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Research Article

An Enhanced Fermatean Fuzzy Composition Relation Based on a Maximum-Average Approach and Its Application in Diagnostic Analysis

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The idea of composition relations on Fermatean fuzzy sets based on the maximum-extreme values approach has been investigated and applied in decision making problems. However, from the perspective of the measure of central tendency, this approach is not reliable because of the information loss occasioned by the use of extreme values. Based on this limitation, we introduce an enhanced Fermatean fuzzy composition relation with a better performance rating based on the maximum-average approach. An easy-to-follow algorithm based on this approach is presented with numerical computations. An application of Fermatean fuzzy composition relations is discussed in diagnostic analysis where diseases and patients are mirrored as Fermatean fuzzy pairs characterized with some related symptoms. To ascertain the veracity of the novel Fermatean fuzzy composition relation, a comparative analysis is presented to showcase the edge of this novel Fermatean fuzzy composition relation over the existing Fermatean fuzzy composition relation.

1. Introduction

Diagnostic analysis of patients' medical samples is a delicate assignment enmeshed with vagueness and hesitation. Many approaches have been posited to ameliorate this problem, like the introduction of fuzzy sets [1]. Though the fuzzy set seems to be promising in tackling uncertainties, it is unreliable because it considers the membership degree Υ (MD) of the case under consideration without minding the possibility of hesitation. Sequel to this weakness, some generalized varieties of fuzzy sets have been put forward such as the intuitionistic fuzzy set (IFS) [2], the Pythagorean fuzzy set (PFS) [3, 4], and the Fermatean fuzzy set (FFS) [5, 6]. By including nonmembership degree Φ (NMD) to Υ of fuzzy set, the idea of IFSs was proposed and applied in numerous applicative areas. Boran and Akay [7] explored pattern recognition using a biparametric similarity measure. Some techniques of similarity measures and distance measures of IFSs have been used to handle pattern recognition problems [8–10]. In [11], a medical diagnosis was carried out based on composite relations. Similarly, in [12, 13], a diagnostic analysis was done based on a similarity measure approach.

The noticeable inadequacy of IFSs is that it only handles the scenario where the summation of membership degree Υ (MD) and nonmembership degree Φ (NMD) is not more than unity. Because of this drawback, intuitionistic fuzzy set of second type (IFSST) was proposed [3, 14], which is widely called Pythagorean fuzzy sets (PFSs) [4]. In PFS, the parameters Υ and Φ are characterized by $\Upsilon + \Phi \geq 1$ such that $\Upsilon^2 + \Phi^2 \leq 1$. PFS has been applied to solve some hands-on problems such as medical diagnosis based on composite relation [15] and other sundry problems [16, 17]. A method for undertaking multi-attribute decision-making (MADM) under interval-valued Pythagorean fuzzy linguistic information was deliberated in [18]. A number of aggregation operators using Einstein t-conorm, Einstein operator, and

Einstein t-norm under the Pythagorean fuzzy environment for decision-making were deliberated in [19, 20]. A new extension of the TOPSIS (technique for order preference by similarity to ideal solution) approach for multiple criteria decision-making (MCDM) with hesitant PFSs was discussed in [21]. Wang and Garg [22] developed some aggregation operators for PFS based on interactive Archimedean norm processes with application to MADM. In [23], a Choquet integral based interval type-2 trapezoidal fuzzy approach was applied in MCDM involving sustainable selection of supplier. A group decision-making approach was discussed in a dynamic feedback mechanism with an attitudinal consensus threshold for minimum adjustment cost [24].

In the same way as IFSs, the construct of PFSs is limited to handle a situation when $\Upsilon = \sqrt[3]{6}/2$ and $\Phi = 1/2$. In a quest to resolve this brainteaser, the intuitionistic fuzzy set of third type (IFSTT) also known as the Fermatean fuzzy set (FFS) was introduced [5, 6]. FFS has a broader scopes include $\Upsilon + \Phi \geq 1$, $\Upsilon^2 + \Phi^2 \geq 1$ and $\Upsilon^3 + \Phi^3 \leq 1$ with the ability to certainly handle indeterminate information in decision making. A number of operators on FFSs were elaborated in [25], and differential calculus of Fermatean fuzzy functions have been introduced [26]. A number of applications of FFSs in MCDM problems based on TOPSIS method, distance measures and certain weighted aggregated operators have been explored [6, 27–29]. Sari et al. [30] studied interval-valued Fermatean fuzzy sets and applied them to capital budgeting techniques. Jeevaraj [31] imposed ordering on interval-valued FFSs with application. A novel decision-making method based on Fermatean fuzzy WASPAS (weighted aggregated sum product assessment) for green construction supplier evaluation was discussed in [32]. In [33], some TOPSIS techniques via Fermatean fuzzy soft sets were discussed with application. Sahoo [34] presented certain score functions on FFSs with application to bride selection. Aydin [35] discussed a fuzzy MCDM method using Fermatean fuzzy theories, and Zhou et al. [36] applied the Fermatean fuzzy ELECTRE (Elimination Et Choix Traduisant la Realite) method to tackle multiple-criteria group decision making. Shahzadi et al. [37] discussed MADM via Fermatean fuzzy Hamacher interactive geometric operators.

The applications of FFSs based on TOPSIS and MCDM methods have been discussed in [33–37]. Some applications of FFSs in the selection of COVID-19 testing centres using aggregation operators, the SAW (simple additive weighting) approach, the VIKOR (VIekriterijumsko KOmpromisno Rangiranje) approach, and the ARAS (additive ratio assessment) approach were considered in [38, 39]. The concept of Fermatean fuzzy composition relations has been studied based on maximum-extreme values with application to diagnostic analysis using simulated data [40].

The idea of composition relation has been presented in intuitionistic fuzzy settings [11], Pythagorean fuzzy setting [41] and Fermatean fuzzy settings [40] based on the maximum-extreme values approach with applications. This approach, though presented in different frameworks, cannot be reliable because it makes use of only minimum and maximum values. This present work puts forward a new composition relation under the Fermatean fuzzy domain based on the maximum-average approach. To express the applicability of the new Fermatean fuzzy composition relation, a case of diagnostic analysis is considered via the approach where diseases and patients are viewed as Fermatean fuzzy pairs. More concepts related to this study have been studied in [42–45].

The specific objectives of the work are to (i) reiterate the max-min-max approach of composition relation [11, 40, 41] in a Fermatean fuzzy setting, (ii) present an enhanced Fermatean fuzzy composition relation based on the max-average approach, (iii) numerically demonstrate the max-min-max approach in conjunction with the new Fermatean fuzzy composition relation, (iv) decide patients' medical status in a Fermatean fuzzy environment based on Fermatean fuzzy composition relations via max-min-max approach and maximum-average approach, respectively, and (v) present a comparative analysis to showcase the edge of the new Fermatean fuzzy composition relation over the approach in [11, 40, 41]. The summary of the paper follows: Section 2 presents the basis of FFSs and the existing Fermatean fuzzy composition relation [40], Section 3 discusses the new Fermatean fuzzy composition relation via the maximum-average approach, Section 4 dwells on diagnostic analysis of patients' medical status where diseases and patients are presented as Fermatean fuzzy values, and Section 5 synopses the paper with recommendations for future work.

2. Fermatean Fuzzy Sets

Some fundamentals of FFSs have been presented in [6, 28, 29, 40]. Let $S \neq \emptyset$ designates a fixed set for this work.

Definition 1. A FFS X in S is a generalized fuzzy set of the form

$$X = \{\langle s, \Upsilon_X(s), \Phi_X(s)\rangle | s \in S\}, \tag{1}$$

where $\Upsilon_X, \Phi_X\colon S \longrightarrow [0,1]$ define MD and NMD of $s \in S$ for $0 \leq \Upsilon_X^3(s) + \Phi_X^3(s) \leq 1$. For a FFS X in S,

$$\Psi_X(s) \in [0,1] = \sqrt[3]{1 - \Upsilon_X^3(s) - \Phi_X^3(s)}, \tag{2}$$

represents the FFS index or hesitation margin of X.

In an IFS, $0 \leq \Upsilon + \Phi \leq 1$, $\Psi = 1 - \Upsilon - \Phi$ and $\Upsilon + \Phi + \Psi = 1$. For PFS, $0 \leq \Upsilon^2 + \Phi^2 \leq 1$, $\Psi = \sqrt{1 - \Upsilon^2 - \Phi^2}$ and $\Upsilon^2 + \Phi^2 + \Psi^2 = 1$. For the case of FFS, we have $0 \leq \Upsilon^3 + \Phi^3 \leq 1$, $\Psi = \sqrt[3]{1 - \Upsilon^3 - \Phi^3}$ and $\Upsilon^3 + \Phi^3 + \Psi^3 = 1$.

Now, some properties of FFSs are presented including equality, inclusion, complement, union, and intersection.

Definition 2. Suppose X and Y in S are FFSs, then

(i) $\overline{X} = \{\langle s, \Phi_X(s), \Upsilon_X(s)\rangle | s \in S\}$

(ii) $X = Y$ iff $\Upsilon_X(s) = \Upsilon_Y(s)$, $\Phi_X(s) = \Phi_Y(s), \forall s \in S$

(iii) $X \subseteq Y$ iff $\Upsilon_X(s) \leq \Upsilon_Y(s)$, $\Phi_X(s) \geq \Phi_Y(s), \forall s \in S$

(iv) $X \prec Y$ iff $\Upsilon_X(s) \leq \Upsilon_Y(s)$, $\Phi_X(s) \leq \Phi_Y(s), \forall s \in S$

(v) $X \cup Y =$
$\{\langle s, \max(\Upsilon_X(s), \Upsilon_Y(s)), \min(\Phi_X(s), \Phi_Y(s))\rangle | s \in S\}$

(vi) $X \cap Y = \{\langle s, \min(\Upsilon_X(s), \Upsilon_Y(s)), \max(\Phi_X(s), \Phi_Y(s))\rangle | s \in S\}$

Now, we present a Fermatean fuzzy pairs (FFPs) thus

Definition 3. FFP is designated by $\langle \alpha, \beta \rangle$ such that $\alpha^3 + \beta^3 \leq 1$ where $\alpha, \beta \in [0,1]$. A FFP evaluates the FFS for which the components (α and β) are interpreted as MD and NMD.

For simplicity sake, we write a FFS $X = \{\langle s, \Upsilon_X(s), \Phi_X(s)\rangle | s \in S\}$ as $X = (\Upsilon_X(s), \Phi_X(s))$.

2.1. Fermatean Fuzzy Composite Relation. Composite relation has been established under IFS, PFS, and FFS [11, 40, 41] to enhance the applications of IFSs, PFSs, and FFSs in decision making. Suppose S_1 and S_2 are two sets. A Fermatean fuzzy relation (FFR) Δ from S_1 to S_2 is a FFS in $S_1 \times S_2$ comprises of MD Υ_Δ and NMD Φ_Δ. A FFR from S_1 to S_2 is denoted by $\Delta(S_1 \longrightarrow S_2)$ or $\Delta \in S_1 \times S_2$.

Definition 4. Suppose Δ_1 and Δ_2 are FFRs in $S_1 \times S_2$ and $S_2 \times S_3$, which can also be written as $\Delta_1(S_1 \longrightarrow S_2)$ and $\Delta_2(S_2 \longrightarrow S_3)$. Then, the Fermatean fuzzy composite relation (FFCR) $\breve{\Delta} = \Delta_1 \circ \Delta_2$ of $S_1 \times S_3$ is defined by

$$\breve{\Delta} = \left\{\langle\langle (s_1, s_3), \Upsilon_{\breve{\Delta}}(s_1, s_3), \Phi_{\breve{\Delta}}(s_1, s_3)\rangle | (s_1, s_3) \in S_1 \times S_3\rangle\right\}, \tag{3}$$

where

$$\begin{aligned} \Upsilon_{\breve{\Delta}}(s_1, s_3) &= \max\Big(\min\Big(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_2, s_3)\Big)\Big) \\ \Phi_{\breve{\Delta}}(s_1, s_3) &= \min\Big(\max\Big(\Phi_{\Delta_1}(s_1, s_2), \Phi_{\Delta_2}(s_2, s_3)\Big)\Big), \end{aligned} \tag{4}$$

$\forall (s_1, s_2) \in S_1 \times S_2$ and $(s_2, s_3) \in S_2 \times S_3$.

Using Definition 4, the FFCR $\breve{\Delta} = \Delta_1 \circ \Delta_2$ is computed by

$$\breve{\Delta} = \Upsilon_{\breve{\Delta}}(s_1, s_3) - \Phi_{\breve{\Delta}}(s_1, s_3)\Psi_{\breve{\Delta}}(s_1, s_3), \quad \forall (s_1, s_3) \in S_1 \times S_3. \tag{5}$$

The composite relation presented in [11, 40, 41] uses the extreme values, i.e., the maximum of the minimum of the membership degrees and the minimum of the maximum of the nonmembership degrees. The result from this approach is not reliable, judging from the knowledge of the measure of central tendency. Because of this limitation, we modify the technique in [11, 40, 41] based on the maximum-average approach.

3. Enhanced Fermatean Fuzzy Composite Relation

This section presents a FFCR based on the maximum of the mean values of membership degrees, and the minimum of the mean values of nonmembership degrees to enhance better performance.

Definition 5. Let Δ_1 and Δ_2 be FFRs of $S_1 \times S_2$ and $S_2 \times S_3$, which can also be written as $\Delta_1(S_1 \longrightarrow S_2)$ and $\Delta_2(S_2 \longrightarrow S_3)$. Then, the FFCR $\breve{\Pi} = \Delta_1 \circ \Delta_2$ of $S_1 \times S_3$ is defined by

$$\breve{\Pi} = \left\{\langle (s_1, s_3), \Upsilon_{\breve{\Pi}}(s_1, s_3), \Phi_{\breve{\Pi}}(s_1, s_3)\rangle | (s_1, s_3) \in S_1 \times S_3\rangle\right\}, \tag{6}$$

where

$$\begin{aligned} \Upsilon_{\breve{\Pi}}(s_1, s_3) &= \max\Big(\text{average}\Big(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_2, s_3)\Big)\Big) \\ \Phi_{\breve{\Pi}}(s_1, s_3) &= \min\Big(\text{average}\Big(\Phi_{\Delta_1}(s_1, s_2), \Phi_{\Delta_2}(s_2, s_3)\Big)\Big), \end{aligned} \tag{7}$$

for $0 \leq \Upsilon^3_{\breve{\Pi}}(s_1, s_3) + \Phi^3_{\breve{\Pi}}(s_1, s_3) \leq 1, \forall (s_1, s_3) \in S_1 \times S_3$. Certainly,

$$\Psi_{\breve{\Pi}}(s_1, s_3) = \sqrt[3]{1 - \Upsilon^3_{\breve{\Pi}}(s_1, s_3) - \Phi^3_{\breve{\Pi}}(s_1, s_3)}. \tag{8}$$

From Definition 5, the new FFCR $\breve{\Pi}$ is computed by

$$\breve{\Pi} = \Upsilon_{\breve{\Pi}}(s_1, s_3) - \Phi_{\breve{\Pi}}(s_1, s_3)\Psi_{\breve{\Pi}}(s_1, s_3), \quad \forall (s_1, s_3) \in S_1 \times S_3. \tag{9}$$

Definition 6. Suppose Δ is a FFR in $S_1 \times S_2$, then the inverse of Δ denoted as Δ^{-1} in $(S_2 \times S_1)$ is defined by

$$\Upsilon_{\Delta^{-1}}(s_2, s_1) = \Upsilon_\Delta(s_1, s_2), \Phi_{\Delta^{-1}}(s_2, s_1) = \Phi_\Delta(s_1, s_2), \quad \forall (s_1, s_2) \in S_1 \times S_2. \tag{10}$$

Definition 7. Suppose Δ_1 and Δ_2 are FFRs in $S_1 \times S_2$, then

(i) $\Delta_1 \leq \Delta_2$ iff $\Upsilon_{\Delta_1}(s_1, s_2) \leq \Upsilon_{\Delta_2}(s_1, s_2)$ and $\Phi_{\Delta_1}(s_1, s_2) \geq \Phi_{\Delta_2}(s_1, s_2), \forall (s_1, s_2) \in S_1 \times S_2$

(ii) $\Delta_1 \prec \Delta_2$ iff $\Upsilon_{\Delta_1}(s_1, s_2) \leq \Upsilon_{\Delta_2}(s_1, s_2)$ and $\Phi_{\Delta_1}(s_1, s_2) \leq \Phi_{\Delta_2}(s_1, s_2), \forall (s_1, s_2) \in S_1 \times S_2$

(iii) $\Delta_1 \wedge \Delta_2 = \left\{\langle (s_1, s_2), \min(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_1, s_2)), \max(\Phi_{\Delta_1}(s_1, s_2), \Phi_{\Delta_2}(s_1, s_2))\rangle\right\}$

(iv) $\Delta_1 \vee \Delta_2 = \{\langle (s_1, s_2), \max(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_1, s_2)), \min(\Phi_{\Delta_1}(s_1, s_2), \Phi_{\Delta_2}(s_1, s_2))\rangle\}$

(v) $\overline{\Delta_1} = \left\{\langle (s_1, s_2), \Phi_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_1}(s_1, s_2)\rangle\right\}$, $\overline{\Delta_2} = \left\{\langle (s_1, s_2), \Phi_{\Delta_2}(s_1, s_2), \Upsilon_{\Delta_2}(s_1, s_2)\rangle\right\}$

Theorem 1. *If Δ_1, Δ_2 and Δ_3 are FFRs in $S_1 \times S_2$, then*

(i) $\Delta_1 \leq \Delta_2 \Rightarrow \Delta_1^{-1} \leq \Delta_2^{-1}$

(ii) $(\Delta_1 \vee \Delta_2)^{-1} = \Delta_1^{-1} \vee \Delta_2^{-1}$

(iii) $(\Delta_1 \wedge \Delta_2)^{-1} = \Delta_1^{-1} \wedge \Delta_2^{-1}$

(iv) $(\Delta_1^{-1})^{-1} = \Delta_1$

(v) $\Delta_1 \wedge (\Delta_2 \vee \Delta_3) = (\Delta_1 \wedge \Delta_2) \vee (\Delta_1 \wedge \Delta_3)$

(vi) $\Delta_1 \vee (\Delta_2 \wedge \Delta_3) = (\Delta_1 \vee \Delta_2) \wedge (\Delta_1 \vee \Delta_3)$

(vii) if $\Delta_1 \geq \Delta_2$ *and* $\Delta_1 \geq \Delta_3$ *then* $\Delta_1 \geq \Delta_2 \vee \Delta_3$

(viii) if $\Delta_1 \leq \Delta_2$ *and* $\Delta_1 \leq \Delta_3$ *then* $\Delta_1 \leq \Delta_2 \wedge \Delta_3$

(ix) If $\Delta_1 \wedge \Delta_2 \leq \Delta_1$ *then* $\Delta_1 \wedge \Delta_2 \leq \Delta_2$

(x) If $\Delta_1 \vee \Delta_2 \geq \Delta_1$ *then* $\Delta_1 \vee \Delta_2 \geq \Delta_2$

Proof. (i) Assume $\Delta_1 \leq \Delta_2$, then

$$\Upsilon_{\Delta_1^{-1}}(s_2, s_1) = \Upsilon_{\Delta_1}(s_1, s_2) \leq \Upsilon_{\Delta_2}(s_1, s_2) = \Upsilon_{\Delta_2^{-1}}(s_2, s_1), \quad (11)$$

and similarly we have

$$\Phi_{\Delta_1^{-1}}(s_2, s_1) = \Phi_{\Delta_1}(s_1, s_2) \geq \Phi_{\Delta_2}(s_1, s_2) = \Phi_{\Delta_2^{-1}}(s_2, s_1), \quad \forall (s_1, s_2) \in S_1 \times S_2. \quad (12)$$

To prove (ii), we have

$$\begin{aligned} \Upsilon_{(\Delta_1 \vee \Delta_2)^{-1}}(s_2, s_1) &= \Upsilon_{\Delta_1 \vee \Delta_2}(s_1, s_2) \\ &= \max\left(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_1, s_2)\right) \\ &= \max\left(\Upsilon_{\Delta_1^{-1}}(s_2, s_1), \Upsilon_{\Delta_2^{-1}}(s_2, s_1)\right) \\ &= \Upsilon_{\Delta_1^{-1} \vee \Delta_2^{-1}}(s_2, s_1), \quad \forall (s_1, s_2) \in S_1 \times S_2. \end{aligned} \quad (13)$$

Similarly, we have

$$\begin{aligned} \Phi_{(\Delta_1 \vee \Delta_2)^{-1}}(s_2, s_1) &= \Phi_{\Delta_1 \vee \Delta_2}(s_1, s_2) \\ &= \min\left(\Phi_{\Delta_1}(s_1, s_2), \Phi_{\Delta_2}(s_1, s_2)\right) \\ &= \min\left(\Phi_{\Delta_1^{-1}}(s_2, s_1), \Phi_{\Delta_2^{-1}}(s_2, s_1)\right) \\ &= \Phi_{\Delta_1^{-1} \vee \Delta_2^{-1}}(s_2, s_1), \quad \forall (s_1, s_2) \in S_1 \times S_2. \end{aligned} \quad (14)$$

The proof of (iii) is similar to (ii). The proof of (iv) follows:

$$\Upsilon_{(\Delta_1^{-1})^{-1}}(s_2, s_1) = \Upsilon_{\Delta_1^{-1}}(s_1, s_2) = \Upsilon_{\Delta_1}(s_2, s_1) = \Upsilon_{\Delta_1}(s_1, s_2). \quad (15)$$

Similarly, $\Phi_{(\Delta_1^{-1})^{-1}}(s_2, s_1) = \Phi_{\Delta_1}(s_1, s_2)$.

Now, we prove (v) as follows:

$$\begin{aligned} \Upsilon_{\Delta_1 \wedge (\Delta_2 \vee \Delta_3)}(s_1, s_2) &= \min\left(\Upsilon_{\Delta_1}(s_1, s_2), \max\left(\Upsilon_{\Delta_2}(s_1, s_2), \Upsilon_{\Delta_3}(s_1, s_2)\right)\right) \\ &= \max\left(\min\left(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_1, s_2)\right), \min\left(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_3}(s_1, s_2)\right)\right) \\ &= \max\left(\Upsilon_{\Delta_1 \wedge \Delta_2}(s_1, s_2), \Upsilon_{\Delta_1 \wedge \Delta_3}(s_1, s_2)\right) = \Upsilon_{(\Delta_1 \wedge \Delta_2) \vee (\Delta_1 \wedge \Delta_3)}(s_1, s_2), \\ &\forall (s_1, s_2) \in S_1 \times S_2. \end{aligned} \quad (16)$$

Similarly, by using Definition 7 we have

$$\Phi_{\Delta_1 \wedge (\Delta_2 \vee \Delta_3)}(s_1, s_2) = \Phi_{(\Delta_1 \wedge \Delta_2) \vee (\Delta_1 \wedge \Delta_3)}(s_1, s_2), \quad \forall (s_1, s_2) \in S_1 \times S_2. \quad (17)$$

The proofs of (vi)–(x) are straightforward. □

Theorem 2. *Suppose we have two FFRs* Δ_1 *in* $S_1 \times S_2$ *and* Δ_2 *in* $S_2 \times S_3$*, respectively, then* $(\Delta_1 \circ \Delta_2)^{-1} = \Delta_1^{-1} \circ \Delta_2^{-1}$.

Proof. Firstly, we show the result with respect to the membership degree. Then,

$$\begin{aligned} \Upsilon_{(\Delta_1 \circ \Delta_2)^{-1}}(s_3, s_1) &= \Upsilon_{\Delta_1 \circ \Delta_2}(s_1, s_3) \\ &= \max\left(\text{average}\left(\Upsilon_{\Delta_1}(s_1, s_2), \Upsilon_{\Delta_2}(s_2, s_3)\right)\right) \\ &= \max\left(\text{average}\left(\Upsilon_{\Delta_1^{-1}}(s_2, s_1), \Upsilon_{\Delta_2^{-1}}(s_3, s_2)\right)\right) \\ &= \max\left(\text{average}\left(\Upsilon_{\Delta_2^{-1}}(s_3, s_2), \Upsilon_{\Delta_1^{-1}}(s_2, s_1)\right)\right) \\ &= \Upsilon_{\Delta_1^{-1} \circ \Delta_2^{-1}}(s_3, s_1), \quad \forall (s_3, s_1) \in S_3 \times S_1. \end{aligned} \quad (18)$$

Similarly, we have

$$\begin{aligned}\Phi_{(\Delta_1\circ\Delta_2)^{-1}}(s_3,s_1) &= \Phi_{\Delta_1\circ\Delta_2}(s_1,s_3)\\ &= \min\big(\text{average}\big(\Phi_{\Delta_1}(s_1,s_2),\Phi_{\Delta_2}(s_2,s_3)\big)\big)\\ &= \min\big(\text{average}\big(\Phi_{\Delta_1^{-1}}(s_2,s_1),\Phi_{\Delta_2^{-1}}(s_3,s_2)\big)\big)\\ &= \min\big(\text{average}\big(\Phi_{\Delta_2^{-1}}(s_3,s_2),\Phi_{\Delta_1^{-1}}(s_2,s_1)\big)\big)\\ &= \Phi_{\Delta_1^{-1}\circ\Delta_2^{-1}}(s_3,s_1),\quad \forall(s_3,s_1)\in S_3\times S_1.\end{aligned} \tag{19}$$

□

Theorem 3. *Suppose Δ is a FFR in $S_1\times S_2$, and Δ_1 is a FFR in $S_2\times S_3$ and $\Delta_2\le\Delta_1$. Then,*

(i) $\Delta_1\circ\Delta=\Delta^\circ\Delta_1$

(ii) $(\Delta_2\circ\Delta_1)\circ\Delta=\Delta_2\circ(\Delta_1\circ\Delta)$

Proof. (i) We show that the FFRs are commutative as follows:

$$\begin{aligned}\Upsilon_{\Delta_1\circ\Delta}(s_1,s_3) &= \max\big(\text{average}\big(\Upsilon_{\Delta}(s_1,s_2),\Upsilon_{\Delta_1}(s_2,s_3)\big)\big)\\ &= \max\big(\text{average}\big(\Upsilon_{\Delta_1}(s_2,s_3),\Upsilon_{\Delta}(s_1,s_2)\big)\big)\\ &= \Upsilon_{\Delta^\circ\Delta_1}(s_1,s_3),\quad \forall(s_1,s_3)\in S_1\times S_3.\end{aligned} \tag{20}$$

Similarly,

$$\begin{aligned}\Phi_{\Delta_1\circ\Delta}(s_1,s_3) &= \min\big(\text{average}\big(\Phi_{\Delta}(s_1,s_2),\Phi_{\Delta_1}(s_2,s_3)\big)\big)\\ &= \min\big(\text{average}\big(\Phi_{\Delta_1}(s_2,s_3),\Phi_{\Delta}(s_1,s_2)\big)\big)\\ &= \Phi_{\Delta^\circ\Delta_1}(s_1,s_3),\quad \forall(s_1,s_3)\in S_1\times S_3.\end{aligned} \tag{21}$$

(ii) Now, we proof the associativity as the FFRs as follows:

$$\begin{aligned}\Upsilon_{(\Delta_2\circ\Delta_1)\circ\Delta}(s_1,s_3) &= \max\big(\text{average}\big(\Upsilon_{\Delta}(s_1,s_2),\Upsilon_{\Delta_2\circ\Delta_1}(s_2,s_3)\big)\big)\\ &= \max\big(\text{average}\big(\Upsilon_{\Delta}(s_1,s_2),\ \max\big(\text{average}\big(\Upsilon_{\Delta_1}(s_2,s_3),\Upsilon_{\Delta_2}(s_2,s_3)\big)\big)\big)\big)\\ &= \max\big(\text{average}\big(\max\big(\text{average}\big(\Upsilon_{\Delta}(s_1,s_2),\Upsilon_{\Delta_1}(s_2,s)\big)\big),\Upsilon_{\Delta_2}(s,s_3)\big)\big)\\ &= \max\big(\text{average}\big(\Upsilon_{\Delta_1\circ\Delta}(s_1,s),\Upsilon_{\Delta_2}(s,s_3)\big)\big)=\Upsilon_{\Delta_2\circ(\Delta_1\circ\Delta)}(s_1,s_3),\end{aligned} \tag{22}$$

for all $(s_1,s_3)\in S_1\times S_3$.

On the contrary, we get

$$\begin{aligned}\Phi_{(\Delta_2\circ\Delta_1)\circ\Delta}(s_1,s_3) &= \min\big(\text{average}\big(\Phi_{\Delta}(s_1,s_2),\Phi_{\Delta_2\circ\Delta_1}(s_2,s_3)\big)\big)\\ &= \min\big(\text{average}\big(\Phi_{\Delta}(s_1,s_2),\ \min\big(\text{average}\big(\Phi_{\Delta_1}(s_2,s_3),\Phi_{\Delta_2}(s_2,s_3)\big)\big)\big)\big)\\ &= \min\big(\text{average}\big(\min\big(\text{average}\big(\Phi_{\Delta}(s_1,s_2),\Phi_{\Delta_1}(s_2,s)\big)\big),\Phi_{\Delta_2}(s,s_3)\big)\big)\\ &= \min\big(\text{average}\big(\Phi_{\Delta_1\circ\Delta}(s_1,s),\Phi_{\Delta_2}(s,s_3)\big)\big)=\Phi_{\Delta_2\circ(\Delta_1\circ\Delta)}(s_1,s_3),\end{aligned} \tag{23}$$

for all $(s_1,s_3)\in S_1\times S_3$. □

Theorem 4. *Let Δ_1 and Δ_2 be FFRs of $S_1\times S_2$, and Δ_3 and Δ_4 be FFRs of $S_2\times S_3$, then we have the following properties:*

(i) $\Delta_1\le\Delta_2\Rightarrow\Delta_1\circ\Delta^*\le\Delta_2\circ\Delta^*$ *for every FFR* Δ^* *in* $S_2\times S_3$

(ii) $\Delta_3\le\Delta_4\Rightarrow\Delta_*\circ\Delta_3\le\Delta_*\circ\Delta_4$ *for every FFR* Δ_* *in* $S_1\times S_2$

(iii) $\Delta_1\prec\Delta_2\Rightarrow\Delta_1\circ\Delta^*\prec\Delta_2\circ\Delta^*$ *for every FFR* Δ^* *in* $S_2\times S_3$

(iv) $\Delta_3\prec\Delta_4\Rightarrow\Delta_*\circ\Delta_3\prec\Delta_*\circ\Delta_4$ *for every FFR* Δ_* *in* $S_1\times S_2$

(v) $\Delta_*\le\Delta^*\Rightarrow\Delta_*\circ\Delta_*\le\Delta^*\circ\Delta^*$ *if* Δ_* *and* Δ^* *are FFRs in* $S_1\times S_1$

Proof. (i) Assume $\Delta_1\le\Delta_2$. Then, $\Upsilon_{\Delta_1}(s_1,s_2)\le\Upsilon_{\Delta_2}(s_1,s_2)$ and $\Phi_{\Delta_1}(s_1,s_2)\ge\Phi_{\Delta_2}(s_1,s_2)$. Now, we have

$$\begin{aligned}\Upsilon_{\Delta_1\circ\Delta^*}(s_1,s_3) &= \max\big(\text{average}\big(\Upsilon_{\Delta^*}(s_1,s_2),\Upsilon_{\Delta_1}(s_2,s_3)\big)\big)\\ &\le \max\big(\text{average}\big(\Upsilon_{\Delta^*}(s_1,s_2),\Upsilon_{\Delta_2}(s_2,s_3)\big)\big)\\ &= \Upsilon_{\Delta_2\circ\Delta^*}(s_1,s_3).\end{aligned} \tag{24}$$

Similarly,

$$\begin{aligned}\Phi_{\Delta_1\circ\Delta^*}(s_1,s_3) &= \min\big(\text{average}\big(\Phi_{\Delta^*}(s_1,s_2),\Phi_{\Delta_1}(s_2,s_3)\big)\big)\\ &\ge \min\big(\text{average}\big(\Phi_{\Delta^*}(s_1,s_2),\Phi_{\Delta_2}(s_2,s_3)\big)\big)\\ &= \Phi_{\Delta_2\circ\Delta^*}(s_1,s_3).\end{aligned} \tag{25}$$

The proofs of (ii)–(iv) are similar.

(v) If $\Delta_* \leq \Delta^*$, then $\Upsilon_{\Delta_*}(s_1, s_1) \leq \Upsilon_{\Delta^*}(s_1, s_1)$ and $\Phi_{\Delta_*}(s_1, s_2) \geq \Phi_{\Delta^*}(s_1, s_2)$. Then,

$$\begin{aligned}\Upsilon_{\Delta_*\circ\Delta_*}(s_1, s_1) &= \max\Big(\text{average}\Big(\Upsilon_{\Delta_*}(s_1, s_1), \Upsilon_{\Delta_*}(s_1, s_1)\Big)\Big)\\ &\leq \max\left(\text{average}\left(\Upsilon_{\Delta^*}(s_1, s_1), \Upsilon_{\Delta^*}(s_1, s_1)\right)\right)\\ &= \Upsilon_{\Delta^*\circ\Delta^*}(s_1, s_1).\end{aligned} \tag{26}$$

Similarly,

$$\begin{aligned}\Phi_{\Delta_*\circ\Delta_*}(s_1, s_1) &= \min\Big(\text{average}\Big(\Phi_{\Delta_*}(s_1, s_1), \Phi_{\Delta_*}(s_1, s_1)\Big)\Big)\\ &\geq \min\left(\text{average}\left(\Phi_{\Delta^*}(s_1, s_1), \Phi_{\Delta^*}(s_1, s_1)\right)\right)\\ &= \Phi_{\Delta^*\circ\Delta^*}(s_1, s_1).\end{aligned} \tag{27}$$

□

Theorem 5. *Let Δ_1, Δ_2 be FFRs in $S_2 \times S_3$, and Δ be FFR in $S_1 \times S_2$. Then, we have*

(i) $(\Delta_1 \vee \Delta_2)\circ\Delta \geq (\Delta_1\circ\Delta)\vee(\Delta_2\circ\Delta)$

(ii) $(\Delta_1 \wedge \Delta_2)\circ\Delta \leq (\Delta_1\circ\Delta)\wedge(\Delta_2\circ\Delta)$

Proof. By Theorem 1, we have $\Delta_1 \vee \Delta_2 \geq \Delta_1$ and $\Delta_1 \vee \Delta_2 \geq \Delta_2$. Thus, $(\Delta_1 \vee \Delta_2)\circ\Delta \geq (\Delta_1\circ\Delta)$ and $(\Delta_1 \vee \Delta_2)\circ\Delta \geq (\Delta_2\circ\Delta)$. Hence, we get $(\Delta_1 \vee \Delta_2)\circ\Delta \geq (\Delta_1\circ\Delta)\vee(\Delta_2\circ\Delta)$, which proves (i). The proof of (ii) is similar. □

Theorem 6. *Suppose Δ_1, Δ_2 are FFRs in $S_2 \times S_3$, and Δ is a FFR in $S_1 \times S_2$. Then, we have*

(i) $(\Delta_1 \vee \Delta_2)\circ\Delta = (\Delta_1\circ\Delta)\vee(\Delta_2\circ\Delta)$

(ii) $(\Delta_1 \wedge \Delta_2)\circ\Delta = (\Delta_1\circ\Delta)\wedge(\Delta_2\circ\Delta)$

Proof. We first proof (i) as follows:

$$\begin{aligned}\Upsilon_{(\Delta_1\vee\Delta_2)\circ\Delta}(s_1, s_3) &= \max\Big(\text{averge}\Big(\Upsilon_{\Delta}(s_1, s_2)\Upsilon_{\Delta_1\vee\Delta_2}(s_2, s_3)\Big)\Big)\\ &= \max\Big(\text{averge}\Big(\Upsilon_{\Delta}(s_1, s_2), \Upsilon_{\Delta_1}(s_2, s_3)\Big), \text{averge}\Big(\Upsilon_{\Delta}(s_1, s_2), \Upsilon_{\Delta_2}(s_2, s_3)\Big)\Big)\\ &= \max\Big(\Upsilon_{\Delta\circ\Delta_1}(s_1, s_3), \Upsilon_{\Delta\circ\Delta_2}(s_1, s_3)\Big) = \Upsilon_{(\Delta_1\circ\Delta)\vee(\Delta_2\circ\Delta)}(s_1, s_3).\end{aligned} \tag{28}$$

Similarly, we obtain

$$\begin{aligned}\Phi_{(\Delta_1\vee\Delta_2)\circ\Delta}(s_1, s_3) &= \min\Big(\text{averge}\Big(\Phi_{\Delta}(s_1, s_2), \Phi_{\Delta_1\vee\Delta_2}(s_2, s_3)\Big)\Big)\\ &= \min\Big(\text{averge}\Big(\Phi_{\Delta}(s_1, s_2), \Phi_{\Delta_1}(s_2, s_3)\Big), \text{averge}\Big(\Phi_{\Delta}(s_1, s_2), \Phi_{\Delta_2}(s_2, s_3)\Big)\Big)\\ &= \min\Big(\Phi_{\Delta\circ\Delta_1}(s_1, s_3), \Phi_{\Delta\circ\Delta_2}(s_1, s_3)\Big) = \Phi_{(\Delta_1\circ\Delta)\vee(\Delta_2\circ\Delta)}(s_1, s_3).\end{aligned} \tag{29}$$

The proof of (ii) is similar. □

3.1. Numerical Illustration of FFCRs.

An example is given to show the supremacy of the new FFCR over the existing FFCR [11, 40, 41].

Given that X and Y are FFSs in $S = \{s_1, s_2, s_3\}$ defined by

$$\begin{aligned}X &= \{\langle s_1, 0.6, 0.2\rangle, \langle s_2, 0.4, 0.6\rangle, \langle s_3, 0.5, 0.3\rangle\},\\ Y &= \{\langle s_1, 0.8, 0.1\rangle, \langle s_2, 0.7, 0.3\rangle, \langle s_3, 0.6, 0.1\rangle\}.\end{aligned} \tag{30}$$

Using the existing FFCR $\breve{\Delta}$, the minimum of the membership degrees between FFSs X and Y for s_1, s_2, s_3 are $0.6, 0.4, 0.5$ implying that

$$\Upsilon_{\breve{\Delta}}(s) = \max(0.6, 0.4, 0.5) = 0.6. \tag{31}$$

Similarly, the maximum of the nonmembership degrees between FFSs X and Y for s_1, s_2, s_3 are $0.2, 0.6, 0.3$, implying that

$$\Phi_{\breve{\Delta}}(s) = \min(0.2, 0.6, 0.3) = 0.2. \tag{32}$$

Now, the FFCR between X and Y is $\breve{\Delta} = 0.6 - (0.2 \times 0.9189) = 0.4162$.

Using the new FFCR $\breve{\Pi}$ in Definition 5, the mean values of the membership degrees between FFSs X and Y for s_1, s_2, s_3 are $0.7, 0.55, 0.55$. Thus,

$$\Upsilon_{\breve{\Pi}}(s) = \max(0.7, 0.55, 0.55) = 0.7. \tag{33}$$

Again, the mean values of the nonmembership degrees between FFSs X and Y for s_1, s_2, s_3 are $0.15, 0.45, 0.2$. Thus,

$$\Phi_{\breve{\Pi}}(s) = \min(0.15, 0.45, 0.2) = 0.15. \tag{34}$$

Again, the mean values of the nonmembership degrees between FFSs X and Y for s_1, s_2, s_3 are $0.15, 0.45, 0.2$. Thus,

$$\Phi_{\breve{\Pi}}(s) = \min(0.15, 0.45, 0.2) = 0.15. \tag{35}$$

The new FFCR between FFSs X and Y is $\breve{\Pi} = 0.7 - (0.15 \times 0.8678) = 0.5698$.

From the results, it is certain that the new FFCR is better than the approach in [11, 40, 41] because the FFCR between X and Y is greater for the new approach (i.e., while the existing approach yields 0.4162, the new approach yields 0.5698). This justifies the advantage of taking the mean values of the parameters of FFS over taking the extreme values.

4. Fermatean Fuzzy Composite Relation in Determination of Patients' Medical Status

This section discusses an application of FFCRs in diagnosis analysis of a patient's medical status using a simulated database of disease diagnosis. For the sake of simulation, take $\mathscr{S}$ as a set of symptoms, $\mathscr{D}$ as a set of diseases, and $\mathscr{P}$ as a set of patients. Then, we represent a medical knowledge in Fermatean fuzzy pairs based on a FFR Δ from $\mathscr{S}$ to $\mathscr{D}$ indicated by $\mathscr{S} \times \mathscr{D}$ to bespeak the grades of association and otherwise between $\mathscr{S}$ and $\mathscr{D}$. In the Fermatean fuzzy medical diagnostic process, the symptoms of the diseases are determined, the medical knowledge of the patients based on Fermatean fuzzy values is formulated, and the diagnosis on the basis of the composition using the existing FFCR and the new FFCR are determined.

4.1. New FFCR between Patients and Diseases. Suppose the medical condition of a patient $\mathscr{P}$ is described in terms of a set of symptoms $\mathscr{S}$, then $\mathscr{P}$ is taken to be assigned a diagnosis based on Π via a FFR Δ_1 from $\mathscr{S}$ to $\mathscr{D}$ designated as $\mathscr{S} \longrightarrow \mathscr{D}$ as simulated by medical knowledge in terms of degrees of association and otherwise.

We construct a FFR Δ_2 from $\mathscr{P}$ to $\mathscr{S}$ represented by $\mathscr{P} \longrightarrow \mathscr{S}$ as Δ_1. Then, the FFCR $\breve{\Pi}$ of Δ_1 and Δ_2 (i.e., $\breve{\Pi} = \Delta_1 \circ \Delta_2$) signifies the medical condition of the patients with regards to the ailments given by the MD and NMD in equation (36).

$$\begin{aligned}\Upsilon_{\breve{\Pi}}(p,d) &= \max\big(\text{average}\big(\Upsilon_{\Delta_2}(p,s), \Upsilon_{\Delta_1}(s,d)\big)\big),\\ \Phi_{\breve{\Pi}}(p,d) &= \min\big(\text{average}\big(\Phi_{\Delta_2}(p,s), \Phi_{\Delta_1}(s,d)\big)\big),\end{aligned} \tag{36}$$

for all the patients and ailments.

The result for which $\breve{\Pi} = \Upsilon_{\breve{\Pi}}(p,d) - \Phi_{\breve{\Pi}}(p,d)\Psi_{\breve{\Pi}}(p,d)$ is the greatest determines the diagnosis of the patient P. For easy computation of the FFCR, the algorithm which describes the step-to-step computational processes of the composite relation between the patients and the diseases is given as follows:

Step 1: Institute a relation between $\mathscr{S}$ and $\mathscr{D}$ as FFPs

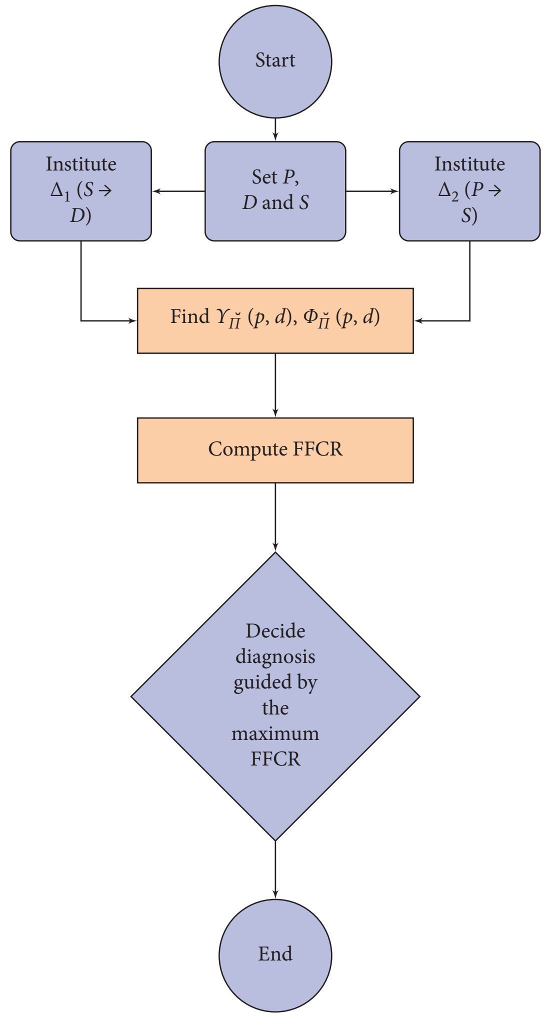
Step 2: Institute a relation between $\mathscr{P}$ and $\mathscr{S}$ as FFPs

Step 3: Find MD and NMD of $\Delta_1 \circ \Delta_2$ between the patients and diseases with respect to the clinical symptoms

Step 4: Calculate FFCR $\breve{\Pi}$ between the patients and diseases using the information from Step 3

Step 5: Decide the diagnosis on the basis of the relation for which the FFCR Π is maximum

The algorithm can be represented as a flowchart.



4.2. Application Example. Assume patients $\mathscr{P} = \{\mathscr{P}_1, \mathscr{P}_2, \mathscr{P}_3, \mathscr{P}_4\}$ visit a medical lab to ascertain their health conditions. After the vital signs of the patients were collected, the following symptoms $\mathscr{S}$, namely, high temperature, headache, stomach pain, cough, and chest pain were observed. From medical knowledge of the consultation, we simulate FFR $\Delta_2(\mathscr{P} \longrightarrow \mathscr{S})$, as shown in Table 1.

After the medical consultations guided by the vital signs, the patients are suspected to be infected by viral fever (V), malaria (M), typhoid fever (T), stomach problem (S), and heart problem (H). Similarly, FFR $\Delta_1(\mathscr{S} \longrightarrow \mathscr{D})$ is given in Table 2. The simulated data in Tables 1 and 2 were used in [11] (S. K. De, R. Biswas, A. R. Roy (2001) An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets and Systems 117(2) 209–213) to demonstrate the application of IFSs in medical diagnosis. However, the data are extended to Fermatean fuzzy values in this work.

Table 1: $\Delta_2 (\mathscr{P} \longrightarrow \mathscr{S})$.

Δ_2	Temp	Headache	Stomach pain	Cough	Chest pain
$\Upsilon_{\mathscr{P}_1}$	0.8000	0.6000	0.2000	0.6000	0.1000
$\Phi_{\mathscr{P}_1}$	0.1000	0.1000	0.8000	0.1000	0.6000
$\Upsilon_{\mathscr{P}_2}$	0.0000	0.4000	0.6000	0.1000	0.1000
$\Phi_{\mathscr{P}_2}$	0.8000	0.4000	0.1000	0.7000	0.8000
$\Upsilon_{\mathscr{P}_3}$	0.8000	0.8000	0.0000	0.2000	0.0000
$\Phi_{\mathscr{P}_3}$	0.1000	0.1000	0.6000	0.7000	0.5000
$\Upsilon_{\mathscr{P}_4}$	0.6000	0.5000	0.3000	0.7000	0.3000
$\Phi_{\mathscr{P}_4}$	0.1000	0.4000	0.4000	0.2000	0.4000

Table 2: $\Delta_1 (\mathscr{S} \longrightarrow \mathscr{D})$.

Δ_1	Temp	Headache	Stomach pain	Cough	Chest pain
Υ_V	0.4000	0.3000	0.1000	0.4000	0.1000
Φ_V	0.0000	0.5000	0.7000	0.3000	0.7000
Υ_M	0.7000	0.2000	0.0000	0.7000	0.1000
Φ_M	0.0000	0.6000	0.9000	0.0000	0.8000
Υ_T	0.3000	0.6000	0.2000	0.2000	0.1000
Φ_T	0.3000	0.1000	0.7000	0.6000	0.9000
Υ_S	0.1000	0.2000	0.8000	0.2000	0.2000
Φ_S	0.7000	0.4000	0.0000	0.7000	0.7000
Υ_H	0.1000	0.0000	0.2000	0.2000	0.8000
Φ_H	0.8000	0.8000	0.8000	0.8000	0.1000

Table 3: MD and NMD for FFCR $\breve{\Pi}$.

$\mathscr{P}$ vs $\mathscr{D}$	V	M	T	S	H
$\mathscr{P}_1$	0.6000	0.7500	0.6000	0.5000	0.4500
	0.0500	0.0500	0.1000	0.2500	0.3500
$\mathscr{P}_2$	0.3500	0.4000	0.5000	0.7000	0.4500
	0.4000	0.3500	0.2500	0.0500	0.4500
$\mathscr{P}_3$	0.6000	0.7500	0.7000	0.5000	0.4500
	0.0500	0.0500	0.1000	0.2500	0.3000
$\mathscr{P}_4$	0.5500	0.7000	0.5500	0.5500	0.5500
	0.0500	0.0500	0.2000	0.2000	0.2500

Table 4: MD and NMD for FFCR $\breve{\Delta}$.

$\mathscr{P}$ vs $\mathscr{D}$	V	M	T	S	H
$\mathscr{P}_1$	0.4000	0.7000	0.6000	0.2000	0.2000
	0.1000	0.1000	0.1000	0.4000	0.6000
$\mathscr{P}_2$	0.3000	0.2000	0.4000	0.6000	0.2000
	0.5000	0.6000	0.4000	0.1000	0.8000
$\mathscr{P}_3$	0.4000	0.7000	0.6000	0.2000	0.2000
	0.1000	0.1000	0.1000	0.4000	0.5000
$\mathscr{P}_4$	0.4000	0.7000	0.5000	0.3000	0.3000
	0.1000	0.1000	0.3000	0.4000	0.4000

By applying our new approach, we get the parameters MD and NMD, as shown in Table 3.

By applying the approach of [11] in a Fermatean fuzzy setting, the parameters MD and NMD of FFCR $\breve{\Delta}$ are computed, as shown in Table 4.

After calculating the indexes of the FFPs, the results for FFCRs using the existing approach [11] are contained in the following matrix:

$$\begin{array}{c} \begin{array}{ccccc} \text{V} & \text{M} & \text{T} & \text{S} & \text{H} \end{array} \\ \begin{bmatrix} 0.3022 & 0.6131 & 0.5078 & -0.1902 & -0.3513 \\ -0.1733 & -0.3513 & 0.0178 & 0.5078 & -0.4264 \\ 0.3022 & 0.6131 & 0.5078 & -0.1902 & -0.2768 \\ 0.3022 & 0.6131 & 0.2161 & -0.0875 & -0.0875 \end{bmatrix} \end{array} \begin{array}{c} \mathcal{P}_1 \\ \mathcal{P}_2 \\ \mathcal{P}_3 \\ \mathcal{P}_4 \end{array}. \quad (37)$$

From the matrix, the following diagnosis are deduced:

(i) Patient $\mathscr{P}_1$ is diagnosed with malaria fever with a reasonable proportion of typhoid fever

(ii) Patient $\mathscr{P}_2$ is diagnosed with stomach problem

(iii) Patient $\mathscr{P}_3$ is diagnosed with malaria fever with a reasonable proportion of typhoid fever

(iv) Patient $\mathscr{P}_4$ is diagnosed with malaria fever

None of the patient is suffering from viral fever and heart problem. $\mathscr{P}_1$ has negative relation with stomach problem and heart problem; $\mathscr{P}_2$ has negative relation with viral fever, malaria fever, and heart problem; $\mathscr{P}_3$ has negative relation with stomach problem and heart problem; and $\mathscr{P}_4$ also has negative relation with stomach problem and heart problem. From the analysis, it is sensible the physician administers the same treatment to patients $\mathscr{P}_1$ and $\mathscr{P}_3$ because they have the same infection load of malaria fever and typhoid fever.

Similarly, the results using the new FFCR $\breve{\Pi}$ are contained in the following matrix:

$$\begin{array}{c} \begin{array}{ccccc} \text{V} & \text{M} & \text{T} & \text{S} & \text{H} \end{array} \\ \begin{bmatrix} 0.5539 & 0.7084 & 0.5078 & 0.2623 & 0.1164 \\ -0.0352 & -0.0352 & 0.2623 & 0.6565 & 0.0292 \\ 0.5539 & 0.7084 & 0.6131 & 0.2623 & 0.1623 \\ 0.5029 & 0.6584 & 0.3624 & 0.3624 & 0.3162 \end{bmatrix} \end{array} \begin{array}{c} \mathcal{P}_1 \\ \mathcal{P}_2 \\ \mathcal{P}_3 \\ \mathcal{P}_4 \end{array}. \quad (38)$$

From the matrix, we obtain the following diagnosis:

(i) Patient $\mathscr{P}_1$ is suffering from malaria fever with a reasonable proportion of viral fever and typhoid fever

(ii) Patient $\mathscr{P}_2$ is suffering from stomach problem only

(iii) Patient $\mathscr{P}_3$ is suffering from malaria fever with a reasonable proportion of typhoid fever and viral fever

(iv) Patient $\mathscr{P}_4$ is suffering from malaria fever with a reasonable proportion of viral fever

In order to show the edge of FFSs over IFSs and PFSs in terms of the ability to restrict uncertainties based on the new composition approach, we make use of the data in Tables 1–3 to compute the composite relation between each patients and diseases. By using the data as intuitionistic fuzzy data, we get the results in the matrix that follows:

$$\begin{array}{c} \begin{array}{ccccc} \text{V} & \text{M} & \text{T} & \text{S} & \text{H} \end{array} \\ \begin{bmatrix} 0.5825 & 0.7400 & 0.5700 & 0.4375 & 0.3800 \\ 0.5825 & 0.3125 & 0.4375 & 0.6875 & 0.4050 \\ 0.5825 & 0.7400 & 0.6800 & 0.4375 & 0.3750 \\ 0.5300 & 0.6875 & 0.5000 & 0.5000 & 0.5000 \end{bmatrix} \end{array} \begin{array}{c} \mathcal{P}_1 \\ \mathcal{P}_2 \\ \mathcal{P}_3 \\ \mathcal{P}_4 \end{array}. \quad (39)$$

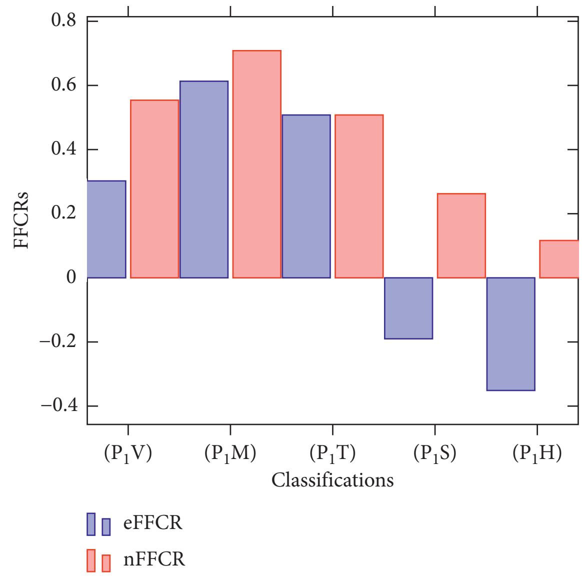


Figure 1: P_1 vs diseases.

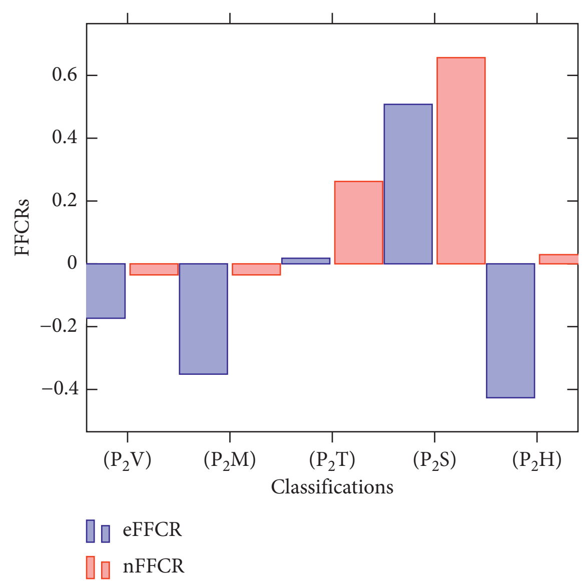


Figure 2: P_2 vs diseases.

From the results using intuitionistic fuzzy data, the following diagnoses are given: patient $\mathscr{P}_1$ is suffering from the same disease as given by our approach; patient $\mathscr{P}_2$ is suffering from stomach problem with a reasonable proportion of viral fever, which is different from the diagnosis of our approach (because of the inability of IFS to reasonably curb uncertainties); patient $\mathscr{P}_3$ is suffering from the same diseases as given by our approach; patient $\mathscr{P}_4$ is suffering from malaria fever with a reasonable proportion of viral fever, and equal proportion of typhoid fever, stomach problem, and heart problem (different from the diagnoses of our approach due to the inability of IFS to reasonably curb uncertainties). Though the values of the composite relation using intuitionistic fuzzy data are greater than our approach, it is certainly because of the inability of IFS to reasonably restrict the uncertainties in the process of diagnosis.

In addition, by using the data as Pythagorean fuzzy data, we get the following results:

$$\begin{array}{c} \begin{array}{ccccc} \mathrm{V} & \mathrm{M} & \mathrm{T} & \mathrm{S} & \mathrm{H} \end{array} \\ \begin{bmatrix} 0.5601 & 0.7170 & 0.5206 & 0.2927 & 0.1624 \\ 0.0112 & 0.1035 & 0.2927 & 0.6644 & 0.1029 \\ 0.5601 & 0.7170 & 0.6293 & 0.2927 & 0.1977 \\ 0.5083 & 0.6644 & 0.3878 & 0.3878 & 0.3508 \end{bmatrix} \end{array} \begin{array}{c} \\ \mathcal{P}_1 \\ \mathcal{P}_2 \\ \mathcal{P}_3 \\ \mathcal{P}_4 \end{array}. \tag{40}$$

Because the concept of PFSs is better than IFSs in terms of the ability to control uncertainties, the diagnoses gotten from the Pythagorean fuzzy data using our approach are the same as with our new Fermatean fuzzy composite relation approach. Though the values of the composite relation using

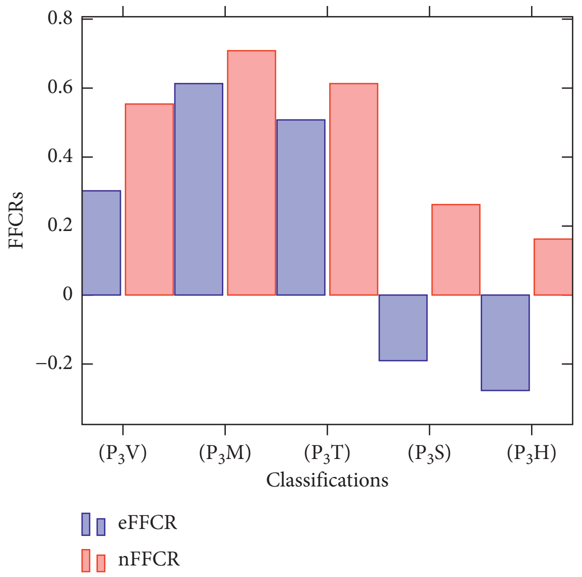


FIGURE 3: P_3 vs diseases.

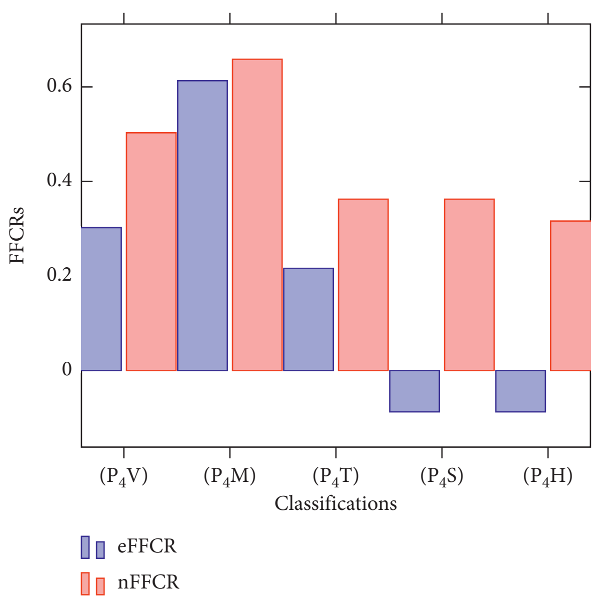


FIGURE 4: P_4 vs diseases.

TABLE 5: Comparative table.

$\mathscr{P}$ vs $\mathscr{D}$	V	M	T	S	H
$\mathscr{P}_1$	0.3022	0.6131	0.5078	−0.1902	−0.3513
	0.5539	**0.7084**	**0.5078**	**0.2623**	**0.1164**
$\mathscr{P}_2$	−0.1733	−0.3513	0.0178	0.5078	−0.4264
	−0.0352	**−0.0352**	**0.2623**	**0.6565**	**0.0292**
$\mathscr{P}_3$	0.3022	0.6131	0.5078	−0.1902	−0.2768
	0.5539	**0.7084**	**0.6131**	**0.2623**	**0.1623**
$\mathscr{P}_4$	0.3022	0.6131	0.2161	−0.0875	−0.0875
	0.5029	**0.6584**	**0.3624**	**0.3624**	**0.3162**

Pythagorean fuzzy data based on our approach are slightly greater than our Fermatean fuzzy data approach, it is certainly because of the failure of PFSs to reasonably restrain the uncertainties of the diagnostic process.

4.3. Comparative Analysis of the FFCRs. To establish the superiority of the new FFCR (nFFCR) over the existing FFCR (eFFCR) [11, 40, 41], a comparative analysis is presented in Figures 1–4 and Table 5.

From Figures 1–4 and Table 5, the new FFCR is superior to the existing FFCR because it provides better relation between the patients and ailments, and thus will guide physician on the suitable treatment. The existing FFCR uses extreme values (the max-min-max approach), whereas the new FFCR uses the maximum-average approach to augment the performance rating. The diagnostic analysis derived from the existing FFCR and the new FFCR are the same, although the new FFCR shows more additional diagnosis with grade of severities. In fact, while the approach in [11, 40, 41] shows that patients $\mathscr{P}_1$ and $\mathscr{P}_3$ should be treated for malaria fever and typhoid fever only, the new approach suggested treatment for viral fever for the patients in addition to malaria fever and typhoid fever. Similarly, while the existing FFCR suggested that patient $\mathscr{P}_4$ should be treated for malaria fever, the new approach included treatment for viral fever.

5. Conclusion

tIn this paper, we have introduced an enhanced FFCR with a better performance rating and applied it in determining the medical diagnosis of certain patients. The modified FFCR was introduced to promote the application of FFSs in decision-making. An algorithm for the modified FFCR was presented to ease computation. To validate the advantage of the new FFCR over the existing approach [11, 40, 41], a comparative analysis was presented in Table 5, from which the modified FFCR approach outperformed the existing approach. Diagnostic analysis on some patients wasconducted based on the modified FFCR where the patients and ailments were presented in FFPs. The medical diagnosis via the modified FFCR will improve suitable drug administration and therapy. The new FFCR will foster further application of FFSs in practical areas of decision making. Although the proposed method enhances reliability with a better performance rating compared to the extreme values method, it has some limitations, which include the following: (i) it cannot be used to model cases involving picture fuzzy information and spherical fuzzy information because it only admits three parameters, and (ii) the maximum-average approach of composite relation cannot be determined by mere inspection like the maximum-extreme values approach.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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