Hindawi
Journal of Mathematics
Volume 2022, Article ID 1436194, 12 pages
https://doi.org/10.1155/2022/1436194

Research Article

Double Domination and Regular Domination in Intuitionistic Fuzzy Hypergraph

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Received 16 April 2022; Revised 30 May 2022; Accepted 14 June 2022; Published 22 July 2022

Academic Editor: Gohar Ali

This study investigates the domination, double domination, and regular domination in intuitionistic fuzzy hypergraph (IFHG), which has enormous application in computer science, networking, chemical, and biological engineering. Few properties of double domination and regular domination of IFHG are established. Furthermore, the definitions of complement and independent set of IFHG are given. The relation between the domination of an IFHG and the independent set of its complement was discussed. Moreover, the application of the double domination in the IFHG was illustrated by determining the containment zones for epidemic situations like COVID-19.

1. Introduction

Presently, the unavailability of complete information arises for complex processes in technology and science features. To handle such situations, in various types of uncertain elements of systems, the mathematical models need to be developed; a vast number of these models are based on an extension of the ordinary set theory to the fuzzy sets.

Zadeh [1] introduced the concept of a fuzzy set in 1965. Zhang et al. [2] analyzed the hesitant fuzzy preference relations. The theory and application of the Intuitionistic Fuzzy Sets were discussed by Atanassov [3]. In defining fuzzy sets, Atanassov added the new component degree of nonmembership to determine Intuitionistic Fuzzy Sets (IFS). The degree of membership and nonmembership is almost independent. However, the sum of these degrees should be less than or equal to one.

Graph theory has enormous applicability in the electrical industry, computer science, system analysis, economics, and biochemistry. Bondy and Murthy [4] discussed several concepts and applications of graph theory. The domination in graphs was examined by Cockayne et al. [5]. The uncertainty of graph-theoretic problems arises in several cases. In such cases, the uncertainty could be dealt with using fuzzy sets and Intuitionistic Fuzzy Sets. Nagoorgani and Sajith Begum [6] defined the degree, order, and size in Intuitionistic Fuzzy Graph (IFG) and extended the properties. New concepts in IFG were initiated by Shao et al. [7]. He elaborated on the application of the IFG in the water supply system. Hypergraphs are a generation of graphs in the case of a set of multiarray relations and have been considered a valuable tool for studying the system's structure. The notion of hypergraph has extended with the fuzzy theory and Intuitionistic Fuzzy Sets as Intuitionistic Fuzzy Hypergraph (IFHG).

Moderson and Nair [8] defined fuzzy hypergraphs. Pradeepa and Vimala [9] examined the regular and totally regular intuitionistic fuzzy hypergraph (IFHG). Yahya and Mohammad Ali [10] described the max product of complement of IFG. Laqman et al. [11] studied the hypergraph representations of complex fuzzy information. Akram and Sarwar [12] discussed the applicability of m-polar fuzzy competition graphs. Akram and Nagoorgani [13] described the strong intuitionistic fuzzy graphs. Several researchers have contributed to the field of IFHG and elaborated its applications [14–17]. Domination in graphs has applicability in problems related to monitoring communication networks and application in LPG supply systems.

Nagoorgani et al. [18] described some exciting properties of the fuzzy dominating set, fuzzy independent set, and fuzzy minimal dominating set. He also established a new type of dominating fuzzy graphs. Domination in fuzzy graphs using strong edges was introduced by Nagoorgani and Chandrasekaran [19]. Somasundaram and Somasundaram [20] gave more concepts of independent domination and connected domination in the fuzzy graph. Nazeer et al. [21] discussed the domination of fuzzy incidence graphs with the algorithm and application for the selection of medical lab. Enriquez et al. [22] discussed the domination in fuzzy directed graphs. Finally, Rana [23] elaborated on a survey on the domination of fuzzy graphs.

Several researchers have examined the theory and application of domination graphs, fuzzy graphs, and IFHG [24–31]. However, no research has been established on domination, double domination, and regular domination in Intuitionistic Fuzzy Hypergraph (IFHG), which has enormous application in computer science, networking, chemical, and biological engineering. This study discussed domination, double domination, and regular domination in Intuitionistic Fuzzy Hypergraph (IFHG). Furthermore, the relation between the domination of an Intuitionistic Fuzzy Hyper Graph and the independent set of its complement was discussed and derived some results with proof and examples. Finally, the application for determining the containment zones for epidemic situations like COVID-19 is given.

2. Basic Definitions and Examples

This section gives some basic definitions of Intuitionistic Fuzzy Hypergraph (IFHG). Also, the definition of the double dominating fuzzy hypergraph, regular dominating, and regular double dominating set complement fuzzy hypergraph are introduced.

Definition 1. Consider the universal set U. The fuzzy set A in U is represented by $A = \{(x, \mu_A(x))\colon (x) > 0, x \in E\}$, where the function $\mu_A\colon U \longrightarrow [0,1]$ is the membership degree of element $x \in U$ in the fuzzy set A.

Definition 2. For a fixed set U be, an Intuitionistic Fuzzy set (IFS) A in U is of the form $A = \{(x, \mu_A(x), \gamma_A(x))\colon (x) > 0, x \in U\}$, where the function $\mu_A\colon U \longrightarrow [0,1]$ and $\gamma_A\colon U \longrightarrow [0,1]$ are determined as the "membership, and non-membership degree of element $x \in U$ respectively" and for every $x \in U$,

$$0 \le \mu_A(x) + \gamma_A(x) \le 1. \tag{1}$$

Definition 3. An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$, where

(i) $V = \{v_1, v_2, \ldots v_m\}$ such that $\mu_1\colon V \longrightarrow [0,1]$ and $\gamma_1\colon V \longrightarrow [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V\,(i = 1, 2, \ldots m)$

(ii) $E \in (V \times V)$, where such that $\mu_1\colon V \longrightarrow [0,1]$ and $\gamma_1\colon V \longrightarrow [0,1]$ are such that

$$\begin{aligned} \mu_2(v_i, v_j) &\le \min[\mu_1(v_i), \mu_1(v_j)],\\ \gamma_2(v_i, v_j) &\le \max[\gamma_1(v_i), \gamma_1(v_j)], \end{aligned} \tag{2}$$

and $0 \le \mu_2(x) + \gamma_2(x) \le 1$ for every $(v_i, v_j) \in E$, where $i = 1, 2 \ldots m,\ j = 1, 2. \ldots n.$

Here, the degree of membership and degree of nonmembership of the vertex v_i is denoted by the triple $(v_i, \mu_{1i}, \gamma_{1i})$. The degree of membership and degree of nonmembership of the edge relation $e_{ij} = (v_i, v_j)$ on V is denoted by the triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$.

Definition 4. The crisp subset of X in which all its elements have nonzero membership degree is defined as the support of the fuzzy set A:

$$\text{supp}\, p(A) = \{x\colon \mu_A(x) > 0\}. \tag{3}$$

Definition 5. An intuitionistic fuzzy hypergraph (IFHG) H is an ordered pair $H = (V, E)$, where

(i) $V = \{v_1, v_2, \ldots v_m\}$ a finite set of vertices

(iii) $E = \{E_1, E_2, \ldots E_m\}$ a finite intuitionistic fuzzy subset of V

(iii) $E_j = \{(v_i, \mu_j(v_i), \gamma_j(v_i))\colon \mu_j(v_i), \gamma_j(v_i) \ge 0\}$ and $0 \le \mu_j(v_i) + \gamma_j(v_i) \le 1\,(j = 1, 2, \ldots m)$

(iv) $E_j \ne \Phi,\ (j = 1, 2, \ldots m)$

(v) $\bigcup_j \text{supp}p(E_j) = V,\ (i = 1, 2 \ldots .m, j = 1, 2, \ldots n)$

Here the edges E_j are intuitionistic fuzzy sets. $\mu_j(v_i)$ and $\gamma_j(v_i)$ denote the degree of membership and degree of nonmembership of the vertex v_i to the edge E_j. Thus, the elements of the incidence matrix of intuitionistic fuzzy hypergraph are of the form $(v_{ij}, \mu_j(v_i), \gamma_j(v_i))$, the sets V and E are crisp sets.

Definition 6. An intuitionistic fuzzy hypergraph $H' = (V', E')$ is said to be an intuitionistic fuzzy subhypergraph (IFHSG) of $= (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$, that is, $\mu_{1i}' \le \mu_{1i}; \gamma_{1i}' \ge \gamma_{1i}$ and $\mu_{2ij}' \le \mu_{2ij}; \gamma_{2ij}' \ge \gamma_{2ij}$ for every $i,\ j = 1, 2, \ldots m$.

Definition 7. The order $O(H)$ of an intuitionistic fuzzy hypergraph H is the number of vertices of the number of hyperedges is called as the size of the intuitionistic fuzzy hypergraph and is denoted by $S(H)$.

Definition 8. The degree of a vertex V in an Intuitionistic fuzzy Hyper graph $H = (V, E)$ is defined as the sum of the weights of the strong edges incident at V. It is denoted by $d_h(v)$. The minimum degree of H is $\delta(H) = \min\{d_h(v)\colon v \in V\}$. The maximum degree of H is $\Delta(H) = \max\{d_h(v)\colon v \in V\}$."

Definition 9. An edge (v_i, v_j) of an IFHG is semi–μ-strong IFHG if $\mu_{2ij} = \mu_{1i}\mu_{1j}$, where, $i, j = 1, 2, \dots m$.

Example 1. From Figure 1and Table 1, semi–μ-strong edges are E_1E_2, E_4, E_5, E_6.

Definition 10. An edge (v_i, v_j) of an IFHG is semi–γ-strong IFHG if $\gamma_{2ij} = \gamma_{1i}\gamma_{1j}$.

Example 2. From Figure 2 and Table 2, semi–γ strong edges of H are. E_2, E_3,

Definition 11. An edge (v_i, v_j) of an IFHG is strong if $\mu_{2ij} = \mu_{1i}\mu_{1j}$ and $\gamma_{2ij} = \gamma_{1i}\gamma_{1j}$, where $i = 1, 2, \dots m$, $j =$ 1,2, ..., n.

Example 3. From Figure 3 and Table 3, the semi–μ-strong edges of H are E_1, E_2, E_3 and semi-γ-strong edges are E_1, E_2, E_3, E_4.

Therefore, the strong edges of H are E_1, E_2, and E_3.

Definition 12. Any two subsets of vertices v_i, v_j of an IFHG are semi–μ-dominating each other if there exist a semi–μ-strong edge (v_i, v_j).

Definition 13. Any two subsets of vertices v_i, v_j of an IFHG are semi–γ-dominating each other if there exist a semi γ-strong edge (v_i, v_j).

Definition 14. Any two subsets of vertices v_i, v_j of an IFHG are dominating each other if there exist a strong edge (v_i, v_j).

Definition 15. Let H be an IFHG, a subset D_h of V is called a dominating set of H, if for every v belongs to V-D_h, there is u belongs to D_h, such that u and v are strong neighbors.

Example 4. From Figure 4 and Table 4, the strong edges of the IFHG H are E_1, E_2, E_5. The dominating sets of IFHG H are $(v_1, v_{3,}v_5), (v_2, v_3, v_6)$.

Definition 16. An IFHG, $H = (V, E)$ is complete μ-strong IFHG, if $\mu_{2ij} = \min(\mu_{1i}\mu_{1j})$ and $\gamma_{2ij} < \max(\gamma_{1i}\gamma_{1j})$, where $(v_i, v_j) \in V$.

Definition 17. An IFHG, $H = (V, E)$ is complete γ-strong IFHG, if $\mu_{2ij} < \min(\mu_{1i}\mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, where $(v_i, v_j) \in V$.

Definition 18. An IFHG, $H = (V, E)$ is complete IFHG, if $\mu_{2ij} = \min(\mu_{1i}\mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}\gamma_{1j})$, where $(v_i, v_j) \in V$.

Definition 19. Let H be an IFHG, a subset S of V is called independent set of H if $\mu_{2ij} \neq \mu_{1i}\mu_{1j}$ and $\gamma_{2ij} \neq \gamma_{1i}\gamma_{1j}$, where $i = 1, 2 \dots m$, $j = 1, 2 \dots .n$.

Example 5. From Figure 5 and Table 5, the strong edges of the IFHG H are E_1, E_3, E_4.

The independent sets of the IFHG H are $\{(v_1, v_{6,}v_7), (v_2, v_7, v_6)....\}$

Definition 20. An independent set S of an IFHG H is said to be a maximal independent set, if for every vertex $v \in V - S$, the set $S \bigcup \{v\}$ is not an independent set.

Definition 21. Two vertices in an IFHG, $H = (V, E)$ are said to be independent set if there is no strong edge between them.

Definition 22. A vertex $u \in V$ of an IFHG, $H = (V, E)$ are isolated vertices if $\mu_2(u, v) = 0$ and $\gamma_2(u, v) = 0$ for all $u \in V$.(i.e.) $N(u) = \varnothing$.

Thus, the other vertices in H should not be dominated by an isolated vertex.

Definition 23. The complement of an IFHG, $H = (V, E)$ is an IFHG $\overline{H} = (\overline{V}, \overline{E})$, where

(i) $\overline{V} = V$

(ii) $\overline{\mu}_{1i} = \mu_{1i}$ and $\overline{\gamma}_{1i} = \gamma_{1i}$ for all $i = 1, 2, \dots m$

(iii) $\overline{\mu}_{2ij} = \min\{(\mu_{1i}, \mu_{1j}) - \mu_{2ij}\}$ and $\overline{\gamma}_{2ij} = \min\{(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij}\gamma_{1j}\}$ for all $i = 1, 2, \dots m$, $j = 1, 2...n$.

Example 6. From Tables 6 and 7, it is found that IFHG in Figure 7 is the complement IFHG of Figure 6.

Definition 24. Let $H = (V, E)$ be an IFHG. A subset D_{2h} of V is a double dominating set of H, if for each vertex in V-D_{2h} is dominated at least by two vertices in D_{2h} is the minimum fuzzy cardinality of all the double dominating set of H is defined as the double domination number of H and denoted by $\gamma_{ddh}(H)$.

Example 7. From Figure 8 and Table 8, the strong arcs are E_1E_2, E_3, E_4, so the double dominating set of the IFHG is $D_{2h} = \{v_2, v_{6,}v_{4,,}v_5\}$, where V-$D_{2h} = \{v_{1,}v_6\}$

Definition 25. A vertex $V \in H$ is an end vertex of IFHG, and it has at most one strong neighbor in H.

Definition 26. A vertex V_j, $(j = 1, 2...n)$ is said to be a cut vertex in IFHG if deleting a vertex V_i where $(i = 1, 2...m)$ reduces the strength of the connectedness between some pair of vertices.

Definition 27. Let u be a vertex in an IFHG, $H = (V, E)$ the set $N(u) = \{v \in V/(u, v)\text{is a strong arc}\}$ is called neighborhood of u.

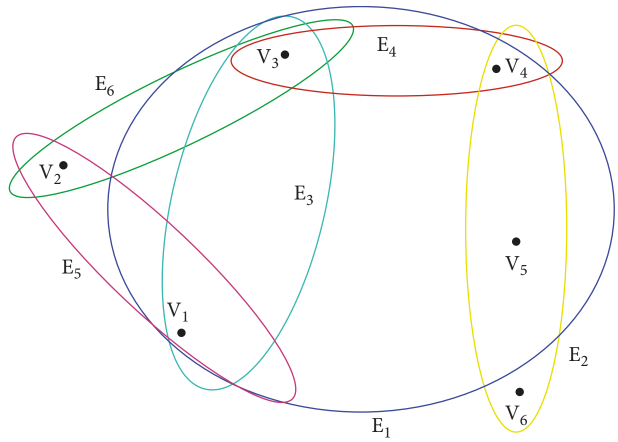

Figure 1: Semi–μ-strong edges of IFHG H.

Table 1: Degree of membership and nonmembership values for vertices and edges of Figure 1.

Vertices v_i	v_1	v_2	v_3	v_4	v_5	v_6
(μ_{1i}, γ_{1i})	(0.2,0.3)	(0.5,0.1)	(0.4,0.3)	(0.2,0.6)	(0.1,0.1)	(0.4,0.1)
Edges E_J	E_1	E_2	E_3	E_4	E_5	E_6
(μ_{2i}, γ_{2i})	(0.0016,0.0054)	(0.008,0.016)	(0.8,0.09)	(0.08,0.18)	(0.1,0.03)	(0.2,0.03)

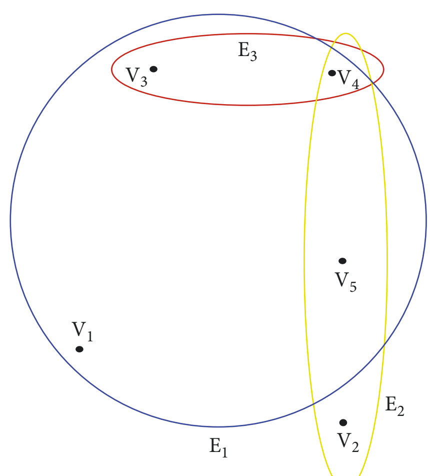

Figure 2: Semi–γ-strong edge of IFHG H.

Table 2: Degree of membership and nonmembership values for vertices and edges of Figure 2.

Vertices v_i	v_1	v_2	v_3	v_4	v_5
(μ_{1i}, γ_{1i})	(0.6,0.1)	(04,0.2)	(0.1,0.2)	(0.3,0.5)	0.3,0.1)
Edges E_J	E_1	E_2	E_3	--	--
(μ_{2i}, γ_{2i})	(0.03,0.7)	(0.036,0.01)	(0.0054,0.001)	--	--

Definition 28. Let $H = (V.E)$ be an IFHG. A set D_{rh} subset of V is called regular intuitionistic fuzzy hyper dominating set if

(i) Every vertex in $V - D_{rh}$ has strong edge to at least one vertex in D_{rh}.

(ii) The hyperdegree of all the vertices in D_{rh} should be same.

Definition 29. Let $H = (V.E)$ be an IFHG. A set D_{r2h} subset of V is called regular double dominating set of intuitionistic fuzzy Hyper graph if

(i) Every vertex in $V - D_{r2h}$ adjacent to at least two vertices in D_{2h}.

(ii) All the vertices in D_{2h} has the same hyperdegree.

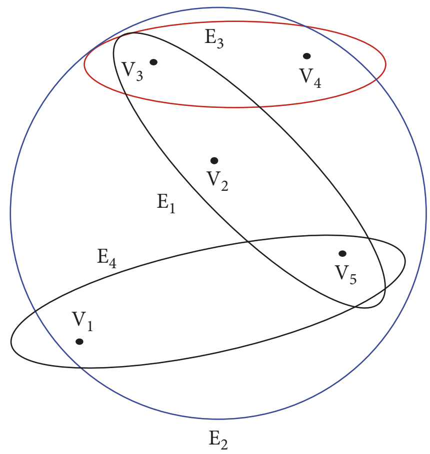

FIGURE 3: Strong edge of IFHG H.

TABLE 3: Degree of membership and nonmembership values for vertices and edges of Figure 3.

Vertices v_i	v_1	v_2	v_3	v_4	v_5
(μ_{1i}, γ_{1i})	(0.03,0.5)	(0.1,0.1)	(0.5,0.4)	(0.7,0.2)	(0.1,0.3)
Edges E_J	E_1	E_2	E_3	E_4	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.005,0.012)	(0.0007,0.0012)	(0.35,0.08)	(0.2,0.15)	--

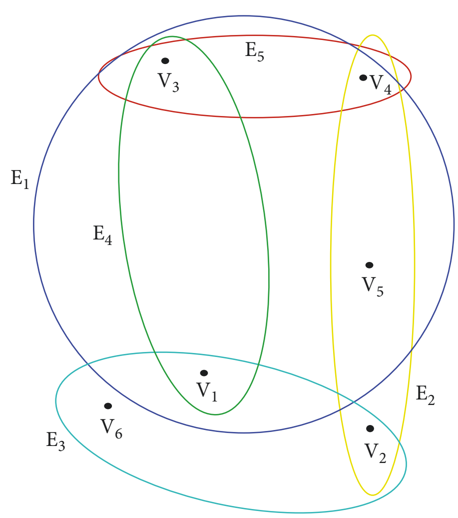

FIGURE 4: Dominating set of IFHG H.

TABLE 4: Degree of membership and nonmembership values for vertices and edges of Figure 4.

Vertices v_i	v_1	v_2	v_3	v_4	v_5	v_6
(μ_{1i}, γ_{1i})	(0.1,0.3)	(0.7,0.1)	(0.5,0.4)	(0.2,0.3)	(0.6,0.3)	(0.1,0.1)
Edges E_J	E_1	E_2	E_3	E_4	E_5	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.006,0.0108)	(0.084,0.009)	(0.007,0.4)	(0.2,0.5)	(0.1,0.12)	--

Example 8. From Figure 9 and Table 9, the strong arcs of the IFHG H are $E_2 E_3, E_4$; hence, the regular double dominating set of H is $D_{r2h} = \{v_1, v_2\}$, here $D_{r2h} = \{v_1, v_2\}$.

Definition 30. The minimum cardinality of regular intuitionistic fuzzy hyper dominating set is regular intuitionistic fuzzy hyper domination number, and it is denoted by $\gamma_{rifh}(H)$.

Definition 31. Let H be an IFHG. A set D_h subset of V is called minimal regular intuitionistic fuzzy hyper dominating set if

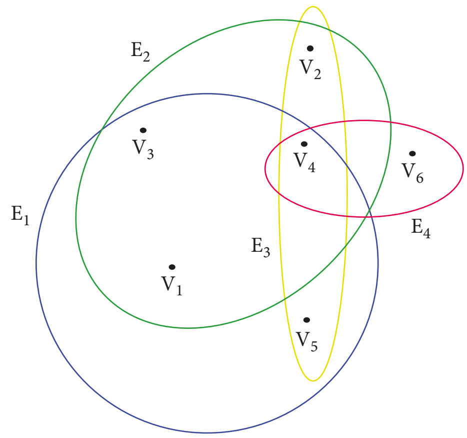

Figure 5: Independent set S of IFHG H.

Table 5: Degree of membership and nonmembership values for vertices and edges of Figure 5.

Vertices v_i	v_1	v_2	v_3	v_4	v_5	v_6
(μ_{1i}, γ_{1i})	(0.2,0.7)	(0.6,0.2)	(0.4,0.3)	(0.3,0.5)	(0.3,0.1)	(0.1,0.1)
Edges E_J	E_1	E_2	E_3	E_4	--	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.0052,0.0105)	(0.1,0.5)	(0.054,0.01)	(0.03,0.05)		

Table 6: Degree of membership and nonmembership values for vertices and edges of Figure 6.

Vertices v_i	v_1	v_2	v_3	v_4	v_5
(μ_{1i}, γ_{1i})	(0.3,0.6)	(0.3,0.4)	(0.5,0.2)	(0.7,0.1)	(0.1,0.1)
Edges E_J	E_1	E_2	E_3	E_4	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.25,0.002)	(0.00315,0.00048)	(0.000315,0.000048)	(0.009,0.024)	

Table 7: Degree of membership and nonmembership values for vertices and edges of Figure 7.

Vertices $\overline{V}_i$	$\overline{v}_1$	$\overline{v}_2$	$\overline{v}_3$	$\overline{v}_4$	$\overline{v}_5$
$(\overline{\mu}_{1i}, \overline{\gamma}_{1i})$	(0.3,0.6)	(0.3,0.4)	(0.5,0.2)	(0.7,0.1)	(0.1,0.1)
Edges $\overline{E}_j$	$\overline{E}_1$	$\overline{E}_2$	$\overline{E}_3$	$\overline{E}_4$	--
$(\overline{\mu}_{2ij}, \overline{\gamma}_{2ij})$	(0.5,0.58)	(0.09,0.36)	(0.0085,0.003)	(0.792,0.7)	

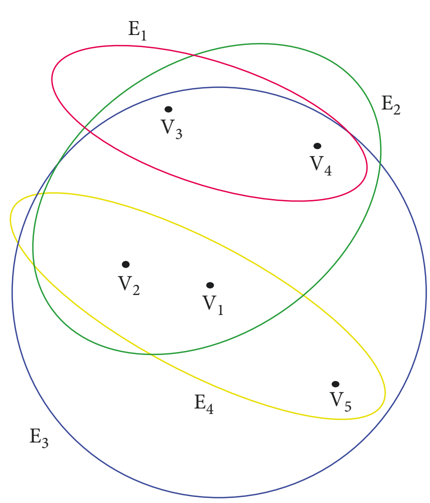

Figure 6: IFHG H.

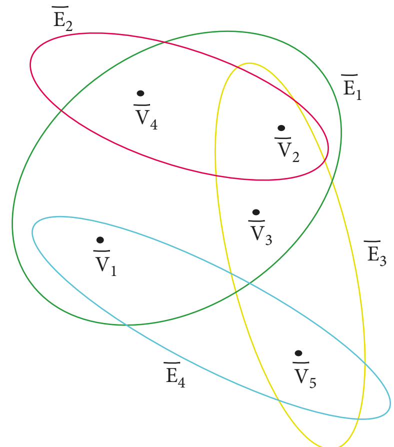

Figure 7: The complement $\overline{H}$ of IFHG H.

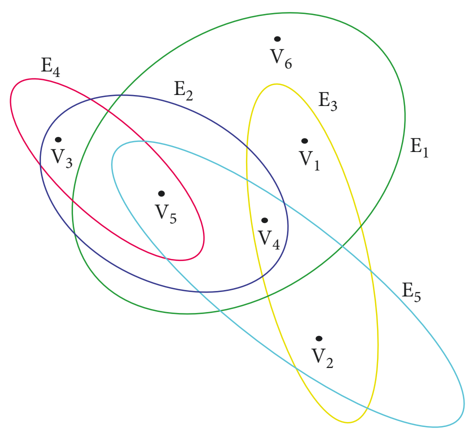

Figure 8: Double dominating set D_{2h} of an IFHG.

Table 8: Degree of membership and nonmembership values for vertices and edges of Figure 8.

Vertices v_i	v_1	v_2	v_3	v_4	v_5	v_6
(μ_{1i}, γ_{1i})	(0.5,0.4)	(0.6,0.4)	(0.11,0.5)	(0.1,0.2)	(0.4,0.4)	(0.3,0.5)
Edges E_J	E_1	E_2	E_3	E_4	E_5	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.006,0.016)	(0.004,0.05)	(0.03,0.032)	(0.04,0.2)	(0.026,0.032)	--

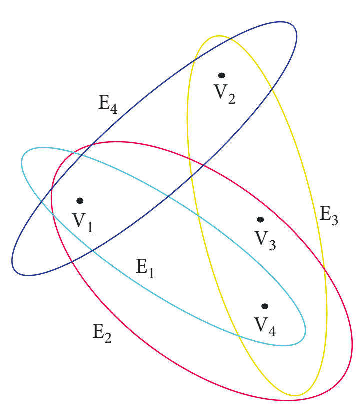

Figure 9: Regular double dominating set D_{r2h} of IFHG H.

Table 9: Degree of membership and nonmembership values for vertices and edges of Figure 9.

Vertices v_i	v_1	v_2	v_3	v_4
(μ_{1i}, γ_{1i})	(0.5,0.5)	(0.7,0.2)	(0.1,0.1)	(0.2,0.3)
Edges E_J	E_1	E_2	E_3	E_4
$(\mu_{2ij}, \gamma_{2ij})$	(0.3,0.15)	(0.001,0.015)	(0.014,0.006)	(0.35,0.1)

(i) Any subset of D_h is not a regular intuitionistic fuzzy hyper dominating set.

(ii) All the vertices in D_h has the same hyperdegree.

Definition 32. An independent set of an IFHG, $H = (V, E)$ is said to be regular independent intuitionistic fuzzy hyper set if a set D_h is subset of V,

(i) $\mu_{2ij} \neq \mu_{1i}\mu_{1j}$ and $\gamma_{2ij} \neq \gamma_{1i}\gamma_{1j}$

(ii) All the vertices in D_h has d hyperdegree and d hyperedges.

Definition 33. Let H be an IFHG. A set D_h subset of V is called maximal regular independent intuitionistic fuzzy hyperset if for every vertex $v \in V - D_h$, the set $D_h \bigcup \{v\}$ is not a regular independent intuitionistic fuzzy hyperset.

3. Results and Proofs

Theorem 1. *The double dominating set of an IFHG exists only if every vertex in $V - D_{2h}$ contains at least two other vertices as strong neighbors.*

Proof. Let D_{2h} is a double dominating set of an IFHG. If there exists a vertex in $V - D_{2h}$ with a single strong neighbor, let it be u and its strong neighbor is v. □

Case 1. If $v \in V - D_{2h}$, then $u \in V - D_{2h}$ has no strong neighbor in D_{2h}, this implies that D_{2h} cannot be a double dominating set.

Case 2. If $V \in D_{2h}$ such that $u \in V - D_{2h}$ has exactly one strong neighbor, again, this implies that D_{2h} cannot be a double dominating set.

We obtain contradiction in both the cases. Hence, there exist at least two strong neighbors for every vertex in $V - D_{2h}$.

Example 9. From Figure 10 and Table 10, the strong arcs of IFHG H are E_1E_3, E_4; hence, $D_{2h} = \{v_2, v_{3,}v_{4,,}v_5\}$, $V - D_{2h} = \{v_1\}$.

v_2 and v_3 has at least two neighbors.

Theorem 2. *The double dominating set D of an IFHG H is the set of all vertices V of that IFGH if and only if all of its vertices are end vertices.*

Proof. Let all the vertices of IFHG H are end vertices. Since H has only end vertices, all of its vertices have exactly one strong neighbor.

Also from Theorem 1, all the vertices in $V - D_{2h}$ should have at least two strong neighbors, which imply that there are no vertices in $V - D_{2h}$ double dominating set.

That is, $V - D_{2h} = \Phi$, Hence, we have $V - D_{2h}$. □

Example 10. From Figure 11 and Table 11, the strong arcs are $E_1E_{3.}$, and all the vertices of H are end vertices. Hence, the double dominating set of H is V itself.

Theorem 3. *Let H be an IFHG, and if the double dominating set D_{2h} of H is an independent set of H, then it was not an independent set of $\overline{H}$.*

Proof. Let D_{2h} be a double dominating set of H. If D_{2h} is an independent set of H. Let $\overline{H}$ be a complement of intuitionistic fuzzy hypergraph H. In $\overline{H} = (\overline{V}, \overline{E})$, where $\overline{V} = V$, $\overline{\mu}_{1i} = \mu_{1i}$ and $\overline{\gamma}_{1i} = \gamma_{1i}$ for all $i = 1, 2, \ldots m$, $\overline{\mu}_{2ij} = \min\{(\mu_{1i}, \mu_{1j}) - \mu_{2ij}\}$ and $\overline{\gamma}_{2ij} = \min\{(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij}\}$, for all $i, = 1, 2, \ldots m$, $j = 1, 2 \ldots n$. Here only the values of hyperedges are changed in $\overline{H}$, which implies most of the adjacent vertices in $\overline{H}$ have strong neighbors and also different double dominating set exists in $\overline{H}$.

Therefore, the same double dominating set D_{2h} in H cannot be an independent set of $\overline{H}$. □

Theorem 4. *A regular independent set is a regular maximal independent set of an IFHG if it is regular independent and dominating set of IFHG.*

Proof. Let D_{2h} be a regular maximal independent set in an IFHG, then for every $u \in V - D_{rh}$ the set $D_{rh} \bigcup \{u\}$ is not an independent set. It is trivial that D_{2h} is regular independent of H. It is enough to prove that D_{2h} Regular dominating set, suppose that D_{2h} is not a regular dominating set, and then there exists at least one vertex $v \in V - D_{rh}$, such that there is no strong neighbor for v in D_{rh}, which implies that $D_{rh} \bigcup \{v\}$ is an independent set of H. This contradicts the fact that D_{rh} is a regular maximal independent set of an IFHG H. Hence, D_h is regular dominating and independent set of IFHG. □

Theorem 5. *A regular dominating set is a regular minimal dominating set of an IFHG if it is regular dominating and independent set of IFHG.*

Proof. Let D_{rh} is regular minimal dominating set of H. It is trivial that D_{rh} is regular dominating of H. It is enough to prove that D_{rh} Regular independent set, suppose that D_{rh} is not a regular independent set, then there exists at least one vertex $v \in D_{rh}$ such that at least one vertex in $D_{rh} - \{v\}$ has a strong hyper-edge to V, which implies $D_{rh} - \{v\}$ is regular dominating set. This contradicts the fact that D_{rh} is a regular minimal dominating set of an IFHG H.

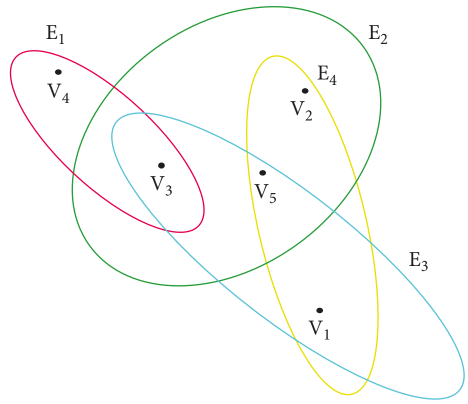

Figure 10: IFHGH.

Table 10: The values for degree of membership and nonmembership of vertices and edges of Figure 10.

Vertices v_i	v_1	v_2	v_3	v_4	v_5
(μ_{1i}, γ_{1i})	((0.1,0.1)	(0.3,0.5)	(0.1,0.3)	(0.5,0.5)	(0.7,0.1)
Edges E_J	E_1	E_2	E_3	E_4	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.05,0.15)	(0.027,0.015)	(0.007,0.003)	(0.021,0.005)	--

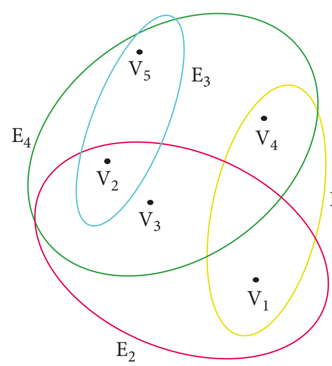

Figure 11: IFGH H with only end nodes.

Table 11: Degree of membership and nonmembership values for vertices and edges of Figure 11.

Vertices v_i	v_1	v_2	v_3	v_4	v_5
(μ_{1i}, γ_{1i})	(0.3,0.6)	(0.2,0.4)	(0.7,0.1)	(0.1,0.8)	(0.1,0.3)
Edges E_J	E_1	E_2	E_3	E_4	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.03,0.48)	(0.942,0.624)	(0.02,0.12)	(0.0014,0.8096)	--

Hence, D_{rh} is regular dominating and independent set of IFHG. □

4. Application of the Present Study

Intuitionistic fuzzy graphs have applicability in several fields. In particular, the domination and double domination in IFHG could be applied in decision-making processes. Xing et al. [30] applied A Choquet integral-based interval Type-2 trapezoidal fuzzy multiple attribute group decision-making for sustainable supplier selection. In this work, we used the double domination of IFHG in decision-making for selecting containment zones. For the past two years, the spread of COVID-19 has been an important issue worldwide. Several countries put complete lockdown to control the spread of the epidemic. However, there are several difficulties with the complete lockdown. The lockdown is now imposed in particular areas (Containment zones) with more active COVID-19 positive cases.

The double domination number of IFHG can be applied to determine the containment zones for the epidemic situation like COVID-19. For example, let a city has recorded five active COVID-19 positive cases and if there are eight zones in the city. Now, draw an intuitionistic fuzzy

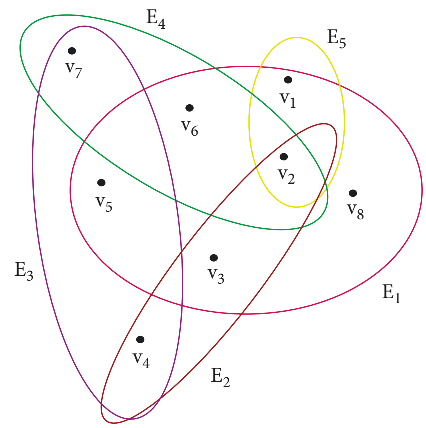

FIGURE 12: Application of double dominating in IFGH.

TABLE 12: Degree of membership and nonmembership values for vertices and edges of Figure 11.

Vertices v_i	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
(μ_{1i}, γ_{1i})	(0.1,0.6)	(0.2,0.5)	(0.7,0.1)	(0.1,0.3)	(0.7,0.3)	(0.6,0.1)	(0.2,0.8)	(0.1,0.1)
Edges E_J	E_1	E_2	E_3	E_4	E_5	--	--	--
$(\mu_{2ij}, \gamma_{2ij})$	(0.000588,0.00009)	(0.014,0.015)	(0.014,0.072)	(0.028,0.12)	(0.02,0.3)	--	--	--

hypergraph by considering the zones as vertices and the travel history of each COVID-19 positive person as hyperedges. The membership value of vertices could be obtained from the combination of data that escalates the rate of spread of the disease, such as population density and unawareness of people about COVID-19 in respective blocks. Further, nonmembership values may be obtained from a combination of data that decrease the spread of COVID-19, such as the administration's precautionary measures and the awareness of the people about COVID-19.

The membership value of edges could be obtained from the combination of data that escalate the rate of spread of the disease by the respective person. Similarly, nonmembership values may be obtained from a combination of data about the person that decreases the spread of COVID-19. It is easy to find the containment zones by finding that minimal double dominating set of the intuitionistic fuzzy hypergraph. Each vertex in the minimal double dominating set has to be set as a containment zone to control the further spread of COVID-19.

From Figure 12 and Table 12, we notice that all the hyperedges of the IFHG are strong edges, and $D_{2h} = \{v_2, v_5\}$ is the double dominating set of the IFHG. Hence, we can make the respective blocks v_2, v_5 of the city as the containment zones for controlling the spread of the epidemic.

5. Conclusion

The present analysis discussed domination, double domination, and regular domination in IFHG, which has enormous application in computer science, networking, and chemical and biological engineering. In particular, the domination and double domination in IFHG could be applied in decision-making processes. The double domination number of IFHG can be applied to determine the containment zones for the epidemic situation like COVID-19. Some important definitions are given with examples. Many real field problems can be solved using this technique of double domination, such as transportation problems, social networking problems, and sports modeling. We illustrated an application to determine the containment zones for epidemic situations like COVID-19 with a minimal double dominating set of IFHG. However, a few limitations are there. Calculating actual value, falsity, and indeterminacy from crisp data is challenging to capture. There are no available methods to find such data. Furthermore, the data relating to the travellers who have been wandering across the blocks has not been fully detailed. More real field problems can be solved in future studies through domination, double domination, and regular domination of IFHG.

The derived results are given below:

(i) The double dominating set of an IFHG exists only if every vertex in $V - D_{2h}$ contains at least two other vertices as strong neighbors.

(ii) The double dominating set D_{2h} of an IFHG H is the set of all vertices that IFGH V if all of its vertices are end vertices.

(iii) Let H be an IFHG, if the double dominating set D_{2h} of H is an independent set of H, then it is not an independent set of $\overline{H}$.

(iv) A regular independent set is a regular maximal independent set of an IFHG if it is a regular independent and dominating set of IFHG.

(v) A regular dominating set is a regular minimal dominating set of an IFHG if it is a regular dominating and independent set of IFHG.

Nomenclature

Notation: Meaning
U: Universal set
A: Fuzzy set
E: Edge set of the graphs
V: Vertices set of the graphs
$\mu_A(x)$: The membership degree of element $x \in U$
$\gamma_A(x)$: The nonmembership degree of element $x \in U$
$\mu_1(v_i)$: The membership degree of a vertex $v_i \in V$
$\gamma_1(v_i)$: The nonmembership degree of a vertex $v_i \in V$
$\mu_2(v_i, v_j)$: The membership degree of an edge $(v_i, v_j) \in E$
$\gamma_2(v_i, v_j)$: The nonmembership degree of an edge $(v_i, v_j) \in E$
IFHG: Intuitionistic Fuzzy Hypergraph
$d_h(v)$: The degree of vertex $v_i \in V$
$\delta(H)$: The minimum degree of the IFHG H
$\Delta(H)$: The maximum degree of IFHG H
D_h: The dominating set of IFHG H
D_{2h}: The double dominating set of IFHG H
D_{rh}: The regular dominating set of IFHG H

Data Availability

No data were used for this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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