

Research Article

On Generalized Topological Indices for Some Special Graphs

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Received 8 February 2022; Revised 14 March 2022; Accepted 16 March 2022; Published 26 April 2022

Academic Editor: Firdous A. Shah

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Topological indices are numeric values associated with a graph and characterize its structure. There are various topological indices in graph theory such as degree-based, distance-based, and counting-related topological indices. Among these indices, degree-based indices are very interesting and studied well in literature. In this work, we studied the generalized form of harmonic, geometric-arithmetic, Kulli-Basava indices, and generalized power-sum-connectivity index for special graph that are bridge graph over path, bridge graph over cycle, bridge graph over complete graph, wheel graph, gear graph, helm graph, and square lattice graph. We found exact values for the stated indices and for the stated special graphs. We also investigated the generalized form of the indices for various properties of alkane isomers, from which we obtained interesting results which are closed to that of experimental obtained results.

1. Introduction

Graph theory, as applied in the field of molecular structures, representing an interdisciplinary science called molecular topology or chemical graph theory. By means of graph theory, statistics, and set theory, we identify structural features involved in structure property activity relationships. Modeling unknown structures and molecules can be classified by means of characterization of chemical structure topologically having the desired properties. In the last few decades, a lot of research studies have been conducted. Numerical value associated with chemical constitutions of chemical structures having many physicochemical properties, biological activity, or chemical reactivity. Topological indices are introduced on the basis of transforming molecular graph into a numeric value which characterizes the topology of a molecular graph. Molecular compound and molecules are often modeled by molecular graphs. For characterization of chemical compound, chemical graph is a model in this regard. A molecular graph is a simple graph where vertices are used for atoms and edges are for bonds. This can be described in many ways like that of by drawing, by means of polynomial, by means of a sequence of numbers, by means of a matrix, or by a derived number which is called

topological index. A numeric value associated with a graph is termed as topological index. Some of the well-known topological indices are degree-based, counting-related, and distance based topological indices. The generalized harmonic index, geometric arithmetic index, Kulli-Basava indices, and generalized power-sum-connectivity index are of the most important. For two isomorphic graph H and G , we have that $\text{Top}(H) = \text{Top}(G)$, where Top is a topological index. The number of edges and vertices of a graph are topological indices [1–5]. Let $G = (V, E)$ be a simple graph having vertex and edge set $V(G)$ and $E(G)$, respectively. For any $v \in V(G)$, the neighbors of v are denoted by $N(v)$ and are defined as $N(v) = \{u \in V(G) | vu \in E(G)\}$. We denote the degree of a vertex u by d_u or $d(u)$ and $S_u = \sum_{v \in N_G(u)} d(v)$ is the degree of neighbors of u . $|V(G)|$ and $|E(G)|$ are the number of vertices and edges of G , respectively. The graph denoted by K_n is called complete graph if all the vertices are connected by an edge [6–8].

In [9], the authors computed (ABC) , (ABC_4) , (GA) , and (GA_5) indices for special graphs that are Cayley tree, square lattice, and complete bipartite graph. In [10], the authors determined hyper-Zagreb index, first multiple Zagreb index, second multiple Zagreb index, Zagreb polynomials, and M-polynomial for nanostructures of bridge graphs. In [11],

the authors introduced Kulli–Basava indices and studied their chemical and mathematical properties that have good response in isomers degeneracy. Also, the authors obtained closed results for the stated indices for some families of graphs and established relations for connecting these indices with other existing indices which are in literature already. In addition, they have found the Kulli–Basava indices for some graph operations. In [12], the author proposed first hyper-Kulli–Basava index and second hyper-Kulli–Basava index and their polynomials of graphs and computed exact results for complete graph, gear graph, and helm graph. Also, the author determined the Kulli–Basava indices of gear graphs and helm graphs. In [13], the authors introduced symmetric division Kulli–Basava index, first and second multiplicative Kulli–Gourava indices, multiplicative F_1 -Kulli–Basava index, and multiplicative (a, b) -Kulli–Basava index of graph. The author computed these indices for regular graph, wheel graph, gear graph, and helm graph. In [14], the authors introduced multiplicative sum-connectivity Kulli–Basava index, multiplicative product connectivity Kulli–Basava index, multiplicative ABC Kulli–Basava index, and multiplicative GA Kulli–Basava index of graphs. The authors computed the multiplicative connectivity Kulli–Basava indices of regular graph, wheel graph, and helm graph. In [15], the authors introduced multiplicative (a, b) -status index for graph. The authors found exact expressions for the multiplicative (a, b) -status index for friendship graphs and wheel graphs. In [16], the author obtained expression for Harmonic index and Randic index of generalized transformation graph G^{xy} and their complement.

In this study, we generalized geometric-arithmetic index and Kulli–Basava indices. Also, we found exact values of generalized harmonic index, generalized geometric-arithmetic index, generalized Kulli–Basava indices, and generalized power-sum-connectivity index for some special graphs that are bridge graph over path, bridge graph over

cycle, bridge graph over complete graph, wheel graph, gear graph, helm graph, and square lattice graph. The generalized power-sum-connectivity index was introduced in [17]. Indices that we studied here in generalized form are the following. Generalized Harmonic index is given by

$$H_\xi(G) = \sum_{pq \in E(G)} 2(d(p) \cdot d(q))^\xi. \quad (1)$$

Generalized geometric-arithmetic index is given by

$$GA_\xi(G) = \sum_{pq \in E(G)} \frac{2(d_p d_q)^\xi}{d_p + d_q}. \quad (2)$$

Generalized Kulli–Basava indices are given by

$$\begin{aligned} KB_1^\xi(G) &= \sum_{rs \in E(G)} (S_e(r) + S_e(s))^\xi, \\ KB_2^\xi(G) &= \sum_{rs \in E(G)} (S_e(r) \cdot S_e(s))^\xi. \end{aligned} \quad (3)$$

where $S_e(r) = \sum_{e \in N_e(r)} d_G(e)$ and $d_G(e) = d_G(a) + d_G(r) - 2$ if $e = ar$. Generalized power-sum-connectivity index is given by

$$Y_\xi(G) = \sum_{gs \in E(G)} (d(g)^{d(g)} + d(s)^{d(s)})^\xi. \quad (4)$$

2. Bridge Graph over Path P_n

Theorem 1 (see [10]). *Let $G_m(P_n, u)$ be a bridge graph over path P_n ; then, the number of vertices in $G_m(P_n, u)$ are mn and number of edges are $mn - 1$.*

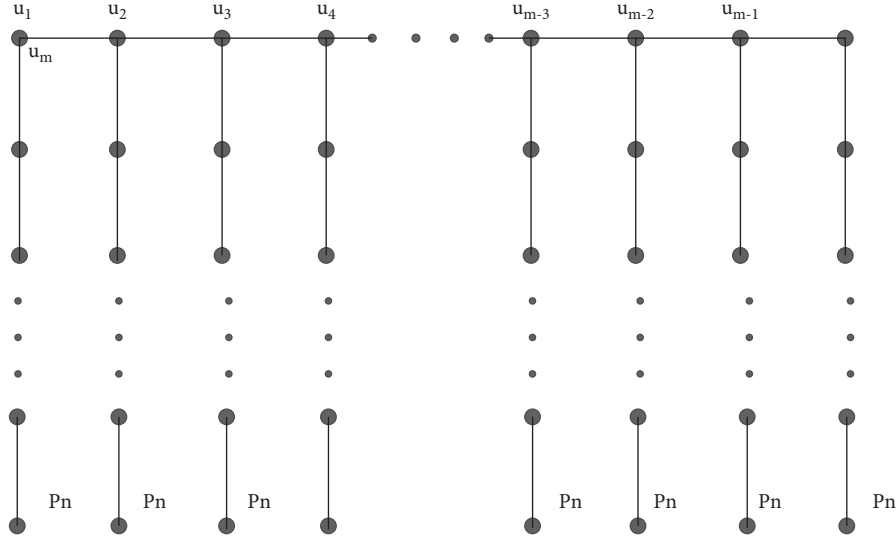
Theorem 2. *Let $G_m(P_n; u)$, $n > 2$ be bridge graph over path P_n . Then,*

$$\begin{aligned} H_\xi(G_m(P_n; u)) &= 2m(3^\xi + 5^\xi + 6^\xi - 3 \cdot 4^\xi + 4^\xi n) + 2^{2\xi+2} - 2^{\xi+1} \cdot 3^{\xi+1}, \\ GA_\xi(G_m(P_n; u)) &= \frac{2^{\xi+1}}{3} \cdot m + 2^{2\xi-1}(mn - 3m + 2) + \frac{2^{\xi+1}3^\xi}{5} \cdot m + 3^{2\xi-1}(m - 3), \\ KB_1^\xi(G_m(P_n; u)) &= m \cdot 5^\xi + m \cdot 7^\xi + m(n - 4) \cdot 8^\xi + 2 \cdot 9^\xi + 4 \cdot (15)^\xi + (m - 4) \cdot (16)^\xi + 2 \cdot (21)^\xi + (m - 5) \cdot (22)^\xi, \\ KB_2^\xi(G_m(P_n; u)) &= m \cdot 4^\xi + m \cdot (12)^\xi + m(n - 4) \cdot (16)^\xi + 2 \cdot (20)^\xi + 4 \cdot (50)^\xi + (m - 4) \cdot (55)^\xi + 2 \cdot (110)^\xi + (m - 5) \cdot (121)^\xi, \\ Y_\xi(G_m(P_n; u)) &= m(5^\xi + 31^\xi + 54^\xi + 8^\xi(n - 3)) + 2^{2\xi}(2^{\xi+1} - 3^{2\xi+1}). \end{aligned} \quad (5)$$

The graph is given in Figure 1.

Proof. Let a path on n vertices be P_n , and $G_m(P_n; u)$ is the bridge graph over path P_n . Using Theorem 1, mn and $mn - 1$ are the vertices and edges, respectively. Making partition between edges of $G_m(P_n; u)$ in such a way that E_1 consists of

edges $\{g, s\}$, where $d(g) = 1$ and $d(s) = 2$, such type of edges are m in number. E_2 consists of edges in which $d(g) = 2$ and $d(s) = 2$, such type of edges are $mn - 3m + 2$ in number which can be calculated as $(n - 1 - 1) + (n - 1 - 1) + (m - 1 - 1)(n - 1 - 2) = 2n - 4 + (m - 2)(n - 3) = mn - 3m + 2$. E_3 consists of edges $\{g, s\}$ in which $d(g) = 2$ and $d(s) = 3$;

FIGURE 1: Bridge graph over path P_n .

these edges are m in numbers. Similarly, E_4 consists of edges $\{g, s\}$, where $d(g) = 3$ and $d(s) = 3$, such edges are $m - 3$ in numbers. Next, we get the following interesting results:

$$H_\xi(G_m(P_n; u)) = \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi + \sum_{gs \in E_3} 2(d(g) + d(s))^\xi + \sum_{gs \in E_4} 2(d(g) + d(s))^\xi$$

$$= 2m(3^\xi + 5^\xi + 6^\xi - 3 \cdot 4^\xi + 4^\xi n) + 2^{2\xi+2} - 2^{\xi+1} \cdot 3^{\xi+1},$$

$$GA_\xi(G_m(P_n; u)) = \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_3} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_4} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)}$$

$$= \frac{2^{\xi+1}}{3} \cdot m + 2^{2\xi-1}(mn - 3m + 2) + \frac{2^{\xi+1} \cdot 3^\xi}{5} \cdot m + 3^{2\xi-1}(m - 3),$$

$$KB_1^\xi(G_m(P_n; u)) = \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) + S_e(s))^\xi$$

$$= m \cdot 5^\xi + m \cdot 7^\xi + m(n - 4) \cdot 8^\xi + 2 \cdot 9^\xi + 4 \cdot (15)^\xi + (m - 4) \cdot (16)^\xi + 2 \cdot (21)^\xi + (m - 5) \cdot (22)^\xi,$$

$$KB_2^\xi(G_m(P_n; u)) = \sum_{gs \in E_1} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) \cdot S_e(s))^\xi = m \cdot 4^\xi$$

$$+ m \cdot (12)^\xi + m(n - 4) \cdot (16)^\xi + 2 \cdot (20)^\xi + 4 \cdot (50)^\xi + (m - 4) \cdot (55)^\xi + 2 \cdot (110)^\xi + (m - 5) \cdot (121)^\xi,$$

$$Y_\xi(G_m(P_n; u)) = \sum_{gs \in E_1} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_2} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_3} (d(g)^{d(g)} + d(s)^{d(s)})^\xi$$

$$+ \sum_{gs \in E_4} (d(g)^{d(g)} + d(s)^{d(s)})^\xi = m(5^\xi + 31^\xi + 54^\xi + 8^\xi(n - 3)) + 2^{2\xi}(2^{\xi+1} - 3^{2\xi+1}).$$

3. Bridge Graph over Cycle C_n

Theorem 3 (see [10]). Let $G_m(C_n, u)$ be a bridge graph over cycle C_n ; then, the number of vertices in $G_m(C_n, u)$ is mn and number of edges is $mn + m - 1$.

Theorem 4. Let $G_m(C_n; u)$, $n > 3$, be bridge graph over cycle C_n ; then,

$$\begin{aligned}
 H_\xi(G_m(C_n; u)) &= m(2^{2\xi+1}n - 2^{2\xi+2} + 2^{\xi+2} \cdot 3^\xi + 2^{3\xi+1}) + 2^3 \cdot 5^\xi + 2^2 \cdot 7^\xi - 2^{\xi+3} \cdot 3^\xi - 2^{3\xi+1} \cdot 3, \\
 GA_\xi(G_m(C_n; u)) &= m\left(2^{2\xi-1}n - 2^{2\xi} + \frac{2^{3\xi+1}}{3} + 2^{4\xi-2}\right) + \frac{2^{\xi+3} \cdot 3^\xi}{5} + \frac{2^{a\xi+2} \cdot 3^\xi}{7} - \frac{2^{3\xi+2}}{3} + 2^{4\xi-2} \cdot 3, \\
 KB_1^\xi(G_m(C_n; u)) &= 4 \cdot (9)^\xi + 2(m-2)(10)^\xi + m(n-4)(8)^\xi + 4(16)^\xi + 4(25)^\xi + 2(m-4)(26)^\xi + 2(39)^\xi + (m-5)(40)^\xi, \\
 KB_2^\xi(G_m(C_n; u)) &= 4 \cdot (20)^\xi + 2(m-2)(24)^\xi + m(n-4)(16)^\xi + 4(55)^\xi + 4(114)^\xi + 2(m-4)(120)^\xi + 2(380)^\xi + (m-5)(400)^\xi, \\
 Y_\xi(G_m(C_n; u)) &= 2^{3\xi}mn - 2^{3\xi+1}m + 2^{2\xi+1} \cdot (65)^\xi m + 2^{9\xi}m + 2^2 \cdot (31)^\xi + 2 \cdot (283)^\xi - 2^{2\xi+2} \cdot (65)^\xi - 2^{9\xi} \cdot 3.
 \end{aligned} \tag{7}$$

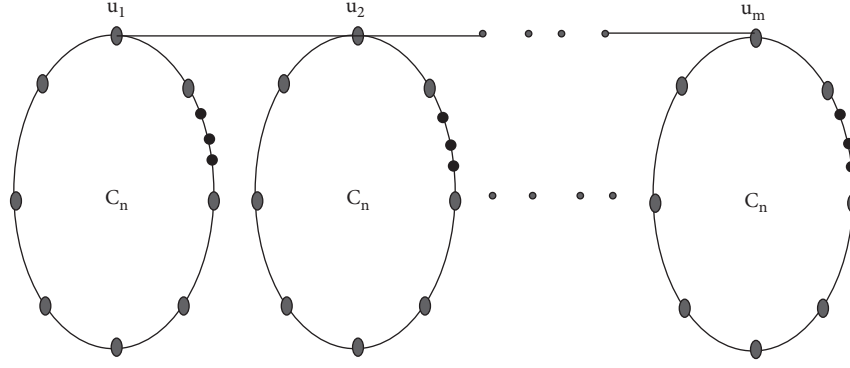
The graph is shown in Figure 2.

Proof. Let cycle on n vertices be C_n and $G_m(C_n; u)$ be the bridge graph. According to Theorem 3, mn and $mn + m - 1$ are vertices and edges, respectively. For computing topological indices for $G_m(C_n; u)$ over C_n , we divide the edge set $E(G_m(C_n; u))$ into five partitions which are based on the degrees of end vertices. First edge partition E_1 contains $mn -$

$2m$ edges gs having $d(g) = 2$ and $d(s) = 2$. E_2 consists of 4 edges gs having $d(g) = 2$ and $d(s) = 3$. E_3 contains $2m - 4$ edges gs having $d(g) = 2$ and $d(s) = 4$. E_4 contains 2 edges gs having $d(g) = 3$ and $d(s) = 4$. E_5 contains $m - 3$ edges gs having $d(r) = 4$ and $d(s) = 4$.

Now, the stated topological indices for $G_m(C_n, u)$ are calculated in the follows:

$$\begin{aligned}
 H_\xi(G_m(C_n; u)) &= \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi + \sum_{gs \in E_3} 2(d(g) + d(s))^\xi + \sum_{gs \in E_4} 2(d(g) + d(s))^\xi \\
 &\quad + \sum_{gs \in E_5} 2(d(g) + d(s))^\xi = m(2^{2\xi+1}n - 2^{2\xi+2} + 2^{\xi+2} \cdot 3^\xi + 2^{3\xi+1}) + 2^3 \cdot 5^\xi + 2^2 \cdot 7^\xi - 2^{\xi+3} \cdot 3^\xi - 2^{3\xi+1} \cdot 3, \\
 GA_\xi(G_m(C_n; u)) &= \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_3} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{rs \in E_4} \frac{2(d_g \cdot d_s)^\xi}{(d_r + d_s)} + \sum_{rs \in E_5} \frac{2(d_g \cdot d_s)^\xi}{(d_r + d_s)} \\
 &= m\left(2^{2\xi-1}n - 2^{2\xi} + \frac{2^{3\xi+1}}{3} + 2^{4\xi-2}\right) + \frac{2^{\xi+3} \cdot 3^\xi}{5} + \frac{2^{a\xi+2} \cdot 3^\xi}{7} - \frac{2^{3\xi+2}}{3} + 2^{4\xi-2} \cdot 3, \\
 KB_1^\xi(G_m(C_n; u)) &= \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) + S_e(s))^\xi \\
 &\quad + \sum_{gs \in E_5} (S_e(g) + S_e(s))^\xi = 4 \cdot (9)^\xi + 2(m-2)(10)^\xi + m(n-4)(8)^\xi + 4(16)^\xi + 4(25)^\xi + 2(m-4)(26)^\xi \\
 &\quad + 2(39)^\xi + (m-5)(40)^\xi,
 \end{aligned}$$

FIGURE 2: Bridge graph over cycle C_n .

$$\begin{aligned}
 KB_2^\xi(G_m(C_n; u)) &= \sum_{gs \in E_1} (S_{(e)}(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_{(e)}(g) \cdot S_e(s))^\xi + \sum_{gs \in E_3} (S_{(e)}(g) \cdot S_e(s))^\xi + \sum_{gs \in E_4} (S_{(e)}(g) \cdot S_e(s))^\xi \\
 &+ \sum_{gs \in E_5} (S_{(e)}(g) \cdot S_e(s))^\xi = 4 \cdot (20)^\xi + 2(m-2)(24)^\xi + m(n-4)(16)^\xi + 4(55)^\xi + 4(114)^\xi \\
 &+ 2(m-4)(120)^\xi + 2(380)^\xi + (m-5)(400)^\xi, \\
 Y_\xi(G_m(C_n; u)) &= \sum_{gs \in E_1} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_2} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_3} (d(g)^{d(g)} + d(s)^{d(s)})^\xi \\
 &+ \sum_{gs \in E_4} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_5} (d(g)^{d(g)} + d(s)^{d(s)})^\xi = 2^{3\xi}mn - 2^{3\xi+1}m + 2^{2\xi+1} \cdot (65)^\xi m + 2^{9\xi}m \\
 &+ 2^2 \cdot (31)^\xi + 2 \cdot (283)^\xi - 2^{2\xi+2} \cdot (65)^\xi - 2^{9\xi} \cdot 3.
 \end{aligned} \tag{8}$$

□

4. Bridge Graph over Complete Graph K_n

Theorem 5. Let $G_m(K_n; u)$ be bridge graph over complete graph K_n with $n > 2$; then,

$$\begin{aligned}
 H_\xi(G_m(K_n; u)) &= 2^2(2n+1)^\xi + 2^{\xi+1}(m-3)(n+1)^\xi + 2^2(n-1)(2n-1)^\xi + 2^{\xi+1}(m-2)(n-1)n^\xi + 2^\xi m(n-2)(n-1)^{\xi+1}, \\
 GA_\xi(G_m(K_n; u)) &= \frac{2^2(n^2+n)^\xi}{(2n+1)} + (m-3)(n+1)^{2\xi-1} + \frac{2^2(n-1)(n^2-n)^\xi}{(2n-1)} + \frac{(m-2)(n-1)(n^2-1)^\xi}{n} + \frac{m(n-2)(n-1)^{2\xi}}{2}, \\
 Y_\xi(G_m(K_n; u)) &= 2(n^n + (n+1)^{(n+1)})^\xi + 2^\xi(m-3)(n+1)^{\xi(n+1)} + 2(n-1)((n-1)^{n-1} + n^n)^\xi \\
 &+ (m-2)(n-1)((n-1)^{(n-1)} + (n+1)^{(n+1)})^\xi + 2^{\xi-1}m(n-2)(n-1)^{\xi(n-1)+1}. \\
 KB_1^\xi(G_m(K_n; u)) &= 2((n+(n-1)(2n-3)) + ((4n-1) + (n-1)(2n-2)))^\xi \\
 &+ 2(((4n-1) + (n-1)(2n-2)) + ((4n) + 2(n-1)^3))^\xi
 \end{aligned}$$

$$\begin{aligned}
& + (m-5) (((4n) + (n-1)(2n-2)) + (4n + (n-1)(2n-1)))^\xi \\
& + (2n-2) (((2n-3) + (n-2)(2n-4)) + ((4n-4) + (n-2)(2n-4)))^\xi \\
& + (2n-2) (((2n-2) + (n-2)(2n-4)) + ((4n-1) + (n-1)(2n-2)))^\xi \\
& + (m-4)(n-1) (((2n-2) + (n-2)(2n-4)) + ((4n) + (n-1)(2n-2)))^\xi \\
& + (n-1)(n-2) (((n-2)(2n-4) + (2n-3)) + ((n-2)(2n-4) + (2n-3)))^\xi \\
& + \frac{(m-2)(n-2)(n-1)}{2} (((n-2)(2n-4) + (2n-2)) + ((n-2)(2n-4) + (2n-2)))^\xi, \\
KB_2^\xi(G_m(K_n; u)) &= 2((n + (n-1)(2n-3)) \cdot ((4n-1) + (n-1)(2n-2)))^\xi \\
& + 2(((4n-1) + (n-1)(2n-2)) \cdot ((4n) + 2(n-1)^3))^\xi \\
& + (m-5) (((4n) + (n-1)(2n-2)) \cdot (4n + (n-1)(2n-1)))^\xi \\
& + (2n-2) (((2n-3) + (n-2)(2n-4)) \cdot ((4n-4) + (n-2)(2n-4)))^\xi \\
& + (2n-2) (((2n-2) + (n-2)(2n-4)) \cdot ((4n-1) + (n-1)(2n-2)))^\xi \\
& + (m-4)(n-1) (((2n-2) + (n-2)(2n-4)) \cdot ((4n) + (n-1)(2n-2)))^\xi \\
& + (n-1)(n-2) (((n-2)(2n-4) + (2n-3)) \cdot ((n-2)(2n-4) + (2n-3)))^\xi \\
& + \frac{(m-2)(n-2)(n-1)}{2} (((n-2)(2n-4) + (2n-2)) \cdot ((n-2)(2n-4) + (2n-2)))^\xi.
\end{aligned} \tag{9}$$

The graph is given in Figure 3.

Proof. Let complete graph of order n be K_n and $G_m(K_n, u)$ be the bridge graph over complete graph. The bridge graph $G_m(K_n, u)$ consists mn vertices and $(mn(n-1) + (2m-2))/2$ edges. For computing topological indices for bridge graph $G_m(K_n; u)$ over complete graph K_n , we divide the edges into five partitions based on degree of end vertices.

The first partition E_1 consists of 2 edges gs such that $d(g) = n$ and $d(s) = n+1$; second edge partition E_2

contains $n-3$ edges gs such that $d(g) = n+1$ and $d(s) = n+1$. The third edge partition E_3 consists of $2n-2$ edges gs such that $d(g) = n-1$ and $d(s) = n$. The fourth edge partition E_4 consists of $(m-2)(n-1)$ edges gs such that $d(g) = n-1$ and $d(s) = n+1$. The last and fifth edge partition consists $(m(n-2)(n-1))/2$ edges gs such that $d(g) = n-1$ and $d(s) = n-1$.

Now, the desired topological indices are the following:

$$\begin{aligned}
H_\xi(G_m(K_n; u)) &= \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi \\
&+ \sum_{gs \in E_3} 2(d(g) + d(s))^\xi + \sum_{gs \in E_4} 2(d(g) + d(s))^\xi + \sum_{gs \in E_5} 2(d(g) + d(s))^\xi \\
&= 2(n + n + 1)^\xi |E_1| + 2(n + 1 + n + 1)^\xi |E_2| + 2(n - 1 + n)^\xi |E_3| + 2(n - 1 + n + 1)^\xi |E_4| + 2(n - 1 + n - 1)^\xi |E_5| \\
&= 2^2(2n + 1)^\xi + 2^{\xi+1}(m-3)(n+1)^\xi + 2^2(n-1)(2n-1)^\xi + 2^{\xi+1}(m-2)(n-1)n^\xi + 2^\xi m(n-2)(n-1)^{\xi+1}, \\
GA_\xi(G_m(K_n; u)) &= \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_3} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_4} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_5} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} = \frac{2^2(n^2 + n)^\xi}{(2n + 1)}
\end{aligned}$$

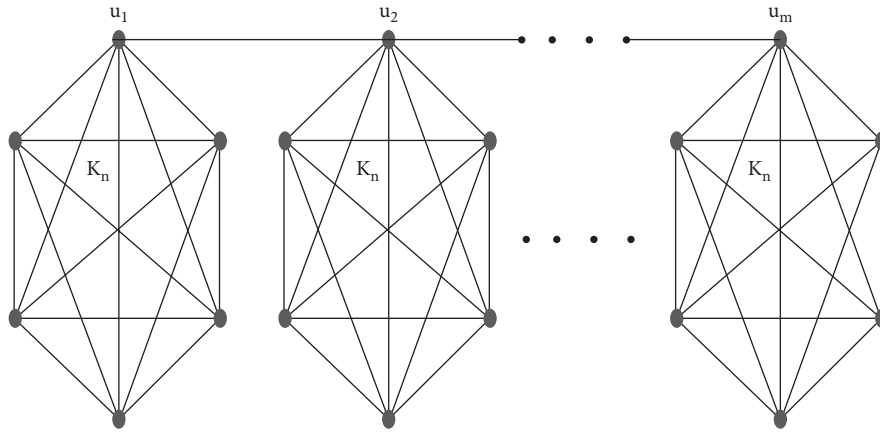
$$\begin{aligned}
& + (m-3)(n+1)^{2\xi-1} + \frac{2^2(n-1)(n^2-n)^\xi}{(2n-1)} + \frac{(m-2)(n-1)(n^2-1)^\xi}{n} + \frac{m(n-2)(n-1)^{2\xi}}{2}, \\
KB_1^\xi(G_m(K_n; u)) &= \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) + S_e(s))^\xi \\
&+ \sum_{gs \in E_5} (S_e(g) + S_e(s))^\xi = 2((n + (n-1)(2n-3)) + ((4n-1) + (n-1)(2n-2)))^\xi \\
&+ 2(((4n-1) + (n-1)(2n-2)) + ((4n) + 2(n-1)^3))^\xi + (m-5)((4n) + (n-1)(2n-2)) + ((4n) + (n-1)(2n-1))^\xi + (2n-2)((2n-3) + (n-2)(2n-4)) + ((4n-4) + (n-2)(2n-4))^\xi + (2n-2)((2n-2) + (n-2)(2n-4)) + ((4n-1) + (n-1)(2n-2))^\xi + (m-4)(n-1)((2n-2) + (n-2)(2n-4)) + ((4n) + (n-1)(2n-2))^\xi + (n-1)(n-2)((n-2)(2n-4) + (2n-3)) + ((n-2)(2n-4) + (2n-3))^\xi \\
&+ \frac{(m-2)(n-2)(n-1)}{2}(((n-2)(2n-4) + (2n-2)) + ((n-2)(2n-4) + (2n-2)))^\xi, KB_2^\xi(G_m(K_n; u)) \\
&= \sum_{gs \in E_1} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) \cdot S_e(s))^\xi \\
&+ \sum_{gs \in E_5} (S_e(g) \cdot S_e(s))^\xi \\
&= 2((n + (n-1)(2n-3)) \cdot ((4n-1) + (n-1)(2n-2)))^\xi \\
&+ 2(((4n-1) + (n-1)(2n-2)) \cdot ((4n) + 2(n-1)^3))^\xi + (m-5)((4n) + (n-1)(2n-2)) \cdot ((4n) + (n-1)(2n-1))^\xi + (2n-2)((2n-3) + (n-2)(2n-4)) \cdot ((4n-4) + (n-2)(2n-4))^\xi + (2n-2)((2n-2) + (n-2)(2n-4)) \cdot ((4n-1) + (n-1)(2n-2))^\xi + (m-4)(n-1)((2n-2) + (n-2)(2n-4)) \cdot ((4n) + (n-1)(2n-2))^\xi + (n-1)(n-2)((n-2)(2n-4) + (2n-3)) \cdot ((n-2)(2n-4) + (2n-3))^\xi \\
&+ \frac{(m-2)(n-2)(n-1)}{2}(((n-2)(2n-4) + (2n-2)) \cdot ((n-2)(2n-4) + (2n-2)))^\xi, Y_\xi(G_m(K_n; u)) \\
&= \sum_{gs \in E_1} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_2} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_3} (d(g)^{d(g)} + d(s)^{d(s)})^\xi \\
&+ \sum_{gs \in E_4} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_5} (d(g)^{d(g)} + d(s)^{d(s)})^\xi, \\
&= 2(n^n + (n+1)^{(n+1)})^\xi + 2^\xi(m-3)(n+1)^{\xi(n+1)} + 2(n-1)((n-1)^{n-1} + n^n)^\xi + (m-2)(n-1)((n-1)^{(n-1)} + (n+1)^{(n+1)})^\xi + 2^{\xi-1}m(n-2)(n-1)^{\xi(n-1)+1}.
\end{aligned}$$

(10)

□

5. Square Lattice, SL_n

Theorem 6. Let SL_n be square lattice with $n \geq 1$; then,

FIGURE 3: Bridge graph over complete graph K_n .

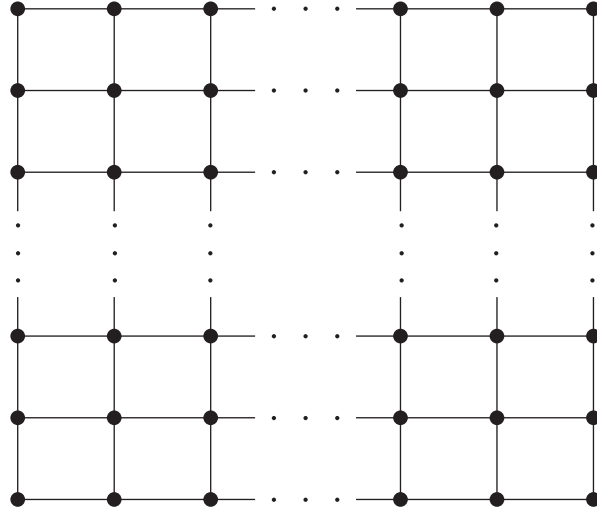
$$\begin{aligned}
 H_\xi(\text{SL}_n) &= 2^4 \cdot 5^\xi + 2^{\xi+1} \cdot 3^\xi (n-3) + 2 \cdot 7^\xi (n-2) + 2^{3\xi+1} (2n^2 - 10n + 12), \\
 GA_\xi(\text{SL}_n) &= \frac{2^{\xi+1} \cdot 3^\xi}{5} (8) + 2^2 \cdot 3^{2\xi-1} (n-3) + \frac{2^{2\xi+3} \cdot 3^\xi (n-2)}{7} + 2^{4\xi-2} (2n^2 - 10n + 12), \\
 KB_1^\xi(\text{SL}_n) &= 2^{\xi+3} \cdot 3^{2\xi} + 0 \cdot 2^3 5^{2\xi} + 2^{\xi+2} (n-5) (13^\xi + 23^\xi) + 2^{\xi+3} \cdot 17^\xi + 2^{2\xi+2} \cdot 3^{2\xi} (n-4) \\
 &\quad + 2^3 \cdot 35^\xi + 2^{4\xi} \cdot 3^\xi (2n^2 - 14n + 24), \\
 KB_2^\xi(\text{SL}_n) &= 2^{3\xi+3} \cdot 3^{2\xi} + 2^{2\xi+3} \cdot 39^\xi + (n-5) (2^2 \cdot 13^{2\xi} + 2^3 \cdot 23^{2\xi}) + 2^{3\xi+3} \cdot 33^\xi + 2^2 \cdot 299^\xi (n-4) \\
 &\quad + 2^{\xi+3} \cdot 253^\xi + 2^{6\xi} \cdot 3^{2\xi} (2n^2 - 14n + 24), \\
 Y_\xi(\text{SL}_n) &= 2^3 \cdot 31^\xi + 2^{2\xi+2} \cdot 3^{2\xi} (n-3) + 2^2 \cdot 283^\xi (n-2) + 2^{9\xi} (2n^2 - 10n + 12).
 \end{aligned} \tag{11}$$

The graph is given in Figure 4.

Proof. Let SL_n be square lattice; then, the number of vertices are n^2 and number of edges are $2n(n-1)$. Here, we divide the edge set into four partitions. First partition E_1 consists of 8 edges gs , such that $d(g) = 2$ and $d(s) = 3$. Second edge

partition E_2 consists of $4(n-3)$ edges gs , such that $d(g) = 3$ and $d(s) = 3$. The third edge partition E_3 consists of $4(n-2)$ edges gs , such that $d(g) = 3$ and $d(s) = 4$. The fourth edge partition E_4 consists of $2n^2 - 10n + 12$ edges gs , such that $d(g) = 4$ and $d(s) = 4$. The desired topological indices for the abovementioned graphs are

$$\begin{aligned}
 H_\xi(\text{SL}_n) &= \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi + \sum_{gs \in E_3} 2(d(g) + d(s))^\xi + \sum_{gs \in E_4} 2(d(g) + d(s))^\xi \\
 &= 2^4 \cdot 5^\xi + 2^{\xi+1} \cdot 3^\xi (n-3) + 2 \cdot 7^\xi (n-2) + 2^{3\xi+1} (2n^2 - 10n + 12), \\
 GA_\xi(\text{SL}_n) &= \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d(s))} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d(s))} + \sum_{gs \in E_3} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d(s))} + \sum_{gs \in E_4} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d(s))} \\
 &= \frac{2(6)^\xi}{5} |E_1| + \frac{2(9)^\xi}{6} |E_2| + \frac{2(12)^\xi}{7} |E_3| + \frac{2(16)^\xi}{8} |E_4| = \frac{2^{\xi+4} \cdot 3^\xi}{5} + 2^2 \cdot 3^{2\xi-1} (n-3) \\
 &\quad + \frac{2^{2\xi+3} \cdot 3^\xi (n-2)}{7} + 2^{4\xi-2} (2n^2 - 10n + 12),
 \end{aligned}$$

FIGURE 4: Square lattice graph, SL_n .

$$\begin{aligned}
 KB_1^\xi(SL_n) &= \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) + S_e(s))^\xi \\
 &= 2^{\xi+3} \cdot 3^{2\xi} + 0 \cdot 2^3 5^{2\xi} + 2^{\xi+2} (n-5) (13^\xi + 23^\xi) + 2^{\xi+3} \cdot 17^\xi + 2^{2\xi+2} \cdot 3^{2\xi} (n-4) + 2^3 \cdot 35^\xi + 2^{4\xi} \cdot 3^\xi (2n^2 - 14n + 24), \\
 KB_2^\xi(SL_n) &= \sum_{gs \in E_1} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_4} (S_e(g) \cdot S_e(s))^\xi \\
 &= 2^{3\xi+3} \cdot 3^{2\xi} + 2^{2\xi+3} \cdot 39^\xi + (n-5) (2^2 \cdot 13^{2\xi} + 2^3 \cdot 23^{2\xi}) + 2^{3\xi+3} \cdot 33^\xi + 2^2 \cdot 299^\xi (n-4) \\
 &\quad + 2^{\xi+3} \cdot 253^\xi + 2^{6\xi} \cdot 3^{2\xi} (2n^2 - 14n + 24), \\
 Y_\xi(SL_n) &= \sum_{gs \in E_1} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_2} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_3} (d(g)^{d(g)} + d(s)^{d(s)})^\xi \\
 &\quad + \sum_{gs \in E_4} (d(g)^{d(g)} + d(s)^{d(s)})^\xi \\
 &= 2^3 \cdot 31^\xi + 2^{2\xi+2} \cdot 3^{2\xi} (n-3) + 2^2 \cdot 283^\xi (n-2) + 2^{9\xi} (2n^2 - 10n + 12).
 \end{aligned}$$

(12)

6. Wheel Graph W_n

Theorem 7. Let W_n be wheel graph having order $n+1$; then,

$$\begin{aligned}
 H_\xi(W_n) &= 2n(2^\xi \cdot 3^\xi + (3+n)^\xi), \\
 GA_\xi(W_n) &= 3^{2\xi-1} n + 2 \cdot 3^\xi n^{\xi+1} (3+n)^{-1}, \\
 KB_1^\xi(W_n) &= \left((18+2n)^\xi + (n^2+2n+9)^\xi \right) n, \\
 KB_2^\xi(W_n) &= (9+n)^{2\xi} n + n^{\xi+1} (n^2+10n+9)^\xi, \\
 Y_\xi(W_n) &= (2^\xi \cdot 3^{3\xi} + (27+n^n)) n.
 \end{aligned} \tag{13}$$

Proof. Let W_n be wheel graph; then, $n+1$ and $2n$ are edges and vertices, respectively. For the desired topological indices, we made two partitions of the edge set that are E_1 and E_2 . The partition E_1 consists of n edges gs such that $d(g) = 3$ and $d(s) = 3$. The second partition E_2 consists of n edges gs such that $d(g) = 3$ and $d(s) = n+1$. The values for the stated indices are calculated in the following:

□

TABLE 1: Experimental values of entropy(S) and the corresponding values of generalized form of harmonic index, geometric arithmetic index, and first Kulli-Basava index of octane isomers.

Alkane	S	H_{ξ}	GA_{ξ}	KB_{ξ}^1
n-octane	111.67	$4.3^{\xi} + 10.4^{\xi}$	$(2^{\xi+2}/3) + 10.4^{\xi-1}$	$2.4^{\xi} + 2.7^{\xi} + 3.8^{\xi}$
2-methylheptane	109.84	$2.3^{\xi} + 10.4^{\xi} + 2.5^{\xi}$	$(2^{\xi}/3) + 6.4^{\xi-1} + 2. (6^{\xi}/5) + 3^{\xi}$	$3^{\xi} + 12^{\xi} + 2.14^{\xi} + 16^{\xi} + 20^{\xi} + 35^{\xi}$
3-methylheptane	111.26	$2^2.3^{\xi} + 6.4^{\xi} + 2^2.5^{\xi}$	$2^{\xi+2} + 4^{\xi} + 3^{\xi} + 2. (6^{\xi}/5)$	$4^{\xi} + 7^{\xi} + 9^{\xi} + 13^{\xi} + 10^{\xi} + 12^{\xi} + 5^{\xi}$
4-methylheptane	109.32	$2^2.3^{\xi} + 6.4^{\xi} + 4.5^{\xi}$	$(2^{\xi+2}/3) + 4^{\xi} + (3^{\xi}/4) + 4.6^{\xi}$	$2.4^{\xi} + 2.8^{\xi} + 10^{\xi} + 2.13^{\xi}$
3-ethylhexane	109.43	$6.3^{\xi} + 6.5^{\xi} + 2.4^{\xi}$	$6. (2^{\xi}/3) + (6^{\xi+1}/5) + 2 \cdot 4^{\xi-1}$	$2.5^{\xi} + 2.13^{\xi} + 14^{\xi} + 8^{\xi} + 4^{\xi}$
2,2-dimethylhexane	103.42	$3.5^{\xi} + 6^{\xi} + 2.4^{\xi} + 3^{\xi}$	$6. (4^{\xi}/5) + 4^{\xi} + (8^{\xi}/3) + (2^{\xi+1}/3)$	$3.1^{\xi} + 6^{\xi} + 19^{\xi} + 10^{\xi} + 7^{\xi} + 4^{\xi}$
2,3-dimethylhexane	108.02	$2 (2^{\xi+2} + 5^{\xi} + 6^{\xi} + 3^{\xi})$	$6. (3^{\xi}/4) + (9^{\xi}/3) + 2.4^{\xi-1} + 2. (6^{\xi}/5) + (2^{\xi+1}/3)$	$2.10^{\xi} + 17^{\xi} + 11^{\xi} + 13^{\xi} + 7^{\xi} + 4^{\xi}$
2,4-dimethylhexane	106.98	$2.3^{\xi} + 6.4^{\xi} + 6.5^{\xi}$	$(2^{\xi+1}/3) + 6. (3^{\xi}/4) + 6. (6^{\xi}/5)$	$5^{\xi} + 2.9^{\xi} + 10^{\xi} + 14^{\xi} + 13^{\xi} + 12^{\xi}$
2,5-dimethylhexane	105.72	$2^3.4^{\xi} + 4.5^{\xi} + 2.4^{\xi}$	$2.3^{\xi} + (4.6^{\xi}/5) + 2.4^{\xi-1}$	$4.9^{\xi} + 2.12^{\xi} + 10^{\xi}$
3,3-dimethylhexane	104.74	$4.3^{\xi} + 4.6^{\xi} + 4.5^{\xi} + 2.4^{\xi}$	$(4.2^{\xi}/3) + (4.8^{\xi}/6) + (4^{\xi+1}/5) + 2 \cdot 4^{\xi-1}$	$4^{\xi} + 6^{\xi} + 9^{\xi} + 2.17^{\xi} + 19^{\xi} + 20^{\xi}$
3,4-dimethylhexane	106.59	$4.3^{\xi} + 4.5^{\xi} + 4.4^{\xi} + 2.6^{\xi}$	$(2^{\xi+2}/3) + (4 \cdot 6^{\xi}/5) + 3^{\xi} + (9^{\xi}/6)$	$2.5^{\xi} + 2.11^{\xi} + 2.13^{\xi} + 18^{\xi}$
2methyl-3ethylpentane	106.06	$4.3^{\xi} + 4.5^{\xi} + 2.6^{\xi} + 4^{\xi+1}$	$(2^{\xi+2}/3) + (4.6^{\xi}/5) + (2.9^{\xi}/6) + (4.3^{\xi}/4)$	$2.5^{\xi} + 2.14^{\xi} + 2.10^{\xi} + 18^{\xi}$
3methyl-3ethylpentane	101.48	$6.3^{\xi} + 6^{\xi+1} + 2.5^{\xi}$	$2^{\xi+1} + 8^{\xi} + (2^{\xi+1}/5)$	$3.6^{\xi} + 3.20^{\xi} + 18^{\xi}$
2,2,3-trimethyl-pentane	101.31	$6.5^{\xi} + 2.7^{\xi} + 2.4^{\xi} + 2.5^{\xi} + 2.3^{\xi}$	$(6.4^{\xi}/5) + (2.12^{\xi}/7) + (2.3^{\xi}/4) + (2.6^{\xi}/5) + (2^{\xi+1}/3)$	$3.16^{\xi} + 2.3^{\xi} + 16^{\xi} + 14^{\xi} + 5^{\xi}$
2,2,4-trimethyl-pentane	104.09	$6.5^{\xi} + 2.6^{\xi} + 2.5^{\xi} + 4^{\xi+1}$	$(6.4^{\xi}/5) + (8^{\xi}/3) + (2.6^{\xi}/5) + 3^{\xi}$	$3.16^{\xi} + 20^{\xi} + 14^{\xi} + 2.9^{\xi}$
2,3,3-trimethyl-pentane	102.06	$2.3^{\xi} + 2.6^{\xi} + 4.5^{\xi} + 2.7^{\xi} + 4^{\xi+1}$	$(2^{\xi+1}/3) + (8^{\xi}/3) + (4^{\xi+1}/5) + (2.12^{\xi}/7) + 3^{\xi}$	$6^{\xi} + 20^{\xi} + 2.18^{\xi} + 24^{\xi} + 2.11^{\xi}$
2,3,4-trimethyl-pentane	102.39	$10.4^{\xi} + 4.6^{\xi}$	$(10.3^{\xi}/4) + (4.9^{\xi}/6)$	$4.10^{\xi} + 2.18^{\xi} + 12^{\xi}$
2,2,3,3-tetramethyl-butane	93.06	$12.5^{\xi} + 2.8^{\xi}$	$(12.4^{\xi}/5) + (2.16^{\xi}/8)$	$6.18^{\xi} + 30^{\xi}$

TABLE 2: Experimental values of entropy(S) and the corresponding values of generalized form of harmonic index, geometric arithmetic index, and first Kulli-Basava index of octane isomers for different values of ξ .

Alkane	S	H_ξ	GA_ξ	KB_1^ξ
n-octane	111.67	111.483 if $\xi = 1.576$	111.665 if $\xi = 2.683$	111.672 if $\xi = 1.4591$
2-methylheptane	109.84	109.843 if $\xi = 1.417$	109.845 if $\xi = 2.5587$	109.882 if $\xi = 0.9875$
3-methylheptane	111.26	111.27 if $\xi = 1.486$	111.27 if $\xi = 2.54$	111.26 if $\xi = 1.2761$
4-methylheptane	109.32	109.389 if $\xi = 1.474$	109.331 if $\xi = 1.749$	109.324 if $\xi = 1.2662$
3-ethylhexane	109.43	109.434 if $\xi = 1.47$	109.43 if $\xi = 2.3807$	109.437 if $\xi = 1.2458$
2,2-dimethylhexane	103.42	103.431 if $\xi = 1.755$	103.426 if $\xi = 2.3423$	103.354 if $\xi = 1.0611$
2,3-dimethylhexane	108.02	108.146 if $\xi = 1.676$	108.032 if $\xi = 2.1141$	108.024 if $\xi = 1.168$
2,4-dimethylhexane	106.98	106.934 if $\xi = 1.392$	106.954 if $\xi = 2.3666$	106.986 if $\xi = 1.1666$
2,5-dimethylhexane	105.72	105.685 if $\xi = 1.387$	105.783 if $\xi = 2.431$	105.599 if $\xi = 1.1778$
3,3-dimethylhexane	104.74	104.749 if $\xi = 1.315$	104.369 if $\xi = 2.2$	04.745 if $\xi = 1.048$
3,4-dimethylhexane	106.59	106.527 if $\xi = 1.384$	106.581 if $\xi = 2.3876$	106.583 if $\xi = 1.1365$
2methyl-3ethylpentane	106.06	106.063 if $\xi = 1.3811$	106.064 if $\xi = 2.246$	106.055 if $\xi = 1.1341$
3methyl-3ethylpentane	101.48	101.408 if $\xi = 1.2911$	101.437 if $\xi = 2.136$	101.405 if $\xi = 1.0199$
2,2,3-trimethyl-pentane	101.31	101.303 if $\xi = 1.247$	101.362 if $\xi = 1.8765$	101.392 if $\xi = 0.984$
2,2,4-trimethyl-pentane	104.09	104.002 if $\xi = 1.267$	104.084 if $\xi = 2.2889$	104.098 if $\xi = 1.0149$
2,3,3-trimethyl-pentane	102.06	102.067 if $\xi = 1.2505$	102.333 if $\xi = 2.08$	102.067 if $\xi = 0.9799$
2,3,4-trimethyl-pentane	102.39	102.391 if $\xi = 1.3043$	102.391 if $\xi = 2.154$	102.172 if $\xi = 1.0581$
2,2,3,3-tetramethyl-butane	93.06	93.062 if $\xi = 1.1184$	93.0975 if $\xi = 2.01$	93.1717 if $\xi = 0.869$

$$H_\xi(W_n) = \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi = 2n(2^\xi \cdot 3^\xi + (3+n)^\xi),$$

$$GA_\xi(W_n) = \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} = 3^{2\xi-1}n + 2 \cdot 3^\xi n^{\xi+1} (3+n)^{-1},$$

$$KB_1^\xi(W_n) = \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi = ((9+n) + (9+n))^\xi |E_1| + ((9+n) + (n(n+1)))^\xi |E_2|$$

$$= \left((18+2n)^\xi + (n^2+2n+9)^\xi \right) n, \quad (14)$$

$$KB_2^\xi(W_n) = \sum_{gs \in E_1} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) \cdot S_e(s))^\xi = ((9+n) \cdot (9+n))^\xi |E_1| + ((9+n) \cdot (n(n+1)))^\xi |E_2|$$

$$= (9+n)^{2\xi} n + n^{\xi+1} (n^2 + 10n + 9)^\xi,$$

$$Y_\xi(W_n) = \sum_{gs \in E_1} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_2} (d(g)^{d(g)} + d(s)^{d(s)})^\xi = (2^\xi \cdot 3^{3\xi} + (27+n^n))n.$$

7. Gear Graph, GR_n

Theorem 8. Let GR_n be gear graph; then,

$$H_\xi(GR_n) = 2^2 \cdot 5^\xi n + 2n(3+n)^\xi,$$

$$GA_\xi(GR_n) = 2^{\xi+2} \cdot 3^\xi \cdot 5^{-1} \cdot n + 2 \cdot 3^\xi \cdot n^{\xi+1} (3+n)^{-1},$$

$$KB_1^\xi(GR_n) = 2n(13+n)^\xi + n(n^2+2n+7)^\xi, \quad (15)$$

$$KB_2^\xi(GR_n) = n(2^{\xi+1} \cdot 3^\xi + n^\xi (n+1)^\xi) (7+n)^\xi,$$

$$Y_\xi(GR_n) = \left(2 \cdot 31^\xi + (27+n^n)^\xi \right) n.$$

Proof. Let GR_n be gear graph; then, $2n+1$ and $3n$ are edges and vertices, respectively. For values of the desired topological indices, we divide the edge set into two partition E_1 and E_2 . The partition E_1 consists of $2n$ edges gs , such that $d(g) = 2$ and $d(s) = 3$. The partition E_2 consists of n edges gs , such that $d(g) = 3$ and $d(s) = n$. For the stated indices, we have the following:

□

TABLE 3: Experimental values of acentric factor and the corresponding values of generalized form of harmonic index, geometric arithmetic index, and first Kulli-Basava index of octane isomers.

Alkane	AcentFac	H_{ξ}	GA_{ξ}	KB_1^{ξ}
n-octane	0.397898	$4.3^{\xi} + 10.4^{\xi}$	$(2^{\xi+2}/3) + 10.4^{\xi-1}$	$2.4^{\xi} + 2.7^{\xi} + 3.8^{\xi}$
2-methylheptane	0.377916	$2.3^{\xi} + 10.4^{\xi} + 2.5^{\xi}$	$(2^{\xi}/3) + 6.4^{\xi-1} + 2 \cdot (6^{\xi}/5) + 3^{\xi}$	$3^{\xi} + 12^{\xi} + 2.14^{\xi} + 16^{\xi} + 20^{\xi} + 35^{\xi}$
3-methylheptane	0.371002	$2^2 \cdot 3^{\xi} + 6.4^{\xi} + 2^2 + 5^{\xi}$	$2^{\xi+2} + 4^{\xi} + 3^{\xi} + 2 \cdot (6^{\xi}/5)$	$4^{\xi} + 7^{\xi} + 9^{\xi} + 13^{\xi} + 10^{\xi} + 12^{\xi} + 5^{\xi}$
4-methylheptane	0.371504	$2^2 \cdot 3^{\xi} + 6.4^{\xi} + 4.5^{\xi}$	$(2^{\xi+2}/3) + 4^{\xi} + (3^{\xi}/4) + 4.6^{\xi}$	$2.4^{\xi} + 2.8^{\xi} + 10^{\xi} + 2.13^{\xi}$
3-ethylhexane	0.362472	$6.3^{\xi} + 6.5^{\xi} + 2.4^{\xi}$	$6 \cdot (2^{\xi}/3) + (6^{\xi+1}/5) + 2.4^{\xi-1}$	$2.5^{\xi} + 2.13^{\xi} + 14^{\xi} + 8^{\xi} + 4^{\xi}$
2,2-dimethylhexane	0.339426	$3.5^{\xi} + 6^{\xi} + 2.4^{\xi} + 3^{\xi}$	$6 \cdot (4^{\xi}/5) + 4^{\xi} + (8^{\xi}/3) + (2^{\xi+1}/3)$	$3.1 \cdot 6^{\xi} + 19^{\xi} + 10^{\xi} + 7^{\xi} + 4^{\xi}$
2,3-dimethylhexane	0.348247	$2(2^{\xi+2} + 5^{\xi} + 6^{\xi} + 3^{\xi})$	$(6.3^{\xi}/4) + (9^{\xi}/3) + 2.4^{\xi-1} + (2.6^{\xi}/5) + (2^{\xi+1}/3)$	$2.10^{\xi} + 17^{\xi} + 11^{\xi} + 13^{\xi} + 7^{\xi} + 4^{\xi}$
2,4-dimethylhexane	0.344223	$2.3^{\xi} + 6.4^{\xi} + 6.5^{\xi}$	$(2^{\xi+1}/3) + 6 \cdot (3^{\xi}/4) + 6 \cdot (6^{\xi}/5)$	$5^{\xi} + 2.9^{\xi} + 10^{\xi} + 14^{\xi} + 13^{\xi} + 12^{\xi}$
2,5-dimethylhexane	0.35683	$2^3 \cdot 4^{\xi} + 4.5^{\xi} + 2.4^{\xi}$	$2.3^{\xi} + (4.6^{\xi}/5) + 2.4^{\xi-1}$	$4.9^{\xi} + 2.12^{\xi} + 10^{\xi}$
3,3-dimethylhexane	0.322596	$4.3^{\xi} + 4.6^{\xi} + 4.5^{\xi} + 2.4^{\xi}$	$(4.2^{\xi}/3) + (4.8^{\xi}/6) + (4^{\xi+1}/5) + 2.4^{\xi-1}$	$4^{\xi} + 6^{\xi} + 9^{\xi} + 2.17^{\xi} + 19^{\xi} + 20^{\xi}$
3,4-dimethylhexane	0.340345	$4.3^{\xi} + 4.5^{\xi} + 4.4^{\xi} + 2.6^{\xi}$	$(2^{\xi+2}/3) + (4.6^{\xi}/5) + 3^{\xi} + (9^{\xi}/6)$	$2.5^{\xi} + 2.11^{\xi} + 2.13^{\xi} + 18^{\xi}$
2methyl-3ethylpentane	0.332433	$4.3^{\xi} + 4.5^{\xi} + 2.6^{\xi} + 4^{\xi+1}$	$(2^{\xi+2}/3) + (4.6^{\xi}/5) + (2.9^{\xi}/6) + (4.3^{\xi}/4)$	$2.5^{\xi} + 2.14^{\xi} + 2.10^{\xi} + 18^{\xi}$
3methyl-3ethylpentane	0.306899	$6.3^{\xi} + 6^{\xi+1} + 2.5^{\xi}$	$2^{\xi+1} + 8^{\xi} + (2^{2\xi+1}/5)$	$3.6^{\xi} \cdot 3.20^{\xi} + 18^{\xi}$
2,2,3-trimethyl-pentane	0.300816	$6.5^{\xi} + 2.7^{\xi} + 2.4^{\xi} + 2.5^{\xi} + 2.3^{\xi}$	$(6.4^{\xi}/5) + (2.12^{\xi}/7) + (2.3^{\xi}/4) + (2.6^{\xi}/5) + (2^{\xi+1}/3)$	$3.16^{\xi} + 23^{\xi} + 16^{\xi} + 14^{\xi} + 5^{\xi}$
2,2,4-trimethyl-pentane	0.30537	$6.5^{\xi} + 2.6^{\xi} + 2.5^{\xi} + 4^{\xi+1}$	$(6.4^{\xi}/5) + (8^{\xi}/3) + (2.6^{\xi}/5) + 3^{\xi}$	$3.16^{\xi} + 20^{\xi} + 14^{\xi} + 2.9^{\xi}$
2,3,3-trimethyl-pentane	0.293177	$2.3^{\xi} + 2.6^{\xi} + 4.5^{\xi} + 2.7^{\xi} + 4^{\xi+1}$	$(2^{\xi+1}/3) + (8^{\xi}/3) + (4^{\xi+1}/5) + (2.12^{\xi}/7) + 3^{\xi}$	$6^{\xi} + 20^{\xi} + 2.18^{\xi} + 24^{\xi} + 2.11^{\xi}$
2,3,4-trimethyl-pentane	0.317422	$10.4^{\xi} + 4.6^{\xi}$	$(10.3^{\xi}/4) + (4.9^{\xi}/6)$	$4.10^{\xi} + 2.18^{\xi} + 12^{\xi}$
2,2,3,3-tetramethyl-butane	0.255294	$12.5^{\xi} + 2.8^{\xi}$	$(12.4^{\xi}/5) + (2.16^{\xi}/8)$	$6.18^{\xi} + 30^{\xi}$

TABLE 4: Experimental values of acentric factor and the corresponding values of generalized form of harmonic index, geometric arithmetic index, and first Kulli–Basava index of octane isomers for different values of ξ .

Alkane	AcentFac	H_ξ	GA_ξ	KB_1^ξ
n-octane	0.397898	0.390726 if $\xi = -2.8$	0.397095 if $\xi = -2.232$	0.393803 if $\xi = -1.63$
2-methylheptane	0.377916	0.377722 if $\xi = -2.87$	0.373743 if $\xi = -1.8$	0.37766 if $\xi = -1.335$
3-methylheptane	0.371002	0.370688 if $\xi = -2.76$	0.377933 if $\xi = -3.52$	0.375643 if $\xi = -1.51$
4-methylheptane	0.371504	0.370266 if $\xi = -2.7609$	0.374663 if $\xi = -2.36$	0.371555 if $\xi = -1.561$
3-ethylhexane	0.362472	0.372775 if $\xi = -2.825$	0.36575 if $\xi = -2.56$	0.362373 if $\xi = -1.571$
2,2-dimethylhexane	0.339426	0.333752 if $\xi = -2.1$	0.339447 if $\xi = -1.91$	0.333496 if $\xi = -1.39$
2,3-dimethylhexane	0.348247	0.341947 if $\xi = -4.61$	0.345979 if $\xi = -2.301$	0.342585 if $\xi = -1.44$
2,4-dimethylhexane	0.344223	0.340783 if $\xi = -2.66$	0.342507 if $\xi = -2.07$	0.344292 if $\xi = -1.366$
2,5-dimethylhexane	0.35683	0.342116 if $\xi = -2.581$	0.358332 if $\xi = -1.78$	0.356664 if $\xi = -1.303$
3,3-dimethylhexane	0.322596	0.337126 if $\xi = -2.6995$	0.324943 if $\xi = -3.12$	0.322704 if $\xi = -1.428$
3,4-dimethylhexane	0.340345	0.345705 if $\xi = -2.75$	0.340694 if $\xi = -1.872$	0.340781 if $\xi = -1.417$
2methyl-3ethylpentane	0.332433	0.332769 if $\xi = -2.78$	0.33118 if $\xi = -1.91$	0.336483 if $\xi = -1.43$
3methyl-3ethylpentane	0.306899	0.307387 if $\xi = -2.88$	0.302726 if $\xi = -2.7801$	0.303201 if $\xi = -1.406$
2,2,3-trimethyl-pentane	0.300816	0.30815 if $\xi = -2.59$	0.300499 if $\xi = -2.5$	0.300757 if $\xi = -1.271$
2,2,4-trimethyl-pentane	0.30537	0.305828 if $\xi = -2.467$	0.307001 if $\xi = -1.674$	0.304683 if $\xi = -1.2201$
2,3,3-trimethyl-pentane	0.293177	0.291389 if $\xi = -2.66$	0.293276 if $\xi = -2.139$	0.295659 if $\xi = -1.26$
2,3,4-trimethyl-pentane	0.317422	0.318999 if $\xi = -2.58$	0.317756 if $\xi = -1.907$	0.315898 if $\xi = -1.26$
2,2,3,3-tetramethyl-butane	0.255294	0.255116 if $\xi = -2.425$	0.255169 if $\xi = -2.245$	0.255542 if $\xi = -1.123$

$$\begin{aligned}
H_\xi(GR_n) &= \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi = 2^2 \cdot 5^\xi n + 2n(3 + n)^\xi, \\
GA_\xi(GR_n) &= \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} = 2^{\xi+2} \cdot 3^\xi \cdot 5^{-1} \cdot n + 2 \cdot 3^\xi \cdot n^{\xi+1} (3 + n)^{-1}, \\
KB_1^\xi(GR_n) &= \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi = 2n(13 + n)^\xi + n(n^2 + 2n + 7)^\xi, \\
KB_2^\xi(GR_n) &= \sum_{gs \in E_1} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) \cdot S_e(s))^\xi = n(2^{\xi+1} \cdot 3^\xi + n^\xi (n + 1)^\xi) (7 + n)^\xi, \\
Y_\xi(GR_n) &= \sum_{gs \in E_1} (d(g)^{(g)} + d(s)^{(d(s))})^\xi + \sum_{gs \in E_2} (d(g)^{(g)} + d(s)^{(d(s))})^\xi = (2 \cdot 31^\xi + (27 + n^n))^\xi.
\end{aligned} \tag{16}$$

8. Helm Graph H_n

Theorem 9. Let H_n be a helm graph; then,

$$\begin{aligned}
H_\xi(H_n) &= (5^\xi + 2^{3\xi} + (4 + n)^\xi) 2n, \\
GA_\xi(H_n) &= (2^{2\xi} \cdot 5^{-1} + 2^{4\xi-3} + 2^{2\xi} \cdot n^\xi (4 + n)^{-1}) 2n, \\
KB_1^\xi(H_n) &= ((20 + n)^\xi + 2^\xi (17 + n)^\xi + (n^2 + 3n + 17)^\xi) n, \\
KB_2^\xi(H_n) &= n(17 + n)^\xi (3^\xi + (17 + n)^\xi + n^\xi (2 + n)^\xi), \\
Y_\xi(H_n) &= (257^\xi + 2^{9\xi} + (256 + n^n)^\xi) n.
\end{aligned} \tag{17}$$

Proof. Let H_n be a helm graph; then, $2n + 1$ and $3n$ are vertices and edges, respectively. For values of the desired topological indices, we divide the edge set into three partitions say E_1 , E_2 , and E_3 . The partition E_1 consists of n edges gs such that $d(g) = 1$ and $d(s) = 4$. The partition E_2 consists of n edges gs such that $d(g) = 4$ and $d(s) = 4$. The partition E_3 consists of n edges gs such that $d(g) = 4$ and $d(s) = n$. The desired values about the stated topological indices are the following:

□

TABLE 5: Experimental values of DHVAP and the corresponding values of generalized form of harmonic index, geometric arithmetic index, and first Kulli-Basava index of octane isomers.

Alkane	DHVAP	H_{ξ}	GA_{ξ}	KB_1^{ξ}
n-octane	9.915	$4.3^{\xi} + 10.4^{\xi}$	$(\frac{2^{\xi+2}}{3}) + 10.4^{\xi-1}$	$2.4^{\xi} + 2.7^{\xi} + 3.8^{\xi}$
2-Methylheptane	9.484	$2.3^{\xi} + 10.4^{\xi} + 2.5^{\xi}$	$(\frac{2^{\xi}}{3}) + 6.4^{\xi-1} + 2.(\frac{6^{\xi}}{5}) + 3^{\xi}$	$3^{\xi} + 12^{\xi} + 2.14^{\xi} + 16^{\xi} + 20^{\xi} + 35^{\xi}$
3-Methylheptane	9.521	$2^2.3^{\xi} + 6.4^{\xi} + 2^2 + 5^{\xi}$	$2^{\xi+2} + 4^{\xi} + 3^{\xi} + 2 \cdot (\frac{6^{\xi}}{5})$	$4^{\xi} + 7^{\xi} + 9^{\xi} + 13^{\xi} + 10^{\xi} + 12^{\xi} + 5^{\xi}$
4-Methylheptane	9.483	$2^2.3^{\xi} + 6.4^{\xi} + 4.5^{\xi}$	$(\frac{2^{\xi+2}}{3}) + 4^{\xi} + (\frac{3^{\xi}}{4}) + 4.6^{\xi}$	$2.4^{\xi} + 2.8^{\xi} + 10^{\xi} + 2.13^{\xi}$
3-Ethylhexane	9.476	$6.3^{\xi} + 6.5^{\xi} + 2.4^{\xi}$	$6.(\frac{2^{\xi}}{3}) + (\frac{6^{\xi+1}}{5}) + 2.4^{\xi-1}$	$2.5^{\xi} + 2.13^{\xi} + 14^{\xi} + 8^{\xi} + 4^{\xi}$
2,2-dimethylhexane	8.915	$3.5^{\xi} + 6^{\xi} + 2.4^{\xi} + 3^{\xi}$	$6.(\frac{4^{\xi}}{5}) + 4^{\xi} + (\frac{8^{\xi}}{3}) + (\frac{2^{\xi+1}}{3})$	$3.1 \ 6^{\xi} + 19^{\xi} + 10^{\xi} + 7^{\xi} + 4^{\xi}$
2,3-dimethylhexane	9.272	$2(2^{\xi+2} + 5^{\xi} + 6^{\xi} + 3^{\xi})$	$(6.3^{\xi}/4) + (9^{\xi}/3) + 2.4^{\xi-1} + (2.6^{\xi}/5) + (\frac{2^{\xi+1}}{3})$	$2.10^{\xi} + 17^{\xi} + 11^{\xi} + 13^{\xi} + 7^{\xi} + 4^{\xi}$
2,4-dimethylhexane	9.029	$2.3^{\xi} + 6.4^{\xi} + 6.5^{\xi}$	$(\frac{2^{\xi+1}}{3}) + 6.(\frac{3^{\xi}}{4}) + 6.(\frac{6^{\xi}}{5})$	$5^{\xi} + 2.9^{\xi} + 10^{\xi} + 14^{\xi} + 13^{\xi} + 12^{\xi}$
2,5-dimethylhexane	9.051	$2^3.4^{\xi} + 4.5^{\xi} + 2.4^{\xi}$	$2.3^{\xi} + (\frac{4.6^{\xi}}{5}) + 2.4^{\xi-1}$	$4.9^{\xi} + 2.12^{\xi} + 10^{\xi}$
3,3-dimethylhexane	8.973	$4.3^{\xi} + 4.6^{\xi} + 4.5^{\xi} + 2.4^{\xi}$	$(\frac{4.2^{\xi}}{3}) + (\frac{4.8^{\xi}}{6}) + (\frac{4^{\xi+1}}{5}) + 2.4^{\xi-1}$	$4^{\xi} + 6^{\xi} + 9^{\xi} + 2.17^{\xi} + 19^{\xi} + 20^{\xi}$
3,4-dimethylhexane	9.316	$4.3^{\xi} + 4.5^{\xi} + 4.4^{\xi} + 2.6^{\xi}$	$(\frac{2^{\xi+2}}{3}) + (\frac{4.6^{\xi}}{5}) + 3^{\xi} + (\frac{9^{\xi}}{6})$	$2.5^{\xi} + 2.11^{\xi} + 2.13^{\xi} + 18^{\xi}$
2methyl-3ethylpentane	9.209	$4.3^{\xi} + 4.5^{\xi} + 2.6^{\xi} + 4^{\xi+1}$	$(\frac{2^{\xi+2}}{3}) + (\frac{4.6^{\xi}}{5}) + (\frac{2.9^{\xi}}{6}) + (\frac{4.3^{\xi}}{4})$	$2.5^{\xi} + 2.14^{\xi} + 2.10^{\xi} + 18^{\xi}$
3methyl-3ethylpentane	9.081	$6.3^{\xi} + 6^{\xi+1} + 2.5^{\xi}$	$2^{\xi+1} + 8^{\xi} + (\frac{2^{2\xi+1}}{5})$	$3.6^{\xi} 3.20^{\xi} + 18^{\xi}$
2,2,3-trimethyl-pentane	8.826	$6.5^{\xi} + 2.7^{\xi} + 2.4^{\xi} + 2.5^{\xi} + 2.3^{\xi}$	$(6.4^{\xi}/5) + (\frac{2.12^{\xi}}{7}) + (\frac{2.3^{\xi}}{4}) + (\frac{2.6^{\xi}}{5}) + (\frac{2^{\xi+1}}{3})$	$3.16^{\xi} + 23^{\xi} + 16^{\xi} + 14^{\xi} + 5^{\xi}$
2,2,4-trimethyl-pentane	8.402	$6.5^{\xi} + 2.6^{\xi} + 2.5^{\xi} + 4^{\xi+1}$	$(\frac{6.4^{\xi}}{5}) + (\frac{8^{\xi}}{3}) + (\frac{2.6^{\xi}}{5}) + 3^{\xi}$	$3.16^{\xi} + 20^{\xi} + 14^{\xi} + 2.9^{\xi}$
2,3,3-trimethyl-pentane	8.897	$2.3^{\xi} + 2.6^{\xi} + 4.5^{\xi} + 2.7^{\xi} + 4^{\xi+1}$	$(\frac{2^{\xi+1}}{3}) + (\frac{8^{\xi}}{3}) + (\frac{4^{\xi+1}}{5}) + (\frac{2.12^{\xi}}{7}) + 3^{\xi}$	$6^{\xi} + 20^{\xi} + 2.18^{\xi} + 24^{\xi} + 2.11^{\xi}$
2,3,4-trimethyl-pentane	9.014	$10.4^{\xi} + 4.6^{\xi}$	$(\frac{10.3^{\xi}}{4}) + (\frac{4.9^{\xi}}{6})$	$4.10^{\xi} + 2.18^{\xi} + 12^{\xi}$
2,2,3,3-tetramethyl-butane	8.41	$12.5^{\xi} + 2.8^{\xi}$	$(\frac{12.4^{\xi}}{5}) + (\frac{2.16^{\xi}}{8})$	$6.18^{\xi} + 30^{\xi}$

TABLE 6: Experimental values of HVP and the corresponding values of generalized form of harmonic index, geometric arithmetic index, and first Kulli-Basava index of octane isomers.

Alkane	HVP	H_k	GA_k	KB_1^k
n-octane	73.19	$4.3^k + 10.4^k$	$(2^{\frac{k+2}{2}}/3) + 10.4^{\frac{k-1}{2}}$	$2.4^k + 2.7^k + 3.8^k$
2-methylheptane	70.3	$2.3^k + 10.4^k + 2.5^k$	$(2^{\frac{k}{2}}/3) + 6.4^{\frac{k-1}{2}} + 2 \cdot (6^{\frac{k}{2}}/5) + 3^k$	$3^k + 12^k + 2.14^k + 16^k + 20^k + 35^k$
3-methylheptane	71.3	$2^2 \cdot 3^k + 6.4^k + 2^2 + 5^k$	$2^{\frac{k+2}{2}} + 4^k + 3^k + 2 \cdot (6^{\frac{k}{2}}/5)$	$4^k + 7^k + 9^k + 13^k + 10^k + 12^k + 5^k$
4-methylheptane	70.91	$2^2 \cdot 3^k + 6.4^k + 4.5^k$	$(2^{\frac{k+2}{2}}/3) + 4^k + (3^{\frac{k}{2}}/4) + 4.6^k$	$2.4^k + 2.8^k + 10^k + 2.13^k$
3-ethylhexane	71.7	$6.3^k + 6.5^k + 2.4^k$	$6 \cdot (2^{\frac{k}{2}}/3) + (6^{\frac{k+1}{2}}/5) + 2.4^{\frac{k-1}{2}}$	$2.5^k + 2.13^k + 14^k + 8^k + 4^k$
2,2-dimethylhexane	67.7	$3.5^k + 6^k + 2.4^k + 3^k$	$6 \cdot (4^{\frac{k}{2}}/5) + 4^k + (8^{\frac{k}{2}}/3) + (2^{\frac{k+1}{2}}/3)$	$3.1 \cdot 6^k + 19^k + 10^k + 7^k + 4^k$
2,3-dimethylhexane	70.2	$2(2^{\frac{k+2}{2}} + 5^k + 6^k + 3^k)$	$(6.3^{\frac{k}{2}}/4) + (9^{\frac{k}{2}}/3) + 2.4^{\frac{k-1}{2}} + (2.6^{\frac{k}{2}}/5) + (2^{\frac{k+1}{2}}/3)$	$2.10^k + 17^k + 11^k + 13^k + 7^k + 4^k$
2,4-dimethylhexane	68.5	$2.3^k + 6.4^k + 6.5^k$	$(2^{\frac{k+1}{2}}/3) + 6 \cdot (3^{\frac{k}{2}}/4) + 6 \cdot (6^{\frac{k}{2}}/5)$	$5^k + 2.9^k + 10^k + 14^k + 13^k + 12^k$
2,5-dimethylhexane	68.6	$2^3 \cdot 4^k + 4.5^k + 2.4^k$	$2.3^k + (4.6^{\frac{k}{2}}/5) + 2.4^{\frac{k-1}{2}}$	$4.9^k + 2.12^k + 10^k$
3,3-dimethylhexane	68.5	$4.3^k + 4.6^k + 4.5^k + 2.4^k$	$(4.2^{\frac{k}{2}}/3) + (4.8^{\frac{k}{2}}/6) + (4^{\frac{k+1}{2}}/5) + 2.4^{\frac{k-1}{2}}$	$4^k + 6^k + 9^k + 2.17^k + 19^k + 20^k$
3,4-dimethylhexane	70.2	$4.3^k + 4.5^k + 4.4^k + 2.6^k$	$(2^{\frac{k+2}{2}}/3) + (4.6^{\frac{k}{2}}/5) + 3^k + (9^{\frac{k}{2}}/6)$	$2.5^k + 2.11^k + 2.13^k + 18^k$
2methyl-3ethylpentane	69.7	$4.3^k + 4.5^k + 2.6^k + 4^{\frac{k+1}{2}}$	$(2^{\frac{k+2}{2}}/3) + (4.6^{\frac{k}{2}}/5) + (2.9^{\frac{k}{2}}/6) + (4.3^{\frac{k}{2}}/4)$	$2.5^k + 2.14^k + 2.10^k + 18^k$
3methyl-3ethylpentane	69.3	$6.3^k + 6^{\frac{k+1}{2}} + 2.5^k$	$2^{\frac{k+1}{2}} + 8^k + (2^{\frac{k+1}{2}}/5)$	$3.6^k + 3.20^k + 18^k$
2,2,3-trimethyl-pentane	67.3	$6.5^k + 2.7^k + 2.4^k + 2.5^k + 2.3^k$	$(6.4^{\frac{k}{2}}/5) + (2.12^{\frac{k}{2}}/7) + (2.3^{\frac{k}{2}}/4) + (2.6^{\frac{k}{2}}/5) + (2^{\frac{k+1}{2}}/3)$	$3.16^k + 2.3^k + 16^k + 14^k + 5^k$
2,2,4-trimethyl-pentane	64.87	$6.5^k + 2.6^k + 2.5^k + 4^{\frac{k+1}{2}}$	$(6.4^{\frac{k}{2}}/5) + (8^{\frac{k}{2}}/3) + (2.6^{\frac{k}{2}}/5) + 3^k$	$3.16^k + 20^k + 14^k + 2.9^k$
2,3,3-trimethyl-pentane	68.1	$2.3^k + 2.6^k + 4.5^k + 2.7^k + 4^{\frac{k+1}{2}}$	$(2^{\frac{k+1}{2}}/3) + (8^{\frac{k}{2}}/3) + (4^{\frac{k+1}{2}}/5) + (2.12^{\frac{k}{2}}/7) + 3^k$	$6^k + 20^k + 2.18^k + 24^k + 2.11^k$
2,3,4-trimethyl-pentane	68.37	$10.4^k + 4.6^k$	$(10.3^{\frac{k}{2}}/4) + (4.9^{\frac{k}{2}}/6)$	$4.10^k + 2.18^k + 12^k$
2,2,3,3-tetramethyl-butane	66.2	$12.5^k + 2.8^k$	$(12.4^{\frac{k}{2}}/5) + (2.16^{\frac{k}{2}}/8)$	$6.18^k + 30^k$

TABLE 7: Experimental values of entropy and the corresponding values of generalized form of second Kulli–Basava index and generalized power-sum-connectivity index of octane isomers.

Alkane	S	KB_2^ξ	Y_ξ
n-octane	111.67	$2.3^\xi + 2.12^\xi + 3.16^\xi$	$2.5^\xi + 5.8^\xi$
2-methylheptane	109.84	$3^\xi + 12^\xi + 2.14^\xi + 16^\xi + 20^\xi + 35^\xi$	$4^\xi + 3.8^\xi + 31^\xi + 2.28^\xi$
3-methylheptane	111.26	$3^\xi + 12^\xi + 20^\xi + 40^\xi + 16^\xi + 32^\xi + 4^\xi$	$2.5^\xi + 2.8^\xi + 2.31^\xi + 28^\xi$
4-methylheptane	109.32	$2.3^\xi + 2.15^\xi + 16^\xi + 2.40^\xi$	$2.5^\xi + 2.8^\xi + 31^\xi + 28^\xi$
3-ethylhexane	109.43	$2.4^\xi + 2.36^\xi + 45^\xi + 15^\xi + 3^\xi$	$3.5^\xi + 3.31^\xi + 8^\xi$
2,2-dimethylhexane	103.42	$3.39^\xi + 78^\xi + 24^\xi + 12^\xi + 3^\xi$	$3.257^\xi + 2.8^\xi + 260^\xi + 5^\xi$
2,3-dimethylhexane	108.02	$2.16^\xi + 72^\xi + 18^\xi + 36^\xi + 12^\xi + 4^\xi$	$3.28^\xi + 54^\xi + 31^\xi + 8^\xi + 5^\xi$
2,4-dimethylhexane	106.98	$4^\xi + 16^\xi + 32^\xi + 48^\xi + 42^\xi + 2.14^\xi$	$5^\xi 3.28^\xi + 3.31^\xi$
2,5-dimethylhexane	105.72	$4.14^\xi + 2.35^\xi + 25^\xi$	$4.28^\xi + 2.28^\xi + 8^\xi$
3,3-dimethylhexane	104.74	$3^\xi + 5^\xi + 18^\xi + 2.52^\xi + 70^\xi + 84^\xi$	$2.5^\xi + 2.260^\xi + 2.257^\xi + 8^\xi$
3,4-dimethylhexane	106.59	$2.4^\xi + 2.18^\xi + 2.36^\xi + 81^\xi$	$2.5^\xi + 2.31^\xi + 2.28^\xi + 54^\xi$
2methyl-3ethylpentane	106.06	$2.4^\xi + 2.40^\xi + 2.16^\xi + 80^\xi$	$2.5^\xi + 2.31^\xi + 54^\xi + 2.28^\xi$
3methyl-3ethylpentane	101.48	$3.5^\xi + 3.75^\xi + 45^\xi$	$3.5^\xi + 3.260^\xi + 257^\xi$
2,2,3-trimethyl-pentane	101.31	$3.39^\xi + 130^\xi + 60^\xi + 40^\xi + 4^\xi$	$3.257^\xi + 283^\xi + 28^\xi + 31^\xi + 5^\xi$
2,2,4-trimethyl-pentane	104.09	$3.39^\xi + 91^\xi + 49^\xi + 2.14^\xi$	$3.257^\xi + 260^\xi + 31^\xi + 2.28^\xi$
2,3,3-trimethyl-pentane	102.06	$5^\xi + 75^\xi + 2.45^\xi + 135^\xi + 2.18^\xi$	$5^\xi + 260^\xi + 2.257^\xi + 283^\xi + 2.28^\xi$
2,3,4-trimethyl-pentane	102.39	$4.16^\xi + 2.80^\xi + 20^\xi$	$5.28^\xi + 2.54^\xi$
2,2,3,3-tetramethyl-butane	93.06	$6.45^\xi + 225^\xi$	$6.257^\xi + 512^\xi$

TABLE 8: Experimental values of acentric factor and the corresponding values of generalized form of second Kulli–Basava index and generalized power-sum-connectivity index of octane isomers.

Alkane	AcentFac	KB_2^ξ	Y_ξ
n-octane	0.397898	$2.3^\xi + 2.12^\xi + 3.16^\xi$	$2.5^\xi + 5.8^\xi$
2-methylheptane	0.377916	$3^\xi + 12^\xi + 2.14^\xi + 16^\xi + 20^\xi + 35^\xi$	$4^\xi + 3.8^\xi + 31^\xi + 2.28^\xi$
3-methylheptane	0.371002	$3^\xi + 12^\xi + 20^\xi + 40^\xi + 16^\xi + 32^\xi + 4^\xi$	$2.5^\xi + 2.8^\xi + 2.31^\xi + 28^\xi$
4-methylheptane	0.371504	$2.3^\xi + 2.15^\xi + 16^\xi + 2.40^\xi$	$2.5^\xi + 2.8^\xi + 31^\xi + 28^\xi$
3-ethylhexane	0.352472	$2.4^\xi + 2.36^\xi + 45^\xi + 15^\xi + 3^\xi$	$3.5^\xi + 3.31^\xi + 8^\xi$
2,2-dimethylhexane	0.339426	$3.39^\xi + 78^\xi + 24^\xi + 12^\xi + 3^\xi$	$3.257^\xi + 2.8^\xi + 260^\xi + 5^\xi$
2,3-dimethylhexane	0.348247	$2.16^\xi + 72^\xi + 18^\xi + 36^\xi + 12^\xi + 4^\xi$	$3.28^\xi + 54^\xi + 31^\xi + 8^\xi + 5^\xi$
2,4-dimethylhexane	0.344223	$4^\xi + 16^\xi + 32^\xi + 48^\xi + 42^\xi + 2.14^\xi$	$5^\xi 3.28^\xi + 3.31^\xi$
2,5-dimethylhexane	0.35683	$4.14^\xi + 2.35^\xi + 25^\xi$	$4.28^\xi + 2.28^\xi + 8^\xi$
3,3-dimethylhexane	0.322596	$3^\xi + 5^\xi + 18^\xi + 2.52^\xi + 70^\xi + 84^\xi$	$2.5^\xi + 2.260^\xi + 2.257^\xi + 8^\xi$
3,4-dimethylhexane	0.340345	$2.4^\xi + 2.18^\xi + 2.36^\xi + 81^\xi$	$2.5^\xi + 2.31^\xi + 2.28^\xi + 54^\xi$
2methyl-3ethylpentane	0.332433	$2.4^\xi + 2.40^\xi + 2.16^\xi + 80^\xi$	$2.5^\xi + 2.31^\xi + 54^\xi + 2.28^\xi$
3methyl-3ethylpentane	0.306899	$3.5^\xi + 3.75^\xi + 45^\xi$	$3.5^\xi + 3.260^\xi + 257^\xi$
2,2,3-trimethyl-pentane	0.300816	$3.39^\xi + 130^\xi + 60^\xi + 40^\xi + 4^\xi$	$3.257^\xi + 283^\xi + 28^\xi + 31^\xi + 5^\xi$
2,2,4-trimethyl-pentane	0.30537	$3.39^\xi + 91^\xi + 49^\xi + 2.14^\xi$	$3.257^\xi + 260^\xi + 31^\xi + 2.28^\xi$
2,3,3-trimethyl-pentane	0.293177	$5^\xi + 75^\xi + 2.45^\xi + 135^\xi + 2.18^\xi$	$5^\xi + 260^\xi + 2.257^\xi + 283^\xi + 2.28^\xi$
2,3,4-trimethyl-pentane	0.317422	$4.16^\xi + 2.80^\xi + 20^\xi$	$5.28^\xi + 2.54^\xi$
2,2,3,3-tetramethyl-butane	0.255294	$6.45^\xi + 225^\xi$	$6.257^\xi + 512^\xi$

$$H_\xi(H_n) = \sum_{gs \in E_1} 2(d(g) + d(s))^\xi + \sum_{gs \in E_2} 2(d(g) + d(s))^\xi + \sum_{gs \in E_3} 2(d(g) + d(s))^\xi = (5^\xi + 2^{3\xi} + (4+n)^\xi)2n,$$

$$GA_\xi(H_n) = \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_1} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} + \sum_{gs \in E_2} \frac{2(d_g \cdot d_s)^\xi}{(d_g + d_s)} = (2^{2\xi} \cdot 5^{-1} + 2^{4\xi-3} + 2^{2\xi} \cdot n^\xi (4+n)^{-1})2n,$$

$$KB_1^\xi(H_n) = \sum_{gs \in E_1} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) + S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) + S_e(s))^\xi$$

$$= ((20+n)^\xi + 2^\xi(17+n)^\xi + (n^2 + 3n + 17)^\xi)n,$$

TABLE 9: Experimental values of DHVAP and the corresponding values of generalized form of second Kulli–Basava index and generalized power-sum-connectivity index of octane isomers.

Alkane	DHVAP	KB_2^ξ	Y_ξ
n-octane	9.915	$2.3^\xi + 2.12^\xi + 3.16^\xi$	$2.5^\xi + 5.8^\xi$
2-methylheptane	9.484	$3^\xi + 12^\xi + 2.14^\xi + 16^\xi + 20^\xi + 35^\xi$	$4^\xi + 3.8^\xi + 31^\xi + 2.28^\xi$
3-methylheptane	9.521	$3^\xi + 12^\xi + 20^\xi + 40^\xi + 16^\xi + 32^\xi + 4^\xi$	$2.5^\xi + 2.8^\xi + 2.31^\xi + 28^\xi$
4-methylheptane	9.483	$2.3^\xi + 2.15^\xi + 16^\xi + 2.40^\xi$	$2.5^\xi + 2.8^\xi + 31^\xi + 28^\xi$
3-ethylhexane	9.476	$2.4^\xi + 2.36^\xi + 45^\xi + 15^\xi + 3^\xi$	$3.5^\xi + 3.31^\xi + 8^\xi$
2,2-dimethylhexane	8.915	$3.39^\xi + 78^\xi + 24^\xi + 12^\xi + 3^\xi$	$3.257^\xi + 2.8^\xi + 260^\xi + 5^\xi$
2,3-dimethylhexane	9.272	$2.16^\xi + 72^\xi + 18^\xi + 36^\xi + 12^\xi + 4^\xi$	$3.28^\xi + 54^\xi + 31^\xi + 8^\xi + 5^\xi$
2,4-dimethylhexane	9.029	$4^\xi + 16^\xi + 32^\xi + 48^\xi + 42^\xi + 2.14^\xi$	$5^\xi 3.28^\xi + 3.31^\xi$
2,5-dimethylhexane	9.051	$4.14^\xi + 2.35^\xi + 25^\xi$	$4.28^\xi + 2.28^\xi + 8^\xi$
3,3-dimethylhexane	8.973	$3^\xi + 5^\xi + 18^\xi + 2.52^\xi + 70^\xi + 84^\xi$	$2.5^\xi + 2.260^\xi + 2.257^\xi + 8^\xi$
3,4-dimethylhexane	9.316	$2.4^\xi + 2.18^\xi + 2.36^\xi + 81^\xi$	$2.5^\xi + 2.31^\xi + 2.28^\xi + 54^\xi$
2methyl-3ethylpentane	9.209	$2.4^\xi + 2.40^\xi + 2.16^\xi + 80^\xi$	$2.5^\xi + 2.31^\xi + 54^\xi + 2.28^\xi$
3methyl-3ethylpentane	9.081	$3.5^\xi + 3.75^\xi + 45^\xi$	$3.5^\xi + 3.260^\xi + 257^\xi$
2,2,3-trimethyl-pentane	8.826	$3.39^\xi + 130^\xi + 60^\xi + 40^\xi + 4^\xi$	$3.257^\xi + 283^\xi + 28^\xi + 31^\xi + 5^\xi$
2,2,4-trimethyl-pentane	8.402	$3.39^\xi + 91^\xi + 49^\xi + 2.14^\xi$	$3.257^\xi + 260^\xi + 31^\xi + 2.28^\xi$
2,3,3-trimethyl-pentane	8.897	$5^\xi + 75^\xi + 2.45^\xi + 135^\xi + 2.18^\xi$	$5^\xi + 260^\xi + 2.257^\xi + 283^\xi + 2.28^\xi$
2,3,4-trimethyl-pentane	9.014	$4.16^\xi + 2.80^\xi + 20^\xi$	$5.28^\xi + 2.54^\xi$
2,2,3,3-tetramethyl-butane	8.41	$6.45^\xi + 225^\xi$	$6.257^\xi + 512^\xi$

TABLE 10: Experimental values of HVAP and the corresponding values of generalized form of second Kulli–Basava index and generalized power-sum-connectivity index of octane isomers.

Alkane	HVAP	KB_2^ξ	Y_ξ
n-octane	73.19	$2.3^\xi + 2.12^\xi + 3.16^\xi$	$2.5^\xi + 5.8^\xi$
2-methylheptane	70.3	$3^\xi + 12^\xi + 2.14^\xi + 16^\xi + 20^\xi + 35^\xi$	$4^\xi + 3.8^\xi + 31^\xi + 2.28^\xi$
3-methylheptane	71.3	$3^\xi + 12^\xi + 20^\xi + 40^\xi + 16^\xi + 32^\xi + 4^\xi$	$2.5^\xi + 2.8^\xi + 2.31^\xi + 28^\xi$
4-methylheptane	70.91	$2.3^\xi + 2.15^\xi + 16^\xi + 2.40^\xi$	$2.5^\xi + 2.8^\xi + 31^\xi + 28^\xi$
3-ethylhexane	71.7	$2.4^\xi + 2.36^\xi + 45^\xi + 15^\xi + 3^\xi$	$3.5^\xi + 3.31^\xi + 8^\xi$
2,2-dimethylhexane	67.7	$3.39^\xi + 78^\xi + 24^\xi + 12^\xi + 3^\xi$	$3.257^\xi + 2.8^\xi + 260^\xi + 5^\xi$
2,3-dimethylhexane	70.2	$2.16^\xi + 72^\xi + 18^\xi + 36^\xi + 12^\xi + 4^\xi$	$3.28^\xi + 54^\xi + 31^\xi + 8^\xi + 5^\xi$
2,4-dimethylhexane	68.5	$4^\xi + 16^\xi + 32^\xi + 48^\xi + 42^\xi + 2.14^\xi$	$5^\xi 3.28^\xi + 3.31^\xi$
2,5-dimethylhexane	68.6	$4.14^\xi + 2.35^\xi + 25^\xi$	$4.28^\xi + 2.28^\xi + 8^\xi$
3,3-dimethylhexane	68.5	$3^\xi + 5^\xi + 18^\xi + 2.52^\xi + 70^\xi + 84^\xi$	$2.5^\xi + 2.260^\xi + 2.257^\xi + 8^\xi$
3,4-dimethylhexane	70.2	$2.4^\xi + 2.18^\xi + 2.36^\xi + 81^\xi$	$2.5^\xi + 2.31^\xi + 2.28^\xi + 54^\xi$
2methyl-3ethylpentane	69.7	$2.4^\xi + 2.40^\xi + 2.16^\xi + 80^\xi$	$2.5^\xi + 2.31^\xi + 54^\xi + 2.28^\xi$
3methyl-3ethylpentane	69.3	$3.5^\xi + 3.75^\xi + 45^\xi$	$3.5^\xi + 3.260^\xi + 257^\xi$
2,2,3-trimethyl-pentane	67.3	$3.39^\xi + 130^\xi + 60^\xi + 40^\xi + 4^\xi$	$3.257^\xi + 283^\xi + 28^\xi + 31^\xi + 5^\xi$
2,2,4-trimethyl-pentane	64.87	$3.39^\xi + 91^\xi + 49^\xi + 2.14^\xi$	$3.257^\xi + 260^\xi + 31^\xi + 2.28^\xi$
2,3,3-trimethyl-pentane	68.1	$5^\xi + 75^\xi + 2.45^\xi + 135^\xi + 2.18^\xi$	$5^\xi + 260^\xi + 2.257^\xi + 283^\xi + 2.28^\xi$
2,3,4-trimethyl-pentane	68.37	$4.16^\xi + 2.80^\xi + 20^\xi$	$5.28^\xi + 2.54^\xi$
2,2,3,3-tetramethyl-butane	66.2	$6.45^\xi + 225^\xi$	$6.257^\xi + 512^\xi$

$$KB_2^\xi(H_n) = \sum_{gs \in E_1} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_2} (S_e(g) \cdot S_e(s))^\xi + \sum_{gs \in E_3} (S_e(g) \cdot S_e(s))^\xi = n(17+n)^\xi (3^\xi + (17+n)^\xi + n^\xi (2+n)^\xi),$$

$$Y_\xi(H_n) = \sum_{gs \in E_1} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_2} (d(g)^{d(g)} + d(s)^{d(s)})^\xi + \sum_{gs \in E_3} (d(g)^{d(g)} + d(s)^{d(s)})^\xi.$$

(18)

9. Chemical Applicability of the Stated Indices in Generalized Form

In quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR),

topological indices with higher correlation factor are of the most interest. In this section of the paper, we are going to discuss the linear regression analysis of the generalized form of the aforementioned topological indices with entropy(S), acentric factor (AcentFac),

□

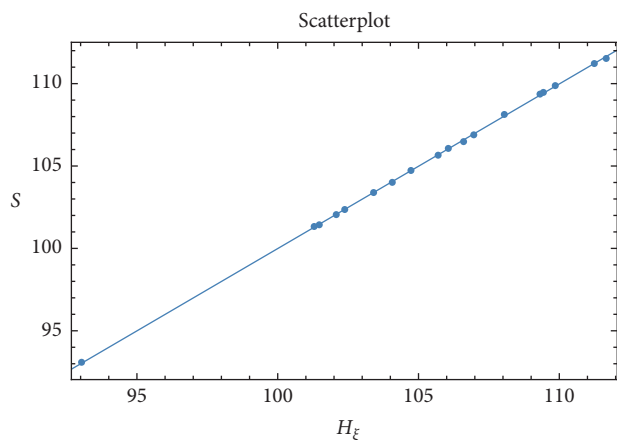


FIGURE 5: Scatter diagram of S on H_ξ superimposed by the fitted regression line.

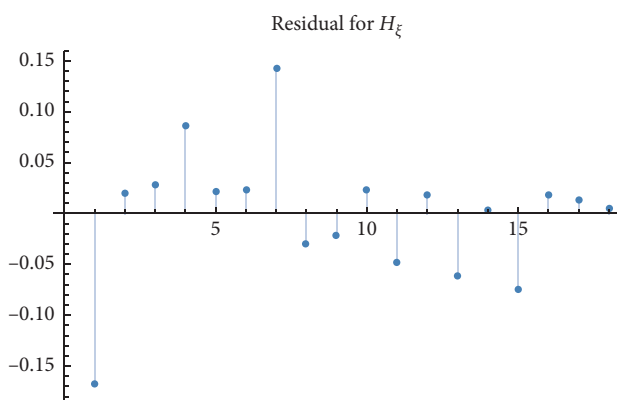


FIGURE 6: Plot shows the residuals for S on H_ξ .

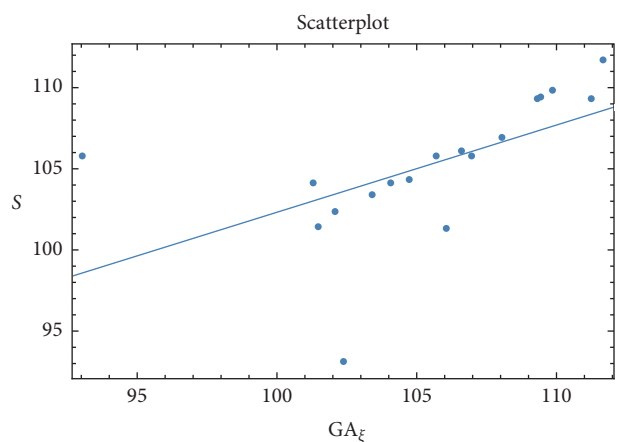


FIGURE 7: Scatter diagram of S on GA_ξ superimposed by the fitted regression line.

enthalpy of vaporization (HVAP), and standard enthalpy of vaporization (DHVAP) of octane isomers, which gives very closed relation regarding different properties of the stated chemicals which are labeled in Tables 1–10, where

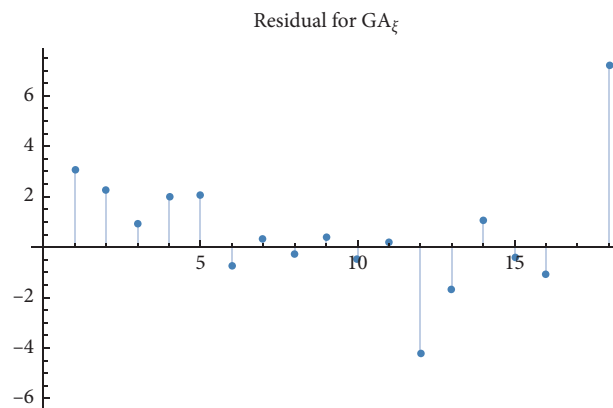


FIGURE 8: Plot shows the residuals for S on GA_ξ .

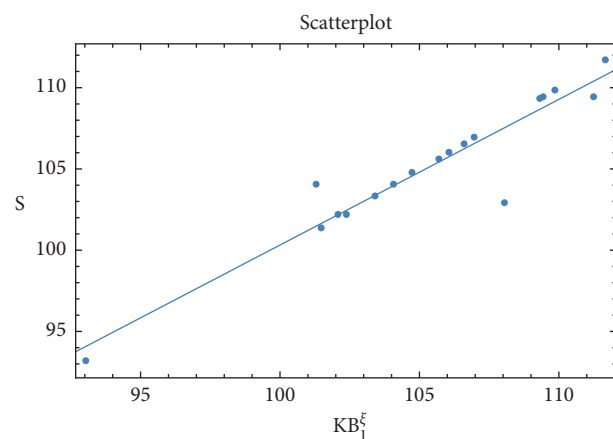


FIGURE 9: Scatter diagram of S on KB_1^ξ superimposed by the fitted regression line.

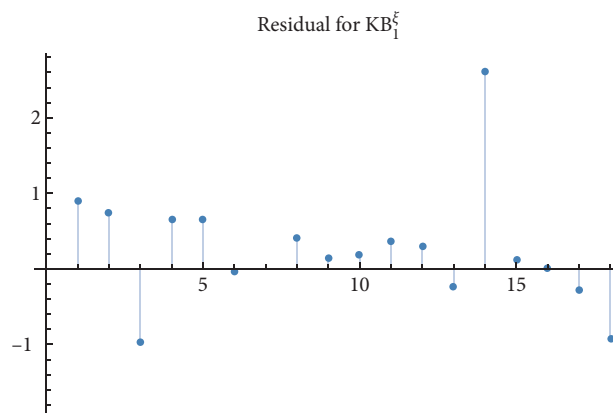


FIGURE 10: Plot shows the residuals for S on KB_1^ξ .

some values are found by plugging different values of ξ in the generalized obtained results. We also presented fitted models for the residuals that are obtained for some of the stated indices. The generalized form of the stated indices is tested using a data of octane isomers that are found in [11]. Statistical calculation of entropy of dominating

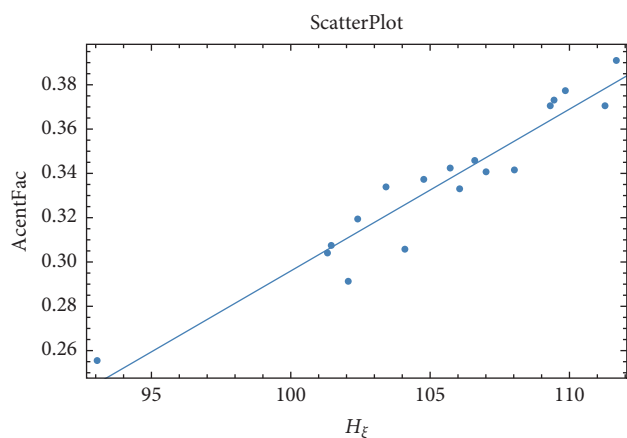


FIGURE 11: Scatter diagram of AcentFac on H_ξ superimposed by the fitted regression line.

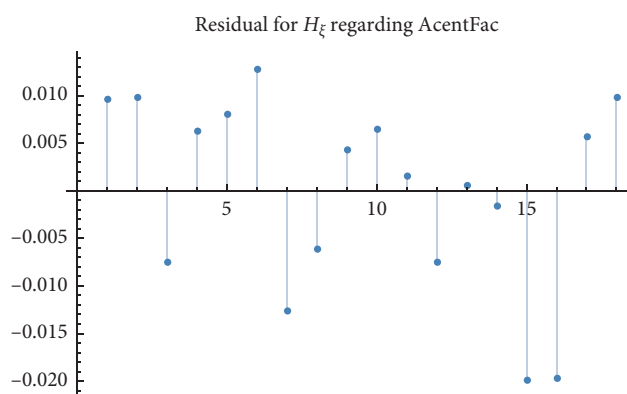


FIGURE 12: Plot shows the residuals for AcentFac on H_ξ .

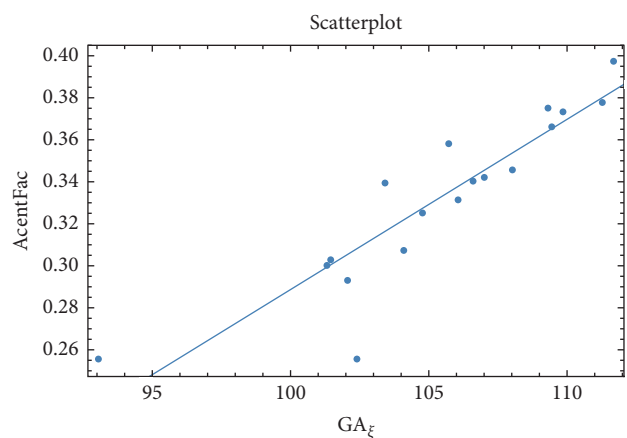


FIGURE 13: Scatter diagram of AcentFac on GA_ξ superimposed by the fitted regression line.

David derived networks can be found in [18], for further study regarding different properties and uses of indices in different chemicals also in other fields of science and technology can be found in the [11, 18–20] and the

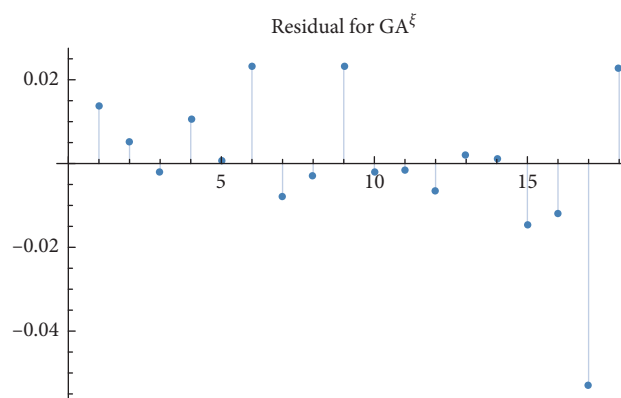


FIGURE 14: Plot shows the residuals for AcentFac on GA_ξ .

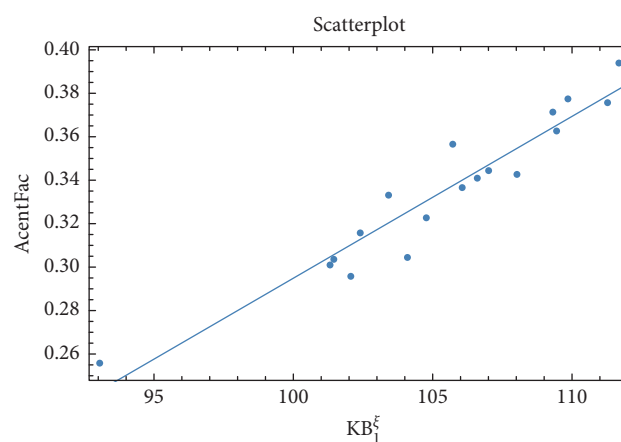


FIGURE 15: Scatter diagram of AcentFac on KB_1^ξ superimposed by the fitted regression line.

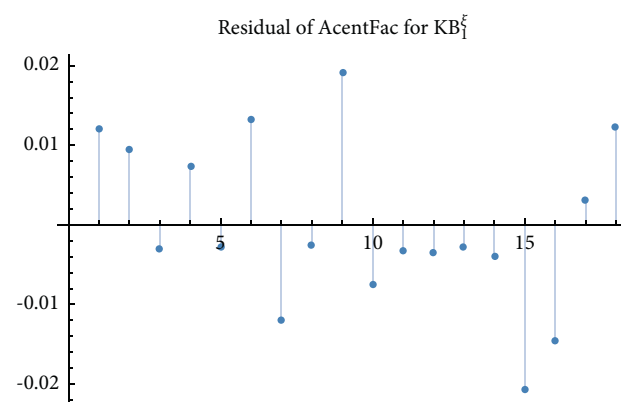


FIGURE 16: Plot for the residuals of AcentFac on KB_1^ξ .

references therein. Using the generalized values from the above table, we get the following interesting approximations for octane isomers which is of the most interest. Figure 5 represents scatter diagram of S on H_ξ superimposed by the fitted regression line. Figure 6 represents

the residuals for S on H_ξ . Figure 7 represents scatter plot of S on GA_ξ superimposed by the fitted regression line. Figure 8 represents the residuals for S on GA_ξ . Figure 9 represents scatter diagram of S on KB_1^ξ superimposed by the fitted regression line. Figure 10 is for the residuals for S on KB_1^ξ .

Figure 11 represents scatter diagram of AcentFac on H_ξ superimposed by the fitted regression line. Figure 12 is for the residuals regarding AcentFac on H_ξ . Figure 13 represents scatter diagram of AcentFac on GA_ξ superimposed by the fitted regression line. Figure 14 is for the residuals for AcentFac on GA_ξ . Figure 15 represents scatter diagram of AcentFac on KB_1^ξ superimposed by the fitted regression line. Figure 16 represents the residuals of AcentFac on KB_1^ξ .

As for different values of ξ , we obtained interesting results regarding different properties of alkanes. The results obtained by means of this generalized forms are closed to that of experimentally obtained values. In the remaining table, we can also find the approximate values of AcentFac, DHVAP, and HVAP. We can find closed values for the desired properties of the stated chemical by assigning different values to ξ . We found some fitted models be using Mathematica for S that are

$$\begin{aligned} S &= 0.074493 + 0.99916H_\xi, \\ S &= 48.5838 + 0.537382GA_\xi, \\ S &= 10.6755 + 0.896433KB_1^\xi. \end{aligned} \quad (19)$$

For AcentFac, we have the following fitted models:

$$\begin{aligned} \text{AcentFac} &= -0.434412 + 0.00730372H_\xi, \\ \text{AcentFac} &= 0.521876 + 0.00810559GA_1^\xi, \\ \text{AcentFac} &= -0.44993 + 0.0074482KB_1^\xi. \end{aligned} \quad (20)$$

For the remaining properties, we can find the fitted models as for some properties are given above in the last two equations.

10. Conclusion

In this study, various degree-based topological indices are studied in their generalized form for various well-known and special graphs that are bridge graph over path, bridge graph over cycle, bridge graph over complete graph, square lattice graph, wheel graph, gear graph, and helm graph. There are various significant applications of these indices in graph theory and chemistry which can be found in literature. We also studied the generalized form of the stated indices for various properties of alkanes which provided good result to the experimental values. In future, one can study this generalized form for other aspects of various chemicals.

Data Availability

All data used in the manuscript are included in the relevant places within the article.

Conflicts of Interest

The authors declare that they have no conflicts of Interest.

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