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Research Article

Multiple Attribute Group Decision-Making Models Using Single-Valued Neutrosophic and Linguistic Neutrosophic Hybrid Element Aggregation Algorithms

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Multiple attribute group decision-making (MAGDM) issues may involve quantitative and qualitative attributes. In inconsistent and indeterminate decision-making issues, current assessment information of quantitative and qualitative attributes with respect to alternatives only contains either numerical neutrosophic values or linguistic neutrosophic values as the single information expression. However, existing neutrosophic techniques cannot perform the mixed information denotation and aggregation operations of numerical neutrosophic values and linguistic neutrosophic values in neutrosophic decision-making issues. To solve the puzzles, this article presents the information denotation, aggregation operations, and MAGDM models of single-valued neutrosophic and linguistic neutrosophic hybrid sets/elements (SVNLNHSs/SVNLHEs) as new techniques to perform MAGDM issues with quantitative and qualitative attributes in the environment of SVNLNHEs. In this study, we first propose a SVNLNHS/SVNLNHE notion that consists of a single-valued neutrosophic element (SVNE) for the quantitative argument and a linguistic neutrosophic element (LNE) for the qualitative argument. According to a linguistic and neutrosophic conversion function and its inverse conversion function, we present some basic operations of single-valued neutrosophic elements and linguistic neutrosophic elements, the SVNLNHE weighted arithmetic mean ($\text{SVNLNHEWAM}_{\text{N}}$) and SVNLNHE weighted geometric mean ($\text{SVNLNHEWGM}_{\text{N}}$) operators (forming SVNEs), and the $\text{SVNLNHEWAM}_{\text{L}}$ and $\text{SVNLNHEWGM}_{\text{L}}$ operators (forming LNEs). Next, MAGDM models are established based on the $\text{SVNLNHEWAM}_{\text{N}}$ and $\text{SVNLNHEWGM}_{\text{N}}$ operators or the $\text{SVNLNHEWAM}_{\text{L}}$ and $\text{SVNLNHEWGM}_{\text{L}}$ operators to realize MAGDM issues with single-valued neutrosophic and linguistic neutrosophic hybrid information, and then their applicability and availability are indicated through an illustrative example in the SVNLNHE circumstance. By comparison with the existing techniques, our new techniques reveal obvious advantages in the mixed information denotation, aggregation algorithms, and decision-making methods in handling MAGDM issues with the quantitative and qualitative attributes in the setting of SVNLNHSs.

1. Introduction

In general, there exist both quantitative attributes and qualitative attributes in multiple attribute (group) decision-making (MADM/MAGDM) issues. In the assessment process, the assessment information of quantitative attributes is usually represented by numerical values because numerical values are more suitable to the denotation form of quantitative arguments, while the assessment information of qualitative attributes is usually assigned by linguistic term values because the linguistic value is more suitable to human judgment and thinking/expression habits. Generally speaking, it is difficult to represent qualitative arguments by numeric values, but they are easily represented by linguistic values. In inconsistent and indeterminate situations, a simplified neutrosophic set (SNS) [1], including an interval-valued neutrosophic set/element (IVNS/IVNE) [2] and a single-valued neutrosophic set/element (SVNS/SVNE) [3], is depicted by the truth, falsity, and indeterminacy membership degrees, while a linguistic neutrosophic set/element (LNS/LNE) [4] is depicted by the truth, falsity, and indeterminacy linguistic values. Since the neutrosophic set theories [5], including SNS, SVNS, IVNS, and LNS, are vital mathematical tools to denote and handle indeterminate and inconsistent issues in the real world, they have been widely applied in decision-making issues [6–14]. In the setting of SNSs, some researchers presented various aggregation operators and their MADM/MAGDM models to solve neutrosophic MADM/MAGDM problems [10, 15–20]. Then, other researchers introduced various extended versions of SNSs, including single-valued neutrosophic rough sets [21], normal neutrosophic sets [22], bipolar neutrosophic sets [23], simplified neutrosophic indeterminate sets [24], and neutrosophic Z-numbers [25], and used them in MADM/MAGDM issues. In the setting of LNEs, some researchers proposed several aggregation operators of LNEs and their MAGDM models to carry out linguistic neutrosophic MAGDM problems [26, 27]. Then, some extended linguistic sets, such as linguistic neutrosophic uncertain sets and linguistic neutrosophic cubic sets, were also presented to perform some linguistic neutrosophic MAGDM problems [28]. Unfortunately, the existing neutrosophic theories and MADM models [28, 29] cannot yet resolve the denotation, operations, and MADM issues of the mixed information of SVNEs and LNEs. However, the existing assessment information of the quantitative or qualitative attributes with respect to alternatives only gives either numerical neutrosophic information or linguistic neutrosophic information as a single information expression. In the case of single-valued neutrosophic and linguistic neutrosophic mixed information, existing neutrosophic technologies cannot represent the mixed information of SVNE and LNE nor can they perform mixed operations of the two. Therefore, the mixed information representation and aggregation operations and decision-making problems pose challenges in this study, which motivates our research to address them. To solve these problems, the aims of this article are as follows: (1) to propose a single-valued neutrosophic and linguistic neutrosophic hybrid set/element (SVNLNHS/SVNLNHE) for the mixed information representation of both SVNE and LNE, (2) to present basic operations of SVNEs and LNEs according to a linguistic and neutrosophic conversion function and its inverse conversion function, (3) to propose the single-valued neutrosophic and linguistic neutrosophic hybrid element weighted arithmetic mean (SVNLNHEWAM$_{\mathrm{N}}$) and single-valued neutrosophic and linguistic neutrosophic hybrid element weighted geometric mean (SVNLNHEWGM$_{\mathrm{N}}$) operators for the aggregated SVNEs and the SVNLNHEWAM$_{\mathrm{L}}$ and SVNLNHEWGM$_{\mathrm{L}}$ operators for the aggregated LNEs, (4) to establish MAGDM models based on the SVNLNHEWAM$_{\mathrm{N}}$ and SVNLNHEWGM$_{\mathrm{N}}$ operators or the SVNLNHEWAM$_{\mathrm{L}}$ and SVNLNHEWGM$_{\mathrm{L}}$ operators in the setting of SVNLNHSs, and (5) to apply the established MAGDM models to an illustrative example on the selection problem of industrial robots that contain both quantitative and qualitative attributes in a SVNLNHS circumstance.

Generally, the main contributions of this article are summarized as follows:

(i) The proposed SVNLNHS/SVNLNHE solves the representation problem of single-valued neutrosophic and linguistic neutrosophic mixed information.

(ii) The proposed weighted aggregation operators of SVNLNHEs based on the linguistic and neutrosophic conversion function and its inverse conversion function provide the effective aggregation algorithms of SVNLNHEs.

(iii) The established MAGDM models can solve MAGDM issues with quantitative and qualitative attributes in a SVNLNHS circumstance.

(iv) The established MAGDM models can solve the selection problem of industrial robots that contain both quantitative and qualitative attributes and show the availability and rationality of the new techniques in a SVNLNHS circumstance.

The remaining structure of this article consists of the following sections. Section 2 reviews the basic concepts and operations of SVNEs and LNEs as the preliminaries of this study. The notions of SVNLNHS and SVNLNHE and some basic operations of SVNEs and LNEs based on the linguistic and neutrosophic conversion function and its inverse conversion function are proposed in Section 3. In Section 4, the SVNLNHEWAM$_{\mathrm{N}}$, SVNLNHEWGM$_{\mathrm{N}}$, SVNLNHEWAM$_{\mathrm{L}}$, and SVNLNHEWGM$_{\mathrm{L}}$ operators are presented in terms of the basic operations of SVNEs and LNEs. In Section 5, two new MAGDM models are established by the SVNLNHEWAM$_{\mathrm{N}}$ and SVNLNHEWGM$_{\mathrm{N}}$ operators or the SVNLNHEWAM$_{\mathrm{L}}$ and SVNLNHEWGM$_{\mathrm{L}}$ operators. Section 6 presents an illustrative example on the selection problem of industrial robots that contains both quantitative and qualitative attributes and then gives a comparative analysis with the existing techniques to show the availability and rationality of the new techniques. Finally, conclusions and future research are summarized in Section 7.

2. Preliminaries of SVNEs and LNEs

This part reviews the basic notions and operations of SVNEs and LNEs.

2.1. Basic Notions and Operations of SVNEs. Set $U = \{u_1, u_2, \ldots, u_m\}$ as a universal set. Then, a SVNS ZN in U can be represented as [1, 3]

$$ZN = \{\langle u_i, x_{ZN}(u_i), y_{ZN}(u_i), z_{ZN}(u_i)\rangle | u_i \in U\}, \tag{1}$$

where $< u_i, x_{ZN}(u_i), y_{ZN}(u_i), z_{ZN}(u_i)>$ $(i = 1, 2, \ldots, m)$ is SVNE in ZN for $u_j \in \mathrm{U}$ and $\mathrm{x}_{\mathrm{ZN}}(\mathrm{u}_{\mathrm{i}}), \mathrm{y}_{\mathrm{ZN}}(\mathrm{u}_{\mathrm{i}}), \mathrm{z}_{\mathrm{ZN}}(\mathrm{u}_{\mathrm{i}}) \in [0, 1]$, and then it is simply denoted as $\mathrm{zn}_{\mathrm{i}} = < \mathrm{x}_{\mathrm{ZNi}}, \mathrm{y}_{\mathrm{ZNi}}, \mathrm{z}_{\mathrm{ZNi}} >$.

For two SVNEs, $\mathrm{zn}_1 = < x_{ZN1}, y_{ZN1}, z_{ZN1}>$, $ZN_2 = < \mathrm{x}_{\mathrm{ZN2}}, \mathrm{y}_{\mathrm{ZN2}}, \mathrm{z}_{\mathrm{ZN}_2}>$, and $\beta > 0$, and their relations are contained as follows [17]:

(1) $zn_1 \oplus zn_2 = \langle x_{ZN1} + x_{ZN2} - x_{ZN1}x_{ZN2}, y_{ZN1}y_{ZN2}, z_{ZN1}z_{ZN2} \rangle$
(2) $zn_1 \otimes zn_2 = \langle x_{ZN1}x_{ZN2}, y_{ZN1} + y_{ZN2} - y_{ZN1}y_{ZN2}, z_{ZN1} + z_{ZN2} - z_{ZN1}z_{ZN2} \rangle$
(3) $\beta \cdot zn_1 = \langle 1-(1-x_{ZN1})^{\beta}, y_{ZN1}^{\beta}, z_{ZN1}^{\beta} \rangle$
(4) $zn_1^{\beta} = \langle x_{ZN1}^{\beta}, 1-(1-y_{ZN1})^{\beta}, 1-(1-z_{ZN1})^{\beta} \rangle$

Suppose that there is a group of SVNEs $zn_i = < x_{ZNi}, y_{ZNi}, z_{ZNi} >$ ($i = 1, 2, \ldots, m$) with their related weights $\beta_i \in [0, 1]$ for $\sum_{i=1}^{m} \beta_i = 1$. Then, the SVNE weighted arithmetic mean (SVNEWAM) operator and the SVNE weighted geometric mean (SVNEWGM) operator are introduced, respectively, as follows [17]:

$$\mathrm{SVNEWAM}(zn_1, zn_2, \ldots, zn_m) = \sum_{i=1}^{m} \beta_i \cdot zn_i = \langle 1 - \prod_{i=1}^{m} (1 - x_{ZNi})^{\beta_i}, \prod_{i=1}^{m} y_{ZNi}^{\beta_i}, \prod_{i=1}^{m} z_{ZNi}^{\beta_i} \rangle, \tag{2}$$

$$\mathrm{SVNEWGM}(zn_1, zn_2, \ldots, zn_m) = \prod_{i=1}^{m} zn_i^{\beta_i} = \langle \prod_{i=1}^{m} x_{ZNi}^{\beta_i}, 1 - \prod_{i=1}^{m} (1 - y_{ZNi})^{\beta_i}, 1 - \prod_{i=1}^{m} (1 - z_{ZNi})^{\beta_i} \rangle. \tag{3}$$

To compare SVNEs, the score and accuracy functions of SVNEs and their ranking laws are introduced below [17].

Set $zn_i = < x_{ZNi}, y_{ZNi}, z_{ZNi} >$ as any SVNE. The score and accuracy functions of zn_i are presented, respectively, as follows:

$$F(zn_i) = \frac{(2 + x_{ZNi} - y_{ZNi} - z_{ZNi})}{3} \text{for } F(zn_i) \in [0, 1], \tag{4}$$

$$G(zn_i) = (x_{ZNi} - z_{ZNi}) \text{for } G(zn_i) \in [-1, 1]. \tag{5}$$

Then, the sorting laws based on the score values of $F(zn_i)$ and the accuracy values of $G(zn_i)$ ($i = 1, 2$) are as follows [17]:

(a) $zn_1 > zn_2$ for $F(zn_1) > F(zn_2)$
(b) $zn_1 > zn_2$ for $F(zn_1) = F(zn_2)$ and $G(zn_1) > G(zn_2)$
(c) $zn_1 = zn_2$ for $F(zn_1) = F(zn_2)$ and $G(zn_1) = G(zn_2)$

2.2. Basic Notions and Operations of LNEs. Let $U = \{u_1, u_2, \ldots, u_m\}$ be a universal set and $S = \{s_p | p = 0, 1, \ldots, r\}$ be a linguistic term set (LTS) with an odd cardinality $r + 1$. Thus, a LNS LH is defined as follows [4]:

$$LH = \left\{ \langle u_i, s_{a(u_i)}, s_{b(u_i)}, s_{c(u_i)} \rangle | u_i \in U \right\}, \tag{6}$$

where $\langle u_i, s_{a(u_i)}, s_{b(u_i)}, s_{c(u_i)} \rangle$ for $u_i \in U$ is LNE in LH and $s_{a(u_i)}, s_{b(u_i)}, s_{c(u_i)} \in S$ are the truth, indeterminacy, and falsity linguistic variables, respectively. For convenience, LNE is simply denoted as $lh_i = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle$.

For two LNEs, $lh_1 = \langle s_{a_1}, s_{b_1}, s_{c_1} \rangle$, $lh_2 = \langle s_{a_2}, s_{b_2}, s_{c_2} \rangle$, , and $\beta > 0$, and their operational relations are as follows [4]:

(1) $lh_1 \oplus lh_2 = \langle s_{a_1 + a_2 - a_1 a_2 / r}, s_{b_1 b_2 / r}, s_{c_1 c_2 / r} \rangle$
(2) $lh_1 \otimes lh_2 = \langle s_{a_1 a_2 / r}, s_{b_1 + b_2 - b_1 b_2 / r}, s_{c_1 + c_2 - c_1 c_2 / r} \rangle$
(3) $\beta \cdot lh_1 = \langle s_{r - r(1 - a_1/r)^{\beta}}, s_{r(b_1/r)^{\beta}}, s_{r(c_1/r)^{\beta}} \rangle$
(4) $lh_1^{\beta} = \langle s_{r(a_1/r)^{\beta}}, s_{r - r(1 - b_1/r)^{\beta}}, s_{r - r(1 - c_1/r)^{\beta}} \rangle$

Suppose that there is a group of LNEs $lh_i = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle$ ($i = 1, 2, \ldots, m$) with their related weights $\beta_i \in [0, 1]$ for $\sum_{i=1}^{m} \beta_i = 1$. Then, the LNE weighted arithmetic mean (LNEWAM) and LNE weighted geometric mean (LNEWGM) operators are introduced as follows [4]:

$$\mathrm{LNEWAM}(lh_1, lh_2, \ldots, lh_m) = \sum_{i=1}^{m} \beta_i \cdot lh_i = \langle s_{r - r\prod_{i=1}^{m}(1 - a_i/r)^{\beta_i}}, s_{r\prod_{i=1}^{m}(b_i/r)^{\beta_i}}, s_{r\prod_{i=1}^{m}(c_i/r)^{\beta_i}} \rangle, \tag{7}$$

$$\mathrm{LNEWGM}(lh_1, lh_2, \ldots, lh_m) = \prod_{i=1}^{m} lh_i^{\beta_i} = \langle s_{r\prod_{i=1}^{m}(a_i/r)^{\beta_i}}, s_{r - r\prod_{i=1}^{m}(1 - b_i/r)^{\beta_i}}, s_{r - r\prod_{i=1}^{m}(1 - c_i/r)^{\beta_i}} \rangle. \tag{8}$$

Set $lh_i = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle$ as any LNE. The score and accuracy functions of lh_i are defined, respectively, as follows [4]:

$$P(lh_i) = \frac{(2r + a_i - b_i - c_i)}{3r} \text{for } P(lh_i) \in [0, 1], \tag{9}$$

$$Q(lh_i) = \frac{(a_i - c_i)}{r} \text{for } Q(lh_i) \in [-1, 1]. \tag{10}$$

Then, the sorting laws based on the score values of $P(lh_i)$ and the accuracy values of $Q(lh_i)$ ($i = 1, 2$) are given as follows [4]:

(a) $lh_1 > lh_2$ for $P(lh_1) > P(lh_2)$
(b) $lh_1 > lh_2$ for $P(lh_1) = P(lh_2)$ and $Q(lh_1) > Q(lh_2)$
(c) $lh_1 = lh_2$ for $P(lh_1) = P(lh_2)$ and $Q(lh_1) = Q(lh_2)$

3. SVNLNHSs and SVNLNHEs

This section proposes SVNLNHS/SVNLNHE for the mixed information representation of both SVNE and LNE and then presents some basic operations of SVNEs and LNEs according to a linguistic and neutrosophic conversion function and its inverse conversion function.

Definition 1. Let $U=\{u_1, u_2, \ldots, u_m\}$ be a universe set and $S=\{s_p|p=0, 1, \ldots, r\}$ be LTS with an odd cardinality $r+1$. Then, a SVNLNHS ML is defined by

$$ML=\{\langle u_i, TL_{ML}(u_i), IL_{ML}(u_i), FL_{ML}(u_i)\rangle | u_i \in U\}, \quad (11)$$

where $TL_{ML}(u_i)$, $IL_{ML}(u_i)$, and $FL_{ML}(u_i)$ are the truth, indeterminacy, and falsity membership functions, and their values are either the fuzzy values for $TL_{ML}(u_i)$, $IL_{ML}(u_i)$, $FL_{ML}(u_i)\in[0, 1]$ or the linguistic values for $TL_{ML}(u_i)$ $IL_{ML}(u_i)$, $FL_{ML}(u_i)\in S$ and $u_i\in U$. Moreover, the SVNLNHS ML is composed of the q SVNEs $zn_i=\langle x_{ZNi}, y_{ZNi}, z_{ZNi}\rangle$ for $x_{ZNi}, y_{ZNi}, z_{ZNi}\in[0, 1]$ $(i=1, 2, \ldots, q)$ and the $m-q$ LNEs $lh_i=\langle s_{a_i}, s_{b_i}, s_{c_i}\rangle$ for $s_{a_i}, s_{b_i}, s_{c_i}\in S$ and a_i, b_i, $c_i\in[0, r]$ $(i=q+1, q+2, \ldots, m)$.

Definition 2. Suppose that ML_1 and ML_2 are two SVNLNHSs, which contain q SVNEs $zn_{1i}=<x_{ZN1i}, y_{ZN1i}, z_{ZN1i}>$ $(i=1, 2, \ldots, q)$ and $m-q$ LNEs $lh_{1i}=\langle s_{a_{1i}}, s_{b_{1i}}, s_{c_{1i}}\rangle$ for $s_{a_{1i}}, s_{b_{1i}}, s_{c_{1i}}\in S$ $(i=q+1, q+2, \ldots, m)$ and q SVNEs $zn_{2i}=<x_{ZN2i}, y_{ZN2i}, z_{ZN2i}>$ $(i=1, 2, \ldots, q)$ and $m-q$ LNEs $lh_{2i}=\langle s_{a_{2i}}, s_{b_{2i}}, s_{c_{2i}}\rangle$ for $s_{a_{2i}}, s_{b_{2i}}, s_{c_{2i}}\in S$ $(i=q+1, q+2, \ldots, m)$. Thus, ML_1 and ML_2 imply the following relations:

(1) $ML_1\subseteq ML_2 \Leftrightarrow zn_{1i}\subseteq zn_{2i}$ $(i=1, 2, \ldots, q)$ and $lh_{1i}\subseteq lh_{2i}$ $(i=q+1, q+2, \ldots, m)$, i.e., $x_{ZN1i}\le x_{ZN2i}$, $y_{ZN2i}\le y_{ZN1i}$, and $z_{ZN2i}\le z_{ZN1i}$ for $i=1, 2, \ldots, q$ and $s_{a_{1i}}\le s_{a_{2i}}$, $s_{b_{1i}}\ge s_{b_{2i}}$, and $s_{c_{1i}}\ge s_{c_{2i}}$ for $i=q+1, q+2, \ldots, m$;

(2) $ML_1=ML_2 \Leftrightarrow zn_{1i}\subseteq zn_{2i}$, $zn_{1i}\supseteq zn_{2i}$, $lh_1\subseteq lh_2$, and $lh_2\subseteq lh_1$, i.e., $x_{ZN1i}=x_{ZN2i}$, $y_{ZN2i}=y_{ZN1i}$, and $z_{ZN2i}=z_{ZN1i}$ for $i=1, 2, \ldots, q$ and $s_{a_{1i}}=s_{a_{2i}}$, $s_{b_{1i}}=s_{b_{2i}}$, and $s_{c_{1i}}=s_{c_{2i}}$ for $i=q+1, q+2, \ldots, m$.

Definition 3. Set $zn_i=<x_{ZNi}, y_{ZNi}, z_{ZNi}>$ and $lh_i=\langle s_{a_i}, s_{b_i}, s_{c_i}\rangle$ as any SVNE and any LNE, respectively. Then, let a linguistic and neutrosophic conversion function be $f(lh_i)=\langle a_i/r, b_i/r, c_i/r\rangle$ for a_i, b_i, $c_i\in[0, r]$, and then its inverse conversion function is $f^{-1}(zn_i)=\langle x_{ZNi}r, y_{ZNi}r, z_{ZNi}r\rangle$ for x_{ZNi}, y_{ZNi}, $z_{ZNi}\in[0, 1]$. Thus, some basic operations of SVNEs and LNEs are given as follows:

(1) $f^{-1}(zn_i)\oplus lh_i=\langle x_{ZNi}r+a_i-x_{ZNi}a_i, y_{ZNi}b_i, z_{ZNi}c_i\rangle$

(2) $zn_i\oplus f(lh_i)=\langle x_{ZNi}+a_i/r-x_{ZNi}a_i/r, y_{ZNi}b_i/r, z_{ZNi}c_i/r\rangle$

(3) $f^{-1}(zn_i)\otimes lh_i=\langle x_{ZNi}a_i, y_{ZNi}r+b_i-y_{ZNi}b_i, z_{ZNi}r+c_i-z_{ZNi}c_i\rangle$

(4) $\beta f^{-1}(zn_i)=\langle r-r(1-x_{ZNi})^{\beta}, r y_{ZNi}^{\beta}, r z_{ZNi}^{\beta}\rangle$ for $\beta>0$

(5) $\beta f(lh_i)=\langle 1-(1-a_i/r)^{\beta}, (b_i/r)^{\beta}, (c_i/r)^{\beta}\rangle$ for $\beta>0$

(6) $(f^{-1}(zn_i))^{\beta}=\langle r x_{ZNi}^{\beta}, r-r(1-y_{ZNi})^{\beta}, r-r(1-z_{ZNi})^{\beta}\rangle$ for $\beta>0$

(7) $f^{\beta}(lh_i)=\langle (a_i/r)^{\beta}, 1-(1-b_i/r)^{\beta}, 1-(1-c_i/r)^{\beta}\rangle$ for $\beta>0$

It is obvious that the operational results of (2), (4), (5), and (8) are LNEs and the operational results of (3) and (7), and (9) are SVNEs.

4. Weighted Arithmetic and Geomatic Mean Operators of SVNLNHEs

This section proposes some weighted aggregation operators of SVNLNHEs corresponding to the linguistic and neutrosophic conversion function and its inverse conversion function, and then indicates their properties.

4.1. Aggregation Operators of SVNLNHEs Corresponding to the Linguistic and Neutrosophic Conversion Function. Let $zn_i=<x_{ZNi}, y_{ZNi}, z_{ZNi}>$ $(i=1, 2, \ldots, q)$ and $lh_i=\langle s_{a_i}, s_{b_i}, s_{c_i}\rangle$ $(i=q+1, q+2, \ldots, m)$ be q SVNEs and $m-q$ LNEs, respectively. Then, based on Definition 3 and the SVNEWAM and SVNEWGM operators of Eqs. (2) and (3) [17], the weighted arithmetic and geomatic mean operators of SVNLNHEs corresponding to the linguistic and neutrosophic conversion function are proposed by the $\mathrm{SVNLNHEWAM_N}$ and $\mathrm{SVNLNHEWGM_N}$ operators,

$$\begin{aligned} SVNLNHEWAM_N\left(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\right) &= \sum_{i=1}^{q}\beta_i\cdot zn_i+\sum_{i=q+1}^{m}\beta_i f(lh_i)\\ &=\langle 1-\prod_{i=1}^{q}(1-x_{ZNi})^{\beta_i}\prod_{i=q+1}^{m}(1-a_i/r)^{\beta_i}, \prod_{i=1}^{q}y_{ZNi}^{\beta_i}\prod_{i=q+1}^{m}(bi/r)^{\beta_i}, \prod_{i=1}^{q}z_{ZNi}^{\beta_i}\prod_{i=q+1}^{m}(c_i/r)^{\beta_i}\rangle \end{aligned} \quad (12)$$

$$\mathrm{SVNLNHEWGM}_N\left(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\right) = \prod_{i=1}^{q} zn_i^{\beta_i} \prod_{i=q+1}^{m} f^{\beta_i}(lh_i)$$
$$= \left\langle \prod_{i=1}^{q} x_{ZNi}^{\beta_i} \prod_{i=q+1}^{m} (a_i/r)^{\beta_i}, 1 - \prod_{i=1}^{q} (1 - y_{ZNi})^{\beta_i} \prod_{i=q+1}^{m} (1 - b_i/r)^{\beta_i}, 1 - \prod_{i=1}^{q} (1 - z_{ZNi})^{\beta_i} \prod_{i=q+1}^{m} (1 - c_i/r)^{\beta_i} \right\rangle, \tag{13}$$

where $\beta_i \in [0, 1]$ is the weight of zn_i $(i = 1, 2, \ldots, q)$ and lh_i $(i = q + 1, q + 2, \ldots, m)$ with $\sum_{i=1}^{m} \beta_i = 1$. Then, the aggregated results of the $\mathrm{SVNLNHEWAM_N}$ and $\mathrm{SVNLNHEWGM_N}$ operators are SVNEs.

Especially, when $q = m$ (without LNEs), the $\mathrm{SVNLNHEWAM_N}$ and $\mathrm{SVNLNHEWGM_N}$ operators are reduced to the SVNEWAM and SVNEWGM operators [17], i.e., Eq. (2) and Eq. (3).

Based on the properties of the SVNEWAM and SVNEWGM operators [17], it is obvious that the $\mathrm{SVNLNHEWAM_N}$ and $\mathrm{SVNLNHEWGM_N}$ operators also contain the following properties:

(1) Idempotency: if $\mathrm{zn_i} = f(\mathrm{lh_i}) = \mathrm{zn}$ for $i = 1, 2, \ldots, q, q + 1, q + 2, \ldots, m$, there are $\mathrm{SVNLNHEWAM}_N$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) = zn$ and $\mathrm{SVNLNHEWGM}_N$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) = zn$.

(2) Boundedness: let $zn^- = \langle \min_i (x_{ZNi}, a_i/r), \max_i (y_{ZNi}, b_i/r), \max_i (z_{ZNi}, c_i/r) \rangle$ and $zn^+ = \langle \max_i (x_{ZNi}, a_i/r), \min_i (y_{ZNi}, b_i/r), \min_i (z_{ZNi}, c_i/r) \rangle$ be the minimum and maximum SVNEs for $i = 1, 2, \ldots, m$, and then there are the inequalities $zn^- \leq \mathrm{SVNLNHEWAM}_N (zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq zn^+$ and $zn^- \leq \mathrm{SVNLNHEWGM}_N$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq zn^+$.

(3) Monotonicity: if the condition of $zn_i \leq zn_i^*$ $(i = 1, 2, \ldots, q)$ and $lh_i \leq lh_i^*$ $(i = q + 1, q + 2, \ldots, \mathrm{m})$ exists, $\mathrm{SVNLNHEWAM}_N (zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq SVNLNHEWAM_N$ $(zn_1^*, zn_2^*, \ldots, zn_q^*, lh_{q+1}^*, lh_{q+2}^*, \ldots, lh_m^*)$ and $\mathrm{SVNLNHEWGM}_N$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq \mathrm{SVNLNHEWGM}_N$ $(zn_1^*, zn_2^*, \ldots, zn_q^*, lh_{q+1}^*, lh_{q+2}^*, \ldots, lh_m^*)$ also exist.

4.2. Aggregation Operators of SVNLNHEs According to the Inverse Conversion Function. Let $zn_i = \langle x_{ZNi}, y_{ZNi}, z_{ZNi} \rangle$ $(i = 1, 2, \ldots, q)$ and $lh_i = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle$ $(i = q + 1, q + 2, \ldots, m)$ be q SVNEs and $m - q$ LNEs, respectively. Then, based on Definition 3 and the LNEWAM and LNEWGM operators of Eqs. (7) and (8) [4], the weighted arithmetic and geomatic mean operators of SVNLNHEs corresponding to the inverse conversion function are proposed by the $\mathrm{SVNLNHEWAM_L}$ and $\mathrm{SVNLNHEWGM_L}$ operators:

$$\mathrm{SVNLNHEWAM}_L\left(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\right) = \sum_{i=1}^{q} \beta_i f^{-1}(zn_i) + \sum_{i=q+1}^{m} \beta_i \cdot lh_i$$
$$= \left\langle r - r\prod_{i=1}^{q} (1 - x_{ZNi})^{\beta_i} \prod_{i=q+1}^{m} (1 - a_i/r)^{\beta_i}, r\prod_{i=1}^{q} y_{ZNi}^{\beta_i} \prod_{i=q+1}^{m} (b_i/r)^{\beta_i}, r\prod_{i=1}^{q} z_{ZNi}^{\beta_i} \prod_{i=q+1}^{m} (c_i/r)^{\beta_i} \right\rangle, \tag{14}$$

$$\mathrm{SVNLNHEWGM}_L\left(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\right) = \prod_{i=1}^{q} \left(f^{-1}(zn_i)\right)^{\beta_i} \prod_{i=q+1}^{m} lh_i^{\beta_i}$$
$$= \left\langle r\prod_{i=1}^{q} x_{ZNi}^{\beta_i} \prod_{i=q+1}^{m} (a_i/r)^{\beta_i}, r - r\prod_{i=1}^{q} (1 - y_{ZNi})^{\beta_i} \prod_{i=q+1}^{m} (1 - b_i/r)^{\beta_i}, r - r\prod_{i=1}^{q} (1 - z_{ZNi})^{\beta_i} \prod_{i=q+1}^{m} (1 - c_i/r)^{\beta_i} \right\rangle, \tag{15}$$

where $\beta_i \in [0, 1]$ is the weight of zn_i $(i = 1, 2, \ldots, q)$ and lh_i $(i = q + 1, q + 2, \ldots, m)$ with $\sum_{i=1}^{m} \beta_i = 1$. Then, the aggregated results of the $\mathrm{SVNLNHEWAM_L}$ and $\mathrm{SVNLNHEWGM_L}$ operators are LNEs.

Especially, when $q = 0$ (without SVNEs), the $\mathrm{SVNLNHEWAM_L}$ and $\mathrm{SVNLNHEWGM_L}$ operators are reduced to the LNEWAM and LNEWGM operators [4], i.e., Eq. (7) and Eq. (8).

Based on the characteristics of the LNEWAM and LNEWGM operators [4], it is obvious that the $\mathrm{SVNLNHEWAM_L}$ and $\mathrm{SVNLNHEWGM_L}$ operators also contain the following characteristics:

(1) Idempotency: if $f^{-1}(\mathrm{zn_i}) = \mathrm{lh_i} = \mathrm{lh}$ for $i = 1, 2, \ldots, q, q + 1, q + 2, \ldots, m$, there are $\mathrm{SVNLNHEWAM}_L$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) = lh$ and $\mathrm{SVNLNHEWGM}_L (zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) = lh$.

(2) Boundedness: let $lh^- = \langle \min_i (rx_{ZNi}, a_i), \max_i (ry_{ZNi}, b_i), \max_i (rz_{ZNi}, c_i) \rangle$ and $lh^+ = \langle \max_i$

(rx_{ZNi}, a_i), $\min_i (ry_{ZNi}, b_i), \min_i (rz_{ZNi}, c_i)\rangle$ be the minimum and maximum LNEs for $i = 1, 2, \ldots, m$, and then there are the inequalities $lh^- \leq$ SVNLNHEWAM$_L$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq lh^+$ and $lh^- \leq$ SVNLNHEWGM$_L$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq lh^+$.

(3) Monotonicity: if the condition of $zn_i \leq zn_i^*$ for $i = 1, 2, \ldots, q$ and $lh_i \leq lh_i^*$ for $i = q+1, q+2, \ldots, m$ exists, SVNLNHEWAM$_L$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq$ SVNLNHEWAM$_L$ $(zn_1^*, zn_2^*, \ldots, zn_q^*, lh_{q+1}^*, lh_{q+2}^*, \ldots, lh_m^*)$ and SVNLNHEWGM$_L$ $(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m) \leq$ SVNLNHEWGM$_L$ $(zn_1^*, zn_2^*, \ldots, zn_q^*, lh_{q+1}^*, lh_{q+2}^*, \ldots, lh_m^*)$ also exist.

5. MAGDM Models in the Environment of SVNLNHSs

In this section, novel MAGDM models are developed in terms of the SVNLNHEWAM$_N$ and SVNLNHEWGM$_N$ operators and the SVNLNHEWAM$_L$ and SVNLNHEWGM$_L$ operators to perform MAGDM issues with quantitative and qualitative attributes in the mixed information environment of SVNEs and LNEs.

Regarding a mixed information MAGDM issue in the circumstance of SVNLNHSs, there exist t alternatives, denoted by a set of them $E = \{E_1, E_2, \ldots E_t\}$, and then they are satisfactorily assessed over m attributes, denoted by a set of them $V = \{v_1, v_2, \ldots, v_q, v_{q+1}, v_{q+2}, \ldots, v_m\}$, which contains q quantitative attributes and $m-q$ qualitative attributes. Then, there is a group of decision makers $G = \{g_1, g_2, \ldots g_e\}$ with their weight vector $\alpha = \{\alpha_1, \alpha_2, \ldots \alpha_e\}$ for $\alpha_k \in [0, 1]$ and $\sum_{k=1}^{e} \alpha_k = 1$. The assessment values of each alternative over the q quantitative attributes are given by the decision makers g_k $(k = 1, 2, \ldots, e)$ and represented by SVNEs $zn_{ji}^k = \langle x_{ZNji}^k, y_{ZNji}^k, z_{ZNji}^k \rangle$ for $x_{ZNji}^k, y_{ZNji}^k, z_{ZNji}^k \in [0, 1]$ $(k = 1, 2, \ldots, e; j = 1, 2, \ldots, t; i = 1, 2, \ldots, q)$, and then the assessment values of each alternative over the $m-q$ qualitative attributes are represented by LNEs $lh_{ji}^k = \langle s_{a_{ji}^k}, s_{b_{ji}^k}, s_{c_{ji}^k} \rangle$ for $s_{a_{ji}^k}, s_{b_{ji}^k}, s_{c_{ji}^k} \in S$ $(k = 1, 2, \ldots, e; j = 1, 2, \ldots, t; i = q+1, q+2, \ldots, m)$ from the LTS $S = \{s_p | p = 0, 1, 2, \ldots, r\}$. Thus, all assessed values can be constructed as the e decision matrices of SVNLNHEs $M_k = (zn_{ji}^k, lh_{ji}^k)_{t\times m}$ $(k = 1, 2, \ldots, e)$. Then, a weight vector $\beta = (\beta_1, \beta_2, \ldots \beta_m)$ is specified to consider the weights β_i of attributes v_i $(i = 1, 2, \ldots, m)$ with $\beta_i \in [0, 1]$ and $\sum_{i=1}^{m} \beta_i = 1$.

Thus, two MAGDM models are developed in terms of the SVNLNHEWAM$_N$ and SVNLNHEWGM$_N$ operators or the SVNLNHEWAM$_L$ and SVNLNHEWGM$_L$ operators to perform MAGDM issues with the mixed evaluation information of SVNEs and LNEs.

Model 1. A MAGDM model using the SVNLNHEWAM$_N$ and SVNLNHEWGM$_N$ operators is developed to perform the MAGDM issue with SVNLNHEs. Its detailed steps are presented as follows:

Step 1: using the SVNEWAM operator of Eq. (2) and the LNEWAM operator of Eq. (7), the decision matrices $M^k = (zn_{ji}^k, lh_{ji}^k)_{t\times m}$ $(k = 1, 2, \ldots, e)$ are aggregated into the overall decision matrix $M = (zn_{ji}, lh_{ji})_{t\times m}$.

Step 2: using the SVNLNHEWAM$_N$ operator of Eq. (12) or the SVNLNHEWGM$_N$ operator of Eq. (13), the aggregated result for E_j $(j = 1, 2, \cdots, t)$ is obtained by the following equation:

$$\begin{aligned} zn_j &= \text{SVNLNHEWAM}_N\left(zn_{j1}, zn_{j2}, \ldots, zn_{jq}, lh_{jq+1}, lh_{jq+2}, \ldots, lh_{jm}\right) = \sum_{i=1}^{q} \beta_i \cdot zn_{ji} + \sum_{i=q+1}^{m} \beta_i f\left(lh_{ji}\right) \\ &= \langle 1 - \prod_{i=1}^{q} \left(1 - x_{ZNji}\right)^{\beta_i} \prod_{i=q+1}^{m} \left(1 - a_{ji}/r\right)^{\beta_i}, \prod_{i=1}^{q} y_{ZNji}^{\beta_i} \prod_{i=q+1}^{m} \left(b_{ji}/r\right)^{\beta_i}, \prod_{i=1}^{q} z_{ZNji}^{\beta_i} \prod_{i=q+1}^{m} \left(c_{ji}/r\right)^{\beta_i} \rangle, \end{aligned} \tag{16}$$

or

$$\begin{aligned} zn_j &= \text{SVNLNHEWGM}_N\left(zn_{j1}, zn_{j2}, \ldots, zn_{jq}, lh_{jq+1}, lh_{jq+2}, \ldots, lh_{jm}\right) = \prod_{i=1}^{q} zn_{ji}^{\beta_i} \prod_{i=q+1}^{m} f^{\beta_i}\left(lh_{ji}\right) \\ &= \langle \prod_{i=1}^{q} x_{ZNji}^{\beta_i} \prod_{i=q+1}^{m} \left(a_{ji}/r\right)^{\beta_i}, 1 - \prod_{i=1}^{q} \left(1 - y_{ZNji}\right)^{\beta_i} \prod_{i=q+1}^{m} \left(1 - b_{ji}/r\right)^{\beta_i}, 1 - \prod_{i=1}^{q} \left(1 - z_{ZNji}\right)^{\beta_i} \prod_{i=q+1}^{m} \left(1 - c_{ji}/r\right)^{\beta_i} \rangle. \end{aligned} \tag{17}$$

Step 3: the score values of $F(zn_j)$ $(j = 1, 2, \ldots, t)$ are given by Eq. (4) and the accuracy values of $G(zn_j)$ $(j = 1, 2, \ldots, t)$ are given by Eq. (5) if necessary.

Step 4: the alternatives are sorted in descending order based on the sorting laws of SVNEs, and the first one is the best choice.

Step 5: end.

Model 2. A MAGDM model using the $\text{SVNLNHEWAM}_\text{L}$ and $\text{SVNLNHEWGM}_\text{L}$ operators is developed to perform the MAGDM issue with SVNLNHEs. Its detailed steps are presented as follows:

Step 1': the same as Step 1.

Step 2': using the $\text{SVNLNHEWAM}_\text{L}$ operator of Eq. (14) or the $\text{SVNLNHEWGM}_\text{L}$ operator of Eq. (15), the aggregated result for $E_j\,(j = 1, 2, \ldots, t)$ is given by the following equation:

$$\begin{aligned} lh_j &= \text{SVNLNHEWAM}_L\left(zn_{j1}, zn_{j2}, \ldots, zn_{jq}, lh_{jq+1}, lh_{jq+2}, \ldots, lh_{jm}\right) = \sum_{i=1}^{q} \beta_i f^{-1}\left(zn_{ji}\right) + \sum_{i=q+1}^{m} \beta_i \cdot lh_{ji} \\ &= \left\langle r - r\prod_{i=1}^{q}\left(1 - x_{ZNji}\right)^{\beta_i} \prod_{i=q+1}^{m}\left(1 - a_{ji}/r\right)^{\beta_i}, r\prod_{i=1}^{q} y_{ZNji}^{\beta_i} \prod_{i=q+1}^{m}\left(b_{ji}/r\right)^{\beta_i}, r\prod_{i=1}^{q} z_{ZNji}^{\beta_i} \prod_{i=q+1}^{m}\left(c_{ji}/r\right)^{\beta_i}\right\rangle, \end{aligned} \tag{18}$$

or

$$\begin{aligned} lh_j &= \text{SVNLNHEWGM}_L\left(zn_{j1}, zn_{j2}, \ldots, zn_{jq}, lh_{jq+1}, lh_{jq+2}, \ldots, lh_{jm}\right) = \prod_{i=1}^{q}\left(f^{-1}\left(zn_{ji}\right)\right)^{\beta_i} \prod_{i=q+1}^{m} lh_{ji}^{\beta_i} \\ &= \left\langle r\prod_{i=1}^{q} x_{ZNji}^{\beta_i} \prod_{i=q+1}^{m}\left(a_{ji}/r\right)^{\beta_i}, r - r\prod_{i=1}^{q}\left(1 - y_{ZNji}\right)^{\beta_i} \prod_{i=q+1}^{m}\left(1 - b_{ji}/r\right)^{\beta_i}, r - r\prod_{i=1}^{q}\left(1 - z_{ZNji}\right)^{\beta_i} \prod_{i=q+1}^{m}\left(1 - c_{ji}/r\right)^{\beta_i}\right\rangle. \end{aligned} \tag{19}$$

Step 3': the score values of $P(lh_j)$ ($j = 1, 2, \ldots, t$) are given by Eq. (9) and the accuracy values of $Q(lh_j)$ ($j = 1, 2, \ldots, t$) are given by Eq. (10) if necessary.

Step 4': the alternatives are sorted in descending order based on the sorting laws of LNEs, and then the first one is the best choice.

Step 5': end.

6. Illustrative Example on the Selection Problem of Industrial Robots Containing Both Quantitative and Qualitative Attributes

This section applies the proposed MAGDM models to an illustrative example on the selection problem of industrial robots that contains both quantitative and qualitative attributes in the circumstance of SVNLNHSs to prove their usefulness, and then gives a comparison with existing techniques to show the availability and rationality of the new techniques.

6.1. Illustrative Example. This subsection applies the proposed MAGDM models to the selection problem of industrial robots containing both quantitative and qualitative attributes to illustrate their application and availability in the circumstance of SVNLNHSs.

Some industrial company wants to buy a type of industrial robots for a manufacturing system. The technical department preliminarily provides four types of industrial robots/alternatives, denoted as their set $E = \{E_1, E_2, E_3, E_4\}$. Then, they must satisfy four requirements/attributes: operating accuracy (v_1), carrying capacity (v_2), control performance (v_3), and operating space and dexterity (v_4). The weight vector of the four attributes is given by $\beta = (0.25, 0.3, 0.25, 0.2)$. Thus, three experts/decision makers are invited to satisfactorily assess each alternative over the four attributes by their truth, falsity, and indeterminacy options/judgments, where the assessment values can be specified in the mixed forms of both the SVNEs $zn_{ji}^k = \langle x_{ZNji}^k, y_{ZNji}^k, z_{ZNji}^k\rangle$ for $x_{ZNji}^k, y_{ZNji}^k, z_{ZNji}^k \in [0, 1]$ ($k = 1, 2, 3$; $i = 1, 2$; $j = 1, 2, 3, 4$) regarding the quantitative attributes v_1 and v_2 and the LNEs $lh_{ji}^k = \langle s_{a_{ji}^k}, s_{b_{ji}^k}, s_{c_{ji}^k}\rangle$ for $s_{a_{ji}^k}, s_{b_{ji}^k}, s_{c_{ji}^k} \in S$ ($k = 1, 2, 3$; $i = 3, 4$; $j = 1, 2, 3, 4$) regarding the qualitative attributes v_3 and v_4 from the LTS $S = \{$very unsatisfactory, unsatisfactory, slight unsatisfactory, medium, slight satisfactory, satisfactory, very satisfactory$\} = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ with $r = 6$. The weight vector of the three decision makers is given by $\alpha = (0.4, 0.35, 0.25)$. Thus, the three decision matrices are constructed as follows:

$$\begin{aligned} M^1 &= \begin{bmatrix} \langle 0.8, 0.1, 0.2\rangle & \langle 0.7, 0.1, 0.1\rangle & \langle s_5, s_2, s_2\rangle & \langle s_5, s_2, s_3\rangle \\ \langle 0.8, 0.2, 0.1\rangle & \langle 0.8, 0.1, 0.3\rangle & \langle s_5, s_1, s_2\rangle & \langle s_4, s_3, s_2\rangle \\ \langle 0.7, 0.1, 0.1\rangle & \langle 0.8, 0.2, 0.2\rangle & \langle s_5, s_3, s_2\rangle & \langle s_4, s_2, s_3\rangle \\ \langle 0.8, 0.2, 0.2\rangle & \langle 0.9, 0.2, 0.3\rangle & \langle s_4, s_1, s_2\rangle & \langle s_5, s_2, s_2\rangle \end{bmatrix}, \\ M^2 &= \begin{bmatrix} \langle 0.7, 0.2, 0.2\rangle & \langle 0.8, 0.1, 0.2\rangle & \langle s_4, s_3, s_1\rangle & \langle s_5, s_2, s_1\rangle \\ \langle 0.8, 0.2, 0.3\rangle & \langle 0.8, 0.2, 0.3\rangle & \langle s_5, s_2, s_1\rangle & \langle s_4, s_1, s_2\rangle \\ \langle 0.8, 0.2, 0.3\rangle & \langle 0.7, 0.1, 0.1\rangle & \langle s_4, s_1, s_2\rangle & \langle s_5, s_2, s_3\rangle \\ \langle 0.9, 0.1, 0.1\rangle & \langle 0.8, 0.2, 0.1\rangle & \langle s_5, s_1, s_3\rangle & \langle s_5, s_1, s_2\rangle \end{bmatrix}, \\ M^3 &= \begin{bmatrix} \langle 0.8, 0.3, 0.1\rangle & \langle 0.8, 0.1, 0.1\rangle & \langle s_5, s_2, s_1\rangle & \langle s_4, s_1, s_1\rangle \\ \langle 0.7, 0.1, 0.2\rangle & \langle 0.9, 0.2, 0.3\rangle & \langle s_4, s_1, s_1\rangle & \langle s_5, s_1, s_1\rangle \\ \langle 0.8, 0.1, 0.1\rangle & \langle 0.8, 0.2, 0.1\rangle & \langle s_5, s_2, s_2\rangle & \langle s_5, s_2, s_3\rangle \\ \langle 0.8, 0.1, 0.1\rangle & \langle 0.8, 0.1, 0.1\rangle & \langle s_5, s_1, s_2\rangle & \langle s_5, s_3, s_2\rangle \end{bmatrix}. \end{aligned} \tag{20}$$

Thus, the two MAGDM models developed can be utilized in the example to perform the MAGDM issue with SVNLNHEs.

Model 1. The MAGDM model using the SVNLNHEWAM$_N$ and SVNLNHEWGM$_N$ operators can be applied in the example, and then its detailed steps are depicted as follows:

Step 1: using the SVNEWAM operator of Eq. (2) and the LNEWAM operator of Eq. (7), the above three decision matrices are aggregated into the following overall decision matrix:

$$M=\begin{bmatrix}\langle 0.7695,0.1677,0.1682\rangle & \langle 0.7648,0.1000,0.1275\rangle & \langle s_{4.7254},s_{2.3050},s_{1.3195}\rangle & \langle s_{4.8108},s_{1.6818},s_{1.5518}\rangle\\ \langle 0.7787,0.1682,0.1747\rangle & \langle 0.8318,0.1516,0.3000\rangle & \langle s_{4.8108},s_{1.2746},s_{1.3195}\rangle & \langle s_{4.3182},s_{1.5518},s_{1.6818}\rangle\\ \langle 0.7648,0.1275,0.1469\rangle & \langle 0.7695,0.1569,0.1320\rangle & \langle s_{4.7254},s_{1.8455},s_{2.0000}\rangle & \langle s_{4.6805},s_{2.0000},s_{3.0000}\rangle\\ \langle 0.8431,0.1320,0.1320\rangle & \langle 0.8484,0.1682,0.1552\rangle & \langle s_{4.6805},s_{1.0000},s_{2.3050}\rangle & \langle s_{5.0000},s_{1.7366},s_{2.0000}\rangle\end{bmatrix}. \quad (21)$$

Step 2: by Eq. (16) or Eq. (17), we give the aggregated values:

$zn_1=<0.7795,\ 0.1958,\ 0.1804>$, $zn_2=<0.7921,\ 0.1884,\ 0.2392>$,

$zn_3=<0.7751,\ 0.2049,\ 0.2231>$, and $zn_4=<0.8290,\ 0.1761,\ 0.2178>$.

Or $zn_1=<0.7789,\ 0.2324,\ 0.1885>$, $zn_2=<0.7876,\ 0.1934,\ 0.2464>$,

$\mathrm{zn}_3=<0.7749,\ 0.2276,\ 0.2754>$, and $\mathrm{zn}_4=<0.8265,\ 0.1850,\ 0.2504>$.

Step 3: by Eq. (4), the score values of $F(\mathrm{z}n_j)$ for E_j ($j=1$, 2, 3, 4) are given below:

$F(\mathrm{z}n_1)=0.8011$, $F(\mathrm{z}n_2)=0.7882$, $F(\mathrm{z}n_3)=0.7824$, and $F(\mathrm{z}n_4)=0.8117$.

Or $F(\mathrm{z}n_1)=0.7860$, $F(\mathrm{z}n_2)=0.7826$, $F(\mathrm{z}n_3)=0.7573$, and $F(\mathrm{z}n_4)=0.7970$.

Step 4: the sorting order of the four alternatives is $E_4>E_1>E_2>E_3$.

Clearly, the sorting orders obtained by the SVNLNHEWAM$_N$ operator of Eq. (16) and the SVNLNHEWGM$_N$ operator of Eq. (17) are identical in this example.

Model 2. The MAGDM model using the SVNLNHEWAM$_L$ and SVNLNHEWGM$_L$ operators can also be applied in the example, and then its detailed steps are depicted as follows:

Step 1': the same as Step 1.

Step 2': by Eq. (18) or Eq. (19), we give the aggregated values:

$lh_1=<s_{4.6773},\ s_{1.1747},\ s_{1.0823}>$, $lh_2=<s_{4.7528},\ s_{1.1302},\ s_{1.4353}>$,

$lh_3=<s_{4.}6508,\ s_{1.2297},\ s_{1.3384}>$, and $lh_4=<s_{4.9739},\ s_{1.0564},\ s_{1.3070}>$,

or $lh_1=<s_{4.6736},\ s_{1.3944},\ s_{1.1311}>$, $lh_2=<s_{4.7255},\ s_{1.1603},\ s_{1.4783}>$,

$lh_3=<s_{4.6494},\ s_{1.3657},\ s_{1.6526}>$, and $lh_4=<s_{4.9591},\ s_{1.1100},\ s_{1.5027}>$.

Step 3': by (9), the score values of $P(lh_j)$ for E_j ($j=1$, 2, 3, 4) are given as follows:

$P(\mathrm{lh}_1)=0.8011$, $P(lh_2)=0.7882$, $P(lh_3)=0.7824$, and $P(lh_4)=0.8117$,

or $P(lh_1)=0.7860$, $P(lh_2)=0.7826$, $P(lh_3)=0.7573$, and $P(lh_4)=0.7970$.

Step 4': the sorting order of the four alternatives is $E_4>E_1>E_2>E_3$.

Hence, the sorting orders obtained by the SVNLNHEWAM$_L$ operator of Eq. (18) and the SVNLNHEWGM$_L$ operator of Eq. (19) are identical in this example.

Obviously, the score values and sorting orders between Model 1 and Model 2 reflect the same results. Moreover, we see that whether SVNEs are converted to LNEs or LNEs to SVNEs in the aggregation operations, their final decision results are actually identical. Thus, decision makers can choose Model 1 or Model 2 in MAGDM applications. Therefore, it is obvious that our new techniques are valid and reasonable.

6.2. Comparative Analysis with the Existing Neutrosophic MAGDM Models. Since the assessed values of SVNLNHEs are given in this illustrative example, the existing neutrosophic MAGDM models [4, 17] cannot deal with this illustrative example in the situation of SVNLNHEs. Then, our new techniques can handle neutrosophic MAGDM issues with SVNEs and/or LNEs and show the following highlights and advantages:

(1) The proposed SVNLNHEs can conveniently denote the mixed information of SVNEs and LNEs regarding the assessment objects of quantitative and qualitative attributes, which is suitable for human judgment and thinking/expression habits, while existing neutrosophic expressions cannot represent SVNLNHE information.

(2) The proposed SVNLNHEWAM$_N$ and SVNLNHEWGM$_N$ operators or the proposed SVNLNHEWAM$_L$ and SVNLNHEWGM$_L$ operators provide the necessary aggregation tools for handling

MAGDM issues in the SVNLNHE circumstance, while the existing SVNEWAM and SVNEWGM operators [17] are only the special cases of the $\mathrm{SVNLNHEWAM_N}$ and $\mathrm{SVNLNHEWGM_N}$ operators, and then the existing LNEWAM and LNEWGM operators [4] are only the special cases of the $\mathrm{SVNLNHEWAM_L}$ and $\mathrm{SVNLNHEWGM_L}$ operators. Furthermore, the various existing aggregation operators cannot aggregate SVNLNHEs.

(3) Since the existing MAGDM models with the single evaluation information of SVNEs or LNEs [4, 17] are the special cases of our new MAGDM models, our new MAGDM models are broader and more versatile than the existing MAGDM models [4, 17]. Furthermore, the various existing MAGDM models cannot carry out MAGDM problems with SVNLNHE information.

Generally, the new techniques solve the SVNLNHE denotation, aggregation operations, and MAGDM issues in the mixed information situation of SVNEs and LNEs. It is clear that our new techniques are very suitable for such decision-making issues with quantitative and qualitative attributes and overcome the defects of the existing decision-making techniques subject to the single evaluation information of SVNEs or LNEs. Therefore, our new techniques reveal obvious superiorities over the existing techniques in the neutrosophic information denotation, aggregation operations, and decision-making methods.

7. Conclusion

Due to the lack of the SVNLNHE denotation, operations, and decision-making models in existing neutrosophic theory and applications, the proposed notion of SVNLNHS/SVNLNHE and the defined linguistic and neutrosophic conversion function solved the hybrid neutrosophic information denotation and operational problems of SVNEs and LNEs. Then, the proposed $\mathrm{SVNLNHEWAM_N}$, $\mathrm{SVNLNHEWGM_N}$, $\mathrm{SVNLNHEWAM_L}$, and $\mathrm{SVNLNHEWGM_L}$ operators provided necessary aggregation algorithms for handling MAGDM issues with SVNLNHEs. The established MAGDM models solved such decision-making issues with quantitative and qualitative attributes in the SVNLNHE circumstance. Since the evaluation values of quantitative and qualitative attributes in the decision-making process are easily represented in SVNEs and LNEs that are given in view of decision makers' preferences/thinking habits, the managerial implications of this original research will be reinforced in neutrosophic decision-making methods and applications. Finally, an illustrative example was given and compared with the existing techniques to show the availability and rationality of the new techniques. Moreover, our new techniques not only overcome the insufficiencies of the existing techniques but also are broader and more versatile than the existing techniques when dealing with MAGDM issues in the setting of SVNLNHEs. However, in this study, the new techniques of the SVNLNHE denotation, aggregation algorithms, and MAGDM models reflected their superiority over existing techniques.

Regarding future research, these new techniques will be further extended to other areas, such as medical diagnosis, slope risk/instability evaluation, default diagnosis, and mechanical concept design, in the mixed information situation of SVNEs and LNEs. Then, we shall also develop more aggregation algorithms, such as Hamacher, Dombi, and Bonferroni aggregation operators, and their applications in clustering analysis, information fusion, image processing, and mine risk/safety evaluation in the mixed information situation of both SVNE and LNE or both IVNE and uncertain LNE.

Abbreviations

IVNS:	Interval-valued neutrosophic set
IVNE:	Interval-valued neutrosophic element
LTS:	Linguistic term set
LNS:	Linguistic neutrosophic set
LNE:	Linguistic neutrosophic element
LNEWAM:	Linguistic neutrosophic element weighted arithmetic mean
LNEWGM:	Linguistic neutrosophic element weighted geometric mean
MADM:	Multiple attribute decision-making
MAGDM:	Multiple attribute group decision-making
SNS:	Simplified neutrosophic set
SVNS:	Single-valued neutrosophic set
SVNE:	Single-valued neutrosophic element
SVNEWAM:	Single-valued neutrosophic element weighted arithmetic mean
SVNEWGM:	Single-valued neutrosophic element weighted geometric mean
SVNLNHS:	Single-valued neutrosophic and linguistic neutrosophic hybrid set
SVNLNHE:	Single-valued neutrosophic and linguistic neutrosophic hybrid element
SVNLNHEWAMN:	Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted arithmetic mean (for the aggregated SVNE)
SVNLNHEWGMN:	Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted geometric mean (for the aggregated SVNE)
SVNLNHEWAML:	Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted arithmetic mean (for the aggregated LNE)
SVNLNHEWGML:	Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted geometric mean (for the aggregated LNE).

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References

[1] J. Ye, "A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets," *Journal of Intelligent and Fuzzy Systems*, vol. 26, pp. 2459–2466, 2014.

[2] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*, Hexis, Frontignan, France, 2005.

[3] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, "Single valued neutrosophic sets," *Multispace and Multistructure*, vol. 4, pp. 410–413, 2010.

[4] Z. Fang and J. Ye, "Multiple attribute group decision-making method based on linguistic neutrosophic numbers," *Symmetry*, vol. 9, no. 7, 2017.

[5] F. Smarandache, *Neutrosophy: Neutrosophic Probability, Set, and Logic*, American Research Press, Rehoboth, DE, USA, 1998.

[6] W. Jiang and Y. Shou, "A novel single-valued neutrosophic set similarity measure and its application in multicriteria decision-making," *Symmetry*, vol. 9, no. 8, p. 127, 2017.

[7] I. Deli and Y. Subas, "A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems," *International Journal of Machine Learning and Cybernetics*, vol. 8, no. 4, pp. 1309–1322, 2017.

[8] C. Liu, "New similarity measures of simplified neutrosophic sets and their applications," *Journal of Information Processing Systems*, vol. 14, no. 3, pp. 790–800, 2018.

[9] M. Akram, S. Naz, and F. Smarandache, "Generalization of maximizing deviation and TOPSIS method for MADM in simplified neutrosophic hesitant fuzzy environment," *Symmetry*, vol. 11, no. 8, 2019.

[10] S. Ashraf, S. Abdullah, S. Zeng, H. Jin, and F. Ghani, "Fuzzy decision support modeling for hydrogen power plant selection based on single valued neutrosophic sine trigonometric aggregation operators," *Symmetry*, vol. 12, no. 2, p. 298, 2020.

[11] H. Garg, "SVNMPR: a new single-valued neutrosophic multiplicative preference relation and their application to decision-making process," *International Journal of Intelligent Systems*, vol. 37, no. 3, pp. 2089–2130, 2022.

[12] I. Gokasar, M. Deveci, and O. Kalan, "CO2 Emission based prioritization of bridge maintenance projects using neutrosophic fuzzy sets based decision making approach," *Research in Transportation Economics*, vol. 91, Article ID 101029, 2022.

[13] M. Deveci, V. Simic, and A. E. Torkayesh, "Remanufacturing facility location for automotive Lithium-ion batteries: an integrated neutrosophic decision-making model," *Journal of Cleaner Production*, vol. 317, Article ID 128438, 2021.

[14] M. S. Bavia, D. Nagarajan, S. Broumi, J. Kavikumar, and A. Rajkumar, "Neutrosophic in multi-criteria decision making for location selection," *Neutrosophic Sets and Systems*, vol. 48, pp. 142–153, 2022.

[15] P. D. Liu and Y. M. Wang, "Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean," *Neural Computing & Applications*, vol. 25, no. 7-8, pp. 2001–2010, 2014.

[16] H. Y. Zhang, J. Q. Wang, and X. H. Chen, "Interval neutrosophic sets and their application in multicriteria decision making problems," *Science World Journal*, vol. 2014, Article ID 645953, 15 pages, 2014.

[17] J. J. Peng, J. Q. Wang, J. Wang, H. Y. Zhang, and X. H. Chen, "Simplified neutrosophic sets and their applications in multicriteria group decision-making problems," *International Journal of Systems Science*, vol. 47, no. 10, pp. 2342–2358, 2016.

[18] H. Garg, "New logarithmic operational laws and their applications to multiattribute decision-making for single-valued neutrosophic numbers," *Cognitive Systems Research*, vol. 52, pp. 931–946, 2018.

[19] S. Ashraf, S. Abdullah, F. Smarandache, and N. Amin, "Logarithmic hybrid aggregation operators based on single valued neutrosophic sets and their applications in decision support systems," *Symmetry*, vol. 11, no. 3, p. 364, 2019.

[20] L. P. Zhou, J. Y. Dong, and S. P. Wan, "Two new approaches for multi-attribute group decision-making with interval-valued neutrosophic Frank aggregation operators and incomplete weights," *IEEE Access*, vol. 7, Article ID 102727, 2019.

[21] H. L. Yang, C. L. Zhang, Z. L. Guo, Y. L. Liu, and X. Liao, "A hybrid model of single valued neutrosophic sets and rough sets: single valued neutrosophic rough set model," *Soft Computing*, vol. 21, pp. 6253–6267, 2017.

[22] R. Sahin, "Normal neutrosophic multiple attribute decision making based on generalized prioritized aggregation operators," *Neural Computing & Applications*, vol. 30, no. 10, pp. 3095–3115, 2018.

[23] L. Wang, H. Y. Zhang, and J. Q. Wang, "Frank Choquet Bonferroni mean operators of bipolar neutrosophic sets and their application to multi-criteria decision making problems," *International Journal of Fuzzy Systems*, vol. 20, no. 1, pp. 13–28, 2018.

[24] S. G. Du, J. Ye, R. Yong, and F. W. Zhang, "Simplified neutrosophic indeterminate decision making method with decision makers' indeterminate ranges," *Journal of Civil Engineering and Management*, vol. 26, no. 6, pp. 590–598, 2020.

[25] S. G. Du, J. Ye, R. Yong, and F. W. Zhang, "Some aggregation operators of neutrosophic Z-numbers and their multicriteria decision making method," *Complex & Intelligent Systems*, vol. 7, no. 1, pp. 429–438, 2021.

[26] P. D. Liu, T. Mahmood, and Q. Khan, "Group decision making based on power Heronian aggregation operators under linguistic neutrosophic environment," *International Journal of Fuzzy Systems*, vol. 20, no. 3, pp. 970–985, 2018.

[27] Y. M. Wang and P. D. Liu, "Linguistic neutrosophic generalized partitioned Bonferroni mean operators and their application to multi-attribute group decision making," *Symmetry*, vol. 10, no. 5, 2018.

[28] M. Xu, R. Yong, and Y. Belayne, "Decision making methods with linguistic neutrosophic information: a review," *Neutrosophic Sets and Systems*, vol. 38, pp. 599–610, 2020.

[29] X. Peng and J. Dai, "A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017," *Artificial Intelligence Review*, vol. 53, no. 1, pp. 199–255, 2020.