Research Article

Multiple Attribute Group Decision-Making Models Using Single-Valued Neutrosophic and Linguistic Neutrosophic Hybrid Element Aggregation Algorithms

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Multiple attribute group decision-making (MAGDM) issues may involve quantitative and qualitative attributes. In inconsistent and indeterminate decision-making issues, current assessment information of quantitative and qualitative attributes with respect to alternatives only contains either numerical neutrosophic values or linguistic neutrosophic values as the single information expression. However, existing neutrosophic techniques cannot perform the mixed information denotation and aggregation operations of numerical neutrosophic values and linguistic neutrosophic values in neutrosophic decision-making issues. To solve the puzzles, this article presents the information denotation, aggregation operations, and MAGDM models of single-valued neutrosophic and linguistic neutrosophic hybrid sets/elements (SVNLNHSs/SVNLNHEs) as new techniques to perform MAGDM issues with quantitative and qualitative attributes in the environment of SVNLNHEs. In this study, we first propose a SVNLNHS/SVNLNHE notion that consists of a single-valued neutrosophic element (SVNE) for the quantitative argument and a linguistic neutrosophic element (LNE) for the qualitative argument. According to a linguistic and neutrosophic conversion function and its inverse conversion function, we present some basic operations of single-valued neutrosophic elements and linguistic neutrosophic elements, the SVNLNHE weighted arithmetic mean (SVNLNHEWAM) and SVNLNHE weighted geometric mean (SVNLNHEWGM) operators (forming SVNEs), and the SVNLNHEWAM and SVNLNHEWGM operators (forming LNEs). Next, MAGDM models are established based on the SVNLNHEWAM and SVNLNHEWGM operators or the SVNLNHEWAM and SVNLNHEWGM operators to realize MAGDM issues with single-valued neutrosophic and linguistic neutrosophic hybrid information, and then their applicability and availability are indicated through an illustrative example in the SVNLNHE circumstance. By comparison with the existing techniques, our new techniques reveal obvious advantages in the mixed information denotation, aggregation algorithms, and decision-making methods in handling MAGDM issues with the quantitative and qualitative attributes in the setting of SVNLNHSs.

1. Introduction

In general, there exist both quantitative attributes and qualitative attributes in multiple attribute (group) decision-making (MADM/MAGDM) issues. In the assessment process, the assessment information of quantitative attributes is usually represented by numerical values because numerical values are more suitable to the denotation form of quantitative arguments, while the assessment information of qualitative attributes is usually assigned by linguistic term values because the linguistic value is more suitable to human judgment and thinking/expression habits. Generally speaking, it is difficult to represent qualitative arguments by numeric values, but they are easily represented by linguistic values. In inconsistent and indeterminate situations, a simplified neutrosophic set (SNS) [1], including an interval-valued neutrosophic set/element (IVNS/IVNE) [2] and a single-valued neutrosophic set/element (SVNS/SVNE) [3],
is depicted by the truth, falsity, and indeterminacy membership degrees, while a linguistic neutrosophic set/element (LNS/LNE) [4] is depicted by the truth, falsity, and indeterminacy linguistic values. Since the neutrosophic set theories [5], including SNS, SVNS, IVNS, and LNS, are vital mathematical tools to denote and handle indeterminate and inconsistent issues in the real world, they have been widely applied in decision-making issues [6–14]. In the setting of SNs, some researchers presented various aggregation operators and their MADM/MAGDM models to solve neutrosophic MADM/MAGDM problems [10, 15–20]. Then, other researchers introduced various extended versions of SNs, including single-valued neutrosophic rough sets [21], normal neutrosophic sets [22], bipolar neutrosophic sets [23], simplified neutrosophic indeterminate sets [24], and neutrosophic Z-numbers [25], and used them in MADM/MAGDM issues. In the setting of LNEs, some researchers proposed several aggregation operators of LNEs and their MAGDM models to carry out linguistic neutrosophic MAGDM problems [26, 27]. Then, some extended linguistic sets, such as linguistic neutrosophic uncertain sets and linguistic neutrosophic cubic sets, were also presented to perform some linguistic neutrosophic MAGDM problems [28]. Unfortunately, the existing neutrosophic theories and MADM models [28, 29] cannot yet resolve the denotation, operations, and MADM issues of the mixed information of SVNEs and LNEs. However, the existing assessment information of the quantitative or qualitative attributes with respect to alternatives only gives either numerical neutrosophic information or linguistic neutrosophic information as a single information expression. In the case of single-valued neutrosophic and linguistic neutrosophic mixed information, existing neutrosophic technologies cannot represent the mixed information of SVNE and LNE nor can they perform mixed operations of the two. Therefore, the mixed information representation and aggregation operations and decision-making problems pose challenges in this study, which motivates our research to address them. To solve these problems, the aims of this article are as follows: (1) to propose a single-valued neutrosophic and linguistic neutrosophic hybrid set/element (SVNLNHS/SVNLNHE) for the mixed information representation of both SVNE and LNE, (2) to present basic operations of SVNEs and LNEs according to a linguistic and neutrosophic conversion function and its inverse conversion function, (3) to propose the single-valued neutrosophic and linguistic neutrosophic hybrid element weighted arithmetic mean (SVNLNHEWAMn) and single-valued neutrosophic and linguistic neutrosophic hybrid element weighted geometric mean (SVNLNHEWGGMn) operators for the aggregated SVNEs and the SVNLNHEWAMl and SVNLNHEWGGMl operators for the aggregated LNEs, (4) to establish MAGDM models based on the SVNLNHEWAMn and SVNLNHEWGGMn operators or the SVNLNHEWAMl and SVNLNHEWGGMl operators in the setting of SVNLNHSs, and (5) to apply the established MAGDM models to an illustrative example on the selection problem of industrial robots that contain both quantitative and qualitative attributes in a SVNLNHS circumstance.

Generally, the main contributions of this article are summarized as follows:

(i) The proposed SVNLNHS/SVNLNHE solves the representation problem of single-valued neutrosophic and linguistic neutrosophic mixed information.

(ii) The proposed weighted aggregation operators of SVNLNHEs based on the linguistic and neutrosophic conversion function and its inverse conversion function provide the effective aggregation algorithms of SVNLNHEs.

(iii) The established MAGDM models can solve MAGDM issues with quantitative and qualitative attributes in a SVNLNHS circumstance.

(iv) The established MAGDM models can solve the selection problem of industrial robots that contain both quantitative and qualitative attributes and show the availability and rationality of the new techniques in a SVNLNHS circumstance.

The remaining structure of this article consists of the following sections. Section 2 reviews the basic concepts and operations of SVNEs and LNEs as the preliminaries of this study. The notions of SVNLNHS and SVNLNHE and some basic operations of SVNEs and LNEs based on the linguistic and neutrosophic conversion function and its inverse conversion function are proposed in Section 3. In Section 4, the SVNLNHEWAMn, SVNLNHEWGGMn, SVNLNHEWAMl, and SVNLNHEWGGMl operators are presented in terms of the basic operations of SVNEs and LNEs. In Section 5, two new MAGDM models are established by the SVNLNHEWAMn and SVNLNHEWGGMn operators or the SVNLNHEWAMl and SVNLNHEWGGMl operators. Section 6 presents an illustrative example on the selection problem of industrial robots that contains both quantitative and qualitative attributes and then gives a comparative analysis with the existing techniques to show the availability and rationality of the new techniques. Finally, conclusions and future research are summarized in Section 7.

2. Preliminaries of SVNEs and LNEs

This part reviews the basic notions and operations of SVNEs and LNEs.

2.1. Basic Notions and Operations of SVNEs. Set \( U = \{u_1, u_2, \ldots, u_m\} \) as a universal set. Then, a SVNS \( ZN \) in \( U \) can be represented as [1, 3]

\[ ZN = \{\langle u_i, x_{ZN}(u_i), y_{ZN}(u_i) \rangle | u_i \in U\}, \]

where \( < u_i, x_{ZN}(u_i), y_{ZN}(u_i) > \ (i = 1, 2, \ldots, m) \) is SVNE in \( ZN \) for \( u_i \in U \) and \( x_{ZN}(u_i), y_{ZN}(u_i), z_{ZN}(u_i) \in [0, 1] \), and then it is simply denoted as \( z_{ni} = < x_{zn1}, y_{zn1}, z_{zn1} >, ZN_i = < x_{zn2}, y_{zn2}, z_{zn2} >, \) and \( \beta > 0 \), and their relations are contained as follows [17]:
2.2. Basic Notions and Operations of LNEs. Let $U = \{u_1, u_2, \ldots, u_m\}$ be a universal set and $S = \{s_p\mid p = 0, 1, \ldots, r\}$ be a linguistic term set (LTS) with an odd cardinality $r + 1$. Thus, a LNS $LH$ is defined as follows [4]:

\[
\text{LNEWAM}(lh_1, lh_2, \ldots, lh_m) = \sum_{i=1}^{m} \beta_i \cdot lh_i = \left\langle \frac{s}{r-r} \right\rangle \left( \prod_{i=1}^{m} (1-a_i/r)^{\beta_i} \cdot \prod_{i=1}^{m} (b_i/r)^{\beta_i} \cdot \prod_{i=1}^{m} (c_i/r)^{\beta_i} \right)
\]

\[
\text{LNEWGM}(lh_1, lh_2, \ldots, lh_m) = \prod_{i=1}^{m} lh_i^{\beta_i} = \left\langle \frac{s}{r-r} \right\rangle \left( \prod_{i=1}^{m} (1-a_i/r)^{\beta_i} \cdot \prod_{i=1}^{m} (b_i/r)^{\beta_i} \cdot \prod_{i=1}^{m} (c_i/r)^{\beta_i} \right)
\]

Set $lh_i = \left\langle s_{a_i}, s_{b_i}, s_{c_i} \right\rangle$ as any LNE. The score and accuracy functions of $lh_i$ are defined, respectively, as follows [4]:

\[
P(lh_i) = \frac{(2r + a_i - b_i - c_i)}{3r}
\]

\[
Q(lh_i) = \frac{(a_i - c_i)}{r}
\]
3. SVNLNHSs and SVNLNHEs

This section proposes SVNLNHS/SVNLNHE for the mixed information representation of both SVNE and LNE and then presents some basic operations of SVNEs and LNEs according to a linguistic and neutrosophic conversion function and its inverse conversion function.

**Definition 1.** Let \( U = \{ u_1, u_2, \ldots, u_m \} \) be a universe set and \( S = \{ s_p \mid p = 0, 1, \ldots, r \} \) be LTS with an odd cardinality \( r + 1 \). Then, a SVNLNHS ML is defined by

\[
ML = \{ \langle u_T, TL_M(u_T), IL_M(u_T), FL_M(u_T) \rangle \mid u_T \in U \},
\]

where \( TL_M(u_T), IL_M(u_T), \) and \( FL_M(u_T) \) are the truth, indeterminacy, and falsity membership functions, and their values are either the fuzzy values for \( x_\text{zoni}, y_\text{zoni}, z_\text{zoni} \) for \( i = 1, 2, \ldots, q \). Moreover, the SVNLNHS-ML is composed of the q SVNEs \( z_{ni} = \langle x_{zoni}, y_{zoni}, z_{zoni} \rangle \) for \( i = 1, 2, \ldots, q \) and the \( m - q \) LNEs \( l_{hi} = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle \) for \( i = q + 1, q + 2, \ldots, m \). Thus, ML1 and ML2 imply the following relations:

1. \( ML_1 \subseteq ML_2 \iff z_{ni1} \subseteq z_{ni2} \) for \( i = 1, 2, \ldots, q \) and \( l_{h1i} \subseteq l_{h2i} \) for \( i = q + 1, q + 2, \ldots, m \);
2. \( ML_1 = ML_2 \iff z_{ni1} = z_{ni2} \) for \( i = 1, 2, \ldots, q \) and \( l_{h1} = l_{h2} \).

**Definition 2.** Suppose that \( ML_1 \) and \( ML_2 \) are two SVNLNHSs, which contain \( q \) SVNEs \( z_{ni1} = \langle x_{zoni1}, y_{zoni1}, z_{zoni1} \rangle \) for \( i = 1, 2, \ldots, q \) and \( m - q \) LNEs \( l_{h1i} = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle \) for \( i = q + 1, q + 2, \ldots, m \). Thus, ML1 and ML2 imply the following relations:

1. \( ML_1 \subseteq ML_2 \iff z_{ni1} \subseteq z_{ni2} \) for \( i = 1, 2, \ldots, q \) and \( l_{h1i} \subseteq l_{h2i} \) for \( i = q + 1, q + 2, \ldots, m \);
2. \( ML_1 = ML_2 \iff z_{ni1} = z_{ni2} \) for \( i = 1, 2, \ldots, q \) and \( l_{h1i} = l_{h2i} \).

**Definition 3.** Let \( z_{ni} = \langle x_{zoni}, y_{zoni}, z_{zoni} \rangle \) and \( l_{hi} = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle \) as any SVNE and any LNE, respectively. Then, let a linguistic and neutrosophic conversion function be

\[
f(l_{hi}) = \langle a_i/r, b_i/r, c_i/r \rangle \text{ for } a_i, b_i, c_i \in [0, r], \text{ and then its inverse conversion function is } f^{-1}(z_{ni}) = \langle x_{zoni}, y_{zoni}, z_{zoni} \rangle \text{ for } x_{zoni}, y_{zoni}, z_{zoni} \in [0, 1].
\]

Thus, some basic operations of SVNEs and LNEs are given as follows:

1. \( f^{-1}(z_{ni}) \otimes l_{hi} = \langle x_{zoni} + a_i - x_{zoni}b_i, y_{zoni}b_i, z_{zoni}c_i \rangle \)
2. \( z_{ni} \otimes f(l_{hi}) = \langle x_{zoni} + a_i - x_{zoni}b_i, y_{zoni}b_i, z_{zoni}c_i \rangle \)
3. \( f^{-1}(z_{ni}) \otimes l_{hi} = \langle x_{zoni} + a_i - x_{zoni}b_i, y_{zoni}b_i, z_{zoni}c_i \rangle \)
4. \( f^{-1}(z_{ni}) \otimes l_{hi} = \langle x_{zoni} + a_i - x_{zoni}b_i, y_{zoni}b_i, z_{zoni}c_i \rangle \)

4. Weighted Arithmetic and Geometric Mean Operators of SVNLNHEs

This section proposes some weighted aggregation operators of SVNLNHEs corresponding to the linguistic and neutrosophic conversion function and its inverse conversion function, and then indicates their properties.

4.1. Aggregation Operators of SVNLNHEs Corresponding to the Linguistic and Neutrosophic Conversion Function. Let \( z_{ni} = \langle x_{zoni}, y_{zoni}, z_{zoni} \rangle \) for \( i = 1, 2, \ldots, q \) and \( l_{hi} = \langle s_{a_i}, s_{b_i}, s_{c_i} \rangle \) for \( i = q + 1, q + 2, \ldots, m \) be q SVNEs and \( m - q \) LNEs, respectively. Then, based on Definition 3 and the SVNEWAM and SVNEWGM operators of Eqs. (2) and (3) [17], the weighted arithmetic and geometric mean operators of SVNLNHEs corresponding to the linguistic and neutrosophic conversion function are proposed by the SVNLNHEWAM \( m \) and SVNLNHEWGM \( m \) operators.
SVNLNHEWAMN(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = \prod_{i=1}^{q} zn_i^\beta_i \prod_{i=q+1}^{m} f^\beta_i (lh_i)

\text{(13)}

where \(\beta_i \in [0, 1]\) is the weight of \(zn_i\) \((i = 1, 2, \ldots, q)\) and \(lh_i\) \((i = q+1, q+2, \ldots, m)\) with \(\sum_{i=1}^{m} \beta_i = 1\). Then, the aggregated results of the SVNLNHEWAMN and SVNLNHEWGML operators are SVNEs.

Especially, when \(q = m\) (without LNEs), the SVNLNHEWAMN and SVNLNHEWGML operators are reduced to the SVNEWAM and SVNEWGM operators [17], i.e., Eq. (2) and Eq. (3).

Based on the properties of the SVNEWAM and SVNEWGM operators [17], it is obvious that the SVNLNHEWAMN and SVNLNHEWGML operators also contain the following characteristics:

1. Idempotency: if \(zn = f(lh) = zn\) for \(i = 1, 2, \ldots, q, q + 1, q + 2, \ldots, m\), there are SVNLNHEWAMN(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = zn and SVNLNHEWGML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = zn.

2. Boundedness: let \(z^n = \langle \min (x_{ZNi}, a_i/r), \max (y_{ZNi}, b_i/r) \rangle\) and \(z^r = (\max (x_{ZNi}, a_i/r), \min (y_{ZNi}, b_i/r))\) be the minimum and maximum SVNEs for \(i = 1, 2, \ldots, m\), and then there are the inequalities

\(zn^r \leq SVNLNHEWAMN(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) \leq zn^r\) and \(zn^r \leq SVNLNHEWGML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) \leq zn^r\).

3. Monotonicity: if the condition of \(zn \leq zn^r\) \((i = 1, 2, \ldots, q)\) and \(lh_i \leq lh_i^r\) \((i = q + 1, q + 2, \ldots, m)\) exists, SVNLNHEWAMN(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) \(\leq SVNLNHEWAMN(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\))\) and SVNLNHEWGML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) \(\leq SVNLNHEWGML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\))\) also exist.

4.2 Aggregation Operators of SVNLNHEs According to the Inverse Conversion Function. Let \(zn = \langle x_{ZNi}, y_{ZNi}, z_{ZNi} \rangle > (i = 1, 2, \ldots, q)\) and \(lh_i = \langle s_i, r_i, s_i \rangle \leq (i = q + 1, q + 2, \ldots, m)\) be q SVNEs and \(m - q\) LNEs, respectively. Then, based on Definition 3 and the LNEWAM and LNEWGM operators of Eqs. (7) and (8) [4], the weighted arithmetic and geometric mean operators of SVNLNHEs corresponding to the inverse conversion function are proposed by the SVNLNHEWAML and SVNLNHEWGML operators:

SVNLNHEWAML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = \sum_{i=1}^{q} \beta_i f^{-1}(zn_i) + \sum_{i=q+1}^{m} \beta_i \cdot lh_i

\text{(14)}

SVNLNHEWGML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = \prod_{i=1}^{q} (f^{-1}(zn_i))^\beta_i \prod_{i=q+1}^{m} lh_i^\beta_i

\text{(15)}

where \(\beta_i \in [0, 1]\) is the weight of \(zn_i\) \((i = 1, 2, \ldots, q)\) and \(lh_i\) \((i = q + 1, q + 2, \ldots, m)\) with \(\sum_{i=1}^{m} \beta_i = 1\). Then, the aggregated results of the SVNLNHEWAML and SVNLNHEWGML operators are LNEs.

Especially, when \(q = 0\) (without SVNEs), the SVNLNHEWAML and SVNLNHEWGML operators are reduced to the LNEWAM and LNEWGM operators [4], i.e., Eq. (7) and Eq. (8).

Based on the characteristics of the LNEWAM and LNEWGM operators [4], it is obvious that the SVNLNHEWAML and SVNLNHEWGML operators also contain the following characteristics:

1. Idempotency: if \(f^{-1}(zn_i) = lh_i = lh\) for \(i = 1, 2, \ldots, q, q + 1, q + 2, \ldots, m\), there are SVNLNHEWAML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = lh and SVNLNHEWGML(\(zn_1, zn_2, \ldots, zn_q, lh_{q+1}, lh_{q+2}, \ldots, lh_m\)) = lh.

2. Boundedness: let \(lh^r = \langle \min (x_{ZNi}, a_i/r), \max (y_{ZNi}, b_i/r) \rangle\) and \(lh^- = \langle \max (z_{ZNi}, c_i/r) \rangle\).
5. MAGDM Models in the Environment of SVNLNHSs

In this section, novel MAGDM models are developed in terms of the SVNLNHEWAM\(_N\) and SVNLNHEWGM\(_N\) operators and the SVNLNHEWAM\(_L\) and SVNLNHEWGM\(_L\) operators to perform MAGDM issues with quantitative and qualitative attributes in the mixed information environment of SVNEs and LNEs.

Regarding a mixed information MAGDM issue in the circumstance of SVNLNHSs, there exist \(t\) alternatives, denoted by a set of them \(E = \{E_1, E_2, \ldots, E_{t}\}\), and then they are satisfactorily assessed over \(m\) attributes, denoted by a set of \(V = \{v_1, v_2, \ldots, v_q, v_q+1, v_{q+2}, \ldots, v_m\}\), which contains \(q\) quantitative attributes and \(m - q\) qualitative attributes. Then, there is a group of decision makers \(G = \{g_1, g_2, \ldots, g_t\}\) with their weight vector \(\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_t\}\) for \(\alpha_i \in [0, 1]\) and \(\sum_{k=1}^{t} \alpha_k = 1\). The assessment values of each alternative over the \(q\) quantitative attributes are given by the decision makers \(g_k (k = 1, 2, \ldots, t)\) and represented by SVNEs \(z_{ni}^{ki} = (x_{ni}^{ki}, y_{ni}^{ki}, z_{ni}^{ki})\) for \(x_{ni}^{ki} \in [0, 1], y_{ni}^{ki} \in [0, 1], z_{ni}^{ki} \in [0, 1]\) and the LTS \(s_{ni}^{ki} = (s_{ni}^{ki}, s_{ni}^{ki}, s_{ni}^{ki})\) for \(s_{ni}^{ki} \in S = \{s_j^{ki} | p = 0, 1, 2, \ldots, r\}\).

Thus, all assessed values can be constructed as the decision matrices of SVNLNHSs \(M_k = (z_{ni}^{ki}, l_{ni}^{ki})_{(i,m)}\) \((k = 1, 2, \ldots, t)\). Then, a weight vector \(\beta = (\beta_1, \beta_2, \ldots, \beta_m)\) is specified to consider the weights \(\beta_i\) of attributes \(v_i (i = 1, 2, \ldots, m)\) with \(\beta_i \in [0, 1]\) and \(\sum_{i=1}^{m} \beta_i = 1\). Thus, two MAGDM models are developed in terms of the SVNLNHEWAM\(_N\) and SVNLNHEWGM\(_N\) operators or the SVNLNHEWAM\(_L\) and SVNLNHEWGM\(_L\) operators to perform MAGDM issues with the mixed evaluation information of SVNEs and LNEs.

Model 1. A MAGDM model using the SVNLNHEWAM\(_N\) and SVNLNHEWGM\(_N\) operators is developed to perform the MAGDM issue with SVNLNHSs. Its detailed steps are presented as follows:

Step 1: using the SVNE operator of Eq. (2) and the LNE operator of Eq. (7), the decision matrices \(M_k = (z_{ni}^{ki}, l_{ni}^{ki})_{(i,m)}\) \((k = 1, 2, \ldots, t)\) are aggregated into the overall decision matrix \(M = (z_{ni}^{ki}, l_{ni}^{ki})_{(i,m)}\).

Step 2: using the SVNLNHEWAM\(_L\) operator of Eq. (12) or the SVNLNHEWGM\(_L\) operator of Eq. (13), the aggregated result for \(E_j (j = 1, 2, \ldots, t)\) is obtained by the following equation:

\[
zn_j = \text{SVNLNHEWAM}_L(z_{nj1}, z_{nj2}, \ldots, z_{njq}, l_{njq+1}, l_{njq+2}, \ldots, l_{njm}) = \sum_{i=1}^{q} \beta_i \cdot z_{nj} + \sum_{i=q+1}^{m} \beta_i f(l_{nj})
\]

or

\[
zn_j = \text{SVNLNHEWGM}_L(z_{nj1}, z_{nj2}, \ldots, z_{njq}, l_{njq+1}, l_{njq+2}, \ldots, l_{njm}) = \sum_{i=1}^{q} \beta_i z_{nj} + \sum_{i=q+1}^{m} \beta_i f(l_{nj})
\]

Step 3: the score values of \(F(z_{nj}) (j = 1, 2, \ldots, t)\) are given by Eq. (4) and the accuracy values of \(G(z_{nj}) (j = 1, 2, \ldots, t)\) are given by Eq. (5) if necessary.

Step 4: the alternatives are sorted in descending order based on the sorting laws of SVNEs, and the first one is the best choice.
Step 5: end.

Model 2. A MAGDM model using the SVNLHGWAM$_L$ and SVNLHGWGM$_Q$ operators is developed to perform the MAGDM issue with SVNLHEs. Its detailed steps are presented as follows:

\[ lh_j = \text{SVNLHGWAM}_L(z_{nj1}, z_{nj2}, \ldots, z_{nj,q}, lh_{jq+1}, lh_{jq+2}, \ldots, lh_{jm}) = \sum_{i=1}^{q} \beta_i f^{-1}(z_{nj}) + \sum_{i=q+1}^{m} \beta_i \cdot lh_{ji} \]

\[ = \langle r - r \prod_{i=1}^{q} \left( 1 - x_{ZNj} \right)^{\beta_i}, r \prod_{i=q+1}^{m} \left( 1 - a_{ji/r} \right)^{\beta_i}, r - r \prod_{i=1}^{q} \left( 1 - y_{ZNj} \right)^{\beta_i}, r \prod_{i=q+1}^{m} \left( 1 - c_{ji/r} \right)^{\beta_i} \rangle \]  

or

\[ lh_j = \text{SVNLHGWGM}_Q(z_{nj1}, z_{nj2}, \ldots, z_{nj,q}, lh_{jq+1}, lh_{jq+2}, \ldots, lh_{jm}) = \prod_{i=1}^{q} \left( f^{-1}(z_{nj}) \right)^{\beta_i} \prod_{i=q+1}^{m} lh_{ji}^{\beta_i} \]

\[ = \langle r \prod_{i=1}^{q} \left( x_{ZNji} \right)^{\beta_i}, r - r \prod_{i=1}^{q} \left( 1 - y_{ZNj} \right)^{\beta_i}, r - r \prod_{i=1}^{q} \left( 1 - y_{ZNj} \right)^{\beta_i}, r - r \prod_{i=1}^{q} \left( 1 - c_{ji/r} \right)^{\beta_i} \rangle \]

Step 3': the score values of \( P(lh_j) \) \((j = 1, 2, \ldots, t)\) are given by Eq. (9) and the accuracy values of \( Q(lh_j) \) \((j = 1, 2, \ldots, t)\) are given by Eq. (10) if necessary.

Step 4': the alternatives are sorted in descending order based on the sorting laws of LNEs, and then the first one is the best choice.

Step 5': end.

6. Illustrative Example on the Selection Problem of Industrial Robots Containing Both Quantitative and Qualitative Attributes

This section applied the proposed MAGDM models to an illustrative example on the selection problem of industrial robots that contains both quantitative and qualitative attributes in the circumstance of SVNLHNSs to prove their usefulness, and then gives a comparison with existing techniques to show the availability and rationality of the new techniques.

6.1. Illustrative Example. This subsection applies the proposed MAGDM models to the selection problem of industrial robots containing both quantitative and qualitative attributes to illustrate their application and availability in the circumstance of SVNLHNSs.

Some industrial company wants to buy a type of industrial robots for a manufacturing system. The technical department preliminarily provides four types of industrial robots/alternatives, denoted as their set \( E = \{ E_1, E_2, E_3, E_4 \} \). Then, they must satisfy four requirements/attributes: operating accuracy \( (v_1) \), carrying capacity \( (v_2) \), control performance \( (v_3) \), and operating space and dexterity \( (v_4) \). The weight vector of the four attributes is given by \( \beta = (0.25, 0.3, 0.25, 0.2) \). Thus, three experts/decision makers are invited to satisfactorily assess each alternative over the four attributes by their truth, falsity, and indeterminacy options/judgments, where the assessment values can be specified in the mixed vector of the three decision makers is given by

\[
\begin{bmatrix}
0.8, 0.1, 0.2 & 0.7, 0.1, 0.1 & 0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 \\
0.7, 0.1, 0.1 & 0.8, 0.1, 0.3 & 0.8, 0.1, 0.2 & 0.8, 0.1, 0.2 \\
0.8, 0.2, 0.2 & 0.9, 0.2, 0.3 & 0.8, 0.1, 0.3 & 0.8, 0.1, 0.3 \\
0.8, 0.2, 0.3 & 0.8, 0.2, 0.3 & 0.7, 0.1, 0.1 & 0.8, 0.2, 0.3 \\
0.9, 0.1, 0.1 & 0.8, 0.2, 0.1 & 0.8, 0.2, 0.1 & 0.8, 0.2, 0.1 \\
0.8, 0.3, 0.1 & 0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 \\
0.7, 0.1, 0.2 & 0.8, 0.2, 0.3 & 0.7, 0.1, 0.2 & 0.7, 0.1, 0.2 \\
0.8, 0.1, 0.1 & 0.8, 0.2, 0.1 & 0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 \\
0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 & 0.8, 0.1, 0.1 \\
\end{bmatrix}
\]
Thus, the two MAGDM models developed can be utilized in the example to perform the MAGDM issue with SVNLNHEs.

Model 1. The MAGDM model using the SVNLNHE-WAML and SVNLNHE-WGML operators can be applied in the example, and then its detailed steps are depicted as follows:

\[ M = \begin{bmatrix}
    (0.7695, 0.1677, 0.1682) & (0.7648, 0.1000, 0.1275) \\
    (0.7787, 0.1682, 0.1747) & (0.8318, 0.1516, 0.3000) \\
    (0.7648, 0.1275, 0.1469) & (0.7695, 0.1569, 0.1320) \\
    (0.8431, 0.1320, 0.1320) & (0.8484, 0.1682, 0.1552)
\end{bmatrix} \]

Step 2: by Eq. (16) or Eq. (17), we give the aggregated values:

\[ z_n = < 0.7795, 0.1958, 0.1804 >, z_{n_2} = < 0.7921, 0.1884, 0.2392 >, z_{n_3} = < 0.7751, 0.2049, 0.2231 >, \] and \[ z_{n_4} = < 0.8290, 0.1761, 0.2178 > . \]

Or \[ z_{n_1} = < 0.7789, 0.2324, 0.1885 >, z_{n_2} = < 0.7876, 0.1934, 0.2464 >, z_{n_3} = < 0.7749, 0.2276, 0.2754 >, \] and \[ z_{n_4} = < 0.8265, 0.1850, 0.2504 > . \]

Step 3: by Eq. (4), the score values of \( F(z_n_j) \) for \( E_{j} (j = 1, 2, 3, 4) \) are given below:

\[ F(z_{n_1}) = 0.8011, \quad F(z_{n_2}) = 0.7882, \quad F(z_{n_3}) = 0.7824, \quad F(z_{n_4}) = 0.8117. \]

Or \[ F(z_{n_1}) = 0.7860, \quad F(z_{n_2}) = 0.7826, \quad F(z_{n_3}) = 0.7573, \] and \[ F(z_{n_4}) = 0.7970. \]

Step 4: the sorting order of the four alternatives is \( E_1 > E_2 > E_3. \)

Clearly, the sorting orders obtained by the SVNLNHE-WAML operator of Eq. (16) and the SVNLNHE-WGML operator of Eq. (17) are identical in this example.

Model 2. The MAGDM model using the SVNLNHE-WAML and SVNLNHE-WGML operators can also be applied in the example, and then its detailed steps are depicted as follows:

Step 1’; the same as Step 1.

Step 2’; by Eq. (18) or Eq. (19), we give the aggregated values:

\[ l_{h_1} = < s_4, 0.6773, s_1, 1.1747, s_1, 0.0823 >, \quad l_{h_2} = < s_4, 0.7528, s_1, 1.3020 >, \]
\[ l_{h_3} = < s_4, 0.6508, s_1, 1.2297, s_1, 1.3386 >, \quad l_{h_4} = < s_4, 0.9739, s_1, 1.0564 >, \]
\[ s_1, 0.3070 >, \]

or \[ l_{h_1} = < s_4, 0.6736, s_1, 1.3944, s_1, 1.3131 >, \quad l_{h_2} = < s_4, 0.7255, s_1, 1.6039 >, \]
\[ s_1, 1.4783 >, \]
\[ l_{h_3} = < s_4, 0.6494, s_1, 1.3657, s_1, 1.6526 >, \quad l_{h_4} = < s_4, 0.9591, s_1, 1.1100 >, \]
\[ s_1, 1.5027 >. \]

Step 3’; by (9), the score values of \( P(l_{h_j}) \) for \( E_{j} (j = 1, 2, 3, 4) \) are given as follows:

\[ P(l_{h_1}) = 0.8011, \quad P(l_{h_2}) = 0.7882, \quad P(l_{h_3}) = 0.7824, \quad P(l_{h_4}) = 0.8117, \]

or \[ P(l_{h_1}) = 0.7860, \quad P(l_{h_2}) = 0.7826, \quad P(l_{h_3}) = 0.7573, \] and \[ P(l_{h_4}) = 0.7970. \]

Step 4’; the sorting order of the four alternatives is \( E_4 > E_1 > E_2 > E_3. \)

Hence, the sorting orders obtained by the SVNLNHE-WAML operator of Eq. (18) and the SVNLNHE-WGML operator of Eq. (19) are identical in this example.

Obviously, the score values and sorting orders between Model 1 and Model 2 reflect the same results. Moreover, we see that whether SVNEs are converted to LNEs or LNEs to SVNEs in the aggregation operations, their final decision results are actually identical. Thus, decision makers can choose Model 1 or Model 2 in MAGDM applications. Therefore, it is obvious that our new techniques are valid and reasonable.

6.2. Comparative Analysis with the Existing Neutrosophic MAGDM Models. Since the assessed values of SVNLNHEs are given in this illustrative example, the existing neutrosophic MAGDM models [4, 17] cannot deal with this illustrative example in the situation of SVNLNHEs. Then, our new techniques can handle neutrosophic MAGDM issues with SVNEs and/or LNEs and show the following highlights and advantages:

(1) The proposed SVNLNHEs can conveniently denote the mixed information of SVNEs and LNEs regarding the assessment objects of quantitative and qualitative attributes, which is suitable for human judgment and thinking/expression habits, while existing neutrosophic expressions cannot represent SVNLNHE information.

(2) The proposed SVNLNHE-WAML and SVNLNHE-WGML operators or the proposed SVNLNHE-WAML and SVNLNHE-WGML operators provide the necessary aggregation tools for handling
MAGDM issues in the SVNLNHE circumstance, while the existing SVNEWAM and SVNEWGM operators [17] are only the special cases of the SVNLNHEWAM and SVNLNHEWGM operators, and then the existing LNEWAM and LNEWGM operators [4] are only the special cases of the SVNLNHEWAML and SVNLNHEWGML operators. Furthermore, the various existing aggregation operators cannot aggregate SVNLNHEs.

(3) Since the existing MAGDM models with the single evaluation information of SVNEs or LNEs [4, 17] are the special cases of our new MAGDM models, our new MAGDM models are broader and more versatile than the existing MAGDM models [4, 17]. Furthermore, the various existing MAGDM models cannot carry out MAGDM problems with SVNLNHE information.

Generally, the new techniques solve the SVNLNHE denotation, aggregation operations, and MAGDM issues in the mixed information situation of SVNEs and LNEs. It is clear that our new techniques are very suitable for such decision-making issues with quantitative and qualitative attributes and overcome the defects of the existing decision-making techniques subject to the single evaluation information of SVNEs or LNEs. Therefore, our new techniques reveal obvious superiorities over the existing techniques in the neutrosophic information denotation, aggregation operations, and decision-making methods.

7. Conclusion

Due to the lack of the SVNLNHE denotation, operations, and decision-making models in existing neutrosophic theory and applications, the proposed notion of SVNLNHS/ SVNLNHE and the defined linguistic and neutrosophic conversion function solved the hybrid neutrosophic information denotation and operational problems of SVNEs and LNEs. Then, the proposed SVNLNHEWAM, SVNLNHEWGM, and SVNLNHEWGM operators provided necessary aggregation algorithms for handling MAGDM issues with SVNLNHEs. The established MAGDM models solved such decision-making issues with quantitative and qualitative attributes in the SVNLNHE circumstance. Since the evaluation values of quantitative and qualitative attributes in the decision-making process are easily represented in SVNEs and LNEs that are given in view of decision makers' preferences/thinking habits, the managerial implications of this original research will be reinforced in neutrosophic decision-making methods and applications. Finally, an illustrative example was given and compared with the existing techniques to show the availability and rationality of the new techniques. Moreover, our new techniques not only overcome the insufficiencies of the existing techniques but also are broader and more versatile than the existing techniques when dealing with MAGDM issues in the setting of SVNLNHEs. However, in this study, the new techniques of the SVNLNHE denotation, aggregation algorithms, and MAGDM models reflected their superiority over existing techniques.

Regarding future research, these new techniques will be further extended to other areas, such as medical diagnosis, slope risk/instability evaluation, default diagnosis, and mechanical concept design, in the mixed information situation of SVNEs and LNEs. Then, we shall also develop more aggregation algorithms, such as Hamacher, Dombi, and Bonferroni aggregation operators, and their applications in clustering analysis, information fusion, image processing, and mine risk/safety evaluation in the mixed information situation of both SVNE and LNE or both IVNE and uncertain LNE.

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IVNS:</td>
<td>Interval-valued neutrosophic set</td>
</tr>
<tr>
<td>IVNE:</td>
<td>Interval-valued neutrosophic element</td>
</tr>
<tr>
<td>LTS:</td>
<td>Linguistic term set</td>
</tr>
<tr>
<td>LNS:</td>
<td>Linguistic neutrosophic set</td>
</tr>
<tr>
<td>LNE:</td>
<td>Linguistic neutrosophic element</td>
</tr>
<tr>
<td>LNEWAM:</td>
<td>Linguistic neutrosophic element weighted arithmetic mean</td>
</tr>
<tr>
<td>LNEWGM:</td>
<td>Linguistic neutrosophic element weighted geometric mean</td>
</tr>
<tr>
<td>MADM:</td>
<td>Multiple attribute decision-making</td>
</tr>
<tr>
<td>MAGDM:</td>
<td>Multiple attribute group decision-making</td>
</tr>
<tr>
<td>SNS:</td>
<td>Simplified neutrosophic set</td>
</tr>
<tr>
<td>SVNS:</td>
<td>Single-valued neutrosophic set</td>
</tr>
<tr>
<td>SVNE:</td>
<td>Single-valued neutrosophic element</td>
</tr>
<tr>
<td>SVNEWAM:</td>
<td>Single-valued neutrosophic element weighted arithmetic mean</td>
</tr>
<tr>
<td>SVNEWGM:</td>
<td>Single-valued neutrosophic element weighted geometric mean</td>
</tr>
<tr>
<td>SVNLNHS:</td>
<td>Single-valued neutrosophic and linguistic neutrosophic hybrid set</td>
</tr>
<tr>
<td>SVNLNHE:</td>
<td>Single-valued neutrosophic and linguistic neutrosophic hybrid element</td>
</tr>
<tr>
<td>SVNLNHEWAMN:</td>
<td>Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted arithmetic mean (for the aggregated SVNE)</td>
</tr>
<tr>
<td>SVNLNHEWGMN:</td>
<td>Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted geometric mean (for the aggregated SVNE)</td>
</tr>
<tr>
<td>SVNLNHEWAML:</td>
<td>Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted arithmetic mean (for the aggregated LNE)</td>
</tr>
<tr>
<td>SVNLNHEWGML:</td>
<td>Single-valued neutrosophic and linguistic neutrosophic hybrid element weighted geometric mean (for the aggregated LNE).</td>
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</tbody>
</table>
Data Availability
No data were used to support this study.

Conflicts of Interest
The authors declare that they have no conflicts of interest regarding the publication of this paper.

References