Hindawi
Journal of Mathematics
Volume 2022, Article ID 1569643, 11 pages
https://doi.org/10.1155/2022/1569643

Research Article

A Study on m-Polar Interval-Valued Intuitionistic Fuzzy Graphs with Application in Management

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Received 12 April 2022; Accepted 7 June 2022; Published 5 July 2022

Academic Editor: Muhammad Kamran Jamil

Interval-valued intuitionistic fuzzy graphs (IVIFGs) have many fields of work in other sciences, including psychology, life sciences, medicine, and social studies, and can help researchers with optimization and save time and money. A very important type of IVIFG is an m-polar IVIFG that can play a significant role in managing and finding effective people in a company or institution. It can also be used to achieve very important results in the field of psychological sciences and social relations. Hence, in this study, we presented the notion of m-polar h-morphism on m-polar IVIFGs. Some elegant theorems on weak isomorphism (WI) and co-weak isomorphism (CWI) are obtained. Likewise, we survey m-polar h-morphism on strong regular (SR) and highly irregular (HI) m-polar-IVIFGs. Management always plays an important role in an organization and can cause its growth and prosperity. But finding a strong leader is a very important issue that needs to be addressed. Therefore, in the last section, we have tried to express the application of the m-polar IVIFG in the management problem.

1. Introduction

Graph theory, as a branch of mathematics, examines topics and the relationships between them. Simultaneously with the widespread influence of this concept in various phenomena, there were many ambiguities in determining the vertices and edges of the graph. Facing ambiguous amounts has made it difficult for researchers. With the definition of the fuzzy set (FS) by Atanassov [1] and the fuzzy graph (FG) by Atanassov and Gargov [2] and Akram et al. [3], a new chapter of work on graphs began by the researchers. The vague problems that preoccupied researchers were now easily solved. This idea soon came to the attention of researchers in this field, and then, different types of FGs were introduced one after another. Akram et al. [4] introduced fuzzy logic. Akram et al. [5] studied new results of regularity in vague graphs (VGs). Azeem and Nadeem [6] introduced the concept of intuitionistic fuzzy set (IFS) as a generalization of FSs. The idea of a bipolar FS was proposed by Azeem et al. [7, 8] in 1994. Bhutani [9] investigated new concepts in bipolar FGs. The interval-valued fuzzy graph (IVFG) was defined by Borzooei et al. [10]. Chen et al. [11] introduced interval-valued intuitionistic fuzzy sets (IVIFSs). Ezhilmaran and Sankar [12, 13] defined m-polar interval-valued intuitionistic fuzzy graphs (IVIFGs) and interval-valued intuitionistic fuzzy competition graphs. Ghorai and Pal [14–16] investigated the vague hypergraphs and Cayley VGs. Kaufman [17, 18] studied new concepts in vague graph structures. Kou et al. [19–21] defined dominating set and equitable dominating set in VGs. Nagoor Gani and Malarvizhi [22, 23] presented total dominating set (DS) and global DS in product VGs. Poulik and Ghorai [24] introduced new concepts in IFG. Poulik and Ghorai [25] investigated global restrained DS and total k-DS in VGs. Some properties of FGs and their types have been discussed by Ghorai and Poulik [26–29]. Ramprasad et al. [30–32] have studied some topics using graphs.

Rao et al. [33] proposed the m-polar FS as an extension of the bipolar FS. This concept was able to study the fuzzy values of several variables on the vertices and edges of a graph simultaneously. Rashmanlou et al. [34, 35] studied the properties of generalized m-polar FGs and m-polar fuzzy planar graphs.

Rosenfeld [36, 37] defined the isomorphism properties on FGs. Seethalakshmi and Gnanajothi [38] introduced WI, CWI, and isomorphism in FGs. Shi and Kosari [39] studied a generic definition of fuzzy morphism between graphs. Shi and Kosari [40] proved that an FG can be associated with a fuzzy group naturally as an automorphism group. Shao [41] presented the notion of f-morphism on IFGs. Talebi [42] studied morphism on m-Polar FG. Talebi [43, 44] has defined morphism on a bipolar IFG and m-polar IFG. Zhang [45] investigated isomorphisms on the L-fuzzy graphs. Some features of isomorphic m-polar FGs were introduced by Zhang [46].

Although IFGs are better at expressing uncertain variables than FGs, they do not perform well in many real-world situations such as IT management. Therefore, when the data come from several factors, it is necessary to use the m-polar IFG. m-polar IFG has better flexibility in the face of problems that cannot be articulated by FG and IFG. IVIFGs have many applications in real systems, especially where the level of information given in the system varies with time and accuracy. m-polarity is used to illustrate real-world phenomena using m-polar fuzzy models in a variety of fields including technology, social networking, and biological sciences. Hence, in this paper, while reviewing the initial definitions, we examine the f-morphism on the m-polar IVIFG. Some of its properties, such as the equivalence relationship, have been studied. The effect of isomorphism and its properties on strong regular and highly irregular m-polar IVIFG has also been investigated. Leadership always plays an important role in an organization and can cause its growth and prosperity. But finding a strong leader is a very important issue that needs to be addressed. Therefore, in this research work, we have tried to determine the leadership of an economic team with the help of an m-polar IVIFG.

2. Preliminaries

In this section, we review the main concepts related to the topic under discussion. In the following definitions, we consider D [0,1] as the set of all subintervals of [0,1], and the elements of this collection are shown in capital letters.

Definition 1. (see [13]). An m-polar IVIFS M on V is described as

$$M = \left\{ \left\langle \left[\varphi_1^{ML}(r), \varphi_1^{MU}(r)\right], \ldots, \left[\varphi_m^{ML}(r), \varphi_m^{MU}(r)\right], \left[\psi_1^{ML}(r), \psi_1^{MU}(r)\right], \ldots, \left[\psi_m^{ML}(r), \psi_m^{MU}(r)\right] \right\rangle \right\}, \tag{1}$$

for all $r \in V$ and or shortly

$$M = \left\{ \left\langle \left[\varphi_i^{ML}(r), \varphi_i^{MU}(r)\right]_{i=1}^{m}, \left[\psi_i^{ML}(r), \psi_i^{MU}(r)\right]_{i=1}^{m} \right\rangle \middle| r \in V, m \in \mathbb{N} \right\}, \tag{2}$$

so that the functions $\varphi_i^M\colon V \longrightarrow D[0,1]$ and $\psi_i^M\colon V \longrightarrow D[0,1]$ show the degree of m-polar MV and m-polar N-MV of the element $r \in V$, respectively. Also,

$$\begin{aligned} 0 \le \varphi_i^{ML}(r) \le \varphi_i^{MU}(r) \le 1, \\ 0 \le \psi_i^{ML}(r) \le \psi_i^{MU}(r) \le 1, \\ 0 \le \varphi_i^{ML}(r) + \psi_i^{MU}(r) \le 1, \quad \forall r \in V. \end{aligned} \tag{3}$$

Definition 2. (see [13]). An m-polar-IVIFG with underlying graph G^* is defined as $G = (V, M, N)$, in which

(i) M is an m-polar-IVIFS on V as

$$M = \left\{ \left\langle \left[\varphi_i^{ML}(r), \varphi_i^{MU}(r)\right]_{i=1}^{m}; \left[\psi_i^{ML}(r), \psi_i^{MU}(r)\right]_{i=1}^{m} \right\rangle \middle| r \in V, m \in \mathbb{N} \right\}. \tag{4}$$

(ii) N is an m-polar-IVIFR on $V \times V$ so that

$$N = \left\{ \langle \left[\varphi_i^{NL}(rs), \varphi_i^{NU}(rs) \right]_{i=1}^{m}; \left[\psi_i^{NL}(rs), \psi_i^{NU}(rs) \right]_{i=1}^{m} \rangle | rs \in E, m \in \mathbb{N} \right\}, \tag{5}$$

and the functions $\varphi_i^N: E \subset V \times V \longrightarrow D[0,1]$ and $\psi_i^N: E \subset V \times V \longrightarrow D[0,1]$ are described by

$$\begin{aligned} \varphi_i^{NL}(rs) &\leq \min\left\{\varphi_i^{ML}(r), \varphi_i^{ML}(s)\right\}, \\ \varphi_i^{NU}(rs) &\leq \min\left\{\varphi_i^{MU}(r), \varphi_i^{MU}(s)\right\}, \\ \varphi_i^{NL}(rs) &\geq \max\left\{\psi_i^{ML}(r), \psi_i^{ML}(s)\right\}, \\ \psi_i^{NU}(rs) &\geq \max\left\{\psi_i^{MU}(r), \psi_i^{MU}(s)\right\}, \end{aligned} \tag{6}$$

such that $0 \leq \varphi_i^{NU}(rs) + \psi_i^{NU}(rs) \leq 1$, $\forall rs \in E$, and $1 \leq i \leq m$.

Definition 3. (see [13]). The m-polar-IVIFG is called to be strong if for $1 \leq i \leq m$,

$$\begin{aligned} \varphi_i^{NL}(rs) &= \min\left\{\varphi_i^{ML}(r), \varphi_i^{ML}(s)\right\}, \\ \varphi_i^{NU}(rs) &= \min\left\{\varphi_i^{MU}(r), \varphi_i^{MU}(s)\right\}, \\ \psi_i^{NL}(rs) &= \max\left\{\psi_i^{ML}(r), \nu_i^{ML}(s)\right\}, \\ \psi_i^{NU}(rs) &= \max\left\{\psi_i^{MU}(r), \psi_i^{MU}(s)\right\}. \end{aligned} \tag{7}$$

All the basic notations are shown in Table 1.

3. h-Morphism on m-Polar IVIFG

In this section, h-Morphism on m-polar IVIFG is introduced and some related results are discussed.

Definition 4. Suppose $G_1 = (V_1, M_1, N_1)$ and $G_2 = (V_2, M_2, N_2)$ are two m-polar-IVIFGs.

A homomorphism h from G_1 to G_2 is a mapping h: $V_1 \longrightarrow V_2$ so that, for each $1 \leq i \leq m$, we have the following:

$$\text{(i)} \begin{cases} \varphi_i^{M_1L}(r) \leq \varphi_i^{M_2L}(h(r)), \\ \varphi_i^{M_1U}(r) \leq \varphi_i^{M_2U}(h(r)), \\ \varphi_i^{M_1L}(r) \geq \psi_i^{M_2L}(h(r)), \\ \psi_i^{M_1U}(r) \geq \psi_i^{M_2U}(h(x)), \text{ for each } r \in V_1. \end{cases}.$$

$$\text{(ii)} \begin{cases} \varphi_i^{N_1L}(rs) \leq \varphi_i^{N_2L}(h(r), h(s)), \\ \varphi_i^{N_1U}(rs) \leq \varphi_i^{N_2U}(h(r), h(s)), \\ \psi_i^{N_1L}(rs) \geq \psi_i^{N_2L}(h(r), h(s)), \\ \psi_i^{N_1U}(rs) \geq \psi_i^{N_2U}(h(r), h(s)), \text{ for each } rs \in E_1. \end{cases}$$

Definition 5. Let G_1 and G_2 be two m-polar-IVIFGs. An isomorphism h from G_1 to G_2 is a bijective mapping (BM) h: $V_1 \longrightarrow V_2$ that holds in the following conditions:

$$\text{(i)} \begin{cases} \varphi_i^{M_1L}(r) = \varphi_i^{M_2L}(h(r)), \\ \varphi_i^{M_1U}(r) = \varphi_i^{M_2U}(h(r)), \\ \psi_i^{M_1L}(r) = \psi_i^{M_2L}(h(r)), \\ \psi_i^{M_1U}(r) = \psi_i^{M_2U}(h(r)), \text{ for each } r \in V_1, 1 \leq i \leq m. \end{cases}$$

(ii) $\{ \varphi_i^{N_1L}(rs) = \varphi_i^{N_2L}(h(r), h(s)), \varphi_i^{N_1U}(rs) = \varphi_i^{N_2U}(h(r), h(s)), \psi_i^{N_1L}(rs) = \psi_i^{N_2L}(h(r), h(s)), \psi_i^{N_1U}(rs) = \psi_i^{N_2U}(h(r), h(s))$, for each $rs \in E_1, 1 \leq i \leq m$.

Definition 6. A WI h from G_1 to G_2 is a BM h: $V_1 \longrightarrow V_2$ that satisfies the following conditions:

(i) h is homomorphism,

$$\text{(ii)} \begin{cases} \varphi_i^{M_1L}(r) = \varphi_i^{M_2L}(h(r)), \\ \varphi_i^{M_1U}(r) = \varphi_i^{M_2U}(h(r)), \\ \psi_i^{M_1L}(r) = \nu_i^{M_2L}(h(r)), \\ \psi_i^{M_1U}(r) = \psi_i^{M_2U}(h(r)), \forall r \in V_1, 1 \leq i \leq m. \end{cases}$$

Definition 7. Let G_1 and G_2 be two m-polar-IVIFGs. A BM h: $V_1 \longrightarrow V_2$ is named m-polar interval-valued intuitionistic fuzzy morphism or h-morphism of m-polar-IVIFG if there are two numbers $l_1 > 0$ and $l_2 > 0$ so that

$$\text{(i)} \begin{cases} \varphi_i^{M_2L}(h(r)) = l_1 \varphi_i^{M_1L}(r), \\ \varphi_i^{M_2U}(h(r)) = l_1 \varphi_i^{M_1U}(r), \\ \psi_i^{M_2L}(h(r)) = l_1 \psi_i^{M_1L}(r), \\ \psi_i^{M_2U}(h(r)) = l_1 \psi_i^{M_1U}(r), \forall r \in V_1, 1 \leq i \leq m. \end{cases}$$

$$\text{(ii)} \begin{cases} \varphi_i^{N_2L}(h(r), h(s)) = l_2 \varphi_i^{N_1L}(rs), \\ \varphi_i^{N_2U}(h(r), h(s)) = l_2 \varphi_i^{N_1U}(rs), \\ \psi_i^{N_2L}(h(r), h(s)) = l_2 \psi_i^{N_1L}(rs), \\ \psi_i^{N_2U}(h(r), h(s)) = l_2 \psi_i^{N_1U}(rs), \forall rs \in E_1, 1 \leq i \leq m. \end{cases}.$$

In this case, h is named an $(l_1, l_2)m$-polar interval-valued intuitionistic h-morphism on G_1 to G_2. If $l_1 = l_2 = l$, then, h is named m-polar interval-valued intuitionistic l-morphism.

Example 1. Consider two m-polar IVIFG $G_1 = (V_1, M_1, N_1)$ and $G_2 = (V_2, M_2, N_2)$ that G_1^* and G_2^* as drawn in Figure 1.

An m-polar IVIFG $G_1 = (V_1, M_1, N_1)$ is shown in Figure 1(a) according to Table 2.

Another m-polar IVIFG $G_1 = (V_2, M_2, N_2)$ is shown in Figure 1(b) according to Table 3.

Here, there exists an m-polar interval-valued intuitionistic h-morphism so that $h(x) = x'$, $h(y) = y'$, $h(z) = z'$, $l_1 = 10$, and $l_2 = 10$.

Theorem 1. *h-morphism is an equivalence relation on a set of m-polar IVIFGs.*

Proof. Consider a set of m-polar IVIFGs. Assume $G_1 \approx G_2$ if there is a $(l_1, l_2)m$-polar interval-valued intuitionistic h-morphism from G_1 to G_2 where both $l_1 \neq 0$ and $l_2 \neq 0$. Consider the identity morphism G_1 to G_1. It is a $(1,1)$-morphism from G_1 to G_1, and so, $\approx$ is reflexive. Let $G_1 \approx G_2$. Then, there is a (l_1, l_2)-morphism from G_1 to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$. So,

Table 1: Some basic notations.

Notation	Meaning
IVIFG	Interval-valued intuitionistic fuzzy graph
WI	Weak isomorphism
CWI	Co-weak isomorphism
SR	Strong regular
HI	Highly irregular
IVIFS	Interval-valued intuitionistic fuzzy set
BM	Bijective mapping
MV	Membership value
SE	Strong edge

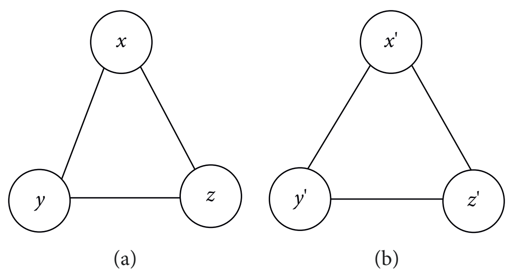

Figure 1: h-morphism of m-polar IVIFGs G_1 and G_2.

Table 2: The degree of MV and N-MV of the nodes and edges in G_1.

		G_1
M_1	x	$\langle[0.02, 0.03], [0.01, 0.04], [0.04, 0.05], [0.02, 0.04]\rangle$
	y	$\langle[0.01, 0.02], [0.03, 0.05], [0.02, 0.05], [0.01, 0.03]\rangle$
	z	$\langle[0.03, 0.04], [0.02, 0.03], [0.01, 0.05], [0.04, 0.06]\rangle$
N_1	xy	$\langle[0.01, 0.02], [0.01, 0.03], [0.04, 0.05], [0.03, 0.04]\rangle$
	xz	$\langle[0.02, 0.03], [0.01, 0.02], [0.04, 0.06], [0.05, 0.06]\rangle$
	yz	$\langle[0.01, 0.02], [0.02, 0.03], [0.03, 0.06], [0.05, 0.06]\rangle$

Table 3: The degree of MV and N-MV of the nodes and edges in G_2.

		G_2
M_2	x'	$\langle[0.2, 0.3], [0.1, 0.4], [0.4, 0.5], [0.2, 0.4]\rangle$
	y'	$\langle[0.1, 0.2], [0.3, 0.5], [0.2, 0.5], [0.1, 0.3]\rangle$
	z'	$\langle[0.3, 0.4], [0.2, 0.3], [0.1, 0.5], [0.4, 0.6]\rangle$
N_2	$x'y'$	$\langle[0.1, 0.2], [0.1, 0.3], [0.4, 0.5], [0.3, 0.4]\rangle$
	$x'z'$	$\langle[0.2, 0.3], [0.1, 0.2], [0.4, 0.6], [0.5, 0.6]\rangle$
	$y'z'$	$\langle[0.1, 0.2], [0.2, 0.3], [0.3, 0.6], [0.5, 0.6]\rangle$

$$\begin{aligned}\varphi_i^{M_2}(h(r)) &= l_1\varphi_i^{M_1}(r), \quad \forall r \in V_1,\\ \varphi_i^{N_2}(h(r), h(s)) &= l_2\varphi_i^{N_1}(rs), \quad \forall rs \in E_1.\end{aligned} \tag{8}$$

Consider h^{-1}: $G_2 \longrightarrow G_1$. Let $m, n \in V_2$. Since h^{-1} is bijective, $m = h(r)$, $n = h(s)$, for some $r, s \in V$. Now,

$$\begin{aligned}\varphi_i^{M_1}\left(h^{-1}(m)\right) &= \varphi_i^{M_1}\left(h^{-1}(h(r))\right) = \varphi_i^{M_1}(r) = \frac{1}{l_1}\varphi_i^{M_2}(h(r)) = \frac{1}{l_1}\varphi_i^{M_2}(m),\\ \varphi_i^{N_1}\left(h^{-1}(m)h^{-1}(n)\right) &= \varphi_i^{N_1}\left(h^{-1}h(r)h^{-1}(f(s))\right) = \varphi_i^{N_1}(rs),\\ &= \frac{1}{l_2}\varphi_i^{N_2}(h(r)h(s)) = \frac{1}{l_2}\varphi_i^{N_2}(mn).\end{aligned} \tag{9}$$

Similarly,

$$\begin{aligned}\psi_i^{M_1}\left(h^{-1}(m)\right) &= \frac{1}{l_1}\psi_i^{M_2}(m),\\ \psi_i^{N_1}\left(h^{-1}(m)h^{-1}(n)\right) &= \frac{1}{l_2}\psi_i^{N_2}(mn).\end{aligned} \tag{10}$$

Therefore, there is $((1/l_1), (1/l_2))$-morphism from G_2 to G_1. Thus, $G_2 \approx G_1$, and so, $\approx$ is symmetric. Let $G_1 \approx G_2$ and $G_2 \approx G_3$. Then, there is a (l_1, l_2)-morphism from G_1 to G_2, say h for some $l_1 \neq 0$ and $l_2 \neq 0$, and there exists (l_3, l_4)-morphism from G_2 to G_3, call g for some $l_3 \neq 0$ and $l_4 \neq 0$. So, for $i = 1, 2, \ldots, m$,

$$\varphi_i^{M_3}(g(r)) = l_3\varphi_i^{M_2}(r), \quad \forall r \in V_2,$$
$$\varphi_i^{N_3}(g(r)g(s)) = l_4\varphi_i^{N_2}(rs), \quad \forall rs \in E_2. \tag{11}$$

Let f: goh: $G_1 \longrightarrow G_3$. Now,

$$\begin{aligned}\varphi_i^{M_3}(f(r)) &= \varphi_i^{M_3}(goh(r)) = \varphi_i^{M_3}(g(h(r))) = l_3\varphi_i^{M_2}(h(r)) = l_3l_1\varphi_i^{M_1}(r),\\ \varphi_i^{N_3}(f(r)f(s)) &= \varphi_i^{N_3}(goh(r)goh(s)) = \varphi_i^{N_3}(g(h(r))g(h(s))),\\ &= l_4\varphi_i^{N_2}(h(r)h(s)) = l_4l_2\varphi_i^{N_1}(rs).\end{aligned} \tag{12}$$

Similarly,

$$\psi_i^{M_3}(f(r)) = l_3l_1\psi_i^{M_1}(r),$$
$$\psi_i^{N_3}(f(r)f(s)) = l_4l_2\psi_i^{N_1}(rs). \tag{13}$$

So, there exists (l_3l_1, l_4l_2) morphism h from G_1 to G_3. Then, $G_1 \approx G_3$ and $\approx$ is transitive. Thus, h-morphism is an equivalence relation in the set of m-polar IVIFGs. □

Theorem 2. *Suppose G_1 and G_2 are two m-polar IVIFGs. Let G_1 be $(l_1, l_2)m$-polar IVIFG morphism to G_2 for $l_1 \neq 0$ and $l_2 \neq 0$. Then, the image of an SE in G_1 is also an SE in G_2 if and only if $l_1 = l_2$.*

Proof. Let rs be a SE in G_1 so that $h(r)h(s)$ is also a SE in G_2. Now as $G_1 \approx G_2$ for $i = 1, 2, \ldots, m$, we have

$$\begin{aligned}l_2\mu_i^{N_1L}(rs) &= \varphi_i^{N_2L}(h(r)h(s)) = \min\left\{\varphi_i^{M_2L}(h(r)), \varphi_i^{M_2L}(h(s))\right\},\\ &= \min\left\{l_1\varphi_i^{M_1L}(r), l_1\varphi_i^{M_1L}(s)\right\} = l_1\min\left\{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\right\},\\ &= l_1\varphi_i^{N_1L}(rs), \quad \forall rs \in E_1.\end{aligned} \tag{14}$$

Hence,

$$l_2\varphi_i^{N_1L}(rs) = l_1\varphi_i^{N_1L}(rs), \quad \forall rs \in E_1. \tag{15}$$

Similarly in other equations,

$$\begin{aligned}l_2\varphi_i^{N_1U}(rs) &= l_1\varphi_i^{N_1U}(rs),\\ l_2\psi_i^{N_1L}(rs) &= l_1\psi_i^{N_1L}(rs),\\ l_2\psi_i^{N_1U}(rs) &= l_1\psi_i^{N_1U}(rs), \quad \forall rs \in E_1.\end{aligned} \tag{16}$$

The equations are true if and only if $l_1 = l_2$. □

Theorem 3. *If an m-polar IVIFG G_1 is CWI to G_2 and G_1 is regular, then, G_2 is also regular.*

Proof. Suppose G_1 is CWI to G_2. Then, there is a BM h: $G_1 \longrightarrow G_2$ so that $i = 1, 2, \ldots, m$ satisfies the following conditions:

$$\begin{aligned}\varphi_i^{N_1L}(rs) &= \varphi_i^{N_2L}(h(r)h(s)),\\ \varphi_i^{N_1U}(rs) &= \varphi_i^{N_2U}(h(r)h(s)),\\ \psi_i^{N_1L}(rs) &= \psi_i^{N_2L}(h(r)h(s)),\\ \psi_i^{N_1U}(rs) &= \psi_i^{N_2U}(h(r)h(s)), \quad \forall rs \in E_1.\end{aligned} \tag{17}$$

As G_1 is regular, for $r \in V_1$, $\sum_{r \neq s}\varphi_i^{N_1L}(rs) = \text{Constant}$. Now,

$$\sum_{h(r)\neq h(s)}\varphi_i^{N_2L}(h(r)h(s)) = \sum_{r\neq s}\varphi_i^{N_1L}(rs) = \text{Constant}. \tag{18}$$

Similarly,

$$\begin{aligned}\sum_{h(r)\neq h(s)}\varphi_i^{N_2U}(h(r)h(s)) &= \sum_{r\neq s}\varphi_i^{N_1U}(rs) = \text{Constant},\\ \sum_{h(r)\neq h(s)}\psi_i^{N_2L}(h(r)h(s)) &= \sum_{r\neq s}\psi_i^{N_1L}(rs) = \text{Constant},\\ \sum_{h(r)\neq h(s)}\psi_i^{N_2U}(h(r)h(s)) &= \sum_{r\neq s}\psi_i^{N_1U}(rs) = \text{Constant}.\end{aligned} \tag{19}$$

Therefore, G_2 is regular. □

Theorem 4. *Let G_1 and G_2 be two m-polar IVIFGs. If G_1 is WI to G_2 and G_1 is strong, then, G_2 is also strong.*

Proof. As G_1 is WI with G_2, then, there exists a WI h: $G_1 \longrightarrow G_2$ which is BM for $i = 1, 2, \ldots, m$ and satisfies the following:

$$\begin{aligned}
&\varphi_i^{M_1L}(r) = \varphi_i^{M_2L}(h(r)),\\
&\varphi_i^{M_1U}(r) = \varphi_i^{M_2U}(h(r)),\\
&\psi_i^{M_1L}(r) = \psi_i^{M_2L}(h(r)),\\
&\psi_i^{M_1U}(r) = \psi_i^{M_2U}(h(r)), \quad \forall r \in V_1,\\
&\varphi_i^{N_1L}(rs) \le \varphi_i^{N_2L}(h(r)h(s)),\\
&\varphi_i^{N_1U}(rs) \le \varphi_i^{N_2U}(h(r)h(s)),\\
&\psi_i^{N_1L}(r) \ge \psi_i^{N_2L}(h(r)h(s)),\\
&\psi_i^{N_1U}(r) \ge \psi_i^{N_2U}(h(r)h(s)), \quad \forall rs \in E_1.
\end{aligned} \tag{20}$$

Since G_1 is strong, $\varphi_i^{N_1L}(rs) = \min\{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\}$. Now, we get

$$\begin{aligned}
\varphi_i^{N_2L}(h(r)h(s)) \ge \varphi_i^{N_1L}(rs) &= \min\{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\},\\
&= \min\{\varphi_i^{M_2L}(h(r)), \varphi_i^{M_2L}(h(s))\}.
\end{aligned} \tag{21}$$

According to the definition, $\varphi_i^{N_2L}(h(r)h(s)) \le \min\{\varphi_i^{M_2L}(h(r)), \varphi_i^{M_2L}(h(s))\}$. Therefore, $\varphi_i^{N_2L}(h(r)h(s)) = \min\{\varphi_i^{M_2L}(h(r)), \varphi_i^{M_2L}(h(s))\}$. Similarly,

$$\begin{aligned}
&\varphi_i^{N_2U}(h(r)h(s)) = \min\{\varphi_i^{M_2U}(h(r)), \varphi_i^{M_2U}(h(s))\},\\
&\psi_i^{N_2L}(h(r)h(s)) = \max\{\psi_i^{A_2L}(h(r)), \psi_i^{M_2L}(h(s))\},\\
&\psi_i^{N_2U}(h(r)h(s)) = \max\{\psi_i^{A_2U}(h(r)), \psi_i^{M_2U}(h(s))\}.
\end{aligned} \tag{22}$$

So, G_2 is strong. □

Theorem 5. *If an m-polar IVIFG G_1 is CWI with an SR m-polar IVIFG G_2, then, G_1 is SR m-polar IVIFG.*

Proof. Since that G_1 is CWI to G_2, there is a CWI h: $G_1 \longrightarrow G_2$ that is BM for $i = 1, 2, \ldots, m$ and satisfies the following:

$$\begin{cases}
\varphi_i^{M_1L}(r) \le \varphi_i^{M_2L}(h(r)),\\
\varphi_i^{M_1U}(rx) \le \varphi_i^{M_2U}(h(r)),\\
\psi_i^{M_1L}(r) \ge \psi_i^{M_2L}(h(r)),\\
\psi_i^{M_1U}(r) \ge \psi_i^{M_2U}(h(r)), \quad \forall r \in V_1,
\end{cases}
\quad
\begin{cases}
\varphi_i^{N_1L}(rs) = \varphi_i^{N_2L}(h(r)h(s)),\\
\varphi_i^{N_1U}(rs) = \varphi_i^{N_2U}(h(r)h(s)),\\
\psi_i^{N_1L}(r) = \psi_i^{N_2L}(h(r)h(s)),\\
\psi_i^{N_1U}(r) = \psi_i^{N_2U}(h(r)h(s)), \quad \forall rs \in E_1.
\end{cases} \tag{23}$$

Now, we get

$$\begin{aligned}
\varphi_i^{N_1L}(rs) = \varphi_i^{N_2L}(h(r), h(s)) &= \min\{\varphi_i^{M_2L}(h(r)), \varphi_i^{M_2L}(h(s))\}\\
&\ge \{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\}.
\end{aligned} \tag{24}$$

But, by the definition, we have

$$\varphi_i^{N_1L}(rs) \le \{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\}. \tag{25}$$

So, $\varphi_i^{N_1L}(rs) = \min\{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\}$. Therefore, G_1 is strong. Also for $r \in V_1$, we have the following:

$$\sum_{r \ne s} \varphi_i^{N_1L}(rs) = \sum_{h(r) \ne h(s)} \varphi_i^{N_2L}(h(r)h(s)) = \text{Constant}. \tag{26}$$

Also, it is constant in other cases because G_2 is regular. Hence, G_1 is regular. □

Theorem 6. *Suppose G_1 and G_2 are two isomorphic m-polar IVIFGs. Then, G_1 is SR if and only if G_2 is SR.*

Proof. Since G_1 is isomorphic with G_2, so there exists an isomorphism h: $G_1 \longrightarrow G_2$ that is BM for $i = 1, 2, \ldots, m$ and satisfies

$$\begin{cases}
\varphi_i^{M_1L}(r) = \varphi_i^{M_2L}(h(r)),\\
\varphi_i^{M_1U}(r) = \varphi_i^{M_2U}(h(r)),\\
\psi_i^{M_1L}(r) = \psi_i^{M_2L}(h(r)),\\
\psi_i^{M_1U}(r) = \psi_i^{M_2U}(h(r)), \quad \forall r \in V_1,
\end{cases}
\quad
\begin{cases}
\varphi_i^{N_1L}(rs) = \varphi_i^{N_2L}(h(r)h(s)),\\
\varphi_i^{N_1U}(rs) = \varphi_i^{N_2U}(h(r)h(s)),\\
\psi_i^{N_1L}(rs) = \psi_i^{N_2L}(h(r)h(s)),\\
\psi_i^{N_1U}(rs) = \psi_i^{N_2U}(h(r)h(s)), \quad \forall rs \in E_1.
\end{cases} \tag{27}$$

Therefore, G_1 is strong if and only if

$$\begin{aligned}
&\varphi_i^{N_1L}(rs) = \min\{\varphi_i^{M_1L}(r), \varphi_i^{M_1L}(s)\},\\
&\varphi_i^{N_1U}(rs) = \min\{\varphi_i^{M_1U}(r), \varphi_i^{M_1U}(s)\},\\
&\psi_i^{N_1L}(rs) = \max\{\psi_i^{M_1L}(r), \psi_i^{M_1L}(s)\},\\
&\psi_i^{N_1U}(rs) = \max\{\psi_i^{M_1U}(r), \psi_i^{M_1U}(s)\},
\end{aligned} \tag{28}$$

if and only if

$$
\begin{aligned}
&\varphi_i^{N_2L}(h(r)h(s)) = \min\left\{\varphi_i^{M_2L}(h(r)), \varphi_i^{M_2L}(h(s))\right\},\\
&\varphi_i^{N_2U}(h(r)h(s)) = \min\left\{\varphi_i^{M_2U}(h(r)), \varphi_i^{M_2U}(h(s))\right\},\\
&\psi_i^{N_2L}(h(r)h(s)) = \max\left\{\psi_i^{M_2L}(h(r)), \psi_i^{M_2L}(h(s))\right\},\\
&\psi_i^{N_2U}(h(r)h(s)) = \max\left\{\psi_i^{M_2U}(h(r)), \psi_i^{M_2U}(h(s))\right\},
\end{aligned}
\tag{29}
$$

if and only if G_2 is strong.

G_1 is regular if and only if, for $r \in V_1$,

$$
\begin{aligned}
&\sum_{r\neq s} \varphi_i^{N_1L}(rs) = \text{Constant},\\
&\sum_{r\neq s} \varphi_i^{N_1U}(rs) = \text{Constant},\\
&\sum_{r\neq s} \psi_i^{N_1L}(rs) = \text{Constant},\\
&\sum_{r\neq s} \psi_i^{N_1U}(rs) = \text{Constant},
\end{aligned}
\tag{30}
$$

if and only if

$$
\begin{aligned}
&\sum_{h(r)\neq h(s)} \varphi_i^{N_2L}(h(r)h(s)) = \text{Constant},\\
&\sum_{h(r)\neq h(s)} \varphi_i^{N_2U}(h(r)h(s)) = \text{Constant},\\
&\sum_{h(r)\neq h(s)} \psi_i^{N_2L}(h(r)h(s)) = \text{Constant},\\
&\sum_{h(r)\neq h(s)} \psi_i^{N_2U}(h(r)h(s)) = \text{Constant},
\end{aligned}
\tag{31}
$$

for all $h(r) \in V_2$, if and only if G_2 is regular. □

Definition 8. Suppose $G = (V, M, N)$ is a connected m-polar IVIFG. Then, G is said to be a HI m-polar IVIFG whenever each node of G is neighbor to nodes with different degrees.

Theorem 7. *For any two isomorphic HI m-polar IVIFGs, the order and the size are the same.*

Proof. Suppose $h: G_1 \longrightarrow G_2$ is an isomorphism between two HI m-polar IVIFGs G_1 and G_2 with the underlying sets V_1 and V_2, respectively; therefore, for $i = 1, 2, \ldots, m$,

$$
\begin{aligned}
&\varphi_i^{M_1L}(r_1) = \varphi_i^{M_2L}(h(r_1)),\\
&\varphi_i^{M_1U}(r_1) = \varphi_i^{M_2U}(h(r_1)),\\
&\psi_i^{M_1L}(r_1) = \psi_i^{M_2L}(h(r_1)),\\
&\psi_i^{M_1U}(r_1) = \psi_i^{M_2U}(h(r_1)), \quad \forall r_1 \in V_1,\\
&\varphi_i^{N_1L}(r_1s_1) = \varphi_i^{N_2L}(h(r_1)h(s_1)),\\
&\varphi_i^{N_1U}(r_1s_1) = \varphi_i^{N_2U}(h(r_1)h(s_1)),\\
&\psi_i^{N_1L}(r_1s_1) = \psi_i^{N_2L}(h(r_1)h(sy_1)),\\
&\psi_i^{N_1U}(r_1s_1) = \psi_i^{N_2U}(h(r_1)h(s_1)), \quad \forall r_1s_1 \in E_1.
\end{aligned}
\tag{32}
$$

So, we get

$$
\begin{aligned}
O(G_1) &= \left\langle \left[\sum_{r_1\in V_1} \varphi_i^{M_1L}(r_1), \sum_{r_1\in V_1} \varphi_i^{M_1U}(r_1)\right]_{i=1}^{m}; \left[\sum_{r_1\in V_1} \psi_i^{M_1L}(r_1), \sum_{r_1\in V_1} \psi_i^{M_1U}(r_1)\right]_{i=1}^{m} \right\rangle,\\
O(G_1) &= \left\langle \left[\sum_{r_1\in V_1} \varphi_i^{M_2L}(h(r_1)), \sum_{r_1\in V_1} \varphi_i^{M_2U}(h(r_1))\right]_{i=1}^{m}; \left[\sum_{r_1\in V_1} \psi_i^{M_2L}(h(r_1)), \sum_{r_1\in V_1} \psi_i^{M_2U}(h(r_1))\right]_{i=1}^{m} \right\rangle\\
&= \left\langle \left[\sum_{r_2\in V_2} \varphi_i^{M_2L}(r_2), \sum_{r_2\in V_2} \varphi_i^{M_2U}(r_2)\right]_{i=1}^{m}; \left[\sum_{r_2\in V_2} \psi_i^{M_2L}(r_2), \sum_{r_2\in V_2} \psi_i^{M_2U}(r_2)\right]_{i=1}^{m} \right\rangle = O(G_2),\\
S(G_1) &= \left\langle \left[\sum_{r_1s_1\in E_1} \varphi_i^{N_1L}(r_1s_1), \sum_{r_1s_1\in E_1} \varphi_i^{N_1U}(r_1s_1)\right]_{i=1}^{m}; \left[\sum_{r_1s_1\in E_1} \psi_i^{N_1L}(r_1s_1), \sum_{r_1s_1\in E_1} \psi_i^{N_1U}(r_1s_1)\right]_{i=1}^{m} \right\rangle\\
&= \left\langle \left[\sum_{r_1s_1\in E_1} \varphi_i^{N_2L}(h(r_1)h(s_1)), \sum_{r_1s_1\in E_1} \varphi_i^{N_2U}(h(r_1)h(s_1))\right]_{i=1}^{m}; \left[\sum_{r_1s_1\in E_1} \psi_i^{N_2L}(h(r_1)h(s_1)), \sum_{r_1s_1\in E_1} \psi_i^{N_2U}(h(r_1)h(s_1))\right]_{i=1}^{m} \right\rangle\\
&= \left\langle \left[\sum_{r_2s_2\in E_2} \varphi_i^{N_2L}(r_2s_2), \sum_{r_2s_2\in E_2} \varphi_i^{N_2U}(r_2s_2)\right]_{i=1}^{m}; \left[\sum_{r_2s_2\in E_2} \psi_i^{N_2L}(r_2s_2), \sum_{r_2s_2\in E_2} \psi_i^{N_2U}(r_2s_2)\right]_{i=1}^{m} \right\rangle = S(G_2).
\end{aligned}
\tag{33}
$$

□

Theorem 8. *If G_1 and G_2 are isomorphic HI m-polar IVIFGs, then the degrees of the corresponding nodes r and h(r) are maintained.*

Proof. Let $h: G_1 \longrightarrow G_2$ be an isomorphism from the HI m-polar IVIFGs G_1 and G_2 with the sets V_1 and V_2, respectively. Therefore, for $i = 1, 2, \ldots, m$,

Table 4: 2-polar IVIF values of managers.

Managers	2-polar IVIF values of managers
CEO	$\langle [0.55, 0.65], [0.60, 0.70], [0.10, 0.20], [0.15, 0.20] \rangle$
Finance	$\langle [0.65, 0.75], [0.70, 0.80], [0.05, 0.10], [0.10, 0.15] \rangle$
Human resources	$\langle [0.65, 0.75], [0.70, 0.80]; [0.10, 0.15], [0.10, 0.20] \rangle$
Purchasing	$\langle [0.55, 0.60], [0.40, 0.50]; [0.10, 0.20], [0.20, 0.25] \rangle$
Production	$\langle [0.55, 0.65], [0.60, 0.75]; [0.15, 0.25], [0.10, 0.20] \rangle$
Sales	$\langle [0.60, 0.65], [0.70, 0.75]; [0.15, 0.20], [0.05, 0.20] \rangle$
Services	$\langle [0.50, 0.60], [0.60, 0.65]; [0.25, 0.30], [0.15, 0.20] \rangle$

$$\begin{aligned}
\varphi_i^{N_1L}(rs) &= \varphi_i^{N_2L}(h(r)h(s)),\\
\varphi_i^{N_1U}(rs) &= \varphi_i^{N_2U}(h(r)h(s)),\\
\psi_i^{N_1L}(rs) &= \psi_i^{N_2L}(h(r)h(s)),\\
\psi_i^{N_1U}(rs) &= \psi_i^{N_2U}(h(r)h(s)), \quad rs \in V_1.
\end{aligned} \tag{34}$$

Therefore,

$$\begin{aligned}
d_{G_1}(r) &= \left\langle \left[\sum_{rs \in V_1 r\neq s} \varphi_i^{N_1L}(rs), \sum_{rs \in V_1 r\neq s} \varphi_i^{N_1U}(rs) \right]_{i=1}^m ; \left[\sum_{rs \in V_1 r\neq s} \psi_i^{N_1L}(rs), \sum_{rs \in V_1 r\neq s} \psi_i^{N_1U}(rs) \right]_{i=1}^m \right\rangle \\
&= \left\langle \left[\sum_{rs \in V_1 r\neq s} \varphi_i^{N_2L}(h(r)h(s)), \sum_{rs \in V_1 r\neq s} \varphi_i^{N_2U}(h(r)h(s)) \right]_{i=1}^m \right\rangle \\
&\cdot \left\langle \left[\sum_{rs \in V_1 r\neq s} \psi_i^{N_2L}(h(r)h(s)), \sum_{rs \in V_1 r\neq s} \psi_i^{N_2U}(h(r)h(s)) \right]_{i=1}^m \right\rangle = d_{G_2}(h(r)).
\end{aligned} \tag{35}$$

It follows that the degrees of the corresponding nodes of G_1 and G_2 are maintained. □

4. Application

Every year, thousands of entrepreneurs start new businesses, hoping to make significant profits in the coming years; unfortunately, according to statistics, half of these businesses are closed and destroyed in less than 5 years due to a lack of leadership skills in the organization. There is a difference between leadership and management of an organization. The manager focuses on the operational process of the organization and the production of the product, determines the strategy, and identifies the solution to the problems. In contrast, the main goal of any leader should be to align the employees of the organization with the vision of the organization.

Power and politics are two important factors in leadership. Power means a person's ability to influence the behavior of others to achieve a goal. Politics is the study of power and influence. When people form a group with a specific purpose, some of them definitely gain power and access to sources of power. The members of the group seek to gain access to these resources and strive to obtain them. As soon as individuals use their power in action and find a way to exercise it, they actually engage in political behavior. They learn how to use their sources of power to achieve their goals. Leaders cannot influence in the upward direction and in the horizontal plane. But this is not true of power.

Consider an entrepreneur appointing a certain number of managers to oversee the specific activities that need to be done and the goals that need to be achieved. Each of them oversees a team that specializes in tasks and focuses on achieving those goals. These managers include the CEO and managers in the areas of finance, human resources, purchasing, production, sales, and services. This entrepreneur intends to choose leadership from these managers. In field surveys, the entrepreneur found the following:

(1) Although financial managers and human resources are under the CEO, there is a lot of competition between them for the CEO

(2) Purchasing, production, and sales managers operate under the supervision of the financial manager

(3) The human resources manager oversees the purchasing, sales, and service managers

(4) The activities of the service manager are under the orders of the production manager

(5) The purchasing manager is responsible for supplying raw materials for the production and service units

(6) Decisions of production and service managers are taken in coordination with the sales manager

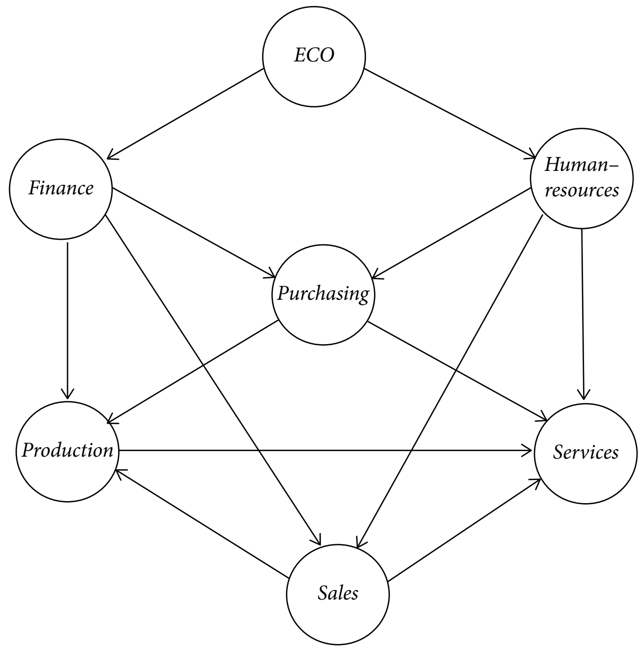

Figure 2: 2-polar IVIFG related to managers.

Table 5: 2-polar IVIF of relationships between managers.

Managers	2-polar IVIF values
CEO—finance	$\langle [0.55, 0.65], [0.60, 0.70]; [0.05, 0.10], [0.10, 0.15] \rangle$
CEO—human resources	$\langle [0.55, 0.65], [0.60, 0.70]; [0.10, 0.15], [0.10, 0.20] \rangle$
Finance—purchasing	$\langle [0.55, 0.60], [0.40, 0.50]; [0.05, 0.10], [0.10, 0.15] \rangle$
Finance—production	$\langle [0.55, 0.65], [0.60, 0.75]; [0.05, 0.10], [0.10, 0.15] \rangle$
Finance—sales	$\langle [0.60, 0.65], [0.70, 0.75]; [0.05, 0.10], [0.05, 0.15] \rangle$
Human resources—purchasing	$\langle [0.55, 0.65], [0.60, 0.75]; [0.10, 0.15], [0.10, 0.20] \rangle$
Human resources—sales	$\langle [0.60, 0.65], [0.70, 0.75]; [0.10, 0.15], [0.05, 0.20] \rangle$
Human resources—services	$\langle [0.50, 0.65], [0.60, 0.65]; [0.10, 0.15], [0.10, 0.20] \rangle$
Purchasing—production	$\langle [0.55, 0.60], [0.40, 0.50]; [0.10, 0.20], [0.10, 0.20] \rangle$
Purchasing—services	$\langle [0.50, 0.60], [0.40, 0.50]; [0.10, 0.20], [0.15, 0.20] \rangle$
Production—services	$\langle [0.50, 0.60], [0.60, 0.65]; [0.15, 0.25], [0.10, 0.20] \rangle$
Sales—production	$\langle [0.55, 0.60], [0.60, 0.75]; [0.15, 0.20], [0.05, 0.20] \rangle$
Sales—services	$\langle [0.50, 0.60], [0.60, 0.65]; [0.15, 0.20], [0.05, 0.20] \rangle$

An m-polar IVIFG can help an entrepreneur determine leadership. For this purpose, based on the surveys conducted for each of the managers, according to their abilities in terms of power and politics in their working group, the 2-polar IVIF values are considered according to Table 4. The numbers in Table 4 show that, for example, the sales manager has 50 to 60 percent power in his work and 25 to 30 percent does not have this power. Also, 70 to 75 percent has a policy, and 15 to 20 percent does not have the necessary policy.

If we consider managers as vertices, then we have a 2-polar IVIFG shown in Figure 2. Since the relationship between managers is considered to be the strongest, so all edges are strong. The values are shown in Table 5.

As can be seen in Figure 2, financial and human resource managers have the most power and policy over other managers and can be leadership options. But since the financial manager has the least degree of hesitation compared to the human resources manager, the best option for leadership is the financial manager.

5. Conclusion

m-polar IVIFG is one of the types of FGs that has better flexibility in dealing with problems that cannot be expressed with FGs and IFGs. It has a wide range of applications in the field of psychological sciences as well as the identification of individuals based on oncological behaviors. In this study, we presented the notion of m-polar h-morphism on m-polar IVIFGs. Some helpful theorems on WI and CWI are obtained. Furthermore, we investigated m-polar h-morphism on SR and HI m-polar-IVIFGs. In the last section, we have tried to express the application of the m-polar IVIFG in the management problem. The results show that the isomorphism in m-polar IVIFG is an equivalence relation so that some structural properties are equivalent in two m-polar

IVIFG. Some parameters, such as order, size, and strong edges, remain the same in two isomorphic m-polar IVIFGs. Some of these features are the same in both WI and CWI. As future work topics, we try to study some features of other types of fuzzy graphs such as m-polar fuzzy graph structure and m-polar cubic fuzzy graph.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
[2] A. Kaufman, *Introduction a la Theorie des Sous-Emsembles Flous*, Masson et Cie, Paris, France, 1973.
[3] A. Rosenfeld, *Fuzzy Graphs, Fuzzy Sets and Their Applications*, Academic Press, New York, NY, USA, 1975.
[4] C. Moraga, “Introduction to fuzzy logic,” *Facta Universitatis – Series: Electronics and Energetics*, vol. 18, no. 2, pp. 319–328, 2005.
[5] R. A. Borzooei, H. Rashmanlou, S. Samanta, and M. Pal, “Regularity of vague graphs,” *Journal of Intelligent and Fuzzy Systems*, vol. 30, no. 6, pp. 3681–3689, 2016.
[6] K. T. Atanassov, “Intuitionistic fuzzy sets,” *Fuzzy Sets and Systems*, vol. 20, pp. 87–96, 1986.
[7] W. R. Zhang, “Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multi agent decision analysis,” in *Proceedings of the First International Joint Conference of The North American Fuzzy Information Processing Society Biannual Conference. The Industrial Fuzzy Control and Intelligence NAFIPS/IFIS/NASA ’94*, vol. 18-21, pp. 305–309, IEEE, San Antonio, TX, USA, December 1994.
[8] W. R. Zhang, “Bipolar fuzzy sets,” *IEEE International Conference Fuzzy System*, vol. 1, pp. 83–840, 1998.
[9] H. Rashmanlou, S. Samanta, M. Pal, and R. A. Borzooei, “A study on bipolar fuzzy graphs,” *Journal of Intelligent and Fuzzy Systems*, vol. 28, no. 2, pp. 571–580, 2015.
[10] J. Hongmei and W. Lianhua, “Interval-valued fuzzy subsemigroups and subgroups associated by interval-valued fuzzy graphs,” in *Proceedings of the 2009 WRI Global Congress on Intelligent Systems*, vol. 1, pp. 484–487, IEEE, Xiamen, China, 2009.
[11] K. T. Atanassov and G. Gargov, “Interval valued intuitionistic fuzzy sets,” *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.
[12] A. A. Talebi, H. Rashmanlou, and S. H. Sadati, “Interval-valued intuitionistic fuzzy competition graph,” *Journal of Multiple-Valued Logic and Soft Computing*, vol. 34, pp. 335–364, 2020.
[13] A. A. Talebi, H. Rashmanlou, and S. H. Sadati, “New concepts on m-polar interval-valued intuitionistic fuzzy graph,” *TWMS Journal of Applied and Engineering Mathematics*, vol. 10, no. 3, pp. 806–818, 2020.
[14] M. Akram, A. N. Gani, and A. B. Saeid, “Vague hyper graphs,” *Journal of Intelligent and Fuzzy Systems*, vol. 26, no. 2, pp. 647–653, 2014.
[15] M. Akram, S. Samanta, and M. Pal, “Cayley vague graphs,” *The Journal of Fuzzy Mathematics*, vol. 25, no. 2, pp. 1–14, 2017.
[16] M. Akram, F. Feng, S. Sarwar, and Y. B. Jun, “Certain types of vague graphs,” *UPB. Science Bulletin Series A*, vol. 76, no. 1, pp. 141–154, 2014.
[17] S. Kosari, Y. Rao, H. Jiang, X. Liu, P. Wu, and Z. Shao, “Vague graph structure with application in medical diagnosis,” *Symmetry*, vol. 12, no. 10, p. 1582, 2020.
[18] Z. Kou, S. Kosari, and M. Akhoundi, “A novel description on vague graph with application in transportation systems,” *Journal of Mathematics*, vol. 2021, p. 11, Article ID 4800499, 2011.
[19] Y. Rao, S. Kosari, and Z. Shao, “Certain properties of vague graphs with a novel application,” *Mathematics*, vol. 8, no. 10, p. 1647, 2020.
[20] Y. Rao, S. Kosari, Z. Shao, R. Cai, and L. Xinyue, “A study on domination in vague incidence graph and its application in medical sciences,” *Symmetry*, vol. 12, no. 11, p. 1885, 2020.
[21] Y. Rao, S. Kosari, Z. Shao, X. Qiang, M. Akhoundi, and X. Zhang, “Equitable domination in vague graphs with application in medical sciences,” *Frontiers Physics*, vol. 9, 2021.
[22] X. Shi and S. Kosari, “Certain properties of domination in product vague graphs with an application in medicine,” *Frontiers Physics*, vol. 9, 2021.
[23] X. Shi and S. Kosari, “New concepts in the vague graph structure with an application in transportation,” *Journal of Function Spaces*, pp. 1–11, 2022.
[24] Z. Shao, S. Kosari, H. Rashmanlou, and M. Shoaib, “New concepts in intuitionistic fuzzy graph with application in water supplier systems,” *Mathematics*, vol. 8, no. 8, p. 1241, 2020.
[25] X. Qiang, M. Akhoundi, Z. Kou, X. Liu, and S. Kosari, “Novel concepts of domination in vague graphs with application in medicine,” *Mathematical Problems in Engineering*, vol. 202110 pages, 2021.
[26] S. Poulik and G. Ghorai, “Estimation of most effected cycles and busiest network route based on complexity function of graph in fuzzy environment,” *Artificial Intelligence Review*, pp. 1–18, 2022.
[27] S. Poulik and G. Ghorai, “New concepts of inverse fuzzy mixed graphs and its application,” *Granular Computing*, vol. 7, no. 3, pp. 549–559, 2021.
[28] S. Poulik and G. Ghorai, “Applications of graph’s complete degree with bipolar fuzzy information,” *Complex Intelligent System*, vol. 8, no. 2, pp. 1115–1127, 2022.
[29] S. Poulik and G. Ghorai, “Determination of journeys order based on graph’s wiener absolute index with bipolar fuzzy information,” *Information Sciences*, vol. 545, no. 2021, pp. 608–619.
[30] M. Azeem and M. F. Nadeem, “Metric-based resolvability of polycyclic aromatic hydrocarbons,” *The European Physical Journal Plus*, vol. 136, no. 4, p. 395, 2021.
[31] M. Azeem, M. F. Nadeem, A. Khalil, and A. Ahmad, “On the bounded partition dimension of some classes of convex polytopes,” *Journal of Discrete Mathematical Sciences and Cryptography*, pp. 1–14, 2021.
[32] H. Raza, M. Waheed, M. K. Jamil, and M. Azeem, “Structures devised by the generalizations of two graph operations and their topological descriptors,” *Main Group Metal Chemistry*, vol. 45, no. 1, pp. 44–56, 2022.
[33] J. Chen, S. Li, S. Ma, and X. Wang, “Fuzzy sets: an extension of bipolar fuzzy sets,” *The Scientific World Journal*, 2014.

[34] G. Ghorai and M. Pal, "On some operations and density of m-polar fuzzy graphs," *Pacific Science Review A: Natural Science and Engineering*, vol. 17, no. 1, pp. 14–22, 2015.

[35] G. Ghorai and M. Pal, "Faces and dual of m-polar fuzzy planar graphs," *Journal of Intelligent and Fuzzy Systems*, vol. 31, no. 3, pp. 2043–2049, 2016.

[36] A. Nagoor Gani and J. Malarvizhi, "Isomorphism properties on strong fuzzy graphs," *International Journal of Algorithms, Computing and Mathematics*, vol. 2, no. 1, pp. 39–47, 2009.

[37] A. Nagoor Gani and J. Malarvizhi, "Isomorphism on fuzzy graphs," *International Journal of Computation Mathematics Science*, vol. 2, no. 4, pp. 200–206, 2008.

[38] K. R. Bhutani, "On automorphisms of fuzzy graphs," *Pattern Recognition Letters*, vol. 9, no. 3, pp. 159–162, 1989.

[39] A. Perchant and I. Bloch, "Fuzzy morphisms between graphs," *Fuzzy Sets and Systems*, vol. 128, no. 2, pp. 149–168, 2002.

[40] P. Bhattacharya, "Some remarks on fuzzy graphs," *Pattern Recognition Letters*, vol. 6, no. 5, pp. 297–302, 1987.

[41] R. Seethalakshmi and R. B. Gnanajothi, "Regularity conditions on an intuitionistic fuzzy graph," *Applied Mathematical Sciences*, vol. 7, no. 105, pp. 5225–5234, 2013.

[42] C. Ramprasad, P. L. N. Varma, S. Satyanarayana, and N. Srinivasarao, "Morphism of m-polar fuzzy graph," *Advances in Fuzzy Systems*, vol. 2017, Article ID 4715421, 9 pages, 2017.

[43] D. Ezhilmaran and K. Sankar, "Morphism of bipolar intuitionistic fuzzy graphs," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 18, no. 5, pp. 605–621, 2015.

[44] K. Sankar and D. Ezhilmaran, "Morphism of m-polar intuitionistic fuzzy graphs," *Journal of Computational and Theoretical Nanoscience*, vol. 15, no. 6, pp. 2277–2282, 2018.

[45] P. Ramachandran and K. v. Thomas, "On isomorphisms of L-fuzzy graphs," *Annals of Fuzzy Mathematics and Informatics*, vol. 11, no. 2, pp. 301–313, 2016.

[46] G. Ghorai and M. Pal, "Some isomorphic properties of m-polar fuzzy graphs with applications," *SpringerPlus*, vol. 5, no. 1, p. 2104, 2016.