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Retraction

Retracted: Generalization of Fuzzy Soft BCK/BCI-Algebras

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] N. Alam, G. Muhiuddin, S. Obeidat, H. N. Zaidi, A. Altaleb, and J. M. Aqib, "Generalization of Fuzzy Soft BCK/BCI-Algebras," *Journal of Mathematics*, vol. 2021, Article ID 9965074, 7 pages, 2021.

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Research Article

Generalization of Fuzzy Soft BCK/BCI-Algebras

N. Alam,[1] G. Muhiuddin ,[2] S. Obeidat,[1] H. N. Zaidi ,[1] A. Altaleb,[1] and J. M. Aqib[3]

[1]*Department of Basic Sciences, Deanship of Preparatory Year, University of Háil, Háil 2440, Saudi Arabia*
[2]*Department of Mathematics, University of Tabuk, Tabuk 71491, Saudi Arabia*
[3]*Department of Software Engineering, College of Computer Science, University of Háil, Háil 2440, Saudi Arabia*

Correspondence should be addressed to G. Muhiuddin; chishtygm@gmail.com

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In this paper, the notions of $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras and $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebras are introduced, and related properties are investigated. Furthermore, relations between fuzzy soft BCK/BCI-algebras and $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras are displayed. Moreover, conditions for an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra to be a fuzzy soft BCK/BCI-algebra are provided. Also, the union, the extended intersection, and the "AND"-operation of two $(\in, \in\vee q)$-fuzzy soft (sub-)BCK/BCI-algebras are discussed, and a characterization of an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra is established.

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1. Introduction

The uncertainty which appeared in economics, engineering, environmental science, medical science, social science, and so on is too complicated to be captured within a traditional mathematical framework. In order to overcome this situation, a number of approaches including fuzzy set theory [1, 2], probability theory, rough set theory [3, 4], vague set theory [5], and the interval mathematics [6] have been developed. The concept of soft set was introduced by Molodtsov [7] as a new mathematical method to deal with uncertainties free from the errors being occurred in the existing theories. Later, Maji et al. [8, 9] defined fuzzy soft sets and also described how soft set theory is applied to the problem of decision making. Study on the soft set theory is currently moving forward quickly. In [10], Jun et al. discussed the intersection-soft filters in R_0-algberas. Roh and Jun [11] studied positive implicative ideals of BCK-algebras based on intersectional soft sets. Roy and Mayi [12] gave results on applying fuzzy soft sets to the problem of decision making. Aygünoğlu and Aygün [13] proposed and investigated the notion of a fuzzy soft group. Furthermore, Jun et al. [14] applied the theory of fuzzy soft sets to BCK/BCI-algebras and introduced the notion of fuzzy soft BCK/BCI-algebras (briefly, FSB-algebras) and related notions. Moreover, Muhiuddin et al. studied and applied the soft set theory to the different algebraic structures on various aspects (see, e.g., [15–23]). Also, some related concepts based on the present work are studied in [24–33].

In this paper, we define the notions of $(\in, \in\vee q)$-FSB-algebras and $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebras. Further, we investigate related properties and consider relations between fuzzy soft BCK/BCI-algebras and $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras. Moreover, we prove that every FSB-algebra over X is an $(\in, \in\vee q)$-FSB-algebra over X and also show by an example that the converse of the aforesaid statement is not true in general. In fact, we provide a condition for an $(\in, \in\vee q)$-FSB-algebra to be a FSB-algebra. In addition, we discuss the union, the extended intersection, and the "AND"-operation of two $(\in, \in\vee q)$-FSB-algebras. Finally, we establish a characterization of an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra. The paper is organized as follows. Section 2 summarizes some definitions and properties related to BCK/BCI-algebras, fuzzy sets, soft sets, and fuzzy soft sets which are needed to develop our main results. In Section 3, the notions of FSB-algebras are studied and the concepts of θ-identity and θ-absolute FSB-algebras are introduced. Section 4 is devoted to the study of $(\in, \in\vee q)$-FSB-algebra. The paper ends with a conclusion and a list of references.

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2. Preliminaries

A BCK/BCI-algebra is the most important class of logical algebras which was introduced by K. Iséki.

By a BCI-algebra, we mean a system $(\tilde{\mathscr{X}};\ *, 0)$, where $\tilde{\mathscr{X}}$ be a nonempty set with a constant 0 and a binary operation $*$ if

(i) $(\forall\varpi, \varrho, \vartheta \in \tilde{\mathscr{X}})(((\varpi * \varrho) * (\varpi * \vartheta)) * (\vartheta * \varrho) = 0)$

(ii) $(\forall\varpi, \varrho \in \tilde{\mathscr{X}})((\varpi * (\varpi * \varrho)) * \varrho = 0)$

(iii) $(\forall\varpi \in \tilde{\mathscr{X}})(\varpi * \varpi = 0)$

(iv) $(\forall\varpi, \varrho \in \tilde{\mathscr{X}})(\varpi * \varrho = 0, \varrho * \varpi = 0 \Rightarrow \varpi = \varrho)$

If a BCI-algebra $\tilde{\mathscr{X}}$ satisfies

(v) $(\forall\varpi \in \tilde{\mathscr{X}})(0 * \varpi = 0)$,

then $\tilde{\mathscr{X}}$ is called a BCK-algebra. Any BCK-algebra $\tilde{\mathscr{X}}$ satisfies

(a1) $(\forall\varpi \in \tilde{\mathscr{X}})(\varpi * 0 = \varpi)$,

(a2) $(\forall\varpi, \varrho, \vartheta \in \tilde{\mathscr{X}})(\varpi \leq \varrho \Longrightarrow \varpi * \vartheta \leq \varrho * \vartheta, \vartheta * \varrho \leq \vartheta * \varpi)$,

(a3) $(\forall\varpi, \varrho, \vartheta \in \tilde{\mathscr{X}})((\varpi * \varrho) * \vartheta = (\varpi * \vartheta) * \varrho)$,

(a4) $(\forall\varpi, \varrho, \vartheta \in \tilde{\mathscr{X}})((\varpi * \vartheta) * (\varrho * \vartheta) \leq \varpi * \varrho)$

where $\varpi \leq \varrho$ if and only if $\varpi * \varrho = 0$.

The following conditions are satisfied in any BCI-algebra $\tilde{\mathscr{X}}$:

(a5) $(\forall\varpi, \varrho, \vartheta \in \tilde{\mathscr{X}})(0 * (0 * ((\varpi * \vartheta) * (\varrho * \vartheta))) = (0 * \varrho) * (0 * \varpi))$.

(a6) $(\forall\varpi, \varrho \in \tilde{\mathscr{X}})(0 * (0 * (\varpi * \varrho)) = (0 * \varrho) * (0 * \varpi))$.

In a BCK/BCI-algebra $\tilde{\mathscr{X}}$, a nonempty subset T of $\tilde{\mathscr{X}}$ is called a BCK/BCI-subalgebra of $\tilde{\mathscr{X}}$ if $\varpi * \varrho \in T\ \forall\, \varpi, \varrho \in T$.

In a BCK/BCI-algebra $\tilde{\mathscr{X}}$, a fuzzy set μ in $\tilde{\mathscr{X}}$ is called a fuzzy BCK/BCI-algebra if it satisfies

$$(\forall\varpi, \varrho \in \tilde{\mathscr{X}})(\mu(\varpi * \varrho) \geq \min\{\mu(\varpi), \mu(\varrho)\}). \tag{1}$$

In a set $\tilde{\mathscr{X}}$, a fuzzy set μ in $\tilde{\mathscr{X}}$ of the form

$$\mu(\vartheta) \coloneqq \begin{cases} t \in (0, 1], & \text{if } \vartheta = \varpi, \\ 0, & \text{if } \vartheta \neq \varpi, \end{cases} \tag{2}$$

is called a fuzzy point with support ϖ and value t and is denoted by ϖ_t.

For a fuzzy set μ in a set $\tilde{\mathscr{X}}$ and a fuzzy point ϖ_t, Pu and Liu [34] presented the symbol $\varpi_t \alpha \mu$, where $\alpha \in \{\in, q, \in\vee q, \in\wedge q\}$. If $\varpi_t \in \mu$ (resp. $\varpi_t\, q\mu$), then we mean $\mu(\varpi) \geq t$ (resp. $\mu(\varpi) + t > 1$), and in this case, ϖ_t is said to belong to (resp. be quasi-coincident with) a fuzzy set μ. If $\varpi_t \in\vee q\mu$ (resp. $\varpi_t \in\wedge q\mu$), then we mean $\varpi_t \in \mu$ or $\varpi_t q\mu$ (resp. $\varpi_t \in \mu$ and $\varpi_t q\mu$).

For an initial universe set U and a set of parameters E, let $P(U)$ denote the power set of U and $\Omega \subset E$. Molodtsov [7] defined the soft set as follows.

Definition 1 (see [7]). A pair (ζ, Ω) is called a soft set over U, where ζ is a function given by

$$\zeta\colon \Omega \longrightarrow P(U). \tag{3}$$

The set $\zeta(\varepsilon)$ for $\varepsilon \in \Omega$ may be considered as the set of ε-approximate elements of the soft set (ζ, Ω). Clearly, a soft set is not a set. We refer the reader to [7] for illustration where several examples are presented.

Let $\mathscr{F}(U)$ denote the set of all fuzzy sets in U.

Definition 2 (see [9]). A pair $(\tilde{\zeta}, \Omega)$ is called a fuzzy soft set over U where $\tilde{\zeta}$ is a mapping given by

$$\tilde{\zeta}\colon \Omega \longrightarrow \mathscr{F}(U). \tag{4}$$

For all $\varpi \in \Omega$, $\tilde{\zeta}[\varpi] \in \mathscr{F}(U)$ and it is called fuzzy value set of parameter ϖ. If $\tilde{\zeta}[\varpi]$, for all $\varpi \in \Omega$, is a crisp subset of U, then $(\tilde{\zeta}, \Omega)$ is degenerated to be the standard soft set. Thus, fuzzy soft sets are a generalization of standard soft sets.

We will use $\mathscr{F}\mathscr{S}(U)$ to denote the set of all fuzzy soft sets over U.

Definition 3 (see [9]). Let $(\tilde{\zeta}, \Omega), (\tilde{\eta}, \Omega) \in \mathscr{F}\mathscr{S}(U)$. The union of $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$ is defined to be the fuzzy soft set $(\tilde{\xi}, \Upsilon)$ satisfying the following conditions:

(i) $\Upsilon = \Omega \cup \mho$,

(ii) for all $\theta \in \Upsilon$,

$$\tilde{\xi}[\theta] = \begin{cases} \tilde{\zeta}[\theta], & \text{if } \theta \in \Omega\backslash\mho, \\ \tilde{\eta}[\theta], & \text{if } \theta \in \mho\backslash\Omega, \\ \tilde{\zeta}[\theta] \cup \tilde{\eta}[\theta], & \text{if } \theta \in \Omega \cap \mho. \end{cases} \tag{5}$$

In this case, we write $(\tilde{\zeta}, \Omega)\,\widetilde{\cup}\,(\tilde{\eta}, \mho) = (\tilde{\xi}, \Upsilon)$.

Definition 4 (see [9]). If $(\tilde{\zeta}, \Omega), (\tilde{\eta}, \mho) \in \mathscr{F}\mathscr{S}(U)$, then "$(\tilde{\zeta}, \Omega)$ AND $(\tilde{\eta}, \mho)$" denoted by $(\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho)$ is defined by

$$(\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho) = (\tilde{\xi}, \Omega \times \mho), \tag{6}$$

where $\tilde{\xi}[\alpha, \beta] = \tilde{\zeta}[\alpha] \cap \tilde{\eta}[\beta]\forall(\alpha, \beta) \in \Omega \times \mho$.

Definition 5 (see [35]). For two soft sets $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$, the extended intersection is the soft set $(\tilde{\xi}, \Upsilon)$ where $\Upsilon = \Omega \cup \mho$, and for every $\theta \in \Upsilon$,

$$\tilde{\xi}[\theta] = \begin{cases} \tilde{\zeta}[\theta], & \text{if } \theta \in \Omega\backslash\mho, \\ \tilde{\eta}[\theta], & \text{if } \theta \in \mho\backslash\Omega, \\ \tilde{\zeta}[\theta] \cap \tilde{\eta}[\theta], & \text{if } \theta \in \Omega \cap \mho. \end{cases} \tag{7}$$

We write $(\tilde{\zeta}, \Omega)\,\widetilde{\cap}_\theta(\tilde{\eta}, \mho) = (\tilde{\xi}, \Upsilon)$.

Definition 6 (see [35]). Let $(\tilde{\zeta}, \Omega), (\tilde{\eta}, \mho) \in \mathscr{F}\mathscr{S}(U)$ such that $\Omega \cap \mho \neq \varnothing$. The restricted intersection of $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$ is denoted by $(\tilde{\zeta}, \Omega)\,\widetilde{\cap}_r(\tilde{\eta}, \mho)$ and is defined as $(\tilde{\zeta}, \Omega)\,\widetilde{\cap}_r(\tilde{\eta}, \mho) = (\tilde{\xi}, \Upsilon)$, where $\Upsilon = \Omega \cap \mho$ and for all $c \in \Upsilon$, $\tilde{\xi}[c] = \tilde{\zeta}[c] \cap \tilde{\eta}[c]$.

3. $(\in, \in\vee q)$-Fuzzy Soft BCK/BCI-Algebras

Definition 7 (see [36]). A fuzzy set μ in $\widetilde{\mathscr{X}}$ is said to be an $(\in, \in\vee q)$-fuzzy subalgebra of $\widetilde{\mathscr{X}}$ if

$$(\forall \hbar, \kappa \in \widetilde{\mathscr{X}})(\forall \omega_1, \omega_2 \in (0,1])\left(\hbar_{\omega_1}, \kappa_{\omega_2} \in \mu \Longrightarrow (\hbar * \kappa)_{\min\{\omega_1,\omega_2\}} \in\vee q\mu\right). \tag{8}$$

Definition 8. Let $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}(\widetilde{\mathscr{X}})$ where $\Omega \subseteq E$. If there exists a parameter $\upsilon \in \Omega$ such that $\widetilde{\zeta}[\upsilon]$ is an $(\in, \in\vee q)$-fuzzy subalgebra of $\widetilde{\mathscr{X}}$, we say that $(\widetilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$ based on a parameter υ. If $(\widetilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$ based on all parameters, we say that $(\widetilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$.

The notion $\mathscr{F}\mathscr{S}_{\mathscr{BCK}/\mathscr{BCI}}\mathscr{A}(\widetilde{\mathscr{X}})$ will be used for the set of all $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras.

Example 1. Let $\widetilde{\mathscr{X}} = \{0, \imath, \mathscr{J}, \ell\}$ be a BCI-algebra with the following table.

$*$	0	$\imath$	$\mathscr{J}$	ℓ
0	0	$\imath$	$\mathscr{J}$	ℓ
$\imath$	$\imath$	0	ℓ	$\mathscr{J}$
$\mathscr{J}$	$\mathscr{J}$	ℓ	0	$\imath$
ℓ	ℓ	$\mathscr{J}$	$\imath$	0

Let $\Omega = \{e_1, e_2, e_3\}$ and let $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}(\widetilde{\mathscr{X}})$. Then, $\widetilde{\zeta}[e_1]$, $\widetilde{\zeta}[e_2]$, and $\widetilde{\zeta}[e_3]$ are fuzzy sets in $\widetilde{\mathscr{X}}$. We define them as follows:

$\widetilde{\zeta}$	0	$\imath$	$\mathscr{J}$	ℓ
e_1	0.6	0.7	0.3	0.3
e_2	0.8	0.2	0.4	0.2
e_3	0.6	0.3	0.3	0.7

Then, $(\widetilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft BCI-algebra over $\widetilde{\mathscr{X}}$.

Proposition 1. *If $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}_{\mathscr{BCK}/\mathscr{BCI}}\mathscr{A}(\widetilde{\mathscr{X}})$, then*

$$(\forall \hbar \in \widetilde{\mathscr{X}})\left(\widetilde{\zeta}[\upsilon](0) \geq \min\left\{\widetilde{\zeta}[\upsilon](\hbar), 0.5\right\}\right), \tag{9}$$

where υ is any parameter in Ω.

Proof. For $\hbar \in \widetilde{\mathscr{X}}$ and $\upsilon \in \Omega$, we have

$$\begin{aligned}\widetilde{\zeta}[\upsilon](0) &= \widetilde{F}[\upsilon](\hbar * \hbar) \geq \min\left\{\widetilde{\zeta}[\upsilon](\hbar), \widetilde{F}[\upsilon](\hbar), 0.5\right\} \\ &= \min\left\{\widetilde{\zeta}[\upsilon](\hbar), 0.5\right\}.\end{aligned} \tag{10}$$

Hence, $\widetilde{\zeta}[\upsilon](0) \geq \min\{\widetilde{F}[\upsilon](\hbar), 0.5\}$ for all $\hbar \in \widetilde{\mathscr{X}}$ and any parameter υ in Ω. □

Theorem 1. *Let $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}_{\mathscr{BCK}/\mathscr{BCI}}\mathscr{A}(\widetilde{\mathscr{X}})$. If $\rho \subseteq \Omega$, then $(\widetilde{\zeta}|_{\rho}, \mho) \in \mathscr{F}\mathscr{S}_{\mathscr{BCK}/\mathscr{BCI}}\mathscr{A}(\widetilde{\mathscr{X}})$.*

Proof (straightforward) □

The following example shows that there exists $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}(\widetilde{\mathscr{X}})$ such that

(i) $(\widetilde{\zeta}, \Omega)$ is not an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$

(ii) There exists a subset ρ of Ω such that $(\widetilde{\zeta}|_{\rho}, \mho)$ is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$

Example 2. Consider a BCK-algebra $\widetilde{\mathscr{X}} = \{\{0, \imath, \mathscr{J}, \kappa, \ell\}$ with the following table.

$*$	0	$\imath$	$\mathscr{J}$	κ	ℓ
0	0	0	0	0	0
$\imath$	$\imath$	0	$\imath$	0	0
$\mathscr{J}$	$\mathscr{J}$	$\mathscr{J}$	0	$\mathscr{J}$	0
κ	κ	κ	κ	0	0
ℓ	ℓ	ℓ	κ	$\mathscr{J}$	0

Let $\Omega = \{e_1, e_2, e_3, e_4, e_5\}$ and let $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}(\widetilde{\mathscr{X}})$. Then, $\widetilde{\zeta}[e_1]$, $\widetilde{\zeta}[e_2]$, $\widetilde{\zeta}[e_3]$, $\widetilde{\zeta}[e_4]$, and $\widetilde{\zeta}[e_5]$ are fuzzy sets in $\widetilde{\mathscr{X}}$. We define them as follows:

$\widetilde{\zeta}$	0	$\imath$	$\mathscr{J}$	κ	ℓ
e_1	0.7	0.6	0.2	0.4	0.2
e_2	0.6	0.3	0.8	0.2	0.4
e_3	0.9	0.4	0.9	0.3	0.3
e_4	0.8	0.1	0.1	0.3	0.8
e_5	0.6	0.4	0.4	0.7	0.4

Then, $(\widetilde{\zeta}, \Omega)$ is not an $(\in, \in\vee q)$-fuzzy soft BCK-algebra over $\widetilde{\mathscr{X}}$ since it is not an $(\in, \in\vee q)$-fuzzy soft BCK-algebra over $\widetilde{\mathscr{X}}$ based on two parameters e_2 and e_4. However, if we take $\rho = \{e_1, e_3, e_5\}$, then $(\widetilde{\zeta}|_{\rho}, \mho)$ is described as follows:

$\widetilde{\zeta}|_{\rho}$	0	$\imath$	$\mathscr{J}$	κ	ℓ
e_1	0.7	0.6	0.2	0.4	0.2
e_3	0.9	0.4	0.9	0.3	0.3
e_5	0.6	0.4	0.4	0.7	0.4

and it is an $(\in, \in\vee q)$-fuzzy soft BCK-algebra over $\widetilde{\mathscr{X}}$.

Theorem 2. *Every fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$ is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathscr{X}}$.*

Proof (straightforward) □

The converse of Theorem 2 is not true as follows.

Example 3. Consider $(\widetilde{\zeta}, \Omega) \in \mathscr{F}\mathscr{S}_{\mathscr{BCI}}\mathscr{A}(\widetilde{\mathscr{X}})$ in Example 1. We know that $(\widetilde{\zeta}, \Omega)$ is not a fuzzy soft BCI-algebra over $\widetilde{\mathscr{X}}$

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since $(\widetilde{\zeta}, \Omega)$ is not a fuzzy soft BCI-algebra over $\widetilde{\mathcal{X}}$ based on the parameter e_1 as $\widetilde{\zeta}[e_1](0) = 0.6 < 0.7 = \widetilde{\zeta}[e_1](a)$.

Lemma 1 (see [36]). *A fuzzy set μ in $\widetilde{\mathcal{X}}$ is an $(\in, \in\vee q)$-fuzzy subalgebra of*

$$\widetilde{\mathcal{X}} \Leftrightarrow (\forall \hbar, \kappa \in \widetilde{\mathcal{X}})(\mu(\hbar * \kappa) \geq \min\{\mu(\hbar), \mu(\kappa), 0.5\}). \tag{11}$$

According to the Lemma 1, the following theorem is straightforward.

Theorem 3. *A fuzzy soft set $(\widetilde{\zeta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$ if and only if*

$$(\forall \hbar, \kappa \in \widetilde{\mathcal{X}})(\forall u \in \Omega)\left(\widetilde{\zeta}[u](\hbar * \kappa) \geq \min\left\{\widetilde{\zeta}[u](\hbar), \widetilde{\zeta}[u](\kappa), 0.5\right\}\right). \tag{12}$$

Theorem 4. *If $(\widetilde{\zeta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$ such that*

$$(\forall v \in \Omega)(\forall \hbar \in \widetilde{\mathcal{X}})(\widetilde{\zeta}[v](\hbar) < 0.5), \tag{13}$$

then $(\widetilde{\zeta}, \Omega)$ is a fuzzy soft BCK/BCI-algebra over $\widetilde{\mathcal{X}}$.

Proof. Let $\hbar, \kappa \in \widetilde{\mathcal{X}}$ and $v \in \Omega$. Since $(\widetilde{\zeta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$, it follows from Theorem 3 and (13) that

$$\begin{aligned} \widetilde{\zeta}[v](\hbar * \kappa) &\geq \min\left\{\widetilde{\zeta}[v](\hbar), \widetilde{\zeta}[v](\kappa), 0.5\right\} \\ &= \min\left\{\widetilde{\zeta}[v](\hbar), \widetilde{\zeta}[v](\kappa)\right\}. \end{aligned} \tag{14}$$

Therefore, $(\widetilde{\zeta}, \Omega)$ is a fuzzy soft BCK/BCI-algebra over $\widetilde{\mathcal{X}}$. □

Theorem 5. *If $(\widetilde{\zeta}, \Omega), (\widetilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$, then the extended intersection of $(\widetilde{\zeta}, \Omega)$ and $(\widetilde{\eta}, \mho)$ is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra over $\widetilde{\mathcal{X}}$.*

Proof. Let $(\widetilde{\zeta}, \Omega)\,\widetilde{\cap}_e\,(\widetilde{\eta}, \mho) = (\widetilde{\xi}, \Upsilon)$ be the extended intersection of $(\widetilde{\zeta}, \Omega)$ and $(\widetilde{\eta}, \mho)$. Then, $\Upsilon = \Omega \cup \mho$. For any $v \in \Upsilon$, if $v \in \Omega \setminus \mho$ (resp. $v \in \mho \setminus \Omega$), then $\widetilde{\xi}[v] = \widetilde{\zeta}[v] \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$ (resp. $\widetilde{\xi}[v] = \widetilde{\eta}[v] \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$). If $\Omega \cap \mho \neq \varnothing$, then $\widetilde{\xi}[v] = \widetilde{\zeta}[v] \cap \widetilde{\eta}[v] \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$ for all $v \in \Omega \cap \mho$ since the intersection of two $(\in, \in\vee q)$-fuzzy BCK/BCI-algebras is an $(\in, \in\vee q)$-fuzzy BCK/BCI-algebra. Therefore, $(\widetilde{\xi}, \Upsilon) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$. □

Corollary 1. *The restricted intersection of two $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras is an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra.*

Theorem 6. *Let $(\widetilde{\zeta}, \Omega), (\widetilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$. If $\Omega \cap \mho = \varnothing$, then the union $(\widetilde{\zeta}, \Omega)\,\widetilde{\cup}\,(\widetilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$.*

Proof. By Definition 3, we can write $(\widetilde{\zeta}, \Omega)\,\widetilde{\cup}\,(\widetilde{\eta}, \mho) = (\widetilde{\xi}, \Upsilon)$, where $\Upsilon = \Omega \cup \mho$ and for all $e \in \Upsilon$,

$$\widetilde{\xi}[e] = \begin{cases} \widetilde{\zeta}[e], & \text{if } e \in \Omega \setminus \mho, \\ \widetilde{\eta}[e], & \text{if } e \in \mho \setminus \Omega, \\ \widetilde{\zeta}[e] \cup \widetilde{\eta}[e], & \text{if } e \in \Omega \cap \mho. \end{cases} \tag{15}$$

Since $v \in \Omega \setminus \mho$ or $v \in \mho \setminus \Omega$ for all $v \in \Upsilon$. If $v \in \Omega \setminus \mho$, then $\widetilde{\xi}[v] = \widetilde{\zeta}[v] \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$ because $(\widetilde{\zeta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$. If $v \in \mho \setminus \Omega$, then $\widetilde{\xi}[v] = \widetilde{\eta}[v] \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$ because $(\widetilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$. Hence, $(\widetilde{\xi}, \Upsilon) = (\widetilde{\zeta}, \Omega)\,\widetilde{\cup}\,(\widetilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$. □

The following illustration shows that Theorem 6 is not valid if $\Omega \cap \mho \neq \varnothing$.

Example 4. Let $\widetilde{\mathcal{X}} = \{0, 1, \imath, \mathcal{J}, \kappa\}$ be a BCI-algebra with the following table.

$*$	0	1	$\imath$	$\mathcal{J}$	κ
0	0	0	$\imath$	$\mathcal{J}$	κ
1	1	0	$\imath$	$\mathcal{J}$	κ
$\imath$	$\imath$	$\imath$	0	κ	$\mathcal{J}$
$\mathcal{J}$	$\mathcal{J}$	$\mathcal{J}$	κ	0	$\imath$
κ	κ	κ	$\mathcal{J}$	$\imath$	0

Consider sets of parameters as follows:

$$\begin{aligned} \Omega &= \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}, \\ \mho &= \{\alpha_3, \alpha_4, \beta_5\}. \end{aligned} \tag{16}$$

Then, Ω and $\mho$ are not disjoint sets of parameters. Let $(\widetilde{\zeta}, \Omega)$ be a fuzzy soft set over $\widetilde{\mathcal{X}}$. Then, $\widetilde{\zeta}[\alpha_1]$, $\widetilde{\zeta}[\alpha_2]$, $\widetilde{\zeta}[\alpha_3]$, and $\widetilde{\zeta}[\alpha_4]$ are fuzzy sets in $\widetilde{\mathcal{X}}$. We define them as follows:

$\widetilde{\zeta}$	0	1	$\imath$	$\mathcal{J}$	κ
α_1	0.7	0.6	0.3	0.3	0.3
α_2	0.6	0.5	0.4	0.2	0.2
α_3	0.8	0.5	0.1	0.3	0.1
α_4	0.5	0.5	0.2	0.2	0.4

Then, $(\widetilde{\zeta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$. Let $(\widetilde{\eta}, \mho) \in \mathcal{FS}(\widetilde{\mathcal{X}})$. Then, $\widetilde{\eta}[\alpha_3]$, $\widetilde{\eta}[\alpha_4]$, and $\widetilde{\eta}[\beta_5]$ are fuzzy sets in $\widetilde{\mathcal{X}}$. We define them as follows:

$\widetilde{\eta}$	0	1	$\imath$	$\mathcal{J}$	κ
α_3	0.8	0.6	0.3	0.1	0.1
α_4	0.7	0.6	0.3	0.3	0.5
β_5	0.9	0.5	0.2	0.4	0.2

Then, $(\widetilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK/BCI}}\mathcal{A}(\widetilde{\mathcal{X}})$, and the union

$$(\widetilde{\zeta}, \Omega)\,\widetilde{\cup}\,(\widetilde{\eta}, \mho) = (\widetilde{\xi}, \Upsilon), \tag{17}$$

of $(\widetilde{\zeta}, \Omega)$ and $(\widetilde{\eta}, \mho)$ is described as follows:

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$\tilde{\xi}$	0	1	$\imath$	$\mathcal{J}$	κ
α_1	0.7	0.6	0.3	0.3	0.3
α_2	0.6	0.5	0.4	0.2	0.2
α_3	0.8	0.6	0.3	0.3	0.1
α_4	0.7	0.6	0.3	0.3	0.5
β_5	0.9	0.5	0.2	0.4	0.2

For a parameter $\alpha_3 \in \Omega \cap \mho$, we have

$$\begin{aligned}\tilde{\xi}[\alpha_3](\mathcal{J} * \imath) &= \left(\tilde{\zeta}[\alpha_3] \cup \tilde{\eta}[\alpha_3]\right)(\mathcal{J} * \imath)\\ &= \left(\tilde{\zeta}[\alpha_3] \cup \tilde{\eta}[\alpha_3]\right)(\kappa)\\ &= \max\left\{\tilde{\zeta}[\alpha_3](\kappa), \tilde{\eta}[\alpha_3](\kappa)\right\}\\ &= \max\{0.1, 0.1\} = 0.1,\\ &\min\left\{\tilde{\xi}[\alpha_3](\mathcal{J}), \tilde{\xi}[\alpha_3](\imath), 0.5\right\}\\ &= \min\left\{\left(\tilde{\zeta}[\alpha_3] \cup \tilde{\eta}[\alpha_3]\right)(\mathcal{J}), \left(\tilde{\zeta}[\alpha_3] \cup \tilde{\eta}[\alpha_3]\right)(\imath), 0.5\right\}\\ &= \min\left\{\max\left\{\tilde{\zeta}[\alpha_3](\mathcal{J}), \tilde{\eta}[\alpha_3](\mathcal{J})\right\}, \max\left\{\tilde{\zeta}[\alpha_3](\mathcal{J}), \tilde{\eta}[\alpha_3](\mathcal{J})\right\}, 0.5\right\}\\ &= \min\{\max\{0.3, 0.1\}, \max\{0.1, 0.3\}, 0.5\}\\ &= 0.3.\end{aligned} \tag{18}$$

Thus, from Theorem 3, $(\tilde{\xi}, \Upsilon) \notin \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$ based on the parameter α_3 and so that $(\tilde{\xi}, \Upsilon) \notin \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$.

Theorem 7. *If* $(\tilde{\zeta}, \Omega), (\tilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$, *then* $(\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$.

Proof. By Definition 4, we have

$$(\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho) = (\tilde{\xi}, \Omega \times \mho), \tag{19}$$

where $\tilde{\xi}[u, v] = \tilde{\zeta}[u] \cap \tilde{\eta}[v], \forall (u, v) \in \Omega \times \rho$. For any $\hbar, \kappa \in \tilde{\mathcal{X}}$, we have

$$\begin{aligned}\tilde{\xi}[u, v](\hbar * \kappa) = (\tilde{\zeta}[u] \cap \tilde{\eta}[v])(\hbar * \kappa) &= \min\left\{\tilde{\zeta}[u](\hbar * \kappa), \tilde{\eta}[v](\hbar * \kappa)\right\}\\ &\geq \min\left\{\min\left\{\tilde{\zeta}[u](\hbar), \tilde{\zeta}[u](\kappa), 0.5\right\}, \min\{\tilde{\eta}[v](\hbar), \tilde{\eta}[v](\kappa), 0.5\}\right\}\\ &= \min\left\{\min\left\{\tilde{\zeta}[u](\hbar), \tilde{\eta}[v](\hbar)\right\}, \min\left\{\tilde{\zeta}[u](\kappa), \tilde{\eta}[v](\kappa)\right\}, 0.5\right\}\\ &= \min\left\{(\tilde{\zeta}[u] \cap \tilde{\eta}[v])(\hbar), (\tilde{\zeta}[u] \cap \tilde{\eta}[v])(\kappa), 0.5\right\}\\ &= \min\left\{\tilde{\xi}[u, v](\hbar), \tilde{\xi}[u, v](\kappa), 0.5\right\}.\end{aligned} \tag{20}$$

Hence, $(\tilde{\xi}, \Omega \times \mho) = (\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$ based on (u, v) by using Theorem 3. Since (u, v) is arbitrary,

$$(\tilde{\xi}, \Omega \times \mho) = (\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}}). \tag{21}$$

□

Definition 9. Let $(\tilde{\zeta}, \Omega), (\tilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$. We say that $(\tilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebra of $(\tilde{\eta}, \mho)$ if

(1) $\Omega \subseteq \rho$,

(2) $\tilde{\zeta}[u]$ is an $(\in, \in\vee q)$-fuzzy sub-BCK/BCI-algebra of $\tilde{\eta}[u]$ for all $u \in \Omega$; that is, $\tilde{\zeta}[u]$ is an $(\in, \in\vee q)$-fuzzy BCK/BCI-algebra satisfying the following condition:

$$(\forall \hbar \in \tilde{\mathcal{X}})(\tilde{\zeta}[u](\hbar) \leq \tilde{\eta}[u](\hbar)). \tag{22}$$

Example 5. Let $(\tilde{\zeta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$ in Example 1. For a subset $\rho = \{e_1, e_3\}$ of Ω, let $(\tilde{\eta}, \mho)$ be fuzzy soft set over $\tilde{\mathcal{X}}$ which is defined as follows:

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$\tilde{\eta}$	0	$\imath$	$\mathcal{J}$	κ
e_1	0.56	0.67	0.23	0.23
e_3	0.56	0.23	0.23	0.67

Then, $(\tilde{\eta}, \mho)$ is an $(\in, \in\vee q)$-fuzzy soft sub-BCI-algebra of $(\tilde{\zeta}, \Omega)$.

Example 6. Let $\tilde{\mathcal{X}} = \{0, 1, 2, 3, 4\}$ be a BCK-algebra with the following Cayley table.

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	0	2
3	3	2	1	0	3
4	4	4	4	4	0

Let $\rho = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters and let $(\tilde{\eta}, \mho) \in \mathcal{FS}(\tilde{\mathcal{X}})$ which is defined as follows:

$\tilde{\eta}$	0	1	2	3	4
e_1	0.6	0.2	0.8	0.2	0.4
e_2	0.7	0.7	0.3	0.3	0.5
e_3	0.8	0.1	0.3	0.1	0.4
e_4	0.6	0.6	0.3	0.3	0.6
e_5	0.9	0.3	0.4	0.3	0.2

Then, $(\tilde{\eta}, \mho) \in \mathcal{FS}_{\mathcal{BCK}}\mathcal{A}(\tilde{\mathcal{X}})$. For a subset $\Omega = \{e_1, e_3, e_4\}$ of ρ, let $(\tilde{\zeta}, \Omega) \in \mathcal{FS}(\tilde{\mathcal{X}})$ defined by

$\tilde{\zeta}$	0	1	2	3	4
e_1	0.56	0.2	0.78	0.2	0.34
e_3	0.78	0.1	0.23	0.1	0.34
e_4	0.56	0.56	0.3	0.3	0.56

Then, $(\tilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft sub-BCK-algebra of $(\tilde{\eta}, \mho)$.

Theorem 8. *Let $(\tilde{\zeta}, \Omega), (\tilde{\eta}, \Omega) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$. If $\tilde{\zeta}[u] \subseteq \tilde{\eta}[u]$ for all $u \in \Omega$, then $(\tilde{\zeta}, \Omega)$ is an $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebra of $(\tilde{\eta}, \Omega)$.*

Proof (straightforward) □

Theorem 9. *Let $(\tilde{\xi}, \Upsilon) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$. If $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$ are $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebras of $(\tilde{\xi}, \Upsilon)$, then so is the extended intersection of $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$.*

Proof. The proof is followed from Theorem 5 and Definition 9. □

Theorem 10. *Let $(\tilde{\xi}, \Upsilon) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$. If $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$ are $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebras of $(\tilde{\xi}, \Upsilon)$, then so is the union of $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$ whenever Ω and ρ are disjoint.*

Proof. The proof is followed from Theorem 6 and Definition 9. □

Theorem 11. *Let $(\tilde{\xi}, \Upsilon) \in \mathcal{FS}_{\mathcal{BCK}/\mathcal{BCI}}\mathcal{A}(\tilde{\mathcal{X}})$. If $(\tilde{\zeta}, \Omega)$ and $(\tilde{\eta}, \mho)$ are $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebras of $(\tilde{\xi}, \Upsilon)$, then $(\tilde{\zeta}, \Omega)\tilde{\wedge}(\tilde{\eta}, \mho)$ is an $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebra of $(\tilde{\xi}, \Upsilon)\tilde{\wedge}(\tilde{\xi}, \Upsilon)$.*

Proof. The proof is followed from Theorem 7 and Definition 9. □

4. Conclusion

In this paper, we introduced the notions of $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras and $(\in, \in\vee q)$-fuzzy soft sub-BCK/BCI-algebras and investigated their related properties. Also, we discussed relations between fuzzy soft BCK/BCI-algebras and $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebras. Moreover, conditions for an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra to be a fuzzy soft BCK/BCI-algebra are provided. Moreover, the union, the extended intersection, and the "AND"-operation of two $(\in, \in\vee q)$-fuzzy soft (sub-) BCK/BCI-algebras are discussed, and a characterization of an $(\in, \in\vee q)$-fuzzy soft BCK/BCI-algebra is established.

We hope that this work will provide a deep impact on the upcoming research in this field and other soft algebraic studies to open up new horizons of interest and innovations. To extend these results, one can further study these notions on different algebras such as rings, hemirings, LA-semigroups, semihypergroups, semihyperrings, BL-algebras, MTL-algebras, R_0-algebras, MV-algebras, EQ-algebras, d-algebras, Q-algebras, and lattice implication algebras. Some important issues for future work are (1) to develop strategies for obtaining more valuable results and (2) to apply these notions and results for studying related notions in other algebraic (soft) structures.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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