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Retraction

Retracted: Graphical Structures of Cubic Intuitionistic Fuzzy Information

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] S. U. Khan, N. Jan, K. Ullah, and L. Abdullah, "Graphical Structures of Cubic Intuitionistic Fuzzy Information," *Journal of Mathematics*, vol. 2021, Article ID 9994977, 21 pages, 2021.

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Research Article

Graphical Structures of Cubic Intuitionistic Fuzzy Information

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The theory developed in this article is based on graphs of cubic intuitionistic fuzzy sets (CIFS) called cubic intuitionistic fuzzy graphs (CIFGs). This graph generalizes the structures of fuzzy graph (FG), intuitionistic fuzzy graph (IFG), and interval-valued fuzzy graph (IVFG). Moreover, several associated concepts are established for CIFG, such as the idea subgraphs, degree of CIFG, order of CIFG, complement of CIFG, path in CIFG, strong CIFG, and the concept of bridges for CIFGs. Furthermore, the generalization of CIFG is proved with the help of some remarks. In addition, the comparison among the existing and the proposed ideas is carried out. Finally, an application of CIFG in decision-making problem is studied, and some future study is proposed.

1. Introduction

Jun et al. [1] proposed cubic set (CS) and started a new research area. A CS is a mixture of two concepts known as fuzzy set (FS) and interval-valued fuzzy set (IVFS). The concept of CS draws the attentions of researchers and some potential works in this direction have been done; for example, the idea of CS was proposed in semigroup theory by Khan et al. [2], as well as some KU-ideal by Yaqoob et al. [3], and KU-algebras are developed for CS by Lu and Ye [4]; the similarity measures of CSs have been proposed and applied in decision-making problem. The framework of cubic neutrosophic sets is proposed by Jun et al. [5], while some pattern recognition problems are solved using neutrosophic sets by Ali et al. [6]. The concept of cubic soft sets was proposed by Muhiuddin and Al-roqi [7], which was further utilized by Muhiuddin et al. [8]. The theory of G-algebras is studied by Jun and Khan in [9] and by Jana and Senapati [10] along with the concepts of ideal in semigroups. Some other works in this direction are given in [11–14].

The theory of intuitionistic fuzzy set (IFS) was developed by Atanassov [15] as a generalization of FS by Rosenfeld [16]. An IFS described the membership and nonmembership degree of an element by two characteristic functions and can model phenomena of yes or no type easily. Garg and Kaur [17] initiated the concept of cubic intuitionistic fuzzy sets (CIFSs) and discussed their properties. Atanassov model of IFS provided a motivation for the concept of intuitionistic fuzzy graphs (IFGs) defined by Parvathi and Karunambigai [18]. The concept of IFG was a generalization of fuzzy graphs (FGs) proposed by Kauffman and Rosenfeld [19, 20] after Zadeh's exemplary work in [16]. FG theory has a potential role in application point of view as described by Chan and Cheung [21] who studied an approach to clustering algorithm using the concepts of FGs. Some FG problems are solved by a novel technique in [22, 23] by discussing the domination of FGs in pattern recognitions. Mathew and Sunitha [24] worked on fuzzy attribute graphs applied to Chinese character recognitions, and Bhattacharya [25] used FGs in image classifications and so forth. For some other works on FG, one may refer to [26–31].

The theory of IFG received great attention as Parvathi and Thamizhendhi [32] introduced the concept of strong IFGs; Akram and Dudek [33] discussed the order, degree, and size of IFGs; Akram and Alshehri [34] developed operations for IFGs; Karunambigai [35] worked on the domination of IFGs; Pasi et al. [36] developed the theory of intuitionistic fuzzy hypergraphs; Karunambigai et al. [37] studied the concepts of trees and cycles for IFGs; Parvathi [38] developed the idea of balanced IFGs, a multicriteria and multiperson decision-making based on IFGs was discussed by Chountas [39]; Akram and Dudek [40] studied constant IFGs; Mathew [41] discussed IF hypergraphs; and the authors of [42] discussed the matrix representation of IFGs. Interval-valued FGs have also been studied extensively after Akram [43] proposed interval-valued FGs, Rashmanlou and Pal [44] discussed the results proposed by [43], complete interval-valued FGs developed interval-valued fuzzy line graphs are discussed by Rashmanlou and Pal [45, 46], and Pramanik et al. [47] proposed balanced interval-valued FGs. Xiao et al. [48] worked on green supplier selection in steel industry with intuitionistic fuzzy Taxonomy method, Zhao et al. [49] proposed an extended CPT-TODIM method for IVIF MAGDM and applied it to urban ecological risk assessment, and Wu et al. [50] presented VIKOR method for financing risk assessment of rural tourism under IVIF environment. Further, for some works on interval-valued FGs, one may refer to [51–55]. Motivated by the existing theory, we proposed the framework of cubic intuitionistic fuzzy sets (CIFSs) and cubic intuitionistic fuzzy graphs (CIFGs). Several graphical and theoretical terms are illustrated with the help of examples and some results.

The manuscript is organized as follows: In Section 1, a brief introduction about existing concepts is presented. In Section 2, some basic definitions from the theories of FG, IFG, and IVFG are defined. The concept of CIFG is proposed in Section 3 along with some other related terms and results including the concepts of subgraphs, degrees, orders, and bridges in CIFGs. Section 4 is based on operations on CIFGs and their results. The applications of CIFG in decision-making problems are discussed in Section 5. Section 6 provides a comparison of CIFG with existing concepts, and Section 7 provides a brief discussion and concluding remarks.

2. Preliminaries

In this section, we introduce some basic concepts about fuzzy set, fuzzy graph, intuitionistic fuzzy set, and intuitionistic fuzzy graph, which provide a base for our graphical work on CIFG. Throughout this manuscript, X denotes the universe of discourse and M, η are considered to be two mappings on $[0,1]$ intervals denoting the membership and nonmembership grades, respectively, of an element.

Definition 1 (see [13]). A *FS* on $\dot{X}$ is defined as $A = \{u, (M_A(u)/u \in \dot{X})\}$, where $M_A(1/2)$ is a map on $[0,1]$.

Definition 2 (see [20]). A pair $\breve{G}^* = (\mathscr{V}, E)$ is known as FG if

(i) $\mathscr{V} = \{M_i\colon i \in I\}$ and $M_1\colon \mathscr{V} \longrightarrow [0,1]$ is the association degree of $M_i \in \mathscr{V}$

(ii) $E = \{(u_i, u_j)\colon (u_i, u_j) \in \mathscr{V} \times \mathscr{V}\}$ and $M_2\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$ where $M_2(u_i, u_j) \le \min[M_1(u_i), M_1(u_j)]$ for all $(u_i, u_j) \in E$.

Definition 3 (see [15]). An IFS A on X is defined as $A = \{\langle\langle u, M_A(u), \eta_A(u)\rangle / u \in \dot{X}\rangle\}$, where M_A and η_A are mappings on 0,1 interval such that $0 \le M_A + \eta_A \le 1$.

Definition 4 (see [18]). A Pair $\breve{G}* = (V, \hat{E})$ is known as IFG if

(i) V is the collection of nodes such that M_1 and η_1 are mappings on unit intervals from V with a condition $0 \le M_1(ui) + \eta_1(ui) \le 1$ for all $u_i \in V$, $i \in I$

(ii) $E \subseteq \mathscr{V} \times \mathscr{V}$, where M_2 and η_2 are mappings that associate some grade to each $(u_i, u_j) \in E$ from $[0,1]$ interval such that $M_2(u_i, u_j) \le \min\{M_1(u_i), M_1(u_j)\}$ and $\eta_2(u_i, u_j) \le \max\{\eta_1(u_i), \eta_1((u_j))\}$ with a condition $0 \le M_2 + \eta_2 \le 1$

Example 1. The graph in Figure 1 is an IFG having four vertices and four edges.

Definition 5 (see [33]). The complement of an IFG $\breve{G}^* = (\mathscr{V}, E)$ is $\breve{G}^{*c} = (\mathscr{V}^c, E^c)$, where

(i) $V_c = V$

(ii) $M_A(u_i)c = M_A(u_i)$, $\eta_A(u_i)c = \eta_A(u_i), \forall\ u_i \in V$

(iii) $M_B(u_i, u_j)^c = \min[M_B(u_i), M_B(u_j)] - M_B(u_i, u_j)$, $\eta_B(u_i, u_j)^c = \max[\eta_B(u_i), \eta_B(u_j)] - \eta_B(u_i, u_j)$, for all $(u_i, u_j) \in E$

Here (u_i, M_A, η_A) represent the vertices and (e_{ij}, M_B, η_B) represent the edges.

Definition 6 (see [32]). A Pair $\breve{G}^* = (\mathscr{V}, E)$ is known as strong IFG if

(i) $\mathscr{V}$ is the collection of nodes such that M_1 and η_1 are mappings on unit intervals from $\mathscr{V}$ with a condition $0 \le M_1(u_i) + \eta_1(u_i) \le 1$ for all $u_i \in \mathscr{V}$ $(i \in I)$

(ii) $E \subseteq \mathscr{V} \times \mathscr{V}$, where M_2 and η_2 are mappings that associate some grade to each $(u_i, u_j) \in E$ from $[0,1]$ interval such that $M_2(u_i, u_j) = \min\{M_1(u_i), M_1(u_j)\}$ and $\eta_2(u_i, u_j) = \max\{\eta_1(u_i), \eta_1(u_j)\}$ with a condition $0 \le M_2 + \eta_2 \le 1$

Remark 1 (see [32]). If $\breve{G}^* = (\mathscr{V}, E)$ is an *IFG*, then by the above definition $(\breve{G}^{*c})^c = \breve{G}^*$ and it is called self-complementary.

Proposition 1 (see [32]). *If $\breve{G}^*$ is strong IFG, then it preserves self-complementary law.*

Example 2. Figures 2(a) and 2(b) provide a verification of Proposition 1.

Clearly $(\breve{G}^{*c})^c = \breve{G}^*$ is self-complementry.

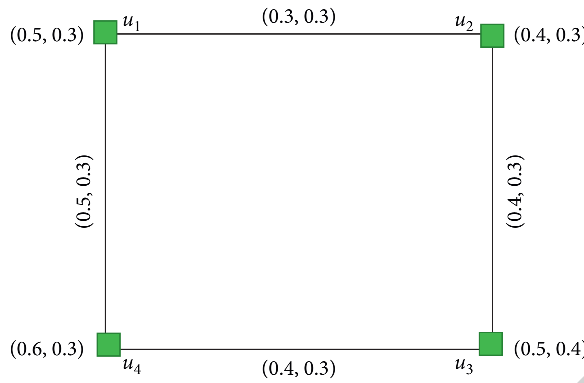
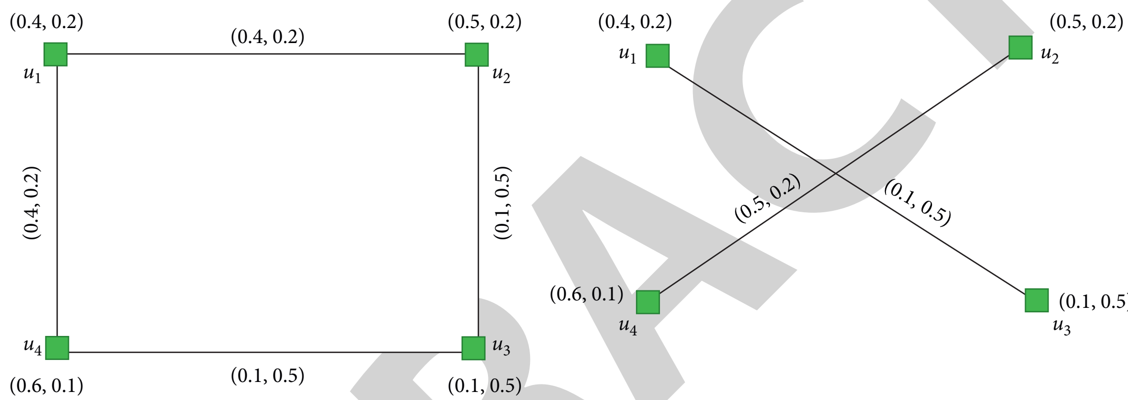


Figure 1: Intuitionistic fuzzy graph.



(a) (b)

Figure 2: (a) Intuitionistic fuzzy graph. (b) Complement of intuitionistic fuzzy graph.

Definition 7 (see [55]). A pair $\check{G} = (A, \mathscr{B})$ of a graph $\check{G}^* = (\mathscr{V}, E)$ is known as IVIFG, where $A = \{([M_{AL}, M_{AU}], [Ŋ_{AL}, Ŋ_{AU}])\}$ is IVFS on $\mathscr{V}$, and $\mathscr{B} = \{([M_{\mathscr{B}L}, M_{\mathscr{B}U}], [Ŋ_{\mathscr{B}L}, Ŋ_{\mathscr{B}U}])\}$ is the IVF relation on E satisfying the following conditions:

(i) $\mathscr{V} = \{u_1, u_2, u_3, \ldots, u_n\}$such that $M_{AL}\colon \mathscr{V} \longrightarrow [0,1], M_{AU}\colon \mathscr{V} \longrightarrow [0,1]$ and $Ŋ_{AL}\colon \mathscr{V} \longrightarrow [0,1]$, $Ŋ_{AU}\colon \mathscr{V} \longrightarrow [0,1]$ represent the degrees of membership and nonmembership of the element $u \in \mathscr{V}$, respectively, and $0 \leq M_A + Ŋ_A \leq 1$ for all $u_i \in \mathscr{V}$ $(i = 1, 2, \ldots, n)$

(ii) The functions $M_{\mathscr{B}L}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$, $M_{\mathscr{B}U}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$, $Ŋ_{\mathscr{B}L}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$, and $Ŋ_{\mathscr{B}U}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$ are such that $M_{\mathscr{B}L}(u, y) \leq \min(M_{AL}(u), M_{AL}(y))$, $Ŋ_{\mathscr{B}L}(u, y) \leq \max(Ŋ_{AL}(u), Ŋ_{AL}(y))$ $M_{\mathscr{B}U}(u, y) \leq \min(M_{AU}(u), M_{AU}(y))$, and $Ŋ_{\mathscr{B}U}(u, y) \leq \max(Ŋ_{AU}(u), Ŋ_{AU}(y))$; $0 \leq M_{\mathscr{B}}(u, y) + Ŋ_{\mathscr{B}}(u, y) \leq 1$ for all $(u_i, y_j) \in E$ $(i, j = 1, 2, \ldots, n)$

Example 3. Let $\check{G}^* = (\mathscr{V}, E)$ be a graph, where $\mathscr{V} = \{u_1, u_2, u_3\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_1\}$ is the set of edges.

3. Cubic Intuitionistic Fuzzy Graphs

In this section, we discussed the basic concept of CIFG-like complement of CIFG, degree of CIFG, and bridge and cut vertex of CIFG with the help of examples and several results (Figures 3 and 4).

Definition 8. A pair $\check{G} = (A, \mathscr{B})$ of a graph $\check{G}^* = (\mathscr{V}, E)$ is known as cubic IFG, where $A = \{([M_{AL}, M_{AU}], [Ŋ_{AL}, Ŋ_{AU}]), (M_A, Ŋ_A)\}$ is a cubic IFS on $\mathscr{V}$, and $\mathscr{B} = \{([M_{\mathscr{B}L}, M_{\mathscr{B}U}], [Ŋ_{\mathscr{B}L}, Ŋ_{\mathscr{B}U}]), (M_{\mathscr{B}}, Ŋ_{\mathscr{B}})\}$ is the cubic IF relation on E satisfying the following conditions:

(iii) $\mathscr{V} = \{u_1, u_2, u_3, \ldots, u_n\}$ such that $M_{AL}\colon \mathscr{V} \longrightarrow [0,1], M_{AU}\colon \mathscr{V} \longrightarrow [0,1]$ and $Ŋ_{AL}\colon \mathscr{V} \longrightarrow [0,1]$, $Ŋ_{AU}\colon \mathscr{V} \longrightarrow [0,1]$ and $M_A\colon \mathscr{V} \longrightarrow [0,1]$, $Ŋ_A\colon \mathscr{V} \longrightarrow [0,1]$⊸ represent the degrees of membership and nonmembership of the element $u \in \mathscr{V}$, respectively, and $0 \leq M_A + Ŋ_A \leq 1$ for all $u_i \in \mathscr{V}$ $(i = 1, 2, \ldots, n)$

(iv) The functions $M_{\mathscr{B}L}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$, $M_{\mathscr{B}U}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$, $Ŋ_{\mathscr{B}L}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$, $Ŋ_{\mathscr{B}U}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$ and $M_{\mathscr{B}}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1], Ŋ_{\mathscr{B}}\colon \mathscr{V} \times \mathscr{V} \longrightarrow [0,1]$ are such that $M_{\mathscr{B}L}(u, y) \leq \min(M_{AL}(u), M_{AL}(y))$, $Ŋ_{\mathscr{B}L}(u, y) \leq$

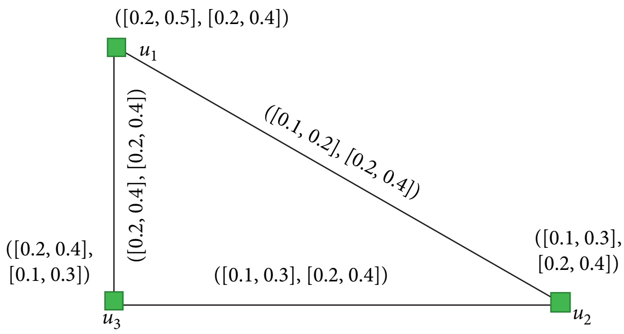


FIGURE 3: Interval-valued intuitionistic fuzzy graph.

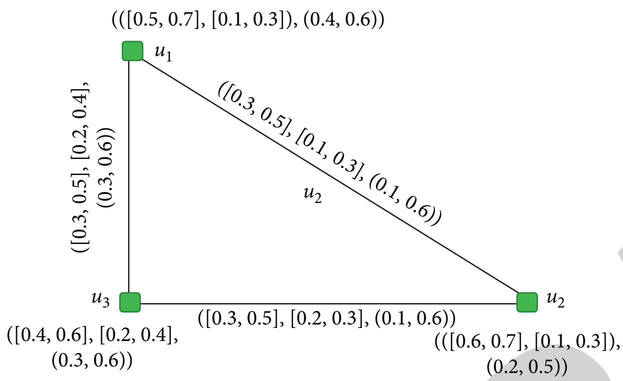


FIGURE 4: Cubic intuitionistic fuzzy graph.

$\max(Ŋ_{AL}(u), Ŋ_{AL}(y))$ $M_{\mathscr{B}U}(u, y) \leq \min(M_{AU}(u), M_{AU}(y))$, and $Ŋ_{\mathscr{B}U}(u, y) \leq \max(Ŋ_{AU}(u), Ŋ_{AU}(y))$; and $M_{\mathscr{B}}(u, y) \leq \min(M_A(u), M_A(y))$ and $Ŋ_{\mathscr{B}}(u, y) \leq \max(M_A(u), M_A(y))$ such that $0 \leq M_{\mathscr{B}}(u, y) + Ŋ_{\mathscr{B}}(u, y) \leq 1$ for all $(u_i, y_j) \in E$ $(i, j = 1, 2, \ldots, n)$

Example 4. Consider a graph $\breve{G}^* = (\mathscr{V}, E)$, where $\mathscr{V} = \{u_1, u_2, u_3\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_1\}$ is the set of edges.

Definition 9. A pair $\breve{G} = (A, \mathscr{B})$ of a graph $\breve{G}^* = (\mathscr{V}, E)$ is known as strong cubic IFG, where $A = \{([M_{AL}, M_{AU}], [Ŋ_{AL}, Ŋ_{AU}]), (M_A, Ŋ_A)\}$ is a cubic IFS on $\mathscr{V}$, and $\mathscr{B} = \{([M_{\mathscr{B}L}, M_{\mathscr{B}U}], [Ŋ_{\mathscr{B}L}, Ŋ_{\mathscr{B}U}]), (M_{\mathscr{B}}, Ŋ_{\mathscr{B}})\}$ is a cubic IF relation on E satisfying the following conditions:

(i) $\mathscr{V} = \{u_1, u_2, u_3, \ldots, u_n\}$ such that $M_{AL}: \mathscr{V} \longrightarrow [0, 1]$, $M_{AU}: \mathscr{V} \longrightarrow [0, 1]$ and $Ŋ_{AL}: \mathscr{V} \longrightarrow [0, 1]$, $Ŋ_{AU}: \mathscr{V} \longrightarrow [0, 1]$ and $M_A: \mathscr{V} \longrightarrow [0, 1]$, $Ŋ_A: \mathscr{V} \longrightarrow [0, 1]$ represent the degrees of membership and nonmembership of the element $u \in \mathscr{V}$, respectively, and $0 \leq M_A + Ŋ_A \leq 1$ for all $u_i \in \mathscr{V}$ $(i = 1, 2, \ldots, n)$

(ii) The functions $M_{\mathscr{B}L}: \mathscr{V} \times \mathscr{V} \longrightarrow [0, 1]$, $M_{\mathscr{B}U}: \mathscr{V} \times \mathscr{V} \longrightarrow [0, 1]$, $Ŋ_{\mathscr{B}L}: \mathscr{V} \times \mathscr{V} \longrightarrow [0, 1]$, $Ŋ_{\mathscr{B}U}: \mathscr{V} \times \mathscr{V} \longrightarrow [0, 1]$, and $M_{\mathscr{B}}: \mathscr{V} \times \mathscr{V} \longrightarrow [0, 1]$, $Ŋ_{\mathscr{B}}: \mathscr{V} \times \mathscr{V} \longrightarrow [0, 1]$ are such that $M_{\mathscr{B}L}(u, y) = \min(M_{AL}(u), M_{AL}(y))$, $Ŋ_{\mathscr{B}L}(u, y) = \max(Ŋ_{AL}(u), Ŋ_{AL}(y))$ $M_{\mathscr{B}U}(u, y) = \min(M_{AU}(u), M_{AU}(y))$, and $Ŋ_{\mathscr{B}U}(u, y) = \max(Ŋ_{AU}(u), Ŋ_{AU}(y))$; and $M_{\mathscr{B}}(u, y) = \min(M_A(u), M_A(y))$ and $Ŋ_{\mathscr{B}}(u, y) = \max(M_A(u), M_A(y))$ such that $0 \leq M_{\mathscr{B}}(u, y) + Ŋ_{\mathscr{B}}(u, y) \leq 1$ for all $(u_i, y_j) \in E$ $(i, j = 1, 2, \ldots, n)$

Definition 10. A cubic IFG $H = (\mathscr{V}^{\curlyvee}, E^{\curlyvee})$ is said to be cubic IFG subgraph of $\breve{G}^* = (\mathscr{V}, E)$ if $\mathscr{V}^{\curlyvee} \subseteq \mathscr{V}$ and $E^{\curlyvee} \subseteq E$. In other words, $[M_{ALi}, M_{AUi}]^{\curlyvee} \leq [M_{ALi}, M_{AUi}]$, $[Ŋ_{ALi}, Ŋ_{AUi}]^{\curlyvee} \leq [Ŋ_{ALi}, Ŋ_{AUi}]$, and $(M_{Ai}, Ŋ_{Ai})^{\curlyvee} \leq (M_{Ai}, Ŋ_{Ai})$ and $[M_{\mathscr{B}Lij}, M_{\mathscr{B}Uij}]^{\curlyvee} \leq [M_{\mathscr{B}Lij}, M_{\mathscr{B}Uij}]$, $[Ŋ_{\mathscr{B}Lij}, Ŋ_{\mathscr{B}Uij}]^{\curlyvee} \leq [Ŋ_{\mathscr{B}Lij}, Ŋ_{\mathscr{B}Uij}]$, and $(M_{\mathscr{B}ij}, Ŋ_{\mathscr{B}ij})^{\curlyvee} \leq (M_{\mathscr{B}ij}, Ŋ_{\mathscr{B}ij})$ for $i, j = 1, 2, \ldots, n$.

Definition 11. The order of cubic IFG $\breve{G}^* = (\mathscr{V}, \hat{E})$ is denoted and defined by

$$O(\breve{G}^*) = \left(\left(\sum_{u \in \mathscr{V}} M_{A\underset{\sim}{L}}(u), \sum_{u \in \mathscr{V}} M_{A\hat{U}}(u), \sum_{u \in \mathscr{V}} Ŋ_{A\underset{\sim}{L}}(u), \sum_{u \in \mathscr{V}} Ŋ_{A\hat{U}}(u)\right), \left(\sum_{u \in \mathscr{V}} M_A(u), \sum_{u \in \mathscr{V}} Ŋ_A(u)\right)\right), \tag{1}$$

and the size of cubic IFG is

$$S(G) = \left(\left(\sum_{\substack{u \neq y \\ u, y \in V}} M_{\mathscr{B}L}(uy) \sum_{\substack{u \neq y \\ u, y \in V}} M_{\mathscr{B}A\hat{U}}(uy), \sum_{\substack{u \neq y \\ u, y \in V}} Ŋ_{\mathscr{B}L}(uy), \sum_{\substack{u \neq y \\ u, y \in V}} Ŋ_{\mathscr{B}A\hat{U}}(uy)\right), \left(\sum_{\substack{u \neq y \\ u, y \in V}} M_{\mathscr{B}}(uy), \sum_{\substack{u \neq y \\ u, y \in V}} Ŋ_{\mathscr{B}}(uy)\right)\right). \tag{2}$$

Definition 12. The degree of a vertex in a cubic IFG $\breve{G}^* = (\mathscr{V}, E)$ is denoted and defined by

$$d(u) = ((dM_{AL}(u), dM_{AU}(u), d\eta_{AU}(u), d\eta_{AU}(u)), (d(M_A)(u), d(\eta_A)(u))), \quad (3)$$

where

$$\begin{aligned} dM_{AL}(u) &= \sum_{\substack{u \neq y \\ u \in V}} M_{\mathscr{B}L}(uy), \\ dM_{AU}(u) &= \sum_{\substack{u \neq y \\ u \in V}} M_{\mathscr{B}U}(uy), \\ d\eta_{AL}(u) &= \sum_{\substack{u \neq y \\ u,y \in V}} \eta_{\mathscr{B}L}(uy), \\ d\eta_{AU}(u) &= \sum_{\substack{u \neq y \\ u,y \in V}} \eta_{\mathscr{B}U}(uy), \\ d(M_A)(u) &= \sum_{\substack{u \neq y \\ u,y \in V}} M_{\mathscr{B}}(uy), \\ d(\eta_A)(u) &= \sum_{\substack{u \neq y \\ u,y \in V}} \eta_{\mathscr{B}}(uy). \end{aligned} \quad (4)$$

Example 5. Let Figure 5 be a graph $\breve{G}^* = (\mathscr{V}, E)$, where $\mathscr{V} = \{u_1, u_2, u_3, u_4\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$ is the set of edges.

The degrees of vertices are

$$\begin{aligned} d(u_1) &= ([0.3, 0.6], [0.5, 0.8], (0.3, 0.8)), \\ d(u_2) &= ([0.4, 0.7], [0.5, 0.8], (0.3, 0.8)), \\ d(u_3) &= ([0.3, 0.7], [0.4, 0.8], (0.2, 0.8)), \\ d(u_4) &= ([0.2, 0.6], [0.4, 0.8], (0.2, 0.9)). \end{aligned} \quad (5)$$

Definition 13. The complement of a cubic IFG $\breve{G} = (\mathrm{A}, \mathscr{B})$ on $\breve{G}^* = (\mathscr{V}, E)$ is defined as follows:

(i) $\overline{A} = A$

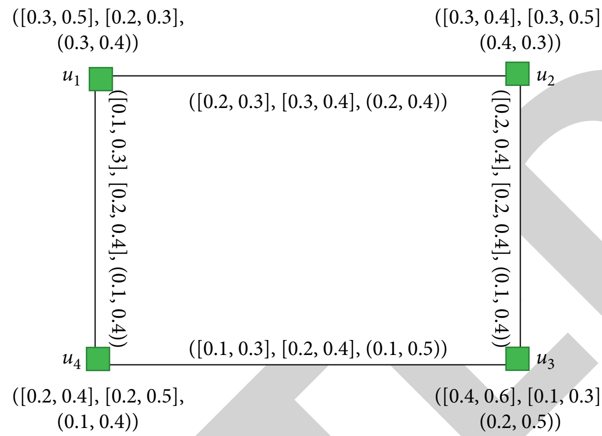


Figure 5: Cubic intuitionistic fuzzy graph.

(ii) $\overline{M_{AL}}(u_i) = M_{AL}(u_i)$, $\overline{M_{AU}}(u_i) = M_{AU}(u_i)$, $\overline{\eta_{AL}}(u_i) = \eta_{AL}(u_i)$, $\overline{\eta_{AU}}(u_i) = \eta_{AU}(u_i)$ and $\overline{M_A}(u_i) = M_A(u_i)$, $\overline{\eta_A}(u_i) = \eta_A(u_i)$ for all $u_i \in \mathscr{V}$

(iii) $\overline{M_{\mathscr{B}L}}(u_i, u_j) = \min[M_{AL}(u_i), M_{AL}(u_j)] - M_{\mathscr{B}U}(u_i, u_j)$, $\overline{M_{\mathscr{B}U}}(u_i, u_j) = \min[M_{AU}(u_i), M_{AU}(u_j)] - M_{\mathscr{B}U}(u_i, u_j)$, $\overline{\eta_{\mathscr{B}L}}(u_i, u_j) = \max[\eta_{AL}(u_i), \eta_{AL}(u_j)] - \eta_{\mathscr{B}U}(u_i, u_j)$, $\overline{\eta_{\mathscr{B}U}}(u_i, u_j) = (1/2)\max[\eta_{AL}(u_i), \eta_{AL}(u_j)] - \eta_{\mathscr{B}U}(u_i, u_j)$ for all $(u_i, u_j) \in E$

Proposition 2. *$\breve{G} = \overline{\overline{\breve{G}}}$ if and only if $\breve{G}$ is strong cubic IF graph.*

Proof. The proof is straightforward. □

Definition 14. A strong IFG is said to be self-complementary if $\breve{G} \cong \overline{\breve{G}}$, where $\overline{\breve{G}}$ is the complement of IFG $\breve{G}$.

Example 6. Let Figures 6 and 7 be two graphs of $\breve{G}^* = (\mathscr{V}, E)$, where $\mathscr{V} = \{u_1, u_2, u_3, u_4\}$ is the set of vertices and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$ is the set of edges.

Clearly $\breve{G} = \overline{\overline{\breve{G}}}$; hence, $\breve{G}$ is self-complementary.

Definition 15. The power of edge relation in a cubic IFG is defined as

$$\begin{aligned} e_{ij}^1 &= \left(e_{ij}, \left(\left(\left[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}\right], \left[\eta_{\mathscr{B}ijL}, \eta_{\mathscr{B}ijU}\right]\right), \left(M_{\mathscr{B}ij}, \eta_{\mathscr{B}ij}\right)\right)\right) \\ e_{ij}^2 &= e_{ij}^* e_{ij} = \left(e_{ij}, \left[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}\right]^2, \left[\eta_{\mathscr{B}ijL}, \eta_{\mathscr{B}ijU}\right]^2, \left(M_{\mathscr{B}ij}^2, \eta_{\mathscr{B}ij}^2\right)\right) \\ e_{ij}^3 &= e_{ij}^* e_{ij}^* e_{ij} = \left(e_{ij}, \left[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}\right]^3, \left[\eta_{\mathscr{B}ijL}, \eta_{\mathscr{B}ijU}\right]^3, \left(M_{\mathscr{B}ij}^3, \eta_{\mathscr{B}ij}^3\right)\right). \end{aligned} \quad (6)$$

Also,

$$e_{ij}^\infty = \left(e_{ij}, \left[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}\right]^\infty, \left[\eta_{\mathscr{B}ijL}, \eta_{\mathscr{B}ijU}\right]^\infty, \left(M_{\mathscr{B}ij}^\infty, \eta_{\mathscr{B}ij}^\infty\right)\right). \quad (7)$$

Here, $[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]^\infty = \max(\{[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]^k\}$, $M_{\mathscr{B}ij}^\infty = \max\{M_{\mathscr{B}ij}^k\})$ and $[\eta_{\mathscr{B}ijL}, \eta_{\mathscr{B}ijU}]^\infty = \min\{[\eta_{\mathscr{B}ijL}, \eta_{\mathscr{B}ijU}]^k\}$, $\eta_{\mathscr{B}ij}^\infty = \min\{\eta_{\mathscr{B}ij}^k]$ are the $M -$ strength

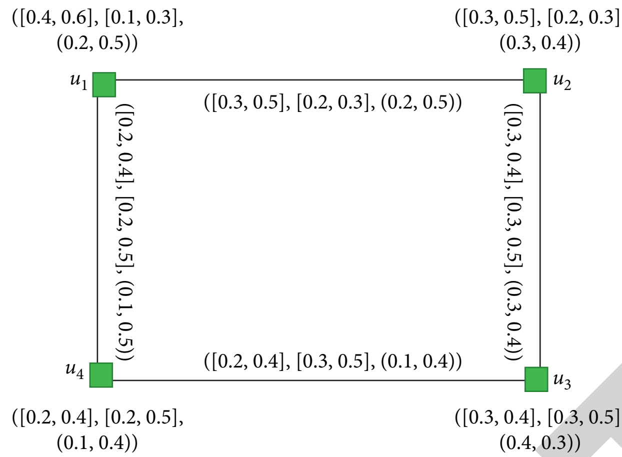


FIGURE 6: Cubic strong intuitionistic fuzzy graph.

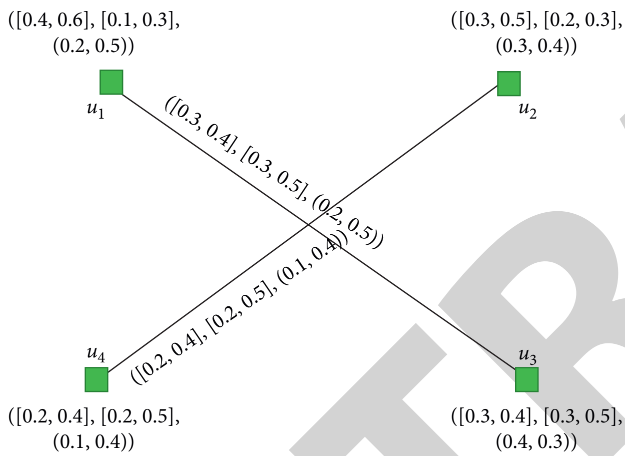


FIGURE 7: Complement of cubic strong intuitionistic fuzzy graph.

and Ŋ – strength of the connectedness between the two vertices (y_i, y_j).

Definition 16. An edge in a cubic IFG $\breve{G}^* = (\mathscr{V}, E)$ is said to be bridge, if deleting that edge reduces the strength of connectedness between some pair of vertices.

Example 7. Let Figure 8 be a graph $\breve{G}^* = (\mathscr{V}, E)$, where $\mathscr{V} = \{u_1, u_2, u_3, u_4\}$ is the set of vertices and $E = \{u_1u_2, u_2 u_3, u_2u_4, u_4u_1\}$ is the set of edges.

The strength of (u_1, u_4) is $([0.1, 0.4], [0.3, 0.5], (0.1, 0.4))$, so (u_1, u_4) is a bridge because when deleteing (u_1, u_4) the strength of the connectedness between u_1 and u_4 is decreased.

Theorem 1. *If $\breve{G}^* = (\mathscr{V}, E)$ is a cubic IFG, then, for any two vertices y_i and y_j, the following are equivalent:*

(i) (y_i, y_j) is a bridge

(ii) $[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]'^{\infty} < [M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}], M'^{\infty}_{\mathscr{B}ij} < M_{\mathscr{B}ij}$ and $[\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}]'^{\infty} > [\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}], \text{Ŋ}'^{\infty}_{\mathscr{B}ij} > \text{Ŋ}_{\mathscr{B}ij}$

(iii) (y_i, y_j) is not an edge of any cycle

Proof. (ii) $\Longrightarrow$ (i).

Consider $[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]'^{\infty} < [M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}], M'^{\infty}_{\mathscr{B}ij} < M_{\mathscr{B}ij}$ and $[\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}]'^{\infty} > [\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}], \text{Ŋ}'^{\infty}_{\mathscr{B}ij} > \text{Ŋ}_{\mathscr{B}ij}$ to show that (y_i, y_j) is a bridge; then $[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]'^{\infty} = [M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]^{\infty} \geq [M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]$, $M'^{\infty}_{\mathscr{B}ij} = M^{\infty}_{\mathscr{B}ij} \geq M_{\mathscr{B}ij}$ and $[\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}]'^{\infty} = [\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}]^{\infty} \leq [\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}], \text{Ŋ}'^{\infty}_{\mathscr{B}ij} = \text{Ŋ}^{\infty}_{\mathscr{B}ij} \leq \text{Ŋ}_{\mathscr{B}ij}$.

$[M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}]'^{\infty} \geq [M_{\mathscr{B}ijL}, M_{\mathscr{B}ijU}], M'^{\infty}_{\mathscr{B}ij} \geq M_{\mathscr{B}ij}$ and $[\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}]'^{\infty} \leq [\text{Ŋ}_{\mathscr{B}ijL}, \text{Ŋ}_{\mathscr{B}ijU}], \text{Ŋ}^{\infty}_{\mathscr{B}ij} \text{Ŋ}'^{\infty}_{\mathscr{B}ij}$, which is a contradiction. Hence, (y_i, y_j) is a bridge.

(i) $\Longrightarrow$ (iii).

Suppose that (y_i, y_j) is a bridge to show that (y_i, y_j) is not an edge of any cycle. If (y_i, y_j) is an edge of cycle, then any path involving the edge (y_i, y_j) can be converted into a path not involving (y_i, y_j) by using the rest of the cycle as a path from y_i to y_j. This implies that (y_i, y_j) cannot be a bridge, which is a contradiction to our supposition. Hence, (y_i, y_j) is not an edge of any cycle.

(iii) $\Longrightarrow$ (i).

The proof is straightforward. □

Definition 17. A vertex u_i in a cubic IFG $\breve{G}^*$ is said to be cut-vertex if deleting a vertex u_i reduces the strength of connectedness between some pair of vertices.

Example 8. Consider a graph $\breve{G}^* = (\mathscr{V}, E)$, where $\mathscr{V} = \{u_1, u_2, u_3, u_4, u_5\}$ is the set of vertices and $E = \{u_1u_2, u_2u_4, u_4u_3, u_4u_5, u_4u_1\}$ is the set of edges.

In Figure 9, u_1 is a cut-vertex.

4. Operations on Cubic IFG

In this section, the operations of CIFG-like Cartesian product of CIFG, union of CIFG, joint operation of CIFG, and so forth with the help of examples are discussed and some interesting results related to these operations are proved.

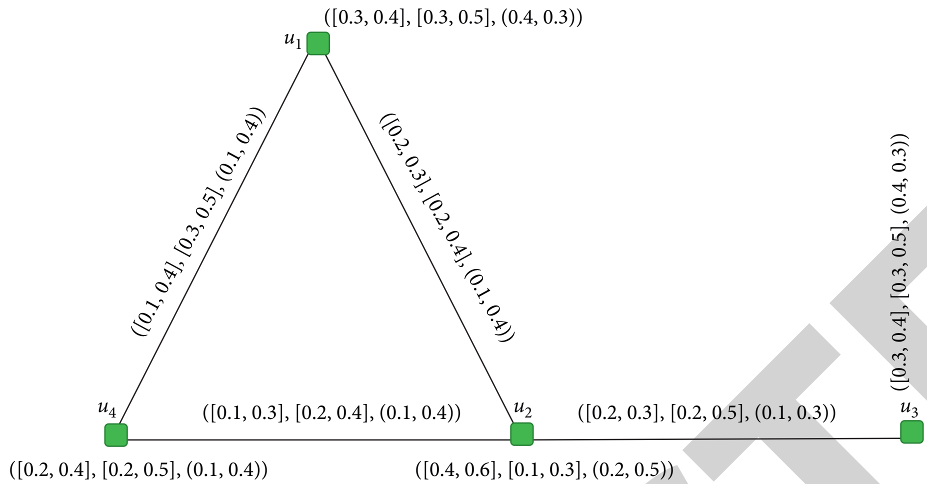


FIGURE 8: Cubic intuitionistic fuzzy graph.

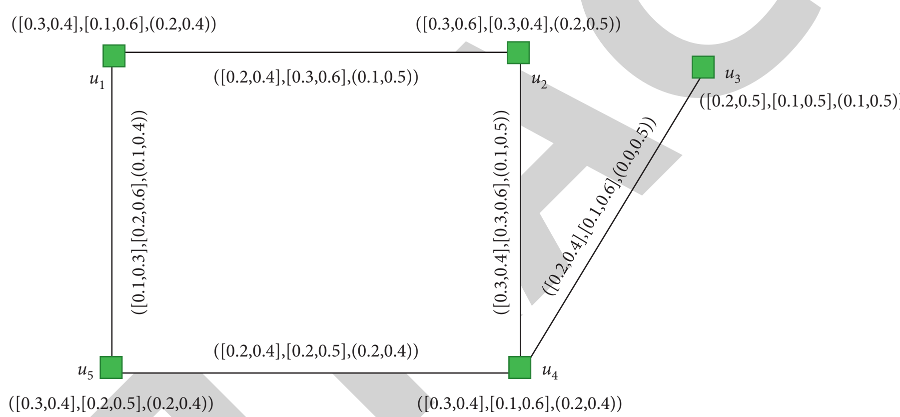


FIGURE 9: Cubic intuitionistic fuzzy graph.

Definition 18. The Cartesian product $\check{G} = \check{G}_1 \times \check{G}_2 = (A_1 \times A_2, \mathscr{B}_1 \times \mathscr{B}_2)$of two cubic IFGs $\check{G}_1 = (A_1, \mathscr{B}_1)$ and $\check{G}_2 = (A_2, \mathscr{B}_2)$ of the graphs $\check{G}_1^* = (\mathscr{V}_1, E_1)$ and $\check{G}_2^* = (\mathscr{V}_2, E_2)$ is defined as follows:

(i)

$$
\begin{aligned}
(M_{A1L} \times M_{A2L})(u_1, u_2) &= \min(M_{A1L}(u_1), M_{A2L}(u_2)),\\
(M_{A1U} \times M_{A2U})(u_1, u_2) &= \min(M_{A1U}(u_1), M_{A2UM}(u_2)),\\
(\text{Ŋ}_{A1L} \times \text{Ŋ}_{A2L})(u_1, u_2) &= \max(\text{Ŋ}_{A1L}(u_1), \text{Ŋ}_{A2L}(u_2)),\\
(\text{Ŋ}_{A1U} \times \text{Ŋ}_{A2U})(u_1, u_2) &= \max(\text{Ŋ}_{A1U}(u_1), \text{Ŋ}_{A2U}(u_2)),\\
(M_{A1} \times M_{A2})(u_1, u_2) &= \min(M_{A1}(u_1), M_{A2}(u_2)),\\
(\text{Ŋ}_{A1} \times \text{Ŋ}_{A2})(u_1, u_2) &= \max(\text{Ŋ}_{A1}(u_1), \text{Ŋ}_{A2}(u_2)), \quad \text{for all } u_1, u_2 \in \mathscr{V}.
\end{aligned}
\tag{8}
$$

(ii)

$$
\begin{aligned}
\left(M_{\mathscr{B}1L}\times M_{\mathscr{B}2L}\right)(u,u_2)(u,y_2)&=\min\left(M_{A1L}(u),M_{\mathscr{B}2L}(u_2y_2)\right),\\
\left(M_{\mathscr{B}1U}\times M_{\mathscr{B}2U}\right)(u,u_2)(u,y_2)&=\min\left(M_{A1U}(u),M_{\mathscr{B}2U}(u_2y_2)\right),\\
\left(\eta_{\mathscr{B}1L}\times \eta_{\mathscr{B}2L}\right)(u,u_2)(u,y_2)&=\max\left(\eta_{A1L}(u),\eta_{\mathscr{B}2L}(u_2y_2)\right),\\
\left(\eta_{\mathscr{B}1U}\times \eta_{\mathscr{B}2U}\right)(u,u_2)(u,y_2)&=\max\left(\eta_{A1U}(u),\eta_{\mathscr{B}2U}(u_2y_2)\right),\\
\left(M_{\mathscr{B}1}\times M_{\mathscr{B}2}\right)(u,u_2)(u,y_2)&=\min\left(M_{A1}(u),M_{\mathscr{B}2}(u_2y_2)\right),\\
\left(\eta_{\mathscr{B}1}\times \eta_{\mathscr{B}2}\right)(u,u_2)(u,y_2)&=\max\left(\eta_{A1}(u),\eta_{\mathscr{B}2}(u_2y_2)\right),\quad \text{for all } u\in\mathscr{V}_1 \text{ and } u_2y_2\in E_2.
\end{aligned}
\tag{9}
$$

(iii)

$$
\begin{aligned}
\left(M_{\mathscr{B}1L}\times M_{\mathscr{B}2L}\right)(u_1,z)(y_1,z)&=\min\left(M_{\mathscr{B}1L}(u_1y_1),M_{A2L}(z)\right),\\
\left(M_{\mathscr{B}1U}\times M_{\mathscr{B}2U}\right)(u_1,z)(y_1,z)&=\min\left(\mathscr{B}_{A1U}(u_1y_1),M_{A2U}(z)\right),\\
\left(\eta_{\mathscr{B}1L}\times \eta_{\mathscr{B}2L}\right)(u_1,z)(y_1,z)&=\max\left(\eta_{\mathscr{B}1L}(u_1y_1),\eta_{A2L}(z)\right),\\
\left(\eta_{\mathscr{B}1U}\times \eta_{\mathscr{B}2U}\right)(u_1,z)(y_1,z)&=\max\left(\eta_{\mathscr{B}1U}(u_1y_1),\eta_{A2U}(z)\right),\\
\left(M_{\mathscr{B}1}\times M_{\mathscr{B}2}\right)(u_1,z)(y_1,z)&=\min\left(M_{\mathscr{B}1}(u_1y_1),M_{A2}(z)\right),\\
\left(\eta_{\mathscr{B}1}\times \eta_{\mathscr{B}2}\right)(u_1,z)(y_1,z)&=\max\left(\eta_{\mathscr{B}1}(u_1y_1),\eta_{A2}(z)\right),\quad \text{for all } z\in\mathscr{V}_2 \text{ and } u_1y_1\in E_1.
\end{aligned}
\tag{10}
$$

Example 9. Let $\breve{G}^* = (\mathscr{V}, E)$ be a graph, where $\mathscr{V}$ is the set of vertices and E is the set of edges; then the product of two cubic IFGs in Figures 10–12 is given below.

Proposition 3. *If $\breve{G}_1$ and $\breve{G}_2$ are strong cubic IFGs, then the Cartesian product $\breve{G}_1\times\breve{G}_2$ is also strong cubic IFG.*

Proof. Suppose that $\breve{G}_1$ and $\breve{G}_2$ are strong cubic IFGs; then there exist $u_i, y_i \in E_i$ such that

$$
\begin{aligned}
M_{\mathscr{B}L}(u_i,y_i)&=\min\left(M_{AL}(u_i),M_{AL}(y_i)\right),\\
\eta_{\mathscr{B}L}(u_i,y_i)&=\max\left(\eta_{AL}(u_i),\eta_{AL}(y_i)\right),\\
M_{\mathscr{B}U}(u_i,y_i)&=\min\left(M_{AU}(u_i),M_{AU}(y_i)\right),\\
\eta_{\mathscr{B}U}(u_i,y_i)&=\max\left(\eta_{AU}(u_i),\eta_{AU}(y_i)\right),\\
M_{\mathscr{B}}(u_i,y_i)&=\min\left(M_{A}(u_i),M_{A}(y)\right),\\
\eta_{\mathscr{B}}(u_i,y_i)&=\max\left(M_{A}(u_i),M_{A}(y_i)\right).
\end{aligned}
\tag{11}
$$

Consider $E=\{(u,u_2)(u,y_2)/u_2\in\mathscr{V}_1,\quad u_2y_2\in E_2\}\cup\{(u_1,z)\ (y_1,z)/z\in\mathscr{V}_2,u_1y_1\in E_1\}$.

Let $(u,u_2)(u,y_2)\in E$; then

$$
\begin{aligned}
\left(M_{\mathscr{B}1L}\times M_{\mathscr{B}2L}\right)(u,u_2)(u,y_2)&=\min\left(M_{A1L}(u),M_{\mathscr{B}2L}(u_2y_2)\right)\\
&=\min\left(M_{A1L}(u),M_{A2L}(u_2),M_{A2L}(y_2)\right).
\end{aligned}
\tag{12}
$$

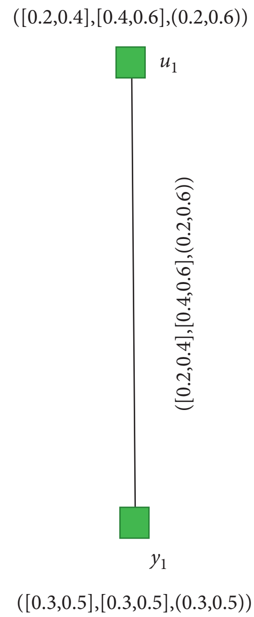


Figure 10: Cubic intuitionistic fuzzy graph.

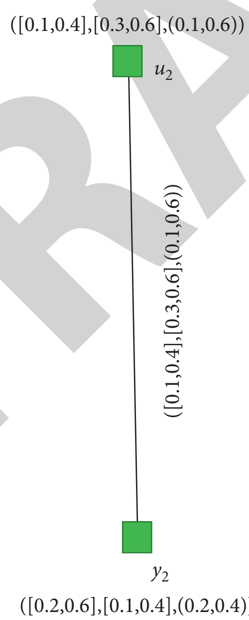


Figure 11: Cubic intuitionistic fuzzy graph.

Similarly,

$$
\begin{aligned}
(M_{\mathscr{B}1L} \times M_{\mathscr{B}2L})(u,u_2)(u,y_2) &= \min(M_{A1L}(u), M_{\mathscr{B}2L}(u_2 y_2)) = \min(M_{A1U}(u), M_{A2U}(u_2), M_{A2U}(y_2)),\\
(M_{A1L} \times M_{A2L})(u_1,u_2) &= \min(M_{A1L}(u_1), M_{A2L}(u_2)),\\
(M_{A1L} \times M_{A2L})(u_1,u_2) &= \min(M_{A1L}(u_1), M_{A2LM}(u_2)),\\
(M_{A1U} \times M_{A2U})(u_1,y_2) &= \min(M_{A1U}(u_1), M_{A2U}(y_2)),\\
(M_{A1U} \times M_{A2U})(u_1,y_2) &= \min(M_{A1U}(u_1), M_{A2U}(y_2)),\\
&= \min((M_{A1U} \times M_{A2U})(u,u_2), (M_{A1U} \times M_{A2U})(u,y_2))\\
&= \min(\min(M_{A1U}(u), M_{A2U}(u_2)), \min(M_{A1U}(u), M_{A2U}(y_2)))\\
&= \min((M_{A1U}(u), M_{A2U}(u_2), M_{A2U}(y_2))).
\end{aligned} \tag{13}
$$

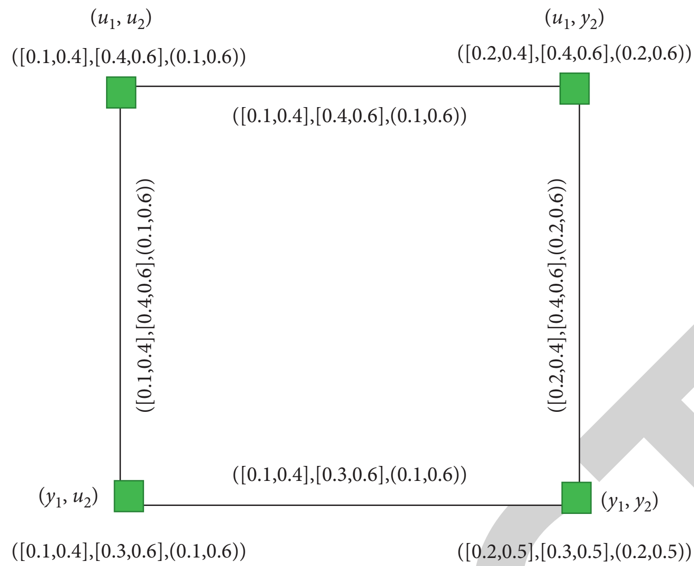


Figure 12: Cartesian product of cubic intuitionistic fuzzy graph.

Hence,

$$\begin{aligned}(M_{\mathscr{B}1L}\times M_{\mathscr{B}2L})(u,u_2)(u,y_2)&=\min\left((M_{A1L}\times M_{A2L})(u,u_2),(M_{A1L}\times M_{A2L})(u,y_2)\right),\\(M_{\mathscr{B}1U}\times M_{\mathscr{B}2U})(u,u_2)(u,y_2)&=\min\left((M_{A1U}\times M_{A2U})(u,u_2),(M_{A1U}\times M_{A2U})(u,y_2)\right).\end{aligned}\tag{14}$$

Similarly, we can show that

$$\begin{aligned}(\eta_{\mathscr{B}1L}\times \eta_{\mathscr{B}2L})(u,u_2)(u,y_2)&=\max\left((\eta_{A1L}\times \eta_{A2L})(u,u_2),(\eta_{A1L}\times \eta_{A2L})(u,y_2)\right),\\(\eta_{\mathscr{B}1U}\times \eta_{\mathscr{B}2U})(u,u_2)(u,y_2)&=\max\left((\eta_{A1U}\times \eta_{A2U})(u,u_2),(\eta_{A1U}\times \eta_{A2U})(u,y_2)\right),\\(M_{\mathscr{B}1}\times M_{\mathscr{B}2})(u,u_2)(u,y_2)&=\min\left((M_{A1}\times M_{A2})(u,u_2),(M_{A1}\times M_{A2})(u,y_2)\right),\\(\eta_{\mathscr{B}1}\times \eta_{\mathscr{B}2})(u,u_2)(u,y_2)&=\max\left((\eta_{A1}\times \eta_{A2})(u,u_2),(\eta_{A1}\times \eta_{A2})(u,y_2)\right).\end{aligned}\tag{15}$$

□

Proposition 4. *If $\breve{G}_1\times\breve{G}_2$ is a strong cubic IFG, then at least $\breve{G}_1$ or $\breve{G}_2$ must be strong.*

Proof. Suppose that $\breve{G}_1$ and $\breve{G}_2$ are not strong cubic IFGs, then there exist $u_i, y_i \in E_i$ such that

$$\begin{aligned}M_{\mathscr{B}L}(u_i,y_i)&<\min(M_{AL}(u_i),M_{AL}(y_i)),\\\eta_{\mathscr{B}L}(u_i,y_i)&>\max(\eta_{AL}(u_i),\eta_{AL}(y_i)),\\M_{\mathscr{B}U}(u_i,y_i)&<\min(M_{AU}(u_i),M_{AU}(y_i)),\\\eta_{\mathscr{B}U}(u_i,y_i)&>\max(\eta_{AU}(u_i),\eta_{AU}(y_i)),\\M_{\mathscr{B}}(u_i,y_i)&<\min(M_{A}(u_i),M_{A}(y)),\\\eta_{\mathscr{B}}(u_i,y_i)&>\max(M_{A}(u_i),M_{A}(y_i)).\end{aligned}\tag{16}$$

Consider $E=\{(u,u_2)(u,y_2)/u_2\in\mathscr{V}_1,\quad u_2y_2\in E_2\}\cup\{(u_1,z)(y_1,z)/z\in\mathscr{V}_2, u_1y_1\in E_1\}$.

Let $(u,u_2)(u,y_2)\in E$, then

$$\begin{aligned}&(M_{\mathscr{B}1L}\times M_{\mathscr{B}2L})(u,u_2)(u,y_2)\\&\quad=\min(M_{A1L}(u),M_{\mathscr{B}2L}(u_2y_2))\\&\quad<\min(M_{A1L}(u),M_{A2L}(u_2),M_{A2L}(y_2)).\end{aligned}\tag{17}$$

Similarly,

$$
\begin{aligned}
(M_{\mathscr{B}1L}\times M_{\mathscr{B}2l})(u,u_2)(u,y_2) &= \min(M_{A1L}(u), M_{\mathscr{B}2L}(u_2y_2)) < \min(M_{A1U}(u), M_{A2U}(u_2), M_{A2U}(y_2)),\\
(M_{A1L}\times M_{A2L})(u_1,u_2) &= \min(M_{A1L}(u_1), M_{A2L}(u_2)),\\
(M_{A1U}\times M_{A2UM})(u_1,u_2) &= \min(M_{A1U}(u_1), M_{A2U}(u_2)),\\
(M_{A1L}\times M_{A2L})(u_1,y_2) &= \min(M_{A1L}(u_1), M_{A2L}(y_2)),\\
(M_{A1U}\times M_{A2U})(u_1,y_2) &= \min(M_{A1U}(u_1), M_{A2U}(y_2))\\
&= \min((M_{A1U}\times M_{A2U})(u,u_2), ((M_{A1U}\times M_{A2U})(u,y_2))\\
&= \min(\min(M_{A1U}(u), M_{A2U}(u_2), \min((M_{A1U}(u), M_{A2U}(y_2)))\\
&= \min(M_{A1U}(u), M_{A2U}(u_2), M_{A2U}(y_2)).
\end{aligned}
\tag{18}
$$

Hence,

$$
\begin{aligned}
(M_{\mathscr{B}1L}\times M_{\mathscr{B}2L})(u,u_2)(u,y_2) &< \min((M_{A1L}\times M_{A2L})(u,u_2), (M_{A1L}\times M_{A2L})(u,y_2)),\\
(M_{\mathscr{B}1U}\times M_{\mathscr{B}2U})(u,u_2)(u,y_2) &< \min((M_{A1U}\times M_{A2U})(u,u_2), (M_{A1U}\times M_{A2U})(u,y_2)).
\end{aligned}
\tag{19}
$$

Similarly, we can show that

$$
\begin{aligned}
(Ŋ_{\mathscr{B}1L}\times Ŋ_{\mathscr{B}2L})(u,u_2)(u,y_2) &> \max((Ŋ_{A1L}\times Ŋ_{A2L})(u,u_2), (Ŋ_{A1L}\times Ŋ_{A2L})(u,y_2)),\\
(Ŋ_{\mathscr{B}1U}\times Ŋ_{\mathscr{B}2U})(u,u_2)(u,y_2) &> \max((Ŋ_{A1U}\times Ŋ_{A2U})(u,u_2), (Ŋ_{A1U}\times Ŋ_{A2U})(u,y_2)),\\
(M_{\mathscr{B}1}\times M_{\mathscr{B}2})(u,u_2)(u,y_2) &< \min((M_{A1}\times M_{A2})(u,u_2), (M_{A1}\times M_{A2})(u,y_2)),\\
(Ŋ_{\mathscr{B}1}\times Ŋ_{\mathscr{B}2})(u,u_2)(u,y_2) &> \max((Ŋ_{A1}\times Ŋ_{A2})(u,u_2), (Ŋ_{A1}\times Ŋ_{A2})(u,y_2)).
\end{aligned}
\tag{20}
$$

Therefore, $\check{G}_1\times\check{G}_2$ is not a strong cubic IFG, which is a contradiction. This completes the proof. □

Definition 19. The composition $\check{G}_1[\check{G}_2] = \check{G}_1\circ\check{G}_2 = (A_1\circ A_2, \mathscr{B}_1\circ\mathscr{B}_2)$ of two cubic IFGs $\check{G}_1 = (A_1,\mathscr{B}_1)$ and $\check{G}_2 = (A_2,\mathscr{B}_2)$ of the graphs $\check{G}_1^* = (\mathscr{V}_1, E_1)$ and $\check{G}_2^* = (\mathscr{V}_2, E_2)$ is defined as follows:

(i)

$$
\begin{aligned}
(M_{A1L}\circ \mathrm{M}_{A2L})(u_1,u_2) &= \min(M_{A1L}(u_1), M_{A2L}(u_2)),\\
(M_{A1U}\circ \mathrm{M}_{A2U})(u_1,u_2) &= \min(M_{A1U}(u_1), M_{A2U}(u_2)),\\
(Ŋ_{A1L}\circ Ŋ_{A2L})(u_1,u_2) &= \max(Ŋ_{A1L}(u_1), Ŋ_{A2L}(u_2)),\\
(Ŋ_{A1U}\circ Ŋ_{A2U})(u_1,u_2) &= \max(Ŋ_{A1U}(u_1), Ŋ_{A2U}(u_2)),\\
(M_{A1}\circ \mathrm{M}_{A2})(u_1,u_2) &= \min(M_{A1}(u_1), M_{A2}(u_2)),\\
(Ŋ_{A1}\circ Ŋ_{A2})(u_1,u_2) &= \max(Ŋ_{A1}(u_1), Ŋ_{A2}(u_2)), \quad \text{for all } u_1,u_2\in\mathscr{V}
\end{aligned}
\tag{21}
$$

(ii)

$$
\begin{aligned}
(M_{\mathscr{B}1L}\circ \mathrm{M}_{\mathscr{B}2L})(u,u_2)(u,y_2) &= \min(M_{A1L}(u), M_{\mathscr{B}2L}(u_2y_2)),\\
(M_{\mathscr{B}1U}\circ \mathrm{M}_{\mathscr{B}2U})(u,u_2)(u,y_2) &= \min(M_{A1U}(u), M_{\mathscr{B}2U}(u_2y_2)),\\
(Ŋ_{\mathscr{B}1L}\circ Ŋ_{\mathscr{B}2L})(u,u_2)(u,y_2) &= \max(Ŋ_{A1L}(u), Ŋ_{\mathscr{B}2L}(u_2y_2)),\\
(Ŋ_{\mathscr{B}1U}\circ Ŋ_{\mathscr{B}2U})(u,u_2)(u,y_2) &= \max(Ŋ_{A1U}(u), Ŋ_{\mathscr{B}2U}(u_2y_2)),\\
(M_{\mathscr{B}1}\circ \mathrm{M}_{\mathscr{B}2})(u,u_2)(u,y_2) &= \min(M_{A1}(u), M_{\mathscr{B}2}(u_2y_2)),\\
(Ŋ_{\mathscr{B}1}\circ Ŋ_{\mathscr{B}2})(u,u_2)(u,y_2) &= \max(Ŋ_{A1}(u), Ŋ_{\mathscr{B}2}(u_2y_2)), \quad \text{for all } u\in\mathscr{V}_1 \text{ and } u_2y_2\in E_2.
\end{aligned}
\tag{22}
$$

(iii)

$$
\begin{aligned}
(M_{\mathscr{B}1L}{}^{\circ}\mathrm{M}_{\mathscr{B}2\mathrm{L}})(u_1,z)(y_1,z) &= \min(M_{\mathscr{B}1L}(u_1y_1), M_{A2L}(z)),\\
(M_{\mathscr{B}1U}{}^{\circ}\mathrm{M}_{\mathscr{B}2\mathrm{U}})(u_1,z)(y_1,z) &= \min(\mathscr{B}_{A1U}(u_1y_1), M_{A2U}(z)),\\
(\text{Ŋ}_{\mathscr{B}1L}{}^{\circ}\text{Ŋ}_{\mathscr{B}2\mathrm{L}})(u_1,z)(y_1,z) &= \max(\text{Ŋ}_{\mathscr{B}1L}(u_1y_1), \text{Ŋ}_{A2L}(z)),\\
(\text{Ŋ}_{\mathscr{B}1U}{}^{\circ}\text{Ŋ}_{\mathscr{B}2\mathrm{U}})(u_1,z)(y_1,z) &= \max(\text{Ŋ}_{\mathscr{B}1U}(u_1y_1), \text{Ŋ}_{A2U}(z)),\\
((M_{\mathscr{B}1}{}^{\circ}\mathrm{M}_{\mathscr{B}2})(u_1,z)(y_1,z) &= \min(M_{\mathscr{B}1}(u_1y_1), M_{A2}(z)),\\
(\text{Ŋ}_{\mathscr{B}1}{}^{\circ}\text{Ŋ}_{\mathscr{B}2})(u_1,z)(y_1,z) &= \max(\text{Ŋ}_{\mathscr{B}1}(u_1y_1), \text{Ŋ}_{A2}(z)), \quad \text{for all } z \in \mathscr{V}_2 \text{ and } u_1y_1 \in E_1.
\end{aligned}
\tag{23}
$$

(iv)

$$
\begin{aligned}
(M_{\mathscr{B}1L}{}^{\circ}\mathrm{M}_{\mathscr{B}2\mathrm{L}})(u_1,u_2)(y_1,y_2) &= \min(M_{A2L}(u_2), M_{A2L}(y_2), M_{\mathscr{B}1L}(u_1y_1)),\\
(M_{\mathscr{B}1U}{}^{\circ}\mathrm{M}_{\mathscr{B}2\mathrm{U}})(u_1,u_2)(y_1,y_2) &= \min(M_{A2U}(u_2), M_{A2U}(y_2), M_{\mathscr{B}1U}(u_1y_1)),\\
(\text{Ŋ}_{\mathscr{B}1L}{}^{\circ}\text{Ŋ}_{\mathscr{B}2\mathrm{L}})(u_1,u_2)(y_1,y_2) &= \max(\text{Ŋ}_{A2L}(u_2), \text{Ŋ}_{A2L}(y_2), \text{Ŋ}_{\mathscr{B}1L}(u_1y_1)),\\
(\text{Ŋ}_{\mathscr{B}1U}{}^{\circ}\text{Ŋ}_{\mathscr{B}2\mathrm{U}})(u_1,u_2)(y_1,y_2) &= \max(\text{Ŋ}_{A2U}(u_2), \text{Ŋ}_{A2U}(y_2), \text{Ŋ}_{\mathscr{B}1U}(u_1y_1)),\\
(M_{\mathscr{B}1}{}^{\circ}\mathrm{M}_{\mathscr{B}2})(u_1,u_2)(y_1,y_2) &= \min(M_{A2}(u_2), M_{A2}(y_2), M_{\mathscr{B}1}(u_1y_1)),\\
(\text{Ŋ}_{\mathscr{B}1}{}^{\circ}\text{Ŋ}_{\mathscr{B}2})(u_1,u_2)(y_1,y_2) &= \max(\text{Ŋ}_{A2}(u_2), \text{Ŋ}_{A2}(y_2), \text{Ŋ}_{\mathscr{B}1}(u_1y_1)), \quad \text{for all } (u_1,u_2)(y_1,y_2) \in E^{\circ} - E.
\end{aligned}
\tag{24}
$$

Example 10. Let $\breve{G}^* = (\mathscr{V}, E)$ be a graph; then the compositions of two cubic IFGs in Figures 13–15 are given as follows.

Proposition 5. *The composition $\breve{G}_1[\breve{G}_2]$ of cubic IFG for the graphs $\breve{G}_1$ and $\breve{G}_2$ of the graphs $\breve{G}_1^*$ and $\breve{G}_2^*$ is a cubic IFG of $\breve{G}_1^*[\breve{G}_2^*]$.*

Proof. The proof is straightforward. □

Definition 20. The union $\breve{G}_1 \cup \breve{G}_2 = (A_1 \cup A_2, \mathscr{B}_1 \cup \mathscr{B}_2)$of two cubic IFGs $\breve{G}_1 = (A_1, \mathscr{B}_1)$ and $\breve{G}_2 = (A_2, \mathscr{B}_2)$ of the graphs $\breve{G}_1^* = (\mathscr{V}_1, E_1)$ and $\breve{G}_2^* = (\mathscr{V}_2, E_2)$ is defined as follows:

(i)

$$
\begin{cases}
(M_{A1L} \cup M_{A2L})(u) = M_{A1L}(u), & \text{if } u \in \mathscr{V}_1 - \mathscr{V}_2,\\
(M_{A1L} \cup M_{A2L})(u) = M_{A2L}(u), & \text{if } u \in \mathscr{V}_2 - \mathscr{V}_1,\\
(M_{A1L} \cup M_{A2L})(u) = \max(M_{A1L}(u), M_{A2L}(u)), & \text{if } u \in \mathscr{V}_1 \cap \mathscr{V}_2.
\end{cases}
\tag{25}
$$

(ii)

$$
\begin{cases}
(M_{A1U} \cup M_{A2U})(u) = M_{A1U}(u), & \text{if } u \in \mathscr{V}_1 - \mathscr{V}_2,\\
(M_{A1U} \cup M_{A2U})(u) = M_{A2U}(u), & \text{if } u \in \mathscr{V}_2 - \mathscr{V}_1,\\
(M_{A1U} \cup M_{A2U})(u) = \max(M_{A1U}(u), M_{A2U}(u)), & \text{if } u \in \mathscr{V}_1 \cap \mathscr{V}_2.
\end{cases}
\tag{26}
$$

(iii)

$$
\begin{cases}
(\text{Ŋ}_{A1L} \cap \text{Ŋ}_{A2L})(u) = \text{Ŋ}_{A1L}(u), & \text{if } u \in \mathscr{V}_1 - \mathscr{V}_2,\\
(\text{Ŋ}_{A1L} \cap \text{Ŋ}_{A2L})(u) = \text{Ŋ}_{A2L}(u), & \text{if } u \in \mathscr{V}_2 - \mathscr{V}_1,\\
(\text{Ŋ}_{A1L} \cap \text{Ŋ}_{A2L})(u) = \min(\text{Ŋ}_{A1L}(u), \text{Ŋ}_{A2L}(u)), & \text{if } u \in \mathscr{V}_1 \cap \mathscr{V}_2.
\end{cases}
\tag{27}
$$

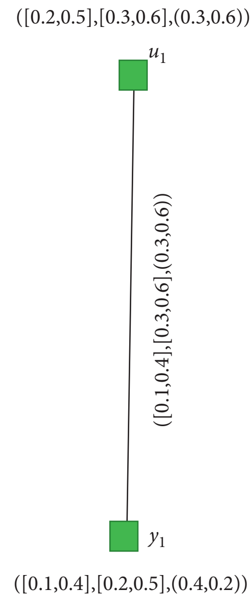


Figure 13: Cubic intuitionistic fuzzy graph.

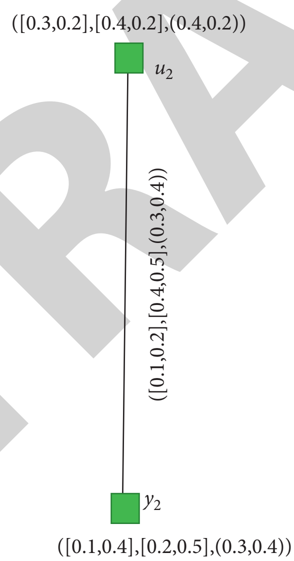


Figure 14: Cubic intuitionistic fuzzy graph.

(iv)

$$\begin{cases} (\text{Ŋ}_{A1U} \cap \text{Ŋ}_{A2U})(u) = \text{Ŋ}_{A1U}(u), & \text{if } u \in \mathscr{V}_1 - \mathscr{V}_2, \\ (\text{Ŋ}_{A1U} \cap \text{Ŋ}_{A2U})(u) = \text{Ŋ}_{A2U}(u) & \text{if } u \in \mathscr{V}_2 - \mathscr{V}_1, \\ (\text{Ŋ}_{A1U} \cap \text{Ŋ}_{A2U})(u) = min(\text{Ŋ}_{A1U}(u), \text{Ŋ}_{A2U}(u)), & \text{if } u \in \mathscr{V}_1 \cap \mathscr{V}_2. \end{cases} \tag{28}$$

(v)

$$\begin{cases} (M_{A1} \cup M_{A2})(u) = M_{A1}(u), & \text{if } u \in \mathscr{V}_1 - \mathscr{V}_2, \\ (M_{A1} \cup M_{A2})(u) = M_{A2}(u), & \text{if } u \in \mathscr{V}_2 - \mathscr{V}_1, \\ (M_{A1} \cup M_{A2})(u) = \max(M_{A1}(u), M_{A2}(u)), & \text{if } u \in \mathscr{V}_1 \cap \mathscr{V}_2. \end{cases} \tag{29}$$

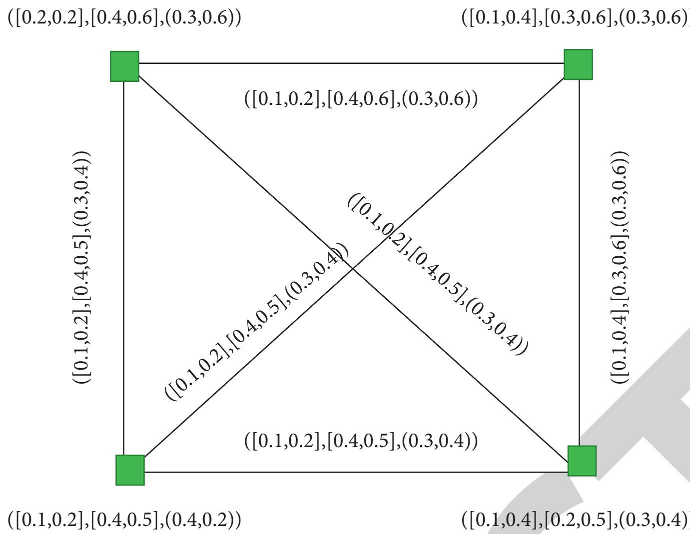


Figure 15: Composition of cubic intuitionistic fuzzy graph.

(vi)

$$\begin{cases} (\eta_{A1} \cap \eta_{A2})(u) = \eta_{A1}(u), & \text{if } u \in \mathscr{V}_1 - \mathscr{V}_2, \\ (\eta_{A1} \cap \eta_{A2})(u) = \eta_{A2}(u), & \text{if } u \in \mathscr{V}_2 - \mathscr{V}_1, \\ (\eta_{A1} \cap \eta_{A2})(u) = \min(\eta_{A1}(u), \eta_{A2}(u)), & \text{if } u \in \mathscr{V}_1 \cap \mathscr{V}_2. \end{cases} \tag{30}$$

(vii)

$$\begin{cases} (M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) = M_{\mathscr{B}1L}(uy), & \text{if } uy \in E_1 - E_2, \\ (M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) = M_{\mathscr{B}2L}(uy), & \text{if } uy \in E_2 - E_1, \\ (M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) = \max(M_{\mathscr{B}1L}(uy), M_{\mathscr{B}2L}(uy)), & \text{if } y \in E_1 \cap E_2. \end{cases} \tag{31}$$

(viii)

$$\begin{cases} (M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy) = M_{\mathscr{B}1U}(uy), & \text{if } uy \in E_1 - E_2, \\ (M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy) = M_{\mathscr{B}2U}(uy), & \text{if } uy \in E_2 - E_1, \\ (M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy) = \max(M_{\mathscr{B}1U}(uy), M_{\mathscr{B}2U}(uy)), & \text{if } uy \in E_1 \cap E_2. \end{cases} \tag{32}$$

(ix)

$$\begin{cases} (\eta_{\mathscr{B}1L} \cap \eta_{\mathscr{B}2L})(uy) = \eta_{\mathscr{B}1L}(uy), & \text{if } uy \in E_1 - E_2, \\ (\eta_{\mathscr{B}1L} \cap \eta_{\mathscr{B}2L})(uy) = \eta_{\mathscr{B}2L}(uy), & \text{if } uy \in E_2 - E_1, \\ (\eta_{\mathscr{B}1L} \cap \eta_{\mathscr{B}2L})(uy) = \min(\eta_{\mathscr{B}1L}(uy), \eta_{\mathscr{B}2L}(uy)), & \text{if } uy \in E_1 \cap E_2. \end{cases} \tag{33}$$

(x)

$$\begin{cases} (\eta_{\mathscr{B}1U} \cap \eta_{\mathscr{B}2U})(uy) = \eta_{\mathscr{B}1U}(uy), & \text{if } uy \in E_1 - E_2, \\ (\eta_{\mathscr{B}1U} \cap \eta_{\mathscr{B}2U})(uy) = \eta_{\mathscr{B}2U}(uy), & \text{if } uy \in E_2 - E_1, \\ (\eta_{\mathscr{B}1U} \cap \eta_{\mathscr{B}2U})(uy) = \min(\eta_{\mathscr{B}1U}(uy), \eta_{\mathscr{B}2U}(uy)), & \text{if } uy \in E_1 \cap E_2. \end{cases} \tag{34}$$

(xi)

$$\begin{cases} (M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) = M_{\mathscr{B}1}(uy), & \text{if } uy \in E_1 - E_2, \\ (M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) = M_{\mathscr{B}2}(uy), & \text{if } uy \in E_2 - E_1, \\ (M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) = \max(M_{\mathscr{B}1}(uy), M_{\mathscr{B}2}(uy)), & \text{if } uy \in E_1 \cap E_2. \end{cases} \tag{35}$$

(xii)

$$\begin{cases} (\text{Ŋ}_{\mathscr{B}1} \cap \text{Ŋ}_{\mathscr{B}2})(uy) = \text{Ŋ}_{\mathscr{B}1}(uy), & \text{if } uy \in E_1 - E_2, \\ (\text{Ŋ}_{\mathscr{B}1} \cap \text{Ŋ}_{\mathscr{B}2})(uy) = \text{Ŋ}_{\mathscr{B}2}(uy), & \text{if } uy \in E_2 - E_1, \\ (\text{Ŋ}_{\mathscr{B}1} \cap \text{Ŋ}_{\mathscr{B}2})(uy) = \min(\text{Ŋ}_{\mathscr{B}1}(uy), \text{Ŋ}_{\mathscr{B}2}(uy)), & \text{if } uy \in E_1 \cap E_2. \end{cases} \tag{36}$$

Example 11. Let $\breve{G}^* = (\mathscr{V}, E)$ be a graph; then the union of two cubic IFGs is given below.

In Figures 16–18 the union of two CIFGs is defined.

Proposition 6. *The union of two cubic IFGs is a cubic IFG.*

Proof. Let $\breve{G}_1 = (A_1, \mathscr{B}_1)$ and $\breve{G}_2 = (A_2, \mathscr{B}_2)$ be the cubic IFGs $\breve{G}_1^*$ and $\breve{G}_2^*$, respectively. Then, we have to prove $\breve{G}_1 \cup \breve{G}_2 = (A_1 \cup A_2, \mathscr{B}_1 \cup \mathscr{B}_2)$is a cubic IFG and of the graphs $\breve{G}_1^* \cup \breve{G}_2^*$. As all the conditions of $A_1 \cup A_2$ are satisfied, we only have to verify the conditions of $\mathscr{B}_1 \cup \mathscr{B}_2$.

First assume that $uy \in E_1 \cap E_2$. Then,

$$\begin{aligned}
(M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) &= \max(M_{\mathscr{B}1L}(uy), M_{\mathscr{B}2L}(uy)) \\
&\leq \max(\min(M_{A1L}(u), M_{A1L}(y)), \min(M_{A2L}(u), M_{A2L}(y))), \\
&= \min(\max(M_{A1L}(u), M_{A2L}(u)), \max(M_{A1L}(y), M_{A2L}(y))), \\
&= \min(M_{A1L} \cup M_{A2L})(u), (M_{A1L} \cup M_{A2L})(y), \\
(M_{\mathscr{B}1U \cup} M_{\mathscr{B}2U})(uy) &= \max(M_{\mathscr{B}1U}(uy), M_{\mathscr{B}2U}(uy)) \\
&\leq \max(\min(M_{A1U}(u), M_{A1U}(y)), \min(M_{A2U}(u), M_{A2L}(y))) \\
&= \min(\max(M_{A1U}(u), M_{A2U}(u)), \max(M_{A1UM}(y), M_{A2U}(y))) \\
&= \min(M_{A1U} \cup M_{A2U})(u), (M_{A1U} \cup M_{A2U})(y), \\
(\text{Ŋ}_{\mathscr{B}1L} \cup \text{Ŋ}_{\mathscr{B}2L})(uy) &= \min(\text{Ŋ}_{\mathscr{B}1L}(uy), \text{Ŋ}_{\mathscr{B}2L}(uy)) \\
&\leq \min(\max(\text{Ŋ}_{A1L}(u), \text{Ŋ}_{A1L}(y)), \max(\text{Ŋ}_{A2L}(u), \text{Ŋ}_{A2L}(y))), \\
&= \max(\min(\text{Ŋ}_{A1L}(u), \text{Ŋ}_{A2L}(u)), \min(\text{Ŋ}_{A1L}(y), \text{Ŋ}_{A2L}(y))), \\
&= \max(\text{Ŋ}_{A1L} \cup \text{Ŋ}_{A2L})(u), (\text{Ŋ}_{A1L} \cup \text{Ŋ}_{A2L})(y), \\
(\text{Ŋ}_{\mathscr{B}1U} \cup \text{Ŋ}_{\mathscr{B}2U})(uy) &= \min(\text{Ŋ}_{\mathscr{B}1U}(uy), \text{Ŋ}_{\mathscr{B}2U}(uy)) \\
&\leq \min(\max(\text{Ŋ}_{A1U}(u), \text{Ŋ}_{A1U}(y)), \max(\text{Ŋ}_{A2U}(u), \text{Ŋ}_{A2U}(y))), \\
&= \max(\min(\text{Ŋ}_{A1U}(u), \text{Ŋ}_{A2U}(u)), \min(\text{Ŋ}_{A1U}(y), \text{Ŋ}_{A2U}(y))), \\
&= \max(\text{Ŋ}_{A1U} \cup \text{Ŋ}_{A2U})(u), (\text{Ŋ}_{A1U} \cup \text{Ŋ}_{A2U})(y), \\
(M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) &= \max(M_{\mathscr{B}1}(uy), M_{\mathscr{B}2}(uy)) \\
&\leq \max(\min(M_{A1}(u), M_{A1}(y)), \min(M_{A2}(u), M_{A2}(y))), \\
&= \min(\max(M_{A1}(u), M_{A2}(u)), \max(M_{A1}(y), M_{A2}(y))), \\
&= \min(M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y), \\
(\text{Ŋ}_{\mathscr{B}1} \cup \text{Ŋ}_{\mathscr{B}2})(uy) &= \min(\text{Ŋ}_{\mathscr{B}1}(uy), \text{Ŋ}_{\mathscr{B}2}(uy)) \\
&\leq \min(\max(\text{Ŋ}_{A1}(u), \text{Ŋ}_{A1}(y)), \max(\text{Ŋ}_{A2}(u), \text{Ŋ}_{A2}(y))) \\
&= \max(\min(\text{Ŋ}_{A1}(u), \text{Ŋ}_{A2}(u)), \min(\text{Ŋ}_{A1}(y), \text{Ŋ}_{A2}(y))) \\
&= \max(\text{Ŋ}_{A1} \cup \text{Ŋ}_{A2})(u), (\text{Ŋ}_{A1} \cup \text{Ŋ}_{A2})(y).
\end{aligned} \tag{37}$$

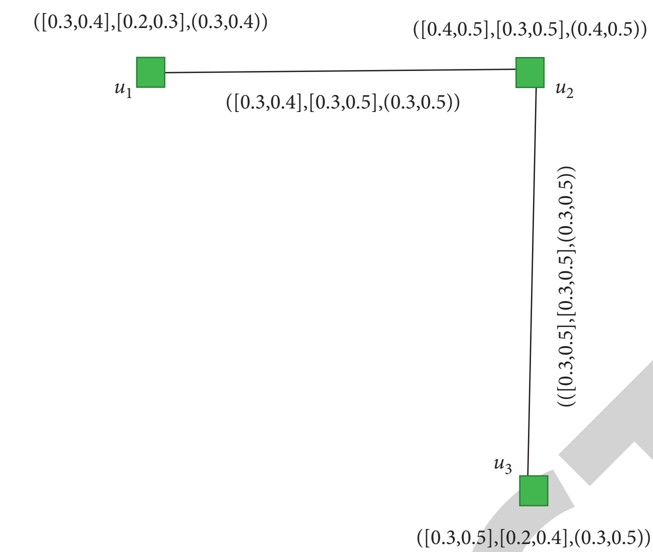


FIGURE 16: Cubic intuitionistic fuzzy graph.

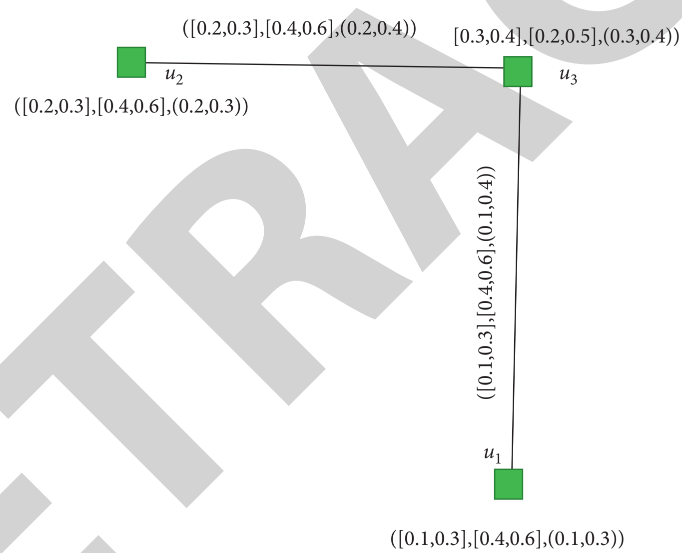


FIGURE 17: Cubic intuitionistic fuzzy graph.

If $uy \in E_1$ and $uy \notin E_2$, then

$$
\begin{aligned}
&(M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) \leq \min((M_{A1L} \cup M_{A2L})(u), (M_{A1L} \cup M_{A2L})(y)),\\
&(M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy) \leq \min((M_{A1U} \cup M_{A2U})(u), (M_{A1U} \cup M_{A2U})(y)),\\
&(\eta_{\mathscr{B}1L} \cup \eta_{\mathscr{B}2L})(uy) \leq \max((\eta_{A1L} \cup \eta_{A2L})(u), (\eta_{A1L} \cup \eta_{A2L})(y)),\\
&(\eta_{\mathscr{B}1U} \cup \eta_{\mathscr{B}2U})(uy) \leq \max((\eta_{A1U} \cup \eta_{A2U})(u), (\eta_{A1U} \cup \eta_{A2U})(y)),\\
&(M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) \leq \min((M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y)),\\
&(\eta_{\mathscr{B}1} \cup \eta_{\mathscr{B}2})(uy) \leq \max((\eta_{A1} \cup \eta_{A2})(u), (\eta_{A1} \cup \eta_{A2})(y)).
\end{aligned}
\tag{38}
$$

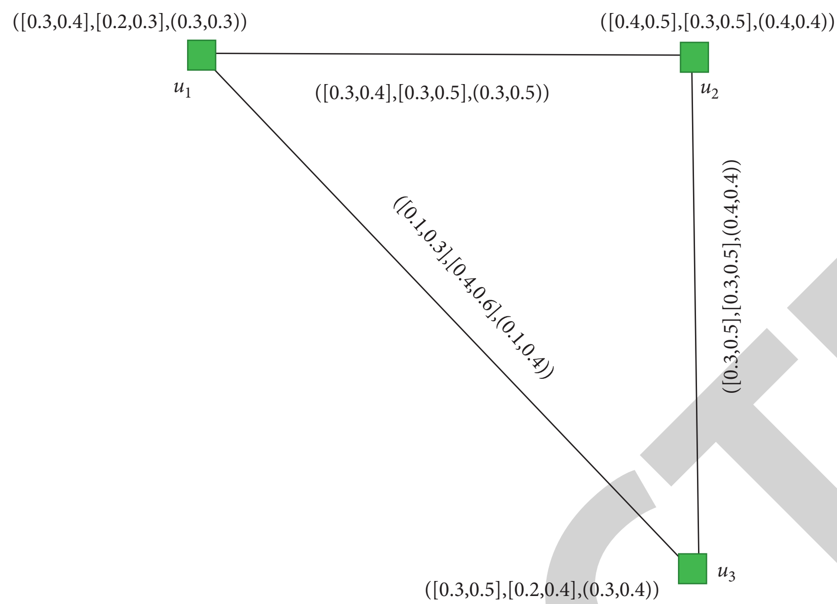


Figure 18: Union of cubic intuitionistic fuzzy graphs.

If $uy \notin E_1$ and $uy \in E_2$, then

$$\begin{aligned}
&(M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) \leq \min((M_{A1L} \cup M_{A2L})(u), (M_{A1L} \cup M_{A2L})(y)),\\
&(M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy) \leq \min((M_{A1U} \cup M_{A2U})(u), (M_{A1U} \cup M_{A2U})(y)),\\
&(\eta_{\mathscr{B}1L} \cup \eta_{\mathscr{B}2L})(uy) \leq \max((\eta_{A1L} \cup \eta_{A2L})(u), (\eta_{A1L} \cup \eta_{A2L})(y)),\\
&(\eta_{\mathscr{B}1U} \cup \eta_{\mathscr{B}2U})(uy) \leq \max((\eta_{A1U} \cup \eta_{A2U})(u), (\eta_{A1U} \cup \eta_{A2U})(y)),\\
&(M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) \leq \min((M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y)),\\
&(\eta_{\mathscr{B}1} \cup \eta_{\mathscr{B}2})(uy) \leq \max((\eta_{A1} \cup \eta_{A2})(u), (\eta_{A1} \cup \eta_{A2})(y)).
\end{aligned} \tag{39}$$

This completes the proof. □

Definition 21. The joint $\breve{G}_1 + \breve{G}_2 = (A_1 + A_2, \mathscr{B}_1 + \mathscr{B}_2)$ of two cubic IFGs $\breve{G}_1 = (A_1, \mathscr{B}_1)$ and $\breve{G}_2 = (A_2, \mathscr{B}_2)$ of the graphs $\breve{G}_1^* = (\mathscr{V}_1, E_1)$ and $\breve{G}_2^* = (\mathscr{V}_2, E_2)$ is defined as follows:

(i)

$$\begin{aligned}
&(M_{A1L} + M_{A2L})(u) = (M_{A1L} \cup M_{A2L})(u),\\
&(M_{A1U} + M_{A2U})(u) = (M_{A1U} \cup M_{A2U})(u)\\
&(\eta_{A1L} + \eta_{A2L})(u) = (\eta_{A1L} \cup \eta_{A2L})(u),\\
&(\eta_{A1U} + \eta_{A2U})(u) = (M_{A1U} \cup M_{A2U})(u),\\
&(M_{A1} + M_{A2})(u) = (M_{A1} \cup M_{A2})(u)\\
&(\eta_{A1} + \eta_{A2})(u) = (\eta_{A1} \cup \eta_{A2})(u).
\end{aligned} \tag{40}$$

If $u \in \mathscr{V}_1 \cup \mathscr{V}_2$,

(ii)

$$\begin{aligned}
&(M_{\mathscr{B}1L} + M_{\mathscr{B}2L})(uy) = (M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy),\\
&(M_{\mathscr{B}1U} + M_{\mathscr{B}2U})(uy) = (M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy),\\
&(\eta_{\mathscr{B}1L} + \eta_{\mathscr{B}2L})(uy) = (\eta_{\mathscr{B}1L} \cup \eta_{\mathscr{B}2L})(uy),\\
&(\eta_{\mathscr{B}1U} + \eta_{\mathscr{B}2U})(uy) = (M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy),\\
&(M_{\mathscr{B}1} + M_{\mathscr{B}2})(uy) = (M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy),\\
&(\eta_{\mathscr{B}1} + \eta_{\mathscr{B}2})(uy) = (\eta_{\mathscr{B}1} \cup \eta_{\mathscr{B}2})(uy)
\end{aligned} \tag{41}$$

$uy \in E_1 \cap E_2$, and then

$$\begin{aligned}
&(M_{\mathscr{B}1L} + M_{\mathscr{B}2L})(uy) = \min(M_{A1L}(u), M_{A2L}(y)),\\
&(M_{\mathscr{B}1U} + M_{\mathscr{B}2U})(uy) = \min(M_{A1U}(u), M_{A2U}(y)),\\
&(\eta_{\mathscr{B}1L} + \eta_{\mathscr{B}2L})(uy) = \max(\eta_{A1L}(u), \eta_{A2L}(y)),\\
&(\eta_{\mathscr{B}1U} + \eta_{\mathscr{B}2U})(uy) = \max(\eta_{A1U}(u), \eta_{A2U}(y)),\\
&(M_{\mathscr{B}1} + M_{\mathscr{B}2})(uy) = \min(M_{A1}(u), M_{A2}(y)),\\
&(\eta_{\mathscr{B}1} + \eta_{\mathscr{B}2})(uy) = \max(\eta_{A1}(u), \eta_{A2}(y)),
\end{aligned} \tag{42}$$

$uy \in E'$, where E' is the set of all edges joining the nodes of $\mathscr{V}_1$ and $\mathscr{V}_2$.

Proposition 7. *The joint of two cubic IFGs is a cubic IFG.*

Proof. Assume that $\breve{G}_1 = (A_1, \mathscr{B}_1)$ and $\breve{G}_2 = (A_2, \mathscr{B}_2)$ are two cubic IFGs of the graphs $\breve{G}_1^* = (\mathscr{V}_1, E_1)$ and $\breve{G}_2^* = (\mathscr{V}_2, E_2)$. Then, we have to prove $\breve{G}_1 + \breve{G}_2 = (A_1 + A_2, \mathscr{B}_1 + \mathscr{B}_2)$ is a cubic IFG. In view of proposition 6 is sufficient to verify the case when $uy \in E\prime$. In this case, we have

$$
\begin{aligned}
(M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) &= \min((M_{A1L}(u)), (M_{A2U}(y)))\\
(M_{\mathscr{B}1L} \cup M_{\mathscr{B}2L})(uy) &= \min((M_{A1L}(u)), (M_{A2U}(y)))\\
&= \min((M_{A1L} + M_{A2L})(u), (M_{A1L} + M_{A2L})(y)),\\
(M_{\mathscr{B}1U} \cup M_{\mathscr{B}2U})(uy) &= \min((M_{A1U}(u)), (M_{A2U}(y)))\\
&\leq \min((M_{A1U} \cup M_{A2U})(u), (M_{A1U} \cup M_{A2U})(y))\\
&= \min((M_{A1U} + M_{A2U})(u), (M_{A1U} + M_{A2U})(y)),\\
(\eta_{\mathscr{B}1L} \cup \eta_{\mathscr{B}2L})(uy) &= \max((\eta_{A1L}(u)), (\eta_{A2L}(y)))\\
&\leq \max((\eta_{A1L} \cup \eta_{A2L})(u), (\eta_{A1L} \cup \eta_{A2L})(y))\\
&= \max((\eta_{A1L} + \eta_{A2L})(u), (\eta_{A1L} + \eta_{A2L})(y)),\\
(\eta_{\mathscr{B}1U} \cup \eta_{\mathscr{B}2U})(uy) &= \max((\eta_{A1U}(u)), (\eta_{A2U}(y)))\\
&\leq \max((\eta_{A1U} \cup \eta_{A2U})(u), (\eta_{A1U} \cup \eta_{A2U})(y))\\
&= \max((\eta_{A1U} + \eta_{A2U})(u), (\eta_{A1U} + \eta_{A2U})(y)),\\
(M_{\mathscr{B}1} \cup M_{\mathscr{B}2})(uy) &= \min((M_{A1}(u)), (M_{A2}(y)))\\
&\leq \min((M_{A1} \cup M_{A2})(u), (M_{A1} \cup M_{A2})(y))\\
&= \min((M_{A1} + M_{A2})(u), (M_{A1} + M_{A2})(y)),\\
(\eta_{\mathscr{B}1} \cup \eta_{\mathscr{B}2})(uy) &= \max((\eta_{A1}(u)), (\eta_{A2}(y)))\\
&\leq \max((\eta_{A1} \cup \eta_{A2})(u), (\eta_{A1} \cup \eta_{A2})(y))\\
&= \max((\eta_{A1} + \eta_{A2})(u), (\eta_{A1} + \eta_{A2})(y)).
\end{aligned}
\tag{43}
$$

This completes the proof. □

5. Application

In this section, we apply the concept of CIFGs in multi-attribute decision-making problem, where the selection of suitable subjects has been carried out.

There are many career options for the students of present times. Moreover, some of the courses are usually chosen where all the available choices remain superior and best choices until a single student has to choose a field of his interest by keeping in view his preferences. At the finishing of college level education requires selecting their first choice of career planning. During this time, pupils must be given enough information about choosing career according to their interest. According to the survey of random sample of 100 pupils of class X carried out in this part, pupils with favour of interests and no favouring of choices of a specific subject up to class X are measured and given below. Based on the data, cubic nonrational fuzzy graph is used as a tool as it makes the level of membership (interval-valued membership) (percentage of students who favour a subject or a pair of subjects) and level of nonmembership (interval-valued nonmembership) (percentage of students who disfavour a subject or a pair of subjects). Employing CIFS, the best subject's combination may be evaluated that are the class having subjects that could be productive to most students and have best academic performance of most of the students.

Let $S = \{$English (E), Language (L), Maths (M), Science (S), Social Sciences (SS)$\}$ be the set of vertices. Tables 1 and 2 illustrate the percentages of students with interest/disinterest towards a subject or a pair of subjects.

Based on the above information, we generate an CIFG as follows (Figure 19).

In every vertex of the graph, the degree of membership shows the percentage of students with zeal for a specific subject and the degree of nonmembership is the percentage of students with no zeal in subject from a random sample of 100 students of class X chosen for survey. Also, the corners of graph of both membership and nonmembership show the favour and disfavour of students to study the combined two subjects at higher secondary corner. From the given graph, the corner $(L - SS)$ possesses high degree of nonmembership, which shows that majority of pupils do not like to study the combined subjects Language and Social Science, and the corner $(M - S)$ possesses high degree of membership, which shows that majority of pupils have zeal for studying the

Table 1: Subject combination.

Subject combination	Interest percentage	Disinterest percentage
E	$[0.3, 0.4], 0.3$	$[0.4, 0.5], 0.7$
L	$[0.2, 0.4], 0.4$	$[0.55, 0.6], 0.6$
M	$[0.2, 0.3], 0.3$	$[0.6, 0.7], 0.5$
S	$[0.1, 0.4], 0.5$	$[0.5, 0.6], 0.4$
SS	$[0.2, 0.3], 0.7$	$[0.3, 0.6], 0.3$

Table 2: Subjects combinations.

Subjects combination	Interest percentage	Disinterest percentage
$E-M$	$[0.2, 0.3], 0.3$	$[0.6, 0.7], 0.7$
$E-L$	$[0.2, 0.4], 0.3$	$[0.55, 0.6], 0.7$
$E-S$	$[0.1, 0.4], 0.3$	$[0.5, 0.6], 0.7$
$E-SS$	$[0.2, 0.3], 0.3$	$[0.4, 0.6], 0.7$
$L-M$	$[0.2, 0.3], 0.3$	$[0.6, 0.7], 0.6$
$L-S$	$[0.1, 0.4], 0.4$	$[0.55, 0.6], 0.6$
$L-SS$	$[0.2, 0.3], 0.4$	$[0.55, 0.6], 0.6$
$M-S$	$[0.1, 0.3], 0.3$	$[0.6, 0.7], 0.5$
$M-SS$	$[0.2, 0.3], 0.3$	$[0.6, 0.7], 0.5$
$S-SS$	$[0.1, 0.3], 0.5$	$[0.5, 0.6], 0.4$

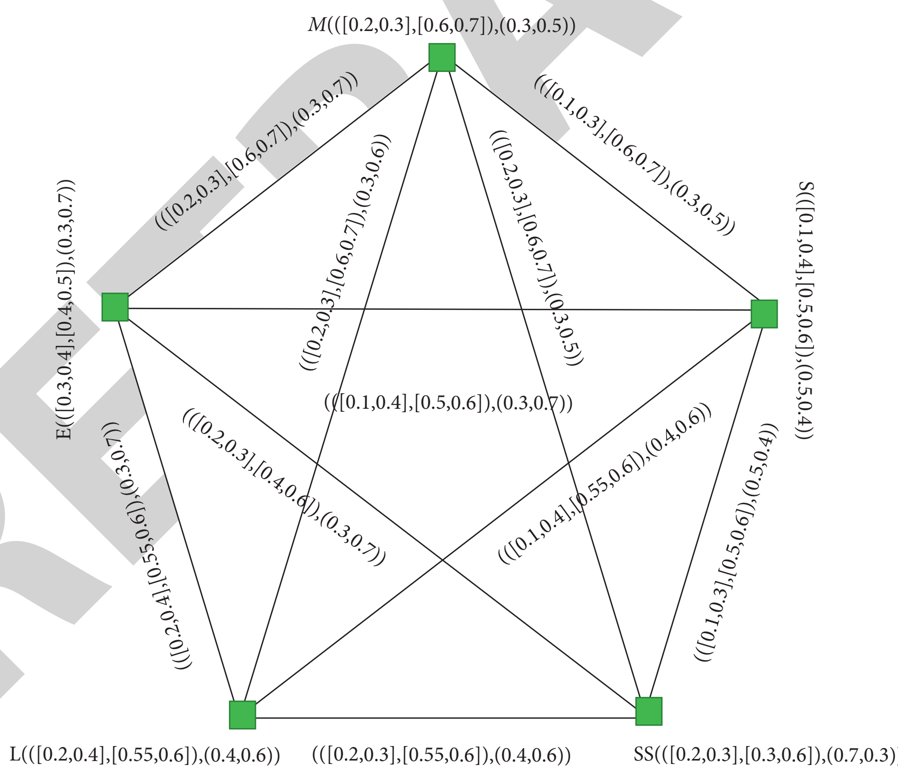


Figure 19: Cubic intuitionistic fuzzy graph.

combined subjects of Math and Science. There is disfavour to study the combined subjects of Tamil and Math, which indicates that these subjects do not require to be combined. Therefore, a high (low) level of membership of any corner shows the high (low) weightage of combined subjects at higher studies.

6. Comparison

Proposition 8. *A cubic IFG is a generalization of cubic FG.*

Proof. Let $\check{G}^* = (\mathscr{V}, E)$ be a cubic IFG. Then if we put the value of nonmembership of the vertex set and edge set as zero in the IVFS and FS, then the cubic IFG reduces to cubic FG. □

Proposition 9. *An IVIFG is a generalization of IVFG.*

Proof. Let $\check{G}^* = (\mathscr{V}, E)$ be an IVIFG. If we put the value of nonmembership of the vertex set and edge set as zero, then the IVIFG reduces to IVFG. □

Proposition 10. *An IFG is a generalization of FG.*

Proof. Let $\check{G}^* = (\mathscr{V}, E)$ be an IFG. If we put the value of nonmembership of the vertex set and edge set as zero, then the IFG reduces to FG. □

7. Conclusion

In this article, we developed a novel concept of CIFG as a generalization of IFGs. The graph theoretic terms like subgraphs, complements, degree of vertices, strength of graphs, paths, and cycle are briefly presented with the help of examples. Some related results and properties of the defined concepts are discussed. The generalization of CIFG is proved by some examples and remarks. A comparison of CIFG with IFG and other related concepts is given. The theory of CIFG is a generalization of IFG and can be applied to many real-life problems such as shortest path problem, communication problem, cluster analysis, and traffic signal problems. In the future, the graphs of the cubic Pythagorean fuzzy sets, cubic q-rung orthopair fuzzy sets, and cubic spherical fuzzy sets can be developed and different aggregation operators are defined for better decision-making.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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