neq \emptyset$. Hence, $x \in \overline{\gamma_{\text{Upper}}}(Y_1)$.

(iii) The proof of this part implies from (i) and (ii). \square

Theorem 3. Let \mathcal{F} be a sm-hyp-group and ξ and γ be the REG-relations on \mathcal{F} such that $\xi \subseteq \gamma$, and let Y_1 be the nonempty subset of \mathcal{F} . Then, $\xi^\tau(Y_1) \neq \gamma^\tau(Y_1)$.

Proof. Since ξ and γ are the REG-relations on \mathcal{F} such that $\xi \subseteq \gamma$, then

$$\begin{aligned} \underline{\gamma}_{\text{Lower}}(Y_1) &\subseteq \underline{\xi}_{\text{Lower}}(Y_1), \\ \overline{\xi}_{\text{Upper}}(Y_1) &\subseteq \overline{\gamma}_{\text{Upper}}(Y_1) \text{ and} \\ \xi^B(Y_1) &\subseteq \gamma^B(Y_1), \end{aligned} \quad (13)$$

which implies that $\xi^\tau(Y_1) \neq \gamma^\tau(Y_1)$. \square

4. T-Structures of R-Sets Based on Hyp-Rings

In this section, we develop some concepts related to topology of R-sets based on hyp-rings.

Definition 7. Let \mathfrak{R} be a hyp-ring, $Y \subseteq \mathfrak{R}$, and \mathcal{F} be a hyperideal of \mathfrak{R} . Then, the (l -) u -approximations and boundary of Y with respect to the hyp-ideal \mathcal{F} are given as follows:

- (i) $\underline{\mathcal{F}}_{\text{Lower}}(Y) = \{x \in \mathfrak{R} : x \oplus \mathcal{F} \subseteq Y\}$
- (ii) $\overline{\mathcal{F}}_{\text{Upper}}(Y) = \{x \in \mathfrak{R} : (x \oplus \mathcal{F}) \cap Y \neq \emptyset\}$
- (iii) $\mathcal{F}^B(Y) = \overline{\mathcal{F}}_{\text{Upper}}(Y) - \underline{\mathcal{F}}_{\text{Lower}}(Y)$

The family of sets

$$\mathcal{F}^\tau(Y) = \{\mathfrak{R}, \emptyset, \underline{\mathcal{F}}_{\text{Lower}}(Y), \overline{\mathcal{F}}_{\text{Upper}}(Y), \mathcal{F}^B(Y)\}, \quad (14)$$

forms a topology on \mathfrak{R} with respect to \mathcal{F} .

Example 2. Let $\mathfrak{R} = \{a_{\mathfrak{R}}, b_{\mathfrak{R}}, c_{\mathfrak{R}}, d_{\mathfrak{R}}, e_{\mathfrak{R}}, f_{\mathfrak{R}}\}$ be a hyp-ring under the binary hyperoperations \oplus and \circ defined in the Cayley (Tables 2 and 3).

Let $\mathcal{F} = \{a_{\mathfrak{R}}, e_{\mathfrak{R}}\}$ be a hyp-ideal of \mathfrak{R} . Consider $Y = \{a_{\mathfrak{R}}, c_{\mathfrak{R}}, d_{\mathfrak{R}}, f_{\mathfrak{R}}\} \subseteq \mathfrak{R}$. Then,

$$\begin{aligned} \underline{\mathcal{F}}_{\text{Lower}}(Y) &= \{c_{\mathfrak{R}}, d_{\mathfrak{R}}\}, \overline{\mathcal{F}}_{\text{Upper}}(Y) = \mathfrak{R}, \\ \mathcal{F}^B(Y) &= \{a_{\mathfrak{R}}, b_{\mathfrak{R}}, e_{\mathfrak{R}}, f_{\mathfrak{R}}\}. \end{aligned} \quad (15)$$

Hence, $\mathcal{F}^\tau(Y) = \{\mathfrak{R}, \emptyset, \{c_{\mathfrak{R}}, d_{\mathfrak{R}}\}, \{a_{\mathfrak{R}}, b_{\mathfrak{R}}, e_{\mathfrak{R}}, f_{\mathfrak{R}}\}\}$, which is clearly a topology on \mathfrak{R} .

Remark 2. Let \mathfrak{R} be a hyp-ring, \mathcal{F} be a hyp-ideal of \mathfrak{R} , and $Y \subseteq \mathfrak{R}$.

- (i) If $\underline{\mathcal{F}}_{\text{Lower}}(Y) = \emptyset$ and $\overline{\mathcal{F}}_{\text{Upper}}(Y) = \mathfrak{R}$, then $\mathcal{F}^\tau(Y) = \{\mathfrak{R}, \emptyset\}$ is called the indiscrete topology on \mathfrak{R} .

- (ii) If $\underline{\mathcal{F}}_{\text{Lower}}(Y) = \overline{\mathcal{F}}_{\text{Upper}}(Y) = Y$, then the topology

$$\begin{aligned} \mathcal{F}^\tau(Y) &= \{\mathfrak{R}, \emptyset, \underline{\mathcal{F}}_{\text{Lower}}(Y)\} \\ &= \{\mathfrak{R}, \emptyset, \overline{\mathcal{F}}_{\text{Upper}}(Y)\} \\ &= \{\mathfrak{R}, \emptyset, Y\}. \end{aligned} \quad (16)$$

- (iii) If $\underline{\mathcal{F}}_{\text{Lower}}(Y) = \emptyset$ and $\overline{\mathcal{F}}_{\text{Upper}}(Y) \neq \mathfrak{R}$, then $\mathcal{F}^\tau(Y) = \{\mathfrak{R}, \emptyset, \overline{\mathcal{F}}_{\text{Upper}}(Y)\}$.

- (iv) If $\underline{\mathcal{F}}_{\text{Lower}}(Y) \neq \emptyset$ and $\overline{\mathcal{F}}_{\text{Upper}}(Y) = \mathfrak{R}$, then $\mathcal{F}^\tau(Y) = \{\mathfrak{R}, \emptyset, \mathcal{F}^B(Y)\}$.

- (v) If $\underline{\mathcal{F}}_{\text{Lower}}(Y) \neq \overline{\mathcal{F}}_{\text{Upper}}(Y)$ where $\underline{\mathcal{F}}_{\text{Lower}}(Y) \neq \emptyset$, then $\mathcal{F}^\tau(Y) = \{\mathfrak{R}, \emptyset, \underline{\mathcal{F}}_{\text{Lower}}(Y), \overline{\mathcal{F}}_{\text{Upper}}(Y), \mathcal{F}^B(Y)\}$ is the discrete topology on \mathfrak{R} .

Theorem 4. Let \mathfrak{R} be a hyp-ring, \mathcal{F} be a hyp-ideal of \mathfrak{R} , and $Y \subseteq \mathfrak{R}$. Then,

- (i) $\underline{\mathcal{F}}_{\text{Lower}}(Y) \subseteq Y \subseteq \overline{\mathcal{F}}_{\text{Upper}}(Y)$
- (ii) $\underline{\mathcal{F}}_{\text{Lower}}(\emptyset) = \emptyset = \overline{\mathcal{F}}_{\text{Upper}}(\emptyset)$
- (iii) $\underline{\mathcal{F}}_{\text{Lower}}(\mathfrak{R}) = \mathfrak{R} = \overline{\mathcal{F}}_{\text{Upper}}(\mathfrak{R})$

Proposition 3. Let \mathfrak{R} be a hyp-ring, \mathcal{F} be a hyp-ideal of \mathfrak{R} , and Y_1 and Y_2 two subsets of \mathfrak{R} such that $Y_1 \subseteq Y_2$. Then,

- (i) $\underline{\mathcal{F}}_{\text{Lower}}(Y_1) \subseteq \underline{\mathcal{F}}_{\text{Lower}}(Y_2)$
- (ii) $\overline{\mathcal{F}}_{\text{Upper}}(Y_1) \subseteq \overline{\mathcal{F}}_{\text{Upper}}(Y_2)$
- (iii) $\mathcal{F}^B(Y_1) \subseteq \mathcal{F}^B(Y_2)$

Theorem 5. Let \mathfrak{R} be a hyp-ring and \mathcal{F} be a hyp-ideal of \mathfrak{R} , and $Y_1, Y_2 \subseteq \mathfrak{R}$ such that $Y_1 \subseteq Y_2$. Then, $\mathcal{F}^\tau(Y_1) \subseteq \mathcal{F}^\tau(Y_2)$.

Proposition 4. Suppose \mathcal{F}, W are two hyp-ideals of \mathfrak{R} such that $\mathcal{F} \subseteq W$, and let Y_1 be the nonempty subset of \mathfrak{R} . Then,

- (i) $\underline{W}_{\text{Lower}}(Y_1) \subseteq \underline{\mathcal{F}}_{\text{Lower}}(Y_1)$
- (ii) $\overline{\mathcal{F}}_{\text{Upper}}(Y_1) \subseteq \overline{W}_{\text{Upper}}(Y_1)$
- (iii) $\mathcal{F}^B(Y_1) \subseteq W^B(Y_1)$

Theorem 6. Let \mathfrak{R} be a hyp-ring and \mathcal{F}, W be the hyp-ideals of \mathfrak{R} such that $\mathcal{F} \subseteq W$ and let Y_1 be the non-empty subset of \mathfrak{R} . Then $\mathcal{F}^\tau(Y_1) \neq W^\tau(Y_1)$.

The following theorem can also be seen in [17].

Theorem 7. Let \mathcal{F} and Y_2 be two hyp-ideals of \mathfrak{R} . Then,

- (i) $\underline{\mathcal{F}}_{\text{Lower}}(Y_2)$ is, if it is nonempty, a hyp-ideal of \mathfrak{R}
- (ii) $\overline{\mathcal{F}}_{\text{Upper}}(Y_2)$ is a hyp-ideal of \mathfrak{R}

Proof

- (i) Suppose $x, y \in \underline{\mathcal{F}}_{\text{Lower}}(Y_2)$ and $r \in \mathfrak{R}$; then,

$$x \oplus \mathcal{F} \subseteq Y_2 \text{ and } y \oplus \mathcal{F} \subseteq Y_2. \quad (17)$$

This implies that $(x \oplus y \oplus \mathcal{F}) \subseteq Y_2$ and $-y \oplus \mathcal{F} \subseteq Y_2$. Also, $(r \circ x \oplus \mathcal{F}) \subseteq Y_2$ and $(x \circ r \oplus \mathcal{F}) \subseteq Y_2$. This implies that

$$x \oplus y \subseteq \underline{\mathcal{F}}_{\text{Lower}}(Y_2) \text{ and } -y \in \underline{\mathcal{F}}_{\text{Lower}}(Y_2). \quad (18)$$

TABLE 2: Tabular form of the hyperoperation “ \oplus ” defined in Example 2.

\oplus	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$f_{\mathfrak{R}}$
$a_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$f_{\mathfrak{R}}$
$b_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$\{a_{\mathfrak{R}}, b_{\mathfrak{R}}\}$	$d_{\mathfrak{R}}$	$\{c_{\mathfrak{R}}, d_{\mathfrak{R}}\}$	$f_{\mathfrak{R}}$	$\{e_{\mathfrak{R}}, f_{\mathfrak{R}}\}$
$c_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$\{a_{\mathfrak{R}}, c_{\mathfrak{R}}, e_{\mathfrak{R}}\}$	$\{b_{\mathfrak{R}}, d_{\mathfrak{R}}, f_{\mathfrak{R}}\}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$
$d_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$\{c_{\mathfrak{R}}, d_{\mathfrak{R}}\}$	$\{b_{\mathfrak{R}}, d_{\mathfrak{R}}, f_{\mathfrak{R}}\}$	\mathfrak{R}	$d_{\mathfrak{R}}$	$\{c_{\mathfrak{R}}, d_{\mathfrak{R}}\}$
$e_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$f_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$\{a_{\mathfrak{R}}, e_{\mathfrak{R}}\}$	$\{b_{\mathfrak{R}}, f_{\mathfrak{R}}\}$
$f_{\mathfrak{R}}$	$f_{\mathfrak{R}}$	$\{e_{\mathfrak{R}}, f_{\mathfrak{R}}\}$	$d_{\mathfrak{R}}$	$\{c_{\mathfrak{R}}, d_{\mathfrak{R}}\}$	$\{b_{\mathfrak{R}}, f_{\mathfrak{R}}\}$	$\{a_{\mathfrak{R}}, b_{\mathfrak{R}}, e_{\mathfrak{R}}, f_{\mathfrak{R}}\}$

TABLE 3: Tabular form of the hyperoperation “ $\hat{\circ}$ ” defined in Example 2.

$\hat{\circ}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$f_{\mathfrak{R}}$
$a_{\mathfrak{R}}$						
$b_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$
$c_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$e_{\mathfrak{R}}$
$d_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$c_{\mathfrak{R}}$	$d_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$f_{\mathfrak{R}}$
$e_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$a_{\mathfrak{R}}$
$f_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$	$e_{\mathfrak{R}}$	$f_{\mathfrak{R}}$	$a_{\mathfrak{R}}$	$b_{\mathfrak{R}}$

Also,

$$r \hat{\circ} x \subseteq \mathcal{F}_{\text{Lower}}(\gamma_2) \text{ and } x \hat{\circ} r \subseteq \mathcal{F}_{\text{Lower}}(\gamma_2). \quad (19)$$

Therefore, $\mathcal{F}_{\text{Lower}}(\gamma_2)$ is a hyp-ideal of \mathfrak{R} .

(ii) Suppose $x, y \in \mathcal{F}_{\text{Upper}}(\gamma_2)$ and $r \in \mathfrak{R}$; then,

$$(x \oplus \mathcal{F}) \cap \gamma_2 \neq \emptyset \text{ and } (y \oplus \mathcal{F}) \cap \gamma_2 \neq \emptyset. \quad (20)$$

So, there exists

$$p \in (x \oplus \mathcal{F}) \cap \gamma_2 \text{ and } q \in (y \oplus \mathcal{F}) \cap \gamma_2. \quad (21)$$

Since γ_2 is a hyp-ideal of \mathfrak{R} , we have $p \oplus q \subseteq \gamma_2$ and $-q \in \gamma_2$; also,

$$p \oplus q \subseteq (x \oplus \mathcal{F}) \oplus (y \oplus \mathcal{F}) = x \oplus y \oplus \mathcal{F} \text{ and } -q \in -y \oplus \mathcal{F}. \quad (22)$$

Hence, $(x \oplus y \oplus \mathcal{F}) \cap \gamma_2 \neq \emptyset$ and $(-y \oplus \mathcal{F}) \cap \gamma_2 \neq \emptyset$, which implies that

$$x \oplus y \subseteq \mathcal{F}_{\text{Upper}}(\gamma_2) \text{ and } -y \in \mathcal{F}_{\text{Upper}}(\gamma_2). \quad (23)$$

Also, we have $r \cdot p \in \gamma_2$ and

$$r \hat{\circ} p \subseteq r \hat{\circ} (x \oplus \mathcal{F}) = (r \hat{\circ} x) \oplus \mathcal{F}. \quad (24)$$

So, $(r \hat{\circ} x \oplus \mathcal{F}) \cap \gamma_2 \neq \emptyset$, which implies $r \hat{\circ} x \subseteq \mathcal{F}_{\text{Upper}}(\gamma_2)$. Similarly, we can prove that $x \hat{\circ} r \subseteq \mathcal{F}_{\text{Upper}}(\gamma_2)$. Therefore, $\mathcal{F}_{\text{Upper}}(\gamma_2)$ is a hyp-ideal of \mathfrak{R} . \square

Theorem 8. Let \mathcal{F} and γ_2 be two hyp-ideals of \mathfrak{R} . Then,

- (i) $\mathcal{F}^B(\gamma_2)$ is not a hyp-ideal of \mathfrak{R} if $\mathcal{F}_{\text{Lower}}(\gamma_2) \neq \emptyset$
- (ii) $\mathcal{F}^B(\gamma_2)$ is a hyp-ideal of \mathfrak{R} if $\mathcal{F}_{\text{Lower}}(\gamma_2) = \emptyset$

Corollary 1. Let \mathcal{F}, γ be two hyp-ideals of \mathfrak{R} . Then,

(i) $\mathcal{Y}_{\text{Lower}}(\mathcal{F})$ is also a hyp-ideal of \mathfrak{R} , where

$$\mathcal{Y}_{\text{Lower}}(\mathcal{F}) \neq \emptyset$$

(ii) $\mathcal{Y}_{\text{Upper}}(\mathcal{F})$ is also a hyp-ideal of \mathfrak{R}

(iii) $\mathcal{Y}^B(\mathcal{F})$ is a hyp-ideal of \mathfrak{R} , when $\mathcal{Y}_{\text{Lower}}(\mathcal{F}) = \emptyset$

Theorem 9. Let \mathfrak{R} and S be two hyp-rings and f be a homomorphism from \mathfrak{R} to S . If γ_1 is a nonempty subset of \mathfrak{R} , then

$$(i) f(\overline{\ker f_{\text{Upper}}(\gamma_1)}) = f(\gamma_1)$$

$$(ii) f(\overline{\ker f_{\text{Lower}}(\gamma_1)}) \subseteq f(\gamma_1)$$

Proof

(i) Since $\gamma_1 \subseteq \overline{\ker f_{\text{Upper}}(\gamma_1)}$, it follows that $f(\gamma_1) \subseteq f(\overline{\ker f_{\text{Upper}}(\gamma_1)})$. Conversely, let $y \in f(\overline{\ker f_{\text{Upper}}(\gamma_1)})$. Then, there exist an element $x \in \overline{\ker f_{\text{Upper}}(\gamma_1)}$ such that $f(x) = y$, so we have $(x \oplus \ker f) \cap \gamma_1 \neq \emptyset$. Then, there exists an element $a \in (x \oplus \ker f) \cap \gamma_1$. Then, $a = x \oplus b$ for some $b \in \ker f$, that is, $x = a - b$. Then, we have

$$\begin{aligned} y &= f(x) = f(a - b) \\ &= f(a) - f(b) \\ &= f(a) \in f(\gamma_1), \end{aligned} \quad (25)$$

and so $f(\overline{\ker f_{\text{Upper}}(\gamma_1)}) = f(\gamma_1)$.

(ii) The proof is easy. \square

5. Conclusion and Future Work

Relations between R-sets, hyp-rings, and topological structures are considered in this paper. In place of universal set, we added sm-hyp-groups and hyp-rings. In future, this work can be extended to soft set theory [30], bipolar fuzzy sets [31], intuitionistic fuzzy sets [32], or neutrosophic sets [33].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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