Hindawi
Journal of Mathematics
Volume 2022, Article ID 1618498, 7 pages
https://doi.org/10.1155/2022/1618498

Research Article

Graph Theory Algorithms of Hamiltonian Cycle from Quasi-Spanning Tree and Domination Based on Vizing Conjecture

T. Anuradha,[1] **T. Lakshmi Surekha**,[1] **Praveena Nuthakki**,[1] **Bullarao Domathoti**,[2] **Ganesh Ghorai**,[3] **and Faria Ahmed Shami**[4]

[1]*Department of Information Technology, Velagapudi Ramakrishna Siddhartha Engineering College, Vijayawada, AP, India*
[2]*Department of Computer Science and Engineering, Jawaharlal Nehru Technological University, Ananthapuram 517501, India*
[3]*Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102, India*
[4]*Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj, Bangladesh*

Correspondence should be addressed to T. Anuradha; anuradha_it@vrsiddhartha.ac.in, Bullarao Domathoti; bullaraodomathoti@gmail.com, and Faria Ahmed Shami; fariashami@bsmrstu.edu.bd

Received 24 February 2022; Accepted 16 July 2022; Published 25 August 2022

Academic Editor: M. T. Rahim

In this study, from a tree with a quasi-spanning face, the algorithm will route Hamiltonian cycles. Goodey pioneered the idea of holding facing 4 to 6 sides of a graph concurrently. Similarly, in the three connected cubic planar graphs with two-colored faces, the vertex is incident to one blue and two red faces. As a result, all red-colored faces must gain 4 to 6 sides, while all obscure-colored faces must consume 3 to 5 sides. The proposed routing approach reduces the constriction of all vertex colors and the suitable quasi-spanning tree of faces. The presented algorithm demonstrates that the spanning tree parity will determine the arbitrary face based on an even degree. As a result, when the Lemmas 1 and 2 theorems are compared, the greedy routing method of Hamiltonian cycle faces generates valuable output from a quasi-spanning tree. In graph idea, a dominating set for a graph $S = (V, E)$ is a subset D of V. The range of vertices in the smallest dominating set for S is the domination number (S). Vizing's conjecture from 1968 proves that the Cartesian fabricated from graphs domination variety is at least as big as their domination numbers production. Proceeding this work, the Vizing's conjecture states that for each pair of graphs S, L.

1. Introduction

Finite integral multipliers are used in the greedy routing algorithm. For the maximal tree, subgraphs are used, in which subgraph is denoted as S. If the edges are suitably labeled, the two trees are distributed among them. Here the variation of a tree's maximum number is established on the vertices n and it is known as the Cayley. In general, a graph S is drawn from the spanning tree vertices. The spanning tree is evaluated by using a single edge S. To define the system's vertex, a diagonal matrix is introduced. The variation between the adjacency matrix and the incidence matrix is determined by the spanning tree. Thus, the subgraph S contains all the vertices, and the diameter for any single tree graph D is denoted as

$$\operatorname{diam}(T(S)) \leq \min\{n-1, m-n+1\}. \tag{1}$$

A spinning graph diameter is determined by one of the two trees, T_1 or T_2, and is denoted as

$$d(T_1, T_2) = n - 1 - |E(T_1) \cap E(T_2)| = \frac{|E(T_1) \Delta E(T_2)|}{2}. \tag{2}$$

The graph tree operation is denoted as T: $S \longrightarrow S$. The subsequent matrix is to combine rows and columns. Next to defining the spanning tree estimation, the product value is obtained. Cauchy-Binet present the estimation. This entire calculation is followed by vertices and adjacency calculations. The edges of the cycle are counted by the sign, and an insertion form appears [1]. To classify subgraphs and establish paths from subgraphs, the edge is connected to the spanning tree. The geometric cycle is not equal to the value of the subgraph.

2. Hamiltonian Cycle from Quasi-Spanning Tree of Faces

There is precisely one Hamiltonian cycle along with no cubic graphs, which is a unique Hamiltonian graph because a minimum of three Hamiltonian cycles are there in a Hamiltonian cubic graph. In 1978, Thomason showed that in a graph with the vertices of the odd degree, in an even number of Hamiltonian cycles, all edges are confined, proving Smith's result [2]. Hence, uniquely Hamiltonian graphs unless even degree vertices and especially k regular exclusively. Odd k does not have Hamiltonian graphs. What is even k? Thomason showed that by Lovász local lemma, k-regular exclusively even $k \geq 300$ does not have Hamiltonian graphs [3], by a cautious option of parameters, theirs statements provide 73 rather of 300. That was modified by Haxell, Seamone, and Verstraete to $k \geq 23$. No 4-regular exclusively Hamiltonian graphs existed assumed by Sheehan. The fact of this assumption would indicate that cycles are the only regular exclusively Hamiltonian graphs, as according to Petersen's 2-factor theorem.

According to Thomason's result, an exclusively Hamiltonian graph has a necessity of minimum of two even degree vertices. This connection between the degree of the graph and either or not it is exclusively Hamiltonian increases several ordinary enquiries, such as either there are any exclusively Hamiltonian graphs of degree 3. Swart and Entringer gave a positive answer to that question by relating in closely cubic graphs an infinite family, that is, graphs along precisely two degrees 4 vertices and all of the other cubic vertices. Fleischer lately demonstrated that there are graphs with every vertex having a degree of 4 or 14 that are uniquely Hamiltonian [4].

Jackson and Bondy examined that an individually Hamiltonian graph of order n consumes minimum one-degree vertex maximum $c\log 28n + 3$, which means the minimum degree is smaller than this number, here $c \approx 2.41$. Jamshed and Abbasi modified that to $\log 2n + 2$, here $c \approx 1.71$. Jackson and Bondy were especially attentive to planar exclusively Hamiltonian graphs in their article. A graph necessity has a minimum of two vertices of degree 2 or 3 that are displayed by them and assumed that all planar individually Hamiltonian graphs must have a minimum of two vertices of degree 2.

Proposition 1. *Given S consumes a Hamiltonian cycle through the exterior red face outdoor, all blue face within, and an edge is shared by no two red faces are together within, then the reduced graph H consumes a face's spanning tree through in D does not contain the external face.*

Proof. S consumes a Hamiltonian cycle through the exterior red face external, every blue faces consistent to vertices in within, and an edge is shared by no two red faces are together insides, if and only if the reduced graph H consumes a face's quasi-spanning tree through in D does not contain the external face. □

Theorem 1. *Assume that all red faces have 6 sides or 4 sides, whereas blue faces have 3 or 5 sides, and that blue faces through 3 sides or 5 sides are adjacent to a minimum of one red face along with 4 sides (no conjecture is created for blue faces by 4, 6, 7, 8, 9, . . . sides). The reduced graph H, which is obtained by crumbling blue faces, then has a correct quasi-spanning tree of faces, prove S a Hamiltonian cycle.*

2.1. We Now Prove the Main Result of This Section. Assume that all of S's red faces have 4 sides or 6 sides, whereas the faces of blue are chance. Assume that the reduced graph H contains a triangle T with a minimum one vertex within, and no triangle in T is none a face (i.e., includes minimum a vertex inside), and that no digon within T is not a face (i.e., includes minimum one vertex within). We shall simplify the inside of the triangle T one step at a time while preserving the property that which is no digon inside of T that is not a face but authorizing the presence of triangles inside of T that are not faced, subject to the succeeding conditions. Handle entire sets of parallel edges like a single edge. Assume T_1 and T_2 are different triangles within of T, along T_1 including T_2 and perhaps T_1 like as T, where T_2 is not a face, and so that there is no triangle T_3 differ from T_1 and T_2 like that T_1 includes T_3 and T_3 includes T_2. Then, we assume that T_2 is a child of T_1. We will need that no triangle T_1 consumes three different children T_2, T_2', and T_2'', any steps in explanation of the inside of the triangle T.

The invariant property of T is that no digon within T is not a face, and no triangle within T consumes three different children.

Lemma 1. *Assume T consumes a minimum of two vertices within and fulfills the invariant property, proving that it is feasible to choose a triangle T' that is to say a face within T and crimple T' into an only vertex so that T even gratify the invariant property [4].*

Proof. Assume that triangle T_1 within T includes a minimum of two vertices and that not any triangle within T_1 is not a face. Take $T_1 = v_1 v_2 v_3$, in T_1, we declare that v_1 consumes a minimum of two different neighbors v_4, v_5. Else, if v_1 consumes no neighbors, so v_1 goes to a triangle within T_1 with an edge $v_2 v_3$ parallel to the side of T_1, which is a contradiction to the hypothesis that has no digon within of T it is nonface, and while v_1 has unique like a neighbor v_4 within of T_1, therefore $v_2 v_3 v_4$ is a triangle within of T_1 it is not a face, which is also a contradiction to the hypothesis.

Then we can select v_4 and v_5 to v_2, v_4, v_5 are successive neighbors of v_1, and crumble the triangle $v_1v_4v_5$. There will be no digons that are not faced as a result of this because like a digon gets here before the crumpling from a triangle which is not a face within T_1, a contradiction to the hypothesis. Inside T_1, though, triangles that do not face may appear. Similar triangles derived from quadrilaterals $v_1v_4v_6v_7$, $v_1v_5v_8v_9$, and $v_4v_5v_{10}v_{11}$. The quadrilaterals $v_1v_5v_8v_9$ can be one of two types: they can contain v_4 or they cannot contain v_4, but they cannot have diagonal edges v_1v_8 or v_5v_9, because then either a triangle this isn't a face was within the quadrilateral, otherwise crumpling the side $v_1v_5$is does not provide the quadrilateral a triangle which is non a face. Such indicates that all like quadrilaterals including v_4 are couple included in all other, and every such quadrilateral that do not comprise v_4 are pairwise included together [9]. The analogous properties prove that for the quadrilaterals $v_1v_4v_6v_7$, However, there is only one kind of these, namely those that contain v_5. Else, $v_6 = v_2$ and we consume the diagonal edge v_2v_4. The quadrilaterals $v_4v_5v_{10}v_{11}$ contain analogous properties, but they are of a unique kind [10, 11], specifically do not comprise v_1, then they are included in the triangle $T_1 = v_1v_2v_3$. A quadrilateral $v_1v_4v_6v_7$ including v_5 essential also include at all quadrilateral $v_1v_5v_8v_9$ that does not contain v_4 and any quadrilateral $v_4v_5v_{10}v_{11}$ that does not contain v_1, and any quadrilateral $v_4v_5v_{10}v_{11}$ that does not contain v_1 must also contain any quadrilateral $v_1v_5v_8v_9$ that contains v_4 [12]. These assurances that these quadrilaterals do not take a main, next crumpling $v_1v_4v_5$, inside T_1, three triangles are not faces and do not conclude together, therefore conserving the property that three children are not taken by triangles [13-15].

For residual case in crumpling a triangle, here is a triangle T_1 which consumes any one child T_2 or two children T_2 and T_3, here together T_2 and T_3 consume precisely one vertex within. Assume T_2 shares no sides through any T_1 or T_3. We must take the quadrilaterals $v_1v_2v_4v_5$, $v_1v_3v_6v_7$, and $v_2v_3v_8v_9$ once more when writing $T_2 = v_1v_2v_3$. Quadrilaterals $v_1v_2v_4v_5$ including v_3, v_3, $v_1v_3v_6v_7$ including v_2, $v_2v_3v_8v_9$ including v_1, and $v_1v_2v_4'v_5'$ not including v_3 may not exist at the same time. For if $v_6 = v_5$, then $v_1v_5v_7$ is not a face and therefore equals T_1, thus v_1 is a vertex of T_1 and the quadrilateral $v_2v_3v_8v_9$ cannot include v_1; while $v_7 = v_4$, $v_1v_5v_4$ is T_1, and the similar argument applies, and while $v_6 = v_4$, then $v_8 = v_5$ and $v_9 = v_7$, so the triangle $v_5v_4v_7$ is T_1, this is not possible because the quadrilateral $v_1v_2v_4'v_5'$ would be inside the triangle $v_1v_2v_7$, it is called a face. As a result of symmetry, we can assume that after identifying v_1 and v_2, there is either no quadrilateral $v_1v_2v_4v_5$ including v_3, otherwise no quadrilateral $v_1v_2v_4v_5$ not including v_3, which will provide an increase to a fresh triangle which is not a face. Crumpling the triangle $v_1v_2v_0$ identifies v_1 and v_2 and creates unique triangles through pairwise confinement introduce the new vertex $v_1 = v_2$, except the triangle T_3, so conserving the property that three children are not taken by triangles. Assume $T_1 = v_1v_2v_3$ shares a single side by T_1, it is a side v_2v_3, then one of the other two sides is not shared by T_3, say the side v_1v_2, and the quadrilaterals $v_1v_2v_4v_5$ unable to include v_3, thus repeatedly we were able to crumple the triangle $v_1v_2v_0$ by v_0 within T_2, producing unique triangles through pairwise confinement introducing the new vertex $v_1 = v_2$, except the triangle T_3, so conserving the property that no triangle consumes three children. While T_2 and T_3 share aside v_1v_3, then every quadrilateral $v_1v_2v_4v_5$ that includes v_3v_3 also includes T_3 [16]. As a result, crumpling $v_1v_2v_0$ with v_0 inside T_2 provides two families of triangles through pairwise confinements concerning $v_1 = v_2$, one including v_3 and the another includingv_3, conserving the property that three children are not taken by triangles.

The succeeding proposition incorporates Herbert Fleischner's result [17]. □

Proposition 2. *Let us consider blue faces remain random and G's red faces get 4 to 6 sides. The reduced graph H has only one triangle which is in the outer layer and it does not have any faces, other than that the H graph has no triangles. H also incorporated no diagonal direction which is not even considered to face. H has a spanning tree face which is triangles and S is said to be Hamiltonian when H contains odd number vertices.*

Proof. While saving the invariant property, collaborate triangle faces into single vertices and redo Lemma 1. The total of vertices stays odd until the outer face remains by reducing the vertices by two. Eventually, a spanning tree is formed by the collaborated triangle. The main observation that results to this result is as follows: □

Lemma 2. *.In Theorem 1, take S as same. If the graph H has triangle T with only one vertex, there is no other triangle inside T, which is not considered to be a face also as it does not have any digons. To find out the acceptable quasi-spanning tree of faces for the graph H', identifying the appropriate quasi-spanning tree face is reduced. By separating all inside vertices (T) and incident edges, it tends to incorporate the look-alike edge inside T to every edge of T, H' obtained from the reduction graph H.*

Proof. As shown in the previous Lemma, by collapsing the triangle faces repeatedly we can wind up a v in T or else make nothing inside T. In a quasi-spanning tree of faces, choose one of the three triangles which imply v, which corresponds to the one in three diagonal directions for the sides of H' in T. And we might either choose triangle T in H' in a face of quasi-spanning trees. When the time T holds an off vertex and which is inside of T, in this scenario the vertex v which is in the T is obtained, and then when the moment T has an even number of vertices and which is in T, in this case, we reached T which has no vertices.

The parity inside the T is represented by the two cases. Initially, if there is a digon named v_1, v_2 has one endpoint which is in T, and to frame a triangle we need to collaborate v_1v_2, the framed triangle does not have any faces out of the quadrilaterals such as $v_{1,}v_{2,}v_{4,}v_5$, again there are a family of two quadrilaterals, consisting of two triangles as $v_{1,}v_{2,}v_{3,}$ and $v_{1,}v_{2,}v_3'$. Quadrilaterals have $v_{3,}$ and v_3'. The quadrilaterals provide triangles with pairwise boundaries of each family, which assures the property invariant that does not have T_1, which is equivalent to T or them have no children. Theorem allows S as connected cubic bipartite planar graph of three nodes. Let us assume, H' be the subgraph of H and reduced graph S is H,

Here we got the results by removing all the possible edges with successive side by side edges. If the graph has one and two and three connected elements since H' has face's spanning trees, then S contains a Hamiltonian cycle. In the occurrence of a single element for H', all faces among three colors of classes are considered.

We demonstrate vertex v inside of t only when there are no digons of v_1, v_2. Which pertained to one of four triangles that share with side T. After that, an appropriate quasi-spanning tree is built, two triangles v_1, v_2, v_3 and v, v_3, v_4 are included in the suitable spanning tree of faces and it does not share its edge. The collaborated triangles which are to remove v, identify v_1 upon v_2 also identifies v_3 with v_4 and convert 5 vertices to only 2 vertices, also change the number of vertices. Hence the complete proof of Lemma is derived.

We can write $T = v_1, v_2, v_3$ when there are no digons inside T initially. There need to be 2 vertices inside of T is present, if not the single vertex which inside T have a degree and it does not have any digons, assuming that the blue face with 3 sides is needed to be adjacent to one red face with at least 4 sides. This indicates v_1 need to have at least two distinct neighbors inside T, if not the case, v_0 is considered to be only one vertex of T, since there are no such triangles as faces. If we calculate v_1, v_2 and v_2, v_3, then v_1 contains a degree of 4. Similarly, edges v_2, v_3 holds at least a degree of 4. Further, there is no such vertex of T that has degree 3 or 5, as all of the blue faces with 3 to 5 sides are close to one red face with 4 sides, as a result, a vertex is considered to be incident to digon. As per Euler's formula, there must be three vertices of degree 4 in T, on the other hand, there are 6 vertices of degree 4, T is present. Let us assume v_0 which is inside T contains four consecutive neighbors: $v_4v_5v_6v_7$. The quadrilateral share one edge with $T = v_1v_2v_3$, as we know T indicates triangle. As v_1, v_3 and v_1, v_2 are getting shared, v_1 has only one adjacent neighbor which is v_0 in T and it has degree 3 and not 4. Let us say v_4, v_7 might be shared with T. In this scenario, make v_0 to an appropriate vertex of quasi and choose the two triangles such as $v_0v_6v_7$ and $v_0v_4v_5$. Here, recognizing v_4 and v_5 detaching v_0 identifying v_6, v_7 and lessens the total number of vertices by 3. The quadrilaterals v_4, v_5, v_8, v_9 have the edge of v_6, v_7 which produces fresh triangles that contain v_6, v_7, v_{10}, v_{11} of quadrilaterals which also gives new triangles that contain v_4 and v_5 of edges. The quadrilaterals $v_6, v_7, v_{10}', v_{11}'$ have edges $v_{4,}v_5$ which gives triangles that are newly created and those triangles having quadrilaterals of v_4, v_5', v_8' does not have the edge $v_6\&v_7$. By recognizing v_4, v_5 and v_6, v_7, we tend to attain two families of newly created triangles with every family giving containment which is considered as pairwise that occurs among its triangles.

This gives that the property does not have T_1 triangle and equal to T or else inside of T having three children. Before proceeding to minimize the number of vertices by T, which increases to two till a single vertex is not inside of T and thus finishes off the proof with variation in parity of numbers inside of T. As recently expressed, this decreases the issue of tracking down an appropriate semitraversing the tree of countenances for H to the assignment of erasing the vertices inside H and interfacing equal edges to the sides of T to acquire H'.

Theorem 1 produces Lemma as a digon is considered as the outer face or else a triangle which has vertices inside of it. There is a triangle that has vertices inside and it does not have any triangles or two vertices of diagon inside or else the diagon contains vertices inside and in the same manner it does not have any triangles or diagons vertices inside. By removing the vertices and adding the same parallel edges to the side of T, this T has vertices inside and it does not have any triangle either. It can be clarified as per Lemma 1. The digon v_1, v_2 have a triangle with vertices inside, when a digon v_1, v_2 has vertices inside but it does not have any triangle and it has a v_0 of a single vertex. Among v_0, v_1 and v_0, v_2, either one considered as a digon; only v_0 had the degree. For instance, it happens when v_0, v_1 is a digon. After removing the vertex v_0, we can moreover choose the digon v_0, v_1 otherwise the triangle v_0, v_1, v_2, that represent also not selecting or choosing the digon v_1, v_2 which has developed a face. When the outer face has no vertices in it and that the graph H is simplified. In such a case, what is considered to complete this process is while selecting the face involved all the vertices in the quasi-spanning tree faces of H and hence Theorem 1 is proved.

Coming up next is a rundown of corollary is an uncommon instance of Theorem 1 that sums up Goodey's outcome to diagrams S with just 4 sides or 6 sides. □

Corollary 1. *Assume S be a 3-connected cubic planar bipartite graph, while the S faces are three colored, through all S vertex incident to a face of all color, since two of the three color classes include only that have 4 sides or 6 sides. The reduced graph H, which is acquired by crumpling the class of the third color, thus includes a correct face's quasi-spanning tree, and hence S is a Hamiltonian cycle.*

2.2. NP Complete and Polynomial Problems. The following result is for a face's spanning tree where the majority of the faces are digons.

Theorem 2. *Consider S stay a 3-connected cubic planar bipartite graph. Assume the reduced graph H for S, and H' the subgraph of H found by eliminating each edge with consecutive parallel edges. H' has a face's spanning tree if it includes one or two or three connected components, and S contains a Hamiltonian cycle. In one of the three color classes, all the faces are squares in the case of a single component for H'.*

Proof. We can take H' be a spanning tree that corresponds to a spanning tree of digons in H, while H' is a single linked component.

We can take a f face of H which takes vertices from together components if H' has two connected components. For the two components of H', we assume two spanning trees of digons, starting with this face f, and enhance that digons are unique at the same time show they do not create a cycle including f. The single face f and the added digons desire eventually span H.

Although H' consumes three connected components, it is possible that H has a face f that touches each three, and we can move from f to two components by examining for the two components, the three spanning trees of digons. Otherwise, we consider the first component, which has faces that contact it, as well as the second and third components, which also contain faces that contact it and the third component. We can select a face f contact the first component and second component, and a face f' contact the first component and third component, so that those two faces do not divide each vertex, thus a cut of H has a minimum of four edges because of 3-connectivity and the reality that at all cut consumes an edge's even number. Initial through those two faces from the three spanning trees we can enhance digons for the three components thus far, a face's spanning tree for H is found since they do not form a cycle.

The result for three connected components applies to four connected components as well, but the result is not valid for five connected components.

Following that, we show how to decide in polynomial time that the reduced graph H consumes a face's spanning tree that is digons or triangles. Simply expands of the result, the case of a face's spanning tree where all but a face's constant number are digons or triangles. □

3. Domination in Graphs

Consider $S = (V, E)$ be a graph through the vertex set V and the boundary set E. If each vertex in s is adjacent to the vertex in s, it is a dominant set of S. The domain number of S, mentioned by $\gamma(S)$ that is called the minimum cardinality of a dominant set of S.

In the investigated branch of the diagram concept, supremacy in diagrams was used. The superiority of the diagrams was utilized in the examined division of the diagram idea. Blending problems with optimal problems, classical problems, and combinatorial problems is a growing principle. It has several applications in a range of fields, including body sciences, engineering, life sciences and society, and so on. The research interest in the graph concept these days is centered on dominance. This is essentially a list of new parameters that may be improved from basic dominance definitions. The NP completeness of elementary domination problems and investigate the relation to another NP completeness by them and action growth in the domination principle.

When in a graph S every vertex is incident on at least one edge in g, the set of edges g is said to cover S. The edge covering a set of a graph S is said to be an edge covering or a cover subgraph or simply a S cover (e.g., a spanning tree in a linked graph is a cover). The example of a computer network over the relation minimum vertex coverage is shown in Figure 1 [5].

3.1. Applications of Domination in Graph. The graph applications of domination have been applied in a variety of

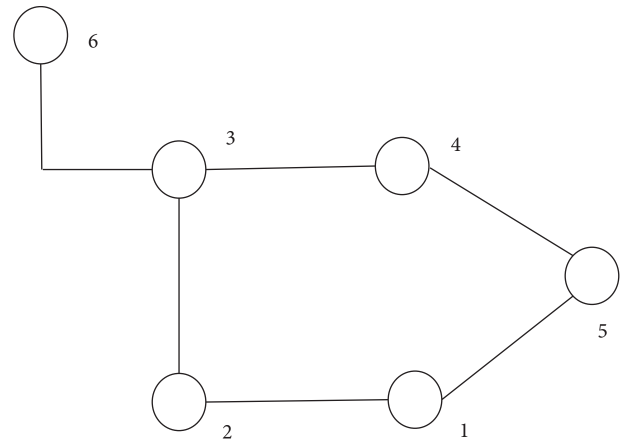

FIGURE 1: The set of vertices $g = \{1,3,4\}$ in S all vertices are cover.

fields. The dominion comes from structural challenges in which there is a constant type of centers (e.g., hearth stations, hospitals) and space must be kept to a minimum. To diminish the number of locations where a surveyor needs to commit to taking peak measurements for a whole area, surveyors use standards of domination.

3.2. Domination Path. A graph containing a dominating path is one where each vertex exterior of P includes a neighbor on P. Let $V(S)$ represents the vertex and $E(S)$ represents a S graph's edge set. $N_S(v)$ represents a vertex's neighborhood v in S and $d_S(v)$ denotes its degree. For $D, T \subseteq V(S) D$, represented by letting $N_S(T) = U_{v \in T} N_S(v) - T$ and letting $N_D(T) = N_S(T) \cap D$ and $d_D(T) = |N_D(T)|$. Likewise, $\delta(S)$ represents the minimum vertex degree and $\Delta(S)$ represents the maximum vertex degree.

Theorem 3. *Since $n \geq 2$, each connected n-vertex graph S along $\delta(S) > (n - 1/3) - 1$ contains a dominating path and proves the inequality is acute.*

Proof. The sharpness structure is declared for $n \equiv 1 \text{mod} 3$. In general, assume Q_i for $k = 1$ the structure be a clique over $\lfloor (n + 2 - i)/3 \rfloor$ vertices, $i \in \{1, 2, 3\}$. Then the three cliques jointly moreover include $n - 1$ vertices, $\delta(G) = \lfloor n - 1/3 \rfloor - 1$. Now presume that S is a connected graph of n-vertex over $\delta(S) \geq (n - 1)/3$ which include no dominating path; find that $n \leq 3t + 3$, here $t = \delta(S)$. Assume first that S is 2-connected. Dirac showed that S essential since containing a cycle over at least min $\{n, 2\delta(S)\}$ vertices. A path over minimum $n - t$ is a dominating path vertex, so we can connect $t < (n/2)$. Once S is 2-connected, we consume $t \geq 2$, and S contain a cycle C of length minimum $2t$. While $V(C)$ dominating path does not have the vertex set, further few vertex u and on C its neighbors are not. Since S is connected, here is the shortest path P_u start at $V(C)$ and end at u. Summing to a path P_u along with C at one end and u's alternative neighbor at the other end (existing then $t \geq 2$) gains a path P along minimum $2t + 3$ vertices. While P it is not that a dominating path, therefore $V(P)$ neglects its neighborhood and some other vertex, it needs $n \geq 3k + 4$, an inconsistency. Therefore, S must include a cut-vertex v. Every component of $S - v$ has minimum t vertices, so $S - v$ has a maximum of three components. Since $S - v$ has maximum $3t + 2$ vertices, consuming three components along minimum t vertices needs one through precisely t vertices. So, a $S - v$ component shall be a complete graph through all vertices adjacent to v. Further two components take order maximum $k + 2$, therefore a vertex w in like a component H is nonadjacent to maximum one other vertex of H, while w is nonadjacent to v. As a result, S contains a dominating path that include v and in this case two vertices each from the two majors $S - v$ components. In the leftover case, $S - v$ consumes two components, though also contains a cut-vertex w, therefore $S - v - w$ include three closely complete components flexible in a dominating path as shown in the above section. While every component of $S - v$ is 2-connected, since all contain a cycle which is spanning or consumes minimum $2t - 2$ vertices, then removing v depart a minimum degree at least $t - 1$. All component contains at most $2t + 2$ vertices because it includes minimum t vertices. We get a path across v that is dominating and neglects very few vertices. On other hand provides a brief proof of nearly the optimal threshold for 2-connected graphs. Dirac's theorem proves too that suppose $\delta(H) > |V(H)|/2$, therefore H is Hamiltonian-connected, sense that some two vertices are a spanning path's terminus. In a P path start at u and end at v and $R \subseteq V(P)$, consider R^+ represent the instant successors set vertices of R with P, and consider R^- represents the set of instant predecessors. We know that $|R^+| = |R^-| = |R|$ once R includes no terminus of P. □

3.3. Vizing's Conjecture in Domination. The comparison of the dimensions of minimal dominant sets and Vizing conjectures in the S and L graphs, in the Cartesian product graph that is called a dominant set. The proof of the Vizing theorem with the use of some colors, every simple non-oriented graph can be multicolored.

Let $S = [V(S), E(S)]$ be determinate. In vertex subsets, P dominates K though $K \subseteq N[P]$, that is, while each K vertex is in P, otherwise is adjacent to a P vertex. P dominates outwardly K, when K, P are separate and P dominate K. The S domain number is the lowest represented cardinal $\gamma(S)$ dominating $V(S)$. Although D dominates $V(S)$, further, D dominates S and this D is a S's dominant set.

Each closed quarter in S must span any dominant set of S. Hence, the domain number of S is minimum similar to the cardinality of whatever set $X \subseteq V(S)$ consuming the characteristics that for different x_1, x_2 in X, and $N[x_1] \cap N[x_2] = \theta$. So, a set X is known as 2-packing and $\sigma(S)$ is represented the maximum cardinality of a 2-packing in S and is named the 2-packing number of S. The independence number of a vertex in S is the maximum cardinality $\sigma(S)$ of an independent set of vertices in S, and the smallest cardinality of a dominant set that is likewise independent is represented $I(S)$.

Assume S is not a complete graph prove for every vertex pair v_1 and v_2 which are not adjacent to S, it is proved that $\gamma(S) - 1 \leq \gamma(S + v_1 v_2) \leq \gamma(S)$. While S has the property that $\gamma(S) - 1 = \gamma(S + v_1 v_2)$ for that pair of nonadjacent vertices, since S is critical concerning the domain (or critical for brevity).

The graph S, which has the domain number u. S is known as a separable graph if all of its vertices can be enclosed by all of its subgraphs.

Theorem 4. *Suppose a decomposable graph S' have a spanning subgraph S, so that $\gamma(S) = \gamma(S')$, then L holds for each graph L, $\gamma(S \times L) = \gamma(S)\gamma(L)$*

Proof. Undirected, finite graphs, coherent, and simple are all considered. Specific, let S denote a graph have the edge set $E = E(S)$ and the vertex set $V = V(S)$. $m, n \in V$ are two vertices and its neighbors, otherwise in case $mn \in E$. The m's open neighborhood belongs to V and it is the m's neighbor set, denoted as $N_s(m)$, whereas the closed neighborhood $N_s[m] = N_s(m) \bigcup \{m\}$. The D' s open neighborhood contained in V and it is the set of all neighbors of vertices in D, denoted as $N_s(D)$, whereas the D' s closed neighborhood is $N_s[D] = N_s(D) \bigcup D$. But S is detached from the context, it perhaps represented by $N(D)$ and $N[D]$ or $N_s(D)$ and $N_s[D]$ correspondingly. The space among two vertices $m, n \in V$ is in S the shortest length (m, n) path and is represented by $d_s(m, n)$. In two graphs, the Cartesian products are $S(V_1, E_1)$ and $L(V_2, E_2)$ represented by $S \times L$, is a vertex-set graph $V_1 \times V_2$ and edge set $E(S \times L) = \{((u_1, v_1), (u_2, v_2))\colon v_1 = v_2 \text{and} (u_1, u_2) \in E_1, \text{or} u_1 = u_2 \text{and} (v_1, v_2) \in E_2\}$.

A subset of vertices $D \subseteq V(S)$ is known as a dominant set of half sum, if $N[D] = V(S)$, and every vertex $u \in D$ a vertex $v \in D$ occurs, thus $d(u, v) \leq 2$. When D is a dominant set of half-sums in the induced subgraph $D \cup T$ of S, a vertex set D semidominates a vertex set T. The semitotal dominance number of S, denoted as $\gamma_{t2}(S)$, is known as a minimum half-sum dominating set size of S. A 2-pack is a subset of S vertices T in which each pair of T' s vertices are a minimum of 3 separate. The maximum 2-pack size of S is known as the 2-pack number [6–8]. □

Theorem 5. *To S, L are all isolate-free graphs. Then,*

$$\gamma_{t2}(S \times L) \geq \rho(S)\gamma_{t2}(L). \tag{3}$$

Proof. Let us take $\{v_1, \ldots, v_p(S)\}$ be a max of 2 packages of graph. Consider without restrictions as $\rho(S) = \gamma(S)$. Every vertex in the graph is at least three far from the packing of vertices. The closed adjacent $N_s[v_i]$ are represented as pairwise disjoint and for $i = 1, \rho(S)$. Consider $\{v_1, \ldots, v_p(S)\}$ is said to be a partition of $V(S)$ just like for $1 \leq \rho(S), N_s[V_i]$. Let B be an $\gamma_{t2}(S \times L)S$-set. For $i = 1, \rho(S)$. Let $B_i = B \cap (V_i \times V(L))$. Moreover, consider a minimum set C_i of vertices $S \times L$ that L_i dominate completely and include as several vertices as feasible in L_i. Further $C_i \subseteq v_i \times V(L)$. Next x is not present in L_i when C_i has a vertex, and x is considered to be the uniquely determined vertex that entirely dominates x' for $x' \in L_i$. Since x' contains neighbors that pertain to L_i, vertices in C_i dominate that all neighbors, even now C_i is a semisumerally dominant set once x is changed to x' in C_i. Hence, vertices set that almost fully dominate L_i and have further vertices in L_i, therefore, called C_i, is an inconsistency. Since $C_i \subseteq L_i$ are subsets, and thus C_i is a partial dominance of L in $S \times L$ persuaded by L_i. Then B_i partially dominance $\{v_i\} \times (L)$, $|B_i| \geq |C_i|$. Therefore, $\gamma_{t2}(S \times L) \geq \rho(S) \sum i = 1|C_i| \geq \rho(S) \sum i = 1\gamma_{t2}(S \times L) = \rho(S)\gamma_{t2}(L)$.

The subtotal must be calculated of the domination number and the results of Vizing's type based on it. Separating minimum half dominating sets into partially dominating sets which is considered as completely dominate. $U = \{u_1, \ldots, u_k\}$ is considered to be a minimum of a semidominant set of graphs S, note that it is suitable for each graph. It might be partitioned into two sets of $X\&Y$. Here X represents vertices set of U that are nearer to anyone vertex of U, on the other hand, Y represented as U on X. Take $\{U_1, \ldots, U_k\}$ as the minimum dominating set of vertices for every graph S, also take $X_i\&1 \leq i \leq k$, and $X_i\&Y_i$ represents partitions in allied and free sets considerably. Therefore, it represents U_i, so as a result, $|X_i|$ is considered to be the max extent for $1 \leq i \leq k$, a maximum relayed semitotal dominant set of S. Maximum of allied partition of the graph S is represented as $\{X_i, Y_i\}$. The set X_i denote a maximum related set of S, and the set Y_i a minimum free set of S. Each maximal related partition of $S\{X, Y\}$ assume $x(S) = |X|$ and $y(s) = |Y|$. □

4. Conclusion

Hamiltonian cycle's quasi-spanning tree of faces is executed in this research. In a cubic bipartite planer graph, a polynomial time technique is utilized to reduce the issues in the minimum quasi spanning tree. For another graph-like products and other domination, numerous researchers have conducted Vizing's conjecture. But still, this conjecture is not yet demonstrated. To prove Vizing's conjecture, a graph theory described one or two conjectures which are still considered as wider problems. To separate free graphs, Vizing types are based on the subtotal of domination number and it proved as well. Vizing's conjecture is said to be true when the polynomial time was positive concerning the particularly build ideal. And here Vizing's conjecture is designed by the graph theory as an appropriate pair.

Data Availability

No data were used to support this study.

Disclosure

An earlier version of the manuscript is presented as preprint in [4] in the link https://theory.stanford.edu/~tomas/barnew.pdf.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

The authors thank VR Siddhartha Engineering College, Vijayawada, Shree Institute of Technical Education, Tirupati, and Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj, Bangladesh.

References

[1] S. Mondal, A. Dey, and T. Pal, "Robust and minimum spanning tree in fuzzy environment," *International Journal of Computing Science and Mathematics*, vol. 1, no. 1, pp. 1–524, 2017.

[2] A. Dey, S. Mondal, and T. Pal, "Robust and minimum spanning tree in fuzzy environment," *International Journal of Computing Science and Mathematics*, vol. 10, no. 5, pp. 513–524, 2019.

[3] A. Dey, R. Pradhan, A. Pal, and T. Pal, "A genetic algorithm for solving fuzzy shortest path problems with interval type-2 fuzzy arc lengths," *Malaysian Journal of Computer Science*, vol. 31, no. 4, pp. 255–270, 2018.

[4] D. Barnette, *Conjecture 5, Recent Progress in Combinatorics*, W. T. Tutte, Ed., p. 343, Academic Press, New York, NY, USA, 1969.

[5] M. A. Henning, "Essential upper bounds on the total domination number," *Discrete Applied Mathematics*, vol. 244, pp. 103–115, 2018.

[6] G. Brinkmann, J. Goedgebeur, and B. D. McKay, "Generation of cubic graphs," *Discrete Mathematics & Theoretical Computer Science*, vol. 13, no. 2, pp. 69–80, 2011.

[7] D. Maji, G. Ghorai, and F. A. Shami, "Some New Upper Bounds for the Y-Index of Graphs," *Journal of Mathematics*, vol. 2022, no. 5, 13 pages, Article ID 4346234, 2022.

[8] D. Maji, G. Ghorai, and Y. U. Gaba, "On the reformulated second Zagreb index of graph operations," *Journal of Chemistry*, vol. 2021, no. 5, 17 pages, Article ID 9289534, 2021.

[9] E. Gaar, A. Wiegele, D. Krenn, and S. Margulies, "An optimization-based sum-of-squares approach to Vizing's conjecture," in *Proceedings of the 2019 on International Symposium on Symbolic and Algebraic Computation*, pp. 155–162, ACM, Beijing, China, July 2019.

[10] R. Davila, T. Kalinowski, and S. Stephen, "A lower bound on the zero forcing number," *Discrete Applied Mathematics*, vol. 250, pp. 363–367, 2018.

[11] R. A. Borzooei and H. Rashmanlou, "Domination in vague graphs and its applications," *Journal of Intelligent and Fuzzy Systems*, vol. 29, no. 5, pp. 1933–1940, 2015.

[12] M. A. Henning and D. F. Rall, "On the total domination number of Cartesian products of graphs," *Graphs and Combinatorics*, vol. 21, no. 1, pp. 63–69, 2005.

[13] B. L. Hartnell and D. F. Rall, "On dominating the Cartesian product of a graph and K_2," *Discussiones Mathematicae Graph Theory*, vol. 24, no. 3, pp. 389–402, 2004.

[14] G. Brinkmann, K. Coolsaet, J. Goedgebeur, and H. M´elot, "House of Graphs: a database of interesting graphs," *Discrete Applied Mathematics*, vol. 161, no. 1-2, pp. 311–314, 2013.

[15] G. Brinkmann and J. Goedgebeur, "Generation of cubic graphs and snarks with large girth," *Journal of Graph Theory*, vol. 86, no. 2, pp. 255–272, 2017.

[16] D. Maji, G. Ghorai, M. K. Mahmood, and M. A. Alam, "On the inverse problem for some topological indices," *Journal of Mathematics*, vol. 2021, no. 1, 8 pages, Article ID 9411696, 2021.

[17] A. Dey, A. Pal, and T. Pal, "Interval type 2 fuzzy set in fuzzy shortest path problem," *Mathematics*, vol. 4, p. 62, 2016.