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Research Article

Some New Results of Interpolative Hardy–Rogers and Ćirić–Reich–Rus Type Contraction

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In this paper, we present new concepts on completeness of Hardy–Rogers type contraction mappings in metric space to prove the existence of fixed points. Furthermore, we introduce the concept of g-interpolative Hardy–Rogers type contractions in b-metric spaces to prove the existence of the coincidence point. Lastly, we add a third concept, interpolative weakly contractive mapping type, Ćirić–Reich–Rus, to show the existence of fixed points. These results are a generalization of previous results, which we have reinforced with examples.

1. Introduction and Preliminaries

The theory of fixed points has known a lot of evolution. It has been given great merit and concern, thanks to its many uses in several fields of mathematics, such as differential equations, graph theory, and nonlinear analysis [1–8]. Besides, the emergence of the fixed theorem with Banach [4] in 1922 on complete normed space was followed by several improvements and generalizations of this theorem on two levels: the first level is related to the applications used, and the second to the spaces used in them. It first knew improvements with Kannan [9] in 1968, and later with other researchers such as Rus, Ćirić, Reich, Hardy, and Rogers. Afterwards, it took another turning with Karapinar [10] in 2018 in a new version, which has made several researchers pursue this field (see [11–19]). Thus, the concept has been applied in various spaces: metric space, b-metric space, rectangular b-metric spaces, and the Branciari distance. More recently, Errai et al. [14] have inserted g-interpolation over Ćirić–Reich–Rus type contraction. They have also introduced the concept of interpolative weakly contractive mapping, which makes us use these two concepts in this paper: the first concept on Hardy–Rogers type contraction and the second on Ćirić–Reich–Rus type contraction as a generalization of the previous findings, reinforced by various examples. This leads us to come up with some remarks. Before starting, we will take some basic concepts that we will use in this article.

Definition 1 (see [20, 21]). Let $s \geq 1$ be a given real number and F be a nonempty set. A function $d\colon \mathsf{F} \times \mathsf{F} \longrightarrow \mathbb{R}^+$ is a b-metric if the following conditions are met for all $v, y, z \in \mathsf{F}$

(b_1) $d(v, y) = 0$ if and only if $v = y$

(b_2) $d(v, y) = d(y, v)$

(b_3) $d(v, z) \leq s[d(v, y) + d(y, z)]$

The pair (F, d) is called a b-metric space.

It is worth mentioning that b-metric spaces are a broader category than metric spaces.

The definitions of b-convergent and b-Cauchy sequences, as well as b-complete b-metric spaces, are defined in the same way as usual metric spaces (see, e.g., [22]).

For the interesting examples and properties of b-metric, see the following papers [23–25] as examples.

Definition 2 (see [26, 27]). Let $\{v_n\}$ be a sequence in a b-metric space (F, d). $S, h\colon \mathsf{F} \longrightarrow \mathsf{F}$ and $v \in \mathsf{F}$. v is said to be coincidence point of pair $\{S, h\}$ if $Sv = hv$.

Definition 3 (see [12, 13]). Let Ψ be the set of all nondecreasing functions ψ: $[0,\infty) \longrightarrow [0,\infty)$, with $\sum_{k=0}^{\infty} \psi^k(t) < \infty$ for all $t > 0$. After that,

(a) $\psi(t) < t$ for each $t > 0$

(b) $\psi(0) = 0$

Remark 1 (see [22]). The following assertions apply in a b-metric space (F, d):

(1) Each b-convergent sequence is a b-Cauchy sequence.

(2) A b-convergent sequence has a unique limit.

(3) In general, a b-metric is not continuous.

To prove our results, the fact in the previous remark necessitates the following lemma regarding b-convergent sequences:

Lemma 1 (see [26]). *Let (F, d) be a b-metric space with $s \geq 1$, and assume that $\{v_n\}$ and $\{y_n\}$ are b-convergent to v, y, respectively, so we have*

$$\frac{1}{s^2} d(v, y) \leq \liminf_{n\longrightarrow\infty} d(v_n, y_n) \leq \limsup_{n\longrightarrow\infty} d(v_n, y_n) \leq s^2 d(v, y). \tag{1}$$

In particular, if $v = y$, then we have $\lim_{n\longrightarrow\infty} d(v_n, y_n) = 0$. In addition, for each $z \in \mathsf{F}$, we have

$$\frac{1}{s} d(v, z) \leq \liminf_{n\longrightarrow\infty} d(v_n, z) \leq \limsup_{n\longrightarrow\infty} d(v_n, z) \leq s d(v, z). \tag{2}$$

Lemma 2. *Let $\{v_n\}$ be a sequence defined on a b-metric space (F, d, s) and meets the conditions:*

(i) $\{v_n\}$ is b-convergent sequence in (F, d, s)

(ii) $d(v_{n+1}, v_n) \leq \psi(d(v_n, v_{n-1})) \leq \psi^2(d(v_{n-1}, v_{n-2})) \leq \ldots \leq \psi^n(d(v_1, v_0))$, where $\psi \in \Psi$

Then, $\{v_n\}$ is a b-Cauchy sequence in (F, d, s).

Proof. Let $p \in \mathbb{N}\backslash\{0\}$; using the triangle inequality of the b-metric space and condition (ii), we have

$$\begin{aligned} d(v_n, v_{n+p}) &\leq s\left[d(v_n, v_{n+1}) + d(v_{n+1}, v_{n+p})\right] \\ &\leq s\left[d(v_n, v_{n+1}) + s d(v_{n+1}, v_{n+2}) + s d(v_{n+2}, v_{n+p})\right] \\ &\vdots \\ &\leq s\psi^n(d(v_0, v_1)) + s^2\psi^{n+1}(d(v_0, v_1)) + \cdots + s^p\psi^{n+p-1}(d(v_0, v_1)) \\ &\leq s^p \sum_{k=n}^{n+p-1} \psi^k(d(v_0, v_1)) \\ &\leq s^p \sum_{k=n}^{\infty} \psi^k(d(v_0, v_1)). \end{aligned} \tag{3}$$

Since $\sum_{k=0}^{\infty} \psi^k(t) < \infty$ for each $t > 0$, then $\lim_{n\longrightarrow\infty} \sum_{k=n}^{\infty} \psi^k(d(v_0, v_1)) = 0$, which implies for any finite integer $p \geq 1$:

$$\lim_{n\longrightarrow\infty} d(v_n, v_{n+p}) = 0. \tag{4}$$

Then, $\{v_n\}$ is a b-Cauchy sequence in (F, d, s). □ □

2. Results

The set of functions ξ: $[0,\infty) \longrightarrow [0,\infty)$ which satisfies $\xi(t) < t$ for all $t > 0$ is denoted by Ξ.

Definition 4. Consider the metric space (F, d). If there exist $\alpha, \eta, \omega, \theta \in (0,1)$ with $\alpha + \eta + \omega + \theta > 1$, the self mapping T: $\mathsf{F} \longrightarrow \mathsf{F}$ is named a ξ-interpolative Hardy–Rogers type contraction, such that

$$d(Tv, Ty) \leq \xi\left([d(v, y)]^{\alpha} [d(v, Tv)]^{\eta} [d(y, Ty)]^{\omega} \left[\frac{d(v, Ty) + d(y, Tv)}{2}\right]^{\theta} \right), \tag{5}$$

for all $v, y \in \mathsf{F}\backslash\mathrm{Fix}(T)$, where $\mathrm{Fix}(T) = \{a \in \mathsf{F} | Ta = a\}$ and $\xi \in \Xi$.

The following is our key finding:

Theorem 1. *In a complete metric space (F, d), a ξ-interpolative Hardy–Rogers type contraction T: $\mathsf{F} \longrightarrow \mathsf{F}$, we assume there exists $v \in \mathsf{F}$ such that $d(v, Tv) < 1$. Then, T has a fixed point in F.*

Proof. Let $\{v_n\}$ be the sequence defined by $v_0 = v$ and $v_{n+1} = Tv_n$ for all integer n. If there exists n_0 such that $v_{n_0} = v_{n_0+1}$, then v_{n_0} is a fixed point of T. The proof is complete. Suppose that $v_{n+1} \neq Tv_n$ for all $n \geq 0$.

By substituting the values $v = v_{n-1}$ and $y = v_n$ in (5), we have

$$\begin{aligned} d(v_n, v_{n+1}) &\leq \xi\left([d(v_{n-1}, v_n)]^{\alpha} [d(v_{n-1}, v_n)]^{\eta} [d(v_n, v_{n+1})]^{\omega} \left[\frac{d(v_{n-1}, v_{n+1}) + d(v_n, v_n)}{2}\right]^{\theta}\right) \\ &\leq \xi\left([d(v_{n-1}, v_n)]^{\alpha+\eta} [d(v_n, v_{n+1})]^{\omega} \left[\frac{d(v_{n-1}, v_n) + d(v_n, v_{n+1})}{2}\right]^{\theta}\right). \end{aligned} \tag{6}$$

Using the fact $\xi(t) < t$ for each $t > 0$, we obtain

$$\begin{aligned} d(v_n, v_{n+1}) &\leq \xi\left([d(v_{n-1}, v_n)]^{\alpha+\eta} [d(v_n, v_{n+1})]^{\omega} \left[\frac{d(v_{n-1}, v_n) + d(v_n, v_{n+1})}{2}\right]^{\theta}\right) \\ &< [d(v_{n-1}, v_n)]^{\alpha+\eta} [d(v_n, v_{n+1})]^{\omega} \left[\frac{d(v_{n-1}, v_n) + d(v_n, v_{n+1})}{2}\right]^{\theta}. \end{aligned} \tag{7}$$

If $d(v_{n-1}, v_n) < d(v_n, v_{n+1})$ for some $n \geq 1$, then

$$\frac{d(v_{n-1}, v_n) + d(v_n, v_{n+1})}{2} \leq d(v_n, v_{n+1}). \tag{8}$$

From (7), we obtain

$$d(v_n, v_{n+1}) < [d(v_{n-1}, v_n)]^{\alpha+\eta} [d(v_n, v_{n+1})]^{\omega+\theta}. \tag{9}$$

Thus,

$$[d(v_n, v_{n+1})]^{1-\omega-\theta} < [d(v_{n-1}, v_n)]^{\alpha+\eta}, \tag{10}$$

which implies

$$[d(v_{n-1}, v_n)]^{1-\omega-\theta} < [d(v_{n-1}, v_n)]^{\alpha+\eta}, \tag{11}$$

which contradicts with $1 - \omega - \theta < \alpha + \eta$ and $d(v_{n-1}, v_n) < 1$. Then, $d(v_{n-1}, v_n) \geq d(v_n, v_{n+1})$ for all $n \geq 1$, and the sequence $(d(v_{n-1}, v_n) + d(v_n, v_{n+1}))/2 \leq d(v_{n-1}, v_n)$. From (7), we deduce

$$[d(v_n, v_{n+1})]^{1-\omega} < [d(v_{n-1}, v_n)]^{\alpha+\eta+\theta} \quad \text{for all } n \geq 1. \tag{12}$$

Since $d(v_0, v_1) < 1$, so there exists a real $l \in (0, 1)$ such that $d(v_0, v_1) \leq l$ and $l = (d(v_0, v_1) + 1)/2$.

By (12), we obtain

$$d(v_1, v_2) \leq [d(v_0, v_1)]^{(\alpha+\eta+\theta)/(1-\omega)} \leq l^{(\alpha+\eta+\theta)/(1-\omega)}. \tag{13}$$

Assume that there exists a real $\rho(n)$ such that

$$d(v_n, v_{n+1}) \leq l^{\rho(n)} \quad \text{for all } n \geq 0. \tag{14}$$

From (12), we deduce

$$[d(v_{n+1}, v_{n+2})]^{1-\omega} < [d(v_n, v_{n+1})]^{\alpha+\eta+\theta} \leq l^{(\alpha+\eta+\theta)\rho(n)}, \tag{15}$$

which gives

$$d(v_{n+1}, v_{n+2}) < l^{\rho(n+1)}, \tag{16}$$

where $\rho(n+1) = ((\alpha + \eta + \theta)/(1 - \omega))\rho(n)$ for all $n \geq 1$ with $\rho(0) = 1$.

Since $(\alpha + \eta + \theta)/(1 - \omega) > 1$; we have $\lim_{n \longrightarrow \infty} \rho(n) = +\infty$.

Consequently,

$$\sum_{n=0}^{+\infty} d(v_n, v_{n+1}) \leq \sum_{n=0}^{+\infty} l^{\rho(n)}, \tag{17}$$

which is convergent, so $\{v_n\}$ is Cauchy sequence in (F, d), and then it converges to some $\nu \in \mathsf{F}$. Suppose that $\nu \neq T\nu$, we find by (5):

$$d(v_{n+1}, Tv) \leq \xi\left([d(v_n, v)]^{\alpha} [d(v_n, v_{n+1})]^{\eta} [d(v, Tv)]^{\omega} \left[\frac{d(v_n, Tv) + d(v, v_{n+1})}{2} \right]^{\theta} \right)$$
$$< [d(v_n, v)]^{\alpha} [d(v_n, v_{n+1})]^{\eta} [d(v, Tv)]^{\omega} \left[\frac{d(v_n, Tv) + d(v, v_{n+1})}{2} \right]^{\theta}. \tag{18}$$

Passing the limit as $n \longrightarrow +\infty$, we get $d(v, Tv) \leq 0$. So $d(v, Tv) = 0$ which is a contradiction. Then, $Tv = v$. □

Example 1. Let $E = [0, 3]$ be endowed with metric μ: $E \times E \longrightarrow [0, \infty)$, defined by

$$\mu(u, v) = \begin{cases} 0, & \text{if } u = v, \\ 2, & \text{if } u, v \in [0, 1] \text{ and } u \neq v, \\ 3, & \text{otherwise.} \end{cases} \tag{19}$$

Let T: $E \longrightarrow E$ be defined as

$$Tu = \begin{cases} \dfrac{1}{3}, & \text{if } u \in [0, 1]; \\ \dfrac{u}{3}, & \text{if } u \in (1, 3]; \end{cases} \tag{20}$$

and the function $\xi(t) = 2/7t^2$ for all $t \in [0, \infty)$.

Choose $\alpha = 0.7$; $\eta = 0.6$; $\omega = 0.8$; and $\theta = 0.5$.

We have $\mu(Tu, Tv) \leq 2$ for all $u, v \in E$.

The following issues are discussed:

First case: if $u, v \in [0, 1]$ or $u = v$ for all $u, v \in [0, 3]$, we have $\xi(t) = 2/7t^2 \geq 0$ for all $t \in [0, \infty)$, and $\mu(Tu, Tv) = 0$ for all $u, v \in [0, 1]$ or $u = v$ for all $u, v \in [0, 3]$.

Consequently, in this case, inequality (5) is satisfied.

Second case: if $u, v \in (1, 3]$ and $u \neq v$, we have

$$\xi\left([\mu(u, v)]^{\alpha} [\mu(u, Tu)]^{\eta} [\mu(v, Tv)]^{\omega} \left[\frac{\mu(u, Tv) + \mu(v, Tu)}{2} \right]^{\theta} \right) = \xi\left(3^{\alpha+\eta+\omega+\theta}\right) = \frac{2}{7} \cdot 3^{5.2} \geq 2. \tag{21}$$

Third case: if $u \in [0, 1]$ and $v \in (1, 3]$ with $u \neq 1/3$, we have

$$\xi\left([\mu(u, v)]^{\alpha} [\mu(u, Tu)]^{\eta} [\mu(v, Tv)]^{\omega} \left[\frac{\mu(u, Tv) + \mu(v, Tu)}{2} \right]^{\theta} \right) = \xi\left(3^{\alpha+\omega} \cdot 2^{\eta-\theta} \cdot 5^{\theta}\right) = \frac{2}{7} \cdot 3^{3} \cdot 2^{0.2} \cdot 5 \geq 2. \tag{22}$$

Fourth case: if $u \in (1, 3]$ and $v \in [0, 1]$ with $v \neq 1/3$, we have

$$\xi\left([\mu(u, v)]^{\alpha} [\mu(u, Tu)]^{\eta} [\mu(v, Tv)]^{\omega} \left[\frac{\mu(u, Tv) + d(v, Tu)}{2} \right]^{\theta} \right) = \xi\left(3^{\alpha+\eta} \cdot 2^{\omega-\theta} \cdot 5^{\theta}\right) = \frac{2}{7} \cdot 3^{2.6} \cdot 2^{0.6} \cdot 5 \geq 2. \tag{23}$$

Hence, in all cases, we have

$$\mu(Tu, Tv) \leq \xi\left([\mu(u, v)]^{\alpha} [\mu(u, Tu)]^{\eta} [\mu(v, Tv)]^{\omega} \left[\frac{\mu(u, Tv) + \mu(v, Tu)}{2} \right]^{\theta} \right). \tag{24}$$

for all $u, v \in [0,3]\backslash\{1/3\}$.

As a result, all the conditions of Theorem 1 are fulfilled, and T has a fixed point, $u = 1/3$.

Example 2. Let $\mathsf{F} = \{a, q, m, r\}$ be endowed with the metric given in the following chart (Table 1).

Consider the self mapping T on F as

$$T: \begin{pmatrix} a & q & m & r \\ q & q & a & r \end{pmatrix}. \tag{25}$$

Take $\xi(t) = (3^t - 1)/(3^t + 1)$, for all $t \in [0, \infty)$; $\alpha = 0.4$; $\eta = 0.8$; $\omega = 0.9$; and $\theta = 0.5$.

We have

$$d(Tu, Tv) \leq \xi\left([d(u,v)]^{\alpha} [d(u,Tu)]^{\eta} [d(v,Tv)]^{\omega} \left[\frac{d(u,Tv) + d(v,Tu)}{2}\right]^{\theta}\right), \tag{26}$$

for all $u, v \in \mathsf{F}\backslash\{q, r\}$.

Then, T has two fixed points, which are q and r.

If we replace $\xi(t) = \lambda t$ with $\lambda \in (0,1)$ in Theorem 1, we get the following corollary.

Corollary 1. *Let (F, d) be a complete metric space and T is self mapping on F such that*

$$d(Tv, Ty) \leq \lambda [d(v,y)]^{\alpha} [d(v,Tv)]^{\eta} [d(y,Ty)]^{\omega} \left[\frac{d(v,Ty) + d(y,Tv)}{2}\right]^{\theta} \tag{27}$$

is satisfied for all $v, y \in \mathsf{F}\backslash Fix(T)$, where $Fix(T) = \{a \in \mathsf{F} | Ta = a\}$ and $\alpha, \eta, \omega, \theta, \lambda \in (0,1)$ such that $\alpha + \eta + \omega + \theta > 1$.

If there exists $v \in \mathsf{F}$ such that $d(x, Tx) < 1$, then T has a fixed point in F.

Definition 5. Let (F, d, s) be a b-metric space and $T, g: \mathsf{F} \longrightarrow \mathsf{F}$ be self mappings on F. We say that T is a g-interpolative Hardy–Rogers type contraction if there exist $\psi \in \Psi$ and $\alpha, \eta, \omega \in (0,1)$ such that

$$d(Tv, Ty) \leq \psi\left([d(gv, gy)]^{\alpha} [d(gv, Tv)]^{\eta} [d(gy, Ty)]^{\omega} \left[\frac{d(gv, Ty) + d(gy, Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right) \tag{28}$$

is satisfied for all $v, y \in \mathsf{F}$ such that $Tv \neq gv, Ty \neq gy$, and $gv \neq gy$.

Theorem 2. *In a b-complete b-metric space (F, d, s), if T is a g-interpolative Hardy–Rogers type contraction such that*

(1) $T\mathsf{F} \subseteq g\mathsf{F}$

(2) $g\mathsf{F}$ is closed

Then, T and g have a coincidence point in F.

Proof. Let $v \in \mathsf{F}$, since $T\mathsf{F} \subseteq g\mathsf{F}$, we can inductively define a sequence $\{v_n\}$ such that

$$v_0 = v, \text{ and } gv_{n+1} = Tv_n, \quad \text{for all integer } n. \tag{29}$$

If there exists $n \in \{0, 1, 2, \ldots\}$ such that $gv_n = Tv_n$, then v_n is a coincidence point of g and T. Assume that $gv_n \neq Tv_n$, for all n. By (28), we obtain

$$\begin{aligned} d(Tv_{n+1},Tv_n) &\le \psi\left([d(gv_{n+1},gv_n)]^{\alpha}[d(gv_{n+1},Tv_{n+1})]^{\eta}[d(gv_n,Tv_n)]^{\omega}\left[\frac{d(gv_{n+1},Tv_n)+d(gv_n,Tv_{n+1})}{2s}\right]^{1-\alpha-\eta-\omega}\right) \\ &= \psi\left([d(Tv_n,Tv_{n-1})]^{\alpha}[d(Tv_n,Tv_{n+1})]^{\eta}[d(Tv_{n-1},Tv_n)]^{\omega}\left[\frac{d(Tv_n,Tv_n)+d(Tv_{n-1},Tv_{n+1})}{2s}\right]^{1-\alpha-\eta-\omega}\right) \\ &\le \psi\left([d(Tv_n,Tv_{n-1})]^{\alpha+\omega}[d(Tv_n,Tv_{n+1})]^{\eta}\left[\frac{d(Tv_{n-1},Tv_n)+d(Tv_n,Tv_{n+1})}{2}\right]^{1-\alpha-\eta-\omega}\right). \end{aligned} \tag{30}$$

Using the fact $\psi(t) < t$ for each $t > 0$, we obtain

$$d(Tv_{n+1},Tv_n) \le [d(Tv_n,Tv_{n-1})]^{\alpha+\omega}[d(Tv_n,Tv_{n+1})]^{\eta}\left[\frac{d(Tv_{n-1},Tv_n)+d(Tv_n,Tv_{n+1})}{2}\right]^{1-\alpha-\eta-\omega}. \tag{31}$$

Suppose that $d(Tv_{n-1},Tv_n) < d(Tv_n,Tv_{n+1})$ for some $n \ge 1$. Then,

$$\frac{d(Tv_{n-1},Tv_n)+d(Tv_n,Tv_{n+1})}{2} \le d(Tv_n,Tv_{n+1}). \tag{32}$$

Thus, from inequality (31), we have

$$d(Tv_{n+1},Tv_n) \le [d(Tv_n,Tv_{n-1})]^{\alpha+\omega}[d(Tv_n,Tv_{n+1})]^{1-\alpha-\omega}. \tag{33}$$

This implies

$$[d(Tv_{n+1},Tv_n)]^{\alpha+\omega} \le [d(Tv_n,Tv_{n-1})]^{\alpha+\omega}. \tag{34}$$

So, we get

$$d(Tv_{n+1},Tv_n) \le d(Tv_n,Tv_{n-1}), \tag{35}$$

which is a contradiction. Thus,

$$d(Tv_{n+1},Tv_n) \le d(Tv_n,Tv_{n-1}), \quad \text{for all } n \ge 1. \tag{36}$$

This means that the positive sequence $\{d(Tv_{n+1},Tv_n)\}$ is monotone decreasing, and consequently, there exists $c \ge 0$ such that $\lim_{n\longrightarrow+\infty} d(Tv_{n+1},Tv_n) = c$.

From (36), we have

$$\begin{aligned} &[d(Tv_n,Tv_{n-1})]^{\alpha+\omega}[d(Tv_n,Tv_{n+1})]^{\eta}\left[\frac{d(Tv_{n-1},Tv_n)+d(Tv_n,Tv_{n+1})}{2}\right]^{1-\alpha-\eta-\omega} \\ &\le [d(Tv_n,Tv_{n-1})]^{\alpha+\omega}[d(Tv_n,Tv_{n-1})]^{\eta}[d(Tv_{n-1},Tv_n)]^{1-\alpha-\eta-\omega} = d(Tv_n,Tv_{n-1}). \end{aligned} \tag{37}$$

So, (31) together with the nondecreasing character of ψ, we obtain

$$d(Tv_{n+1},Tv_n) \le \psi(d(Tv_n,Tv_{n-1})). \tag{38}$$

Similar to the previous method, we find

$$d(Tv_{n+1},Tv_n) \le \psi(d(Tv_n,Tv_{n-1})) \le \psi^2(d(Tv_{n-1},Tv_{n-2})) \le \ldots \le \psi^n(d(Tv_1,Tv_0)). \tag{39}$$

Letting $n \longrightarrow \infty$ in (39) and using the fact $\lim_{n\longrightarrow\infty}\psi^n(t) = 0$ for each $t > 0$, we deduce that $c = 0$, that is,

$$\lim_{n\longrightarrow\infty} d(Tv_{n+1},Tv_n) = 0. \tag{40}$$

By Lemma 2, we deduce that $\{Tv_n\}$ is a b-Cauchy sequence, and consequently, $\{gv_n\}$ is also a b-Cauchy sequence. Let $z \in \digamma$ such that,

$$\lim_{n\longrightarrow\infty} d(Tv_n,z) = \lim_{n\longrightarrow\infty} d(gv_{n+1},z) = 0. \tag{41}$$

And, since $z \in g\digamma$, there exists $u \in \digamma$ such that $z = gu$. We claim that u is a coincidence point of g and T. Thus, if we assume that $gu \neq Tu$, we obtain

$$\begin{aligned} d(Tv_n, Tu) &\leq \psi\left([d(gv_n, gu)]^{\alpha} [d(gv_n, Tv_n)]^{\eta} [d(gu, Tu)]^{\omega} \left[\frac{d(gv_n, Tu) + d(gu, Tv_n)}{2s} \right]^{1-\alpha-\eta-\omega} \right) \\ &< [d(gv_n, gu)]^{\alpha} [d(gv_n, Tv_n)]^{\eta} [d(gu, Tu)]^{\omega} \left[\frac{d(gv_n, Tu) + d(gu, Tv_n)}{2s} \right]^{1-\alpha-\eta-\omega}. \end{aligned} \tag{42}$$

Consequently,

$$\frac{1}{s} d(z, Tu) \leq [sd(z, gu)]^{\alpha} \left[s^2 d(z, z) \right]^{\eta} [d(gu, Tu)]^{\omega} \left[\frac{d(z, Tu) + d(gu, z)}{2} \right]^{1-\alpha-\eta-\omega} = 0, \tag{43}$$

which is a contradiction. This implies that

$$Tu = z = gu. \tag{44}$$

Then, u is a coincidence point in $\digamma$ of T and g. □

Example 3. Let the set $\digamma = \{a, r, i, m\}$ and a function d: $\digamma \times \digamma \longrightarrow [0, \infty)$ defined as follows (Table 2)

One can check that the function d is a b-metric for $s = 2$ by doing a simple calculation. Self-mappings g, T on $\digamma$ are defined as

$$g: \begin{pmatrix} a & r & i & m \\ a & m & i & i \end{pmatrix}, T: \begin{pmatrix} a & r & i & m \\ i & m & m & i \end{pmatrix}. \tag{45}$$

Choose $\alpha = 0.4; \eta = 0.1; \omega = 0.7;$ and $\psi(t) = (5^t - 1)/(5^t + 1)$ for all $t \in [0, \infty)$.

For all $u, v \in \digamma \backslash \{r, m\}$, it is obvious that g, T fulfills (28). Furthermore, r and m are two coincidence points of g and T.

Example 4. Let $\digamma = [0, +\infty)$ and d: $\digamma \times \digamma \longrightarrow [0, \infty)$ be defined by

$$d(v, y) = \begin{cases} (v + y)^2, & \text{if } v \neq y; \\ 0, & \text{if } v = y. \end{cases} \tag{46}$$

Then, $(\digamma, d)$ is a complete b-metric space.

Define two self-mappings T and g on $\digamma$ by $g(v) = v^2$, for all $v \in \digamma$ and

$$Tx = \begin{cases} 1, & \text{if } v \in [0, 2]; \\ e^{-v}, & \text{if } v \in (2, +\infty). \end{cases} \tag{47}$$

T is a g-interpolative Hardy–Rogers type contraction for $\alpha = 0.3$; $\eta = 0.6$; and $\omega = 0.4$. Let $\psi(t) = 3/5t^2$, for all $t \in [0, +\infty)$.

Hence, the following issues are discussed.

First case: if $v, y \in [0, 2]$ or $v = y$ for all $v \in [0, +\infty)$, this is obvious.

Second case: if $v \in [0, 2] \backslash \{1\}$ and $y \in (2, +\infty)$.

Consider

$$M(v, y) = [d(gv, gy)]^{\alpha} [d(gv, Tv)]^{\eta} [d(gy, Ty)]^{\omega} \left[\frac{d(gv, Ty) + d(gy, Tv)}{2s} \right]^{1-\alpha-\eta-\omega}. \tag{48}$$

We have

$$d(Tv, Ty) = \left(1 + e^{-v}\right)^2 \leq \left(1 + e^{-2}\right)^2 \tag{49}$$

and

$$\begin{aligned}\psi(M(v,y)) &= \psi\left([d(gv,gy)]^{\alpha}[d(gv,Tv)]^{\eta}[d(gy,Ty)]^{\omega}\left[\frac{d(gv,Ty)+d(gy,Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right)\\ &= \psi\left(\left(v^2+y^2\right)^{2\alpha}\left(v^2+1\right)^{2\eta}\left(y^2+e^{-y}\right)^{2\omega}\left[\frac{\left(v^2+e^{-v}\right)^2+\left(y^2+1\right)^2}{4}\right]^{1-\alpha-\eta-\omega}\right)\\ &\geq \psi\left(4^{2\alpha}\cdot 1^{2\eta}\cdot 4^{2\omega}\left(\frac{5^2}{4}\right)^{1-\alpha-\eta-\omega}\right)\\ &= \psi\left(4^{1.7}\cdot 5^{-0.6}\right)\\ &= 3\cdot 4^{3.4}\cdot 5^{-2.2}\geq\left(1+e^{-2}\right)^2.\end{aligned} \tag{50}$$

Therefore,

$$d(Tv,Ty)\leq\psi\left([d(gv,gy)]^{\alpha}[d(gv,Tv)]^{\eta}[d(gy,Ty)]^{\omega}\left[\frac{d(gv,Ty)+d(gy,Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right). \tag{51}$$

Third case: if $v\in(2,+\infty)$ and $y\in[0,2]\backslash\{1\}$.
We have

$$d(Tv,Ty)=\left(e^{-v}+1\right)^2\leq\left(1+e^{-2}\right)^2, \tag{52}$$

and

$$\begin{aligned}\psi(M(v,y)) &= \psi\left([[d(gv,gy)]^{\alpha}[d(gv,Tv)]^{\eta}[d(gy,Ty)]^{\omega}\left[\frac{d(gv,Ty)+d(gy,Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right)\\ &= \psi\left(\left(v^2+y^2\right)^{2\alpha}\left(v^2+e^{-v}\right)^{2\eta}\left(y^2+1\right)^{2\omega}\left[\frac{\left(v^2+1\right)^2+\left(y^2+e^{-v}\right)^2}{4}\right]^{1-\alpha-\eta-\omega}\right)\\ &\geq \psi\left(4^{2\alpha}\cdot 4^{2\eta}\cdot 1^{2\omega}\left[\frac{5^2+0^2}{4}\right]^{1-\alpha-\eta-\omega}\right)\\ &= 3\cdot 4^{4.2}\cdot 5^{-2.2}\\ &\geq\left(1+e^{-2}\right)^2.\end{aligned} \tag{53}$$

Therefore,

$$d(Tv,Ty)\leq\psi\left([d(gv,gy)]^{\alpha}[d(gv,Tv)]^{\eta}[d(gy,Ty)]^{\omega}\left[\frac{d(gv,Ty)+d(gy,Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right). \tag{54}$$

Fourth case: if $v,y\in(2,+\infty)$ and $v\neq y$, we have

$$d(Tv,Ty)=\left(e^{-v}+e^{-y}\right)^2\leq 2e^{-4}. \tag{55}$$

Using the property of ψ, we get

$$\begin{aligned}\psi(M(v,y)) &= \psi\left([d(gv,gy)]^{\alpha}[d(gv,Tv)]^{\eta}[d(gy,Ty)]^{\omega}\left[\frac{d(gv,Ty)+d(gy,Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right)\\ &= \psi\left(\left(v^2+y^2\right)^{2\alpha}\left(v^2+e^{-v}\right)^{2\eta}\left(y^2+e^{-y}\right)^{2\omega}\left[\frac{\left(v^2+e^{-y}\right)^2+\left(y^2+e^{-v}\right)^2}{4}\right]^{1-\alpha-\eta-\omega}\right)\\ &\geq \psi\left(8^{2\alpha}\cdot 4^{2\eta}\cdot 4^{2\omega}\left[\frac{4^2+4^2}{4}\right]^{1-\alpha-\eta-\omega}\right)\\ &= \frac{3}{5}\cdot 2^{9.8} \geq 2e^{-4}.\end{aligned} \tag{56}$$

Therefore,

$$d(Tv,Ty) \leq \psi\left([d(gv,gy)]^{\alpha}[d(gv,Tv)]^{\eta}[d(gy,Ty)]^{\omega}\left[\frac{d(gv,Ty)+d(gy,Tv)}{2s}\right]^{1-\alpha-\eta-\omega}\right). \tag{57}$$

For all $v, y \in \mathsf{F}\backslash\{1\}$, it is obvious that g, T fulfills (28). Furthermore, one is a coincidence point of g and T.

The two previous examples lead us to the following remark.

Remark 2. In the Theorem 2, T and g do not need a fixed point, just as T and g accept a coincidence point and are not necessarily unique.

Definition 6. In a metric space (F, d), we say that a self mapping $T\colon \mathsf{F} \longrightarrow \mathsf{F}$ is an interpolative weakly contractive mapping type Ćirić–Reich–Rus, if there exists a constant $\alpha, \eta \in (0,1)$ such that

$$\zeta(d(Tv,Ty)) \leq \zeta(R(v,y)) - \varphi(R(v,y)), \tag{58}$$

for all $v, y \in \mathsf{F}\backslash\mathrm{Fix}(T)$, where

$$\begin{aligned}&\mathrm{Fix}(T) = \{a \in X | Ta = a\},\\ &R(v,y) = [d(v,y)]^{\alpha}[d(v,Tv)]^{\eta}[d(y,Ty)]^{1-\alpha-\eta},\end{aligned} \tag{59}$$

$\varphi\colon [0,\infty) \longrightarrow [0,\infty)$ is a lower semicontinuous function with $\varphi(t) = 0$ if and only if $t = 0$, $\zeta\colon [0,\infty) \longrightarrow [0,\infty)$ is a continuous monotone nondecreasing function with $\zeta(t) = 0$ if and only if $t = 0$.

Theorem 3. *In a complete metric space (F, d), if $T\colon \mathsf{F} \longrightarrow \mathsf{F}$ is a interpolative weakly contractive mapping type Ćirić–Reich–Rus, then T has a fixed point.*

Proof. For any $v_0 \in \mathsf{F}$, we consider a sequence $\{v_n\}$ by $v = v_0$ and $v_{n+1} = Tv_n$, $n = 0, 1, 2, \ldots$

If there exists $n_0 \in \mathbb{N}$ such that $v_{n_0+1} = v_{n_0}$, then v_{n_0} is clearly a fixed point in F. Otherwise, $v_{n+1} \neq v_n$ for each $n \geq 0$.

Substituting $v = v_n$ and $y = v_{n-1}$ in (58), we obtain that

$$\begin{aligned}\zeta(d(v_{n+1},v_n)) &\leq \zeta\left([d(v_n,v_{n-1})]^{\alpha}[d(v_n,v_{n+1})]^{\eta}[d(v_{n-1},v_n)]^{1-\alpha-\eta}\right)\\ &\quad - \varphi\left([d(v_n,v_{n-1})]^{\alpha}[d(v_n,v_{n+1})]^{\eta}[d(v_{n-1},v_n)]^{1-\alpha-\eta}\right)\\ &\leq \zeta\left([d(v_n,v_{n-1})]^{1-\eta}[d(v_n,v_{n+1})]^{\eta}\right).\end{aligned} \tag{60}$$

Using property of function ζ, we get

$$d(v_{n+1},v_n) \leq [d(v_n,v_{n-1})]^{1-\eta}[d(v_n,v_{n+1})]^{\eta}. \tag{61}$$

We derive

$$[d(v_{n+1},v_n)]^{1-\eta} \leq [d(v_n,v_{n-1})]^{1-\eta}. \tag{62}$$

Therefore,

$$d(v_{n+1},v_n) \leq d(v_n,v_{n-1}), \quad \text{for all } n \geq 1. \tag{63}$$

It follows that the positive sequence $\{d(v_{n+1},v_n)\}$ is decreasing. Eventually, there exists $c \geq 0$ such that $\lim_{n\longrightarrow\infty} d(v_{n+1},v_n) = c$.

Taking $n \longrightarrow \infty$ in inequality (60), we obtain

$$\zeta(c) \leq \zeta(c) - \varphi(c). \tag{64}$$

We deduce that $c = 0$. Hence,

$$\lim_{n\longrightarrow\infty} d(v_{n+1},v_n) = 0. \tag{65}$$

Therefore, $\{v_n\}$ is a Cauchy sequence. Suppose not, then there exists a real number $\varepsilon > 0$, for any $k \in \mathbb{N}$, $\exists m_k \geq n_k \geq k$ such that

$$d\left(v_{m_k}, v_{n_k}\right) \geq \varepsilon \text{ and } d\left(v_{m_k-1}, v_{n_k}\right) < \varepsilon. \tag{66}$$

Putting $v = v_{n_k-1}$ and $y = v_{m_k-1}$ in (58) and using (66), we obtain

Table 1: Table of metric d for Example 2.

$d(x,y)$	a	q	m	r
a	0	0.2	3	1
q	0.2	0	2.8	0.8
m	3	2.8	0	2
r	1	0.8	2	0

Table 2: Table of metric d for Example 3.

$d(x,y)$	a	r	i	m
a	0	4	49/16	9/4
r	4	0	1/16	1/4
i	49/16	1/16	0	1/16
m	9/4	1/4	1/16	0

$$\zeta(\varepsilon) \le \zeta\left(d\left(v_{m_k}, v_{n_k}\right)\right) \le \zeta\left(R\left(v_{m_k-1}, v_{n_k-1}\right)\right) - \varphi\left(R\left(v_{m_k-1}, v_{n_k-1}\right)\right), \tag{67}$$

where

$$R\left(v_{m_k-1}, v_{n_k-1}\right) = \left[d\left(v_{m_k-1}, v_{n_k-1}\right)\right]^{\alpha}\left[d\left(v_{m_k-1}, v_{m_k}\right)\right]^{\eta}\left[d\left(v_{n_k-1}, v_{n_k}\right)\right]^{1-\alpha-\eta} \tag{68}$$

and

$$d\left(v_{m_k-1}, v_{n_k-1}\right) \le d\left(v_{m_k-1}, v_{n_k}\right) + d\left(v_{n_k}, v_{n_k-1}\right) \le \varepsilon + d\left(v_{n_k}, v_{n_k-1}\right). \tag{69}$$

Letting $k \longrightarrow \infty$ and using (65), we conclude

$$\lim_{k\longrightarrow\infty} R\left(v_{m_k-1}, v_{n_k-1}\right) = 0. \tag{70}$$

Then,

$$\zeta(\varepsilon) \le \zeta(0) - \varphi(0) = 0, \tag{71}$$

which is contradiction with $\varepsilon > 0$. Thus, $\{v_n\}$ is a Cauchy sequence, and since (F, d) is complete, we obtain $\tau \in \mathsf{F}$ such that $\lim_{n\longrightarrow\infty} d(v_n, \tau) = 0$ and assume that $T\tau \ne \tau$, we have

$$\zeta(d(v_{n+1}, T\tau)) \le \zeta(R(v_n, \tau)) - \varphi(R(v_n, \tau)) \quad \text{for all } n, \tag{72}$$

where

$$R(v_n, \tau) = [d(v_n, \tau)]^{\alpha}[d(v_n, v_{n+1})]^{\eta}[d(\tau, T\tau)]^{1-\alpha-\eta}. \tag{73}$$

Using (65), we get

$$\lim_{n\longrightarrow\infty} R(v_n, \tau) = 0. \tag{74}$$

Letting $n \longrightarrow \infty$ in (72), we get

$$\zeta(d(\tau, T\tau)) \le \zeta(0) - \varphi(0) = 0, \tag{75}$$

which is a contradiction. Thus, $T\tau = \tau$. □ □

Example 5. Consider the space $\mathsf{F} = \{a, r, i, m\}$ equipped with the metric defined by the values of the following table (Table 3).

Consider the self mapping T on F as

$$T: \begin{pmatrix} a & r & i & m \\ i & m & i & m \end{pmatrix}. \tag{76}$$

For $\zeta(t) = e^t - 1$ and $\varphi(t) = (3/2)^t - 1$ for all $t \in [0, \infty)$, taking $\alpha = 0.2$ and $\eta = 0.7$, we have

$$\zeta(d(Tu, Tv)) \le \zeta\left([d(u,v)]^{\alpha}[d(u,Tu)]^{\eta}[d(v,Tv)]^{1-\alpha-\eta}\right) - \varphi\left([d(u,v)]^{\alpha}[d(u,Tu)]^{\eta}[d(v,Tv)]^{1-\alpha-\eta}\right), \tag{77}$$

for all $u, v \in \mathsf{F}\backslash\{i, m\}$.

Then, T possesses two fixed points: i and m.

Example 6. Let the set $\Lambda = [0, 5]$ and a function $\varrho: \Lambda \times \Lambda \longrightarrow [0, \infty)$ be defined as follows:

$$\varrho(v, y) = \begin{cases} 0, & \text{if } v = y, \\ 5, & \text{if } v, y \in [0, 1) \text{ and } v \ne y, \\ 3, & \text{otherwise.} \end{cases} \tag{78}$$

Then, (Λ, ϱ) is a complete metric space.

Let $T: \Lambda \longrightarrow \Lambda$ be defined as

Table 3: Table of metric d for Example 5.

$d(x,y)$	a	r	i	m
a	0	2	7	8
r	2	0	5	6
i	7	5	0	1
m	8	6	1	0

$$Tx = \begin{cases} 0, & \text{if } v \in [0,1); \\ 3, & \text{if } v \in [1,5]. \end{cases} \tag{79}$$

Choose $\zeta(t) = t^2$ and $\varphi(t) = 1/3t$ for all $t \in [0,+\infty)$, taking $\alpha = 0.4$ and $\eta = 0.3$, the following issues are discussed:

First case: if $v = y$ or $v, y \in (0,1)$, or $v, y \in [1,5]\backslash\{3\}$ with $v \neq y$, this is obvious.

Second case: If $v \in (0,1)$ and $y \in [1,5]\backslash\{3\}$, we have

$$\zeta(\varrho(Tv,Ty)) = \zeta(\varrho(0,3)) = \zeta(3) = 9, \tag{80}$$

$$[\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta} = 3^{\alpha}\cdot 5^{\eta}\cdot 3^{1-\alpha-\eta} = 3\cdot\left(\frac{5}{3}\right)^{\eta}. \tag{81}$$

Therefore,

$$\begin{aligned}\zeta\left([\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta}\right) &- \varphi\left([\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta}\right) \\ &= \left(\frac{5}{3}\right)^{\eta}\left[9\cdot\left(\frac{5}{3}\right)^{\eta} - 1\right] \geq 9 = \zeta(3) = \zeta(\varrho(Tv,Ty)).\end{aligned} \tag{82}$$

Third case: if $v \in [1,5]\backslash\{3\}$ and $y \in (0,1)$, we have

$$\zeta(\varrho(Tv,Ty)) = \zeta(\varrho(3,0)) = 9 \tag{83}$$

and

$$[\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta} = 3^{\alpha}\cdot 3^{\eta}\cdot 5^{1-\alpha-\eta} = 5\cdot\left(\frac{3}{5}\right)^{\alpha+\eta}. \tag{84}$$

Thus,

$$\begin{aligned}\zeta\left([\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta}\right) &- \varphi\left([\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta}\right) \\ &= \left(\frac{3}{5}\right)^{\alpha+\eta}\left[25\cdot\left(\frac{3}{5}\right)^{\alpha+\eta} - \frac{5}{3}\right] \geq 9 = \zeta(3) = \zeta(\varrho(Tv,Ty)).\end{aligned} \tag{85}$$

Hence,

$$\zeta(d(Tu,Tv)) \leq \zeta\left([\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta}\right) - \varphi\left([\varrho(v,y)]^{\alpha}[\varrho(v,Tv)]^{\eta}[\varrho(y,Ty)]^{1-\alpha-\eta}\right), \tag{86}$$

for all $v, y \in \Lambda\backslash\{0,3\}$.

Then, T possesses two fixed points: 0 and 3.

The two previous examples lead us to the next remark.

Remark 3. If T is an interpolative weakly contractive mapping type Ćirić–Reich–Rus, T accepts a fixed point that is not necessarily a single one.

We have the following corollary if $\zeta(t) = t$ in Theorem 3:

Corollary 2 *Let (F, d) be complete metric space and $T: \mathsf{F} \longrightarrow \mathsf{F}$ self mapping on F. If there exists a constant $\alpha, \eta \in (0,1)$ such that*

$$d(Tv,Ty) \leq [d(v,y)]^{\alpha}[d(v,Tv)]^{\eta}[d(y,Ty)]^{1-\alpha-\eta} - \varphi\left([d(v,y)]^{\alpha}[d(v,Tv)]^{\eta}[d(y,Ty)]^{1-\alpha-\eta}\right), \tag{87}$$

for all $v, y \in \mathsf{F}$ *and* $v \neq Tv, y \neq Ty$, *where* $\varphi: [0, \infty) \longrightarrow [0, \infty)$ *is a lower semicontinuous function with* $\varphi(t) = 0$ *if and only if* $t = 0$, *then* T *has a fixed point.*

If we use $\varphi(t) = (1 - \lambda)t$ for a constant $\lambda, \eta \in (0, 1)$ in Corollary 2, then we get the following corollary.

Corollary 3 *Let* (F, d) *be complete metric space and* $T: \mathsf{F} \longrightarrow \mathsf{F}$ *self mapping on* F. *If there exists a constant* $\alpha, \eta \in (0, 1)$ *such that*

$$d(Tv, Ty) \leq \lambda [d(v, y)]^{\alpha} [d(v, Tv)]^{\eta} [d(y, Ty)]^{1-\alpha-\eta}, \tag{88}$$

for all $v, y \in \mathsf{F}$ *and* $v \neq Tv, y \neq Ty$, *then* T *has a fixed point.*

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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