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Research Article

Complex-Valued Migrativity of Complex Fuzzy Operations

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Complex fuzzy sets (CFSs), as an important extension of fuzzy sets, have been investigated in the literature. Operators of CFSs are of high importance. In addition, α−migrativity for various fuzzy operations on [0, 1] has been well discussed, where α is a real number and $\alpha \in [0,1]$. Thus, this paper studies α−migrativity for binary functions on the unit circle of the complex plane $\mathbf{O}$, where α is a complex number and $\alpha \in \mathbf{O}$. In particular, we show that a binary function is α−migrativity for all $\alpha \in \mathbf{O}$ if and only if it is α−migrativity for all $\alpha \in [0,1] \cup \overline{\mathbf{O}}$, where $\overline{\mathbf{O}}$ is the boundary point subset of $\mathbf{O}$. Finally, we discuss the relationship between migrativity and rotational invariance of binary operators on $\mathbf{O}$.

1. Introduction

Complex fuzzy sets (CFSs) were introduced by Ramot et al. [1, 2], whose membership degree is a complex number on the unit disc of the complex plane $\mathbf{O}$, where $\mathbf{O} = \{\alpha \in \mathbb{C} \|\alpha| \leq 1\}$. Operations are of high importance in the theory of CFSs. Various concepts and properties have been developed for complex fuzzy operations. Dick [3] introduced the rotational invariance of operators of CFSs. Dai [4, 5] generalized Dick's works on rotational invariance and order induced by algebraic product operation. Zhang et al. [6] studied operation properties and δ-equalities of CFSs. Dick, Yager, and Yazdanbahksh [7] gave some complex fuzzy operations based on Pythagorean fuzzy operations, which was developed by Liu et al. [8]. Then Dick [9] considered complex fuzzy S-implications. Hu et al. [10–13] discussed orthogonality preserving operators and parallelity preserving operators of CFSs.

The α-migrativity [14] as an important property of binary fuzzy operators has been discussed in the cases of overlap/grouping functions [15, 16], uninorms [17–22], triangular subnorm [23], t-norms [24], nullnorm [25], copulas [26, 27], and aggregation functions [28–30]. In the aforementioned migrative functions, their research domain is limited to real numbers on [0, 1]. For example, a binary function $f\colon \mathbf{I}^2 \longrightarrow \mathbf{I}$ is migrative if $f(\alpha x, y) = f(\mu, \alpha \nu)$ holds for all $\mu, \nu \in \mathbf{I}$ and $\alpha \in \mathbf{I}$, where $\mathbf{I} = [0,1]$.

This paper focuses on the α-migrativity of complex fuzzy binary operations, i.e., functions $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$, where $\alpha \in \mathbf{O}$ is a complex number. Moreover, since a CFS is composed of a magnitude term and a phase term, we consider magnitude-migrativity and phase-migrativity, which respectively limits $\alpha \in \mathbf{I}$ and $\alpha \in \overline{\mathbf{O}}$, where $\overline{\mathbf{O}}$ is the boundary point subset of $\mathbf{O}$, i.e., $\overline{\mathbf{O}} = \{\alpha \in \mathbb{C} \|\alpha| = 1\}$.

As far as we know, migrativity including magnitude-migrativity and phase-migrativity of complex fuzzy operations have not been studied yet. Moreover, we note that phase-migrativity and rotational invariance [3, 4] of complex fuzzy operations are similar with respect to angle rotation operations. It is essential to straighten out the relationship between phase-migrativity and rotational invariance for complex fuzzy operations.

This article is structured as follows: in Section 2, we introduce the concepts of migrativity, magnitude-migrativity, and phase-migrativity for complex fuzzy binary operations. In Section 3, we give characterizations of these migrativity properties of complex fuzzy binary operations. In Section 4, the relationship between rotational invariance and migrativity is studied. In Section 5, concluding remarks are given.

2. Migrativity

Definition 1. Consider a fixed point $\alpha \in \mathbf{O}$, a binary operation $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ is said to be α−migrative if

$$f(\alpha\mu, \nu) = f(\mu, \alpha\nu), \quad \text{for all } \mu, \nu \in \mathbf{O}. \tag{1}$$

Note that α− migrativity refers to a fixed complex number α. This can be generalized as follows:

Definition 2. A binary operation $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ is said to be migrative if and only if (briefly, iff)

$$f(\alpha\mu, \nu) = f(\mu, \alpha\nu), \quad \text{for all } \mu, \nu \in \mathbf{O} \text{ and } \alpha \in \mathbf{O}. \tag{2}$$

A complex vector includes the amplitude term and the phase part. So, we introduce the following concepts:

Definition 3. A binary operation $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ is said to be amplitude-migrative iff

$$f(r\mu, \nu) = f(\mu, r\nu), \quad \text{for all } \mu, \nu \in \mathbf{O} \text{ and } r \in \mathbf{I}. \tag{3}$$

Definition 4. A binary operation $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ is said to be phase-migrative if and only if

$$f\left(e^{j\theta}\mu, \nu\right) = f\left(\mu, e^{j\theta}\nu\right), \quad \text{for all } \mu, \nu \in \mathbf{O} \text{ and } \theta \in \mathbb{R}, \tag{4}$$

where $j = \sqrt{-1}$.

Note that phase-migrativity means α-migrativity for all $\alpha \in \overline{\mathbf{O}}$.

Theorem 1. *A binary operation $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ is migrative iff, for all $r \in \mathbf{I}$ and $\theta \in \mathbb{R}$, it holds that*

$$\left.\begin{aligned} f\left(e^{j\theta}\mu, \nu\right) &= f\left(\mu, e^{j\theta}\nu\right) \\ f(r\mu, \nu) &= f(\mu, r\nu) \end{aligned}\right\}. \tag{5}$$

Proof. ($\Rightarrow$) Trivial.

($\Leftarrow$) For any $\alpha \in \mathbf{O}$, denote $\alpha = r_\alpha \cdot e^{j\theta_\alpha}$ where $r_\alpha \in \mathbf{I}$. Then $f(\alpha\mu, \nu) = f(r_\alpha \cdot e^{j\theta_\alpha}\mu, \nu) = f(e^{j\theta_\alpha}\mu, r_\alpha \cdot \nu) = f(\mu, e^{j\theta_\alpha} \cdot r_\alpha\nu) = f(\mu, \alpha\nu)$. □

For a complex fuzzy binary function f, as shown in Figure 1(a) and 1(b), if it is phase-migrative, then we have $\beta_1 = \beta_2$ for any θ and inputs $\mu, \nu \in \mathbf{O}$.

A binary operation is migrative if and only if it is amplitude-migrative and phase-migrative. From this result, we have the following result: □

Corollary 1. *Let $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a binary operation. Then the following statements are equivalent.*

(1) $f(\alpha\mu, \nu) = f(\mu, \alpha\nu)$, for all $\mu, \nu \in \mathbf{O}$ and $\alpha \in \mathbf{O}$;

(2) $f(\alpha\mu, \nu) = f(\mu, \alpha\nu)$, for all $\mu, \nu \in \mathbf{O}$ and $\alpha \in \mathbf{I} \cup \overline{\mathbf{O}}$.

Note that f is α−migrative for all $\alpha \in \mathbf{O}$ if and only if it is α−migrative for all $\alpha \in \mathbf{I} \cup \overline{\mathbf{O}}$. This is very interesting because $\mathbf{I} \cup \overline{\mathbf{O}}$ is a proper subset of $\mathbf{O}$, i.e., $(\mathbf{I} \cup \overline{\mathbf{O}}) \subsetneq \mathbf{O}$, and the size of $\mathbf{I} \cup \overline{\mathbf{O}}$ is much smaller than that of $\mathbf{O}$. Obviously, in the above corollary, $\mathbf{I} \cup \overline{\mathbf{O}}$ could be replaced by other subsets, such as $[-1, 0] \cup \overline{\mathbf{O}}$.

Example 1. The operations $f_1, f_2, f_3\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ are respectively defined by

$$\begin{aligned} f_1(\mu, \nu) &= \mu \cdot \nu, \\ f_2(\mu, \nu) &= |\mu| \cdot \nu, \\ f_3(\mu, \nu) &= |\mu| \cdot \mu \cdot \nu. \end{aligned} \tag{6}$$

Obviously, f_1 is migrative. Interestingly, for all $r \in \mathbf{I}$, we have $f_2(r\mu, \nu) = f_2(\mu, r\nu)$. Thus f_2 is amplitude-migrative. Similarly, for all $\theta \in \mathbb{R}$, we have $f_3(e^{j\theta}\mu, \nu) = f_3(\mu, e^{j\theta}\nu)$. Thus, f_3 is phase-migrative. But f_2 is not phase-migrative, f_3 is not amplitude-migrative, thus, they are not migrative.

3. Characterization of Migrativity

One of the important results of migrative real-valued functions is the following theorem:

Theorem 2 (see [28]). *A binary operation $f\colon \mathbf{I}^2 \longrightarrow \mathbf{I}$ is migrative iff there exists a function $g\colon \mathbf{I} \longrightarrow \mathbf{I}$ such that $f(\mu, \nu) = g(xy)$ for all $\mu, \nu \in \mathbf{I}$.*

This result is not true for amplitude-migrative (or phase-migrative) functions (see Example 1), but it is true for migrative complex-valued functions.

Theorem 3. *A binary operation $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ is migrative iff there exists a function $f\colon \mathbf{O} \longrightarrow \mathbf{O}$ such that $f(\mu, \nu) = g(xy)$ for all $\mu, \nu \in \mathbf{O}$.*

Proof. ($\Leftarrow$) If g exists, then $f(\alpha\mu, \nu) = g(\alpha\mu\nu) = f(\mu, \alpha\nu)$.

($\Rightarrow$) If f is migrative, then $f(\mu, \nu) = f(\mu \cdot 1, \nu) = f(1, \mu\nu)$, thus, $g(\mu\nu) = f(1, \mu\nu)$ is the function.

In this way, the function g is the migrative generator of the migrative binary operation f.

The following result is immediate: □

Theorem 4. *Let $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a migrative binary operation. Then*

(1) $f(1, 1) = 1$ if and only of $g(1) = 1$;

(2) $f(0, 0) = 0$ if and only of $g(0) = 0$;

Example 2. We give some migrative functions and their migrative generators.

(1) *The migrative generator of $f(\mu, \nu) = \mu\nu$ is $g(\mu) = \mu$;*

(2) *The migrative generator of $f(\mu, \nu) = |\mu\nu|$ is $g(\mu) = |\mu|$;*

(3) *The migrative generator of $f(\mu, \nu) = 1 - |\mu\nu|$ is $g(\mu) = 1 - |\mu|$.*

Moreover, we have the following results.

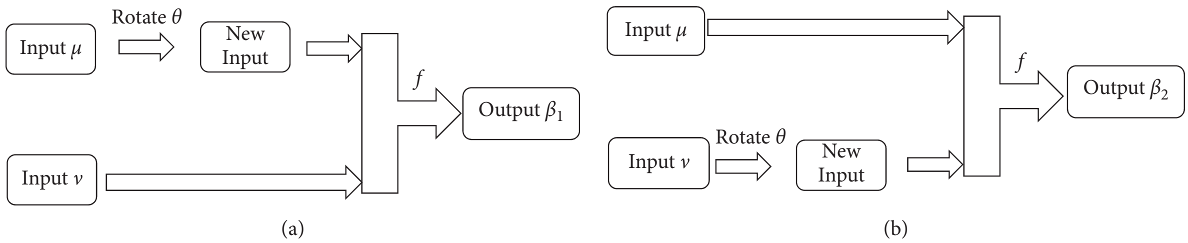

Figure 1: Phase-migrative function (a) $f(\mu \cdot e^{j\theta}, \nu)$ and (b) $f(\mu, \nu \cdot e^{j\theta})$.

Theorem 5. *Let $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a migrative function. Then f is commutative, i.e., $f(\mu, \nu) = f(\nu, \mu)$.*

Proof. If f is migrative, then $f(\mu, \nu) = f(\mu \cdot 1, \nu) = f(1, \mu\nu) = f(\nu \cdot 1, \mu) = f(\nu, \mu)$ for all $\mu, \nu \in \mathbf{O}$.

This result is not true for amplitude-migrative (or phase-migrative) functions (see Example 1). The following result is true even for amplitude-migrative (or phase-migrative) functions. □

Theorem 6. *If a binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is amplitude-migrative (or phase-migrative), then for all $\mu, \nu \in \mathbf{O}$,*

(1) $f(-\mu, -\nu) = f(\mu, \nu)$;

(2) $f(-\mu, \nu) = f(\mu, -\nu)$.

Proof. Here we only give the proof of (1). If f is amplitude-migrative, then

$$f(-\mu, -\nu) = f(\mu, (-1)(-1)\nu) = f(\mu, \nu), \tag{7}$$

for all $\mu, \nu \in \mathbf{O}$.

If f is phase-migrative, then

$$f(-\mu, -\nu) = f\left(e^{j\pi}\mu, e^{j\pi}\nu\right) = f\left(\mu, e^{j\pi}e^{j\pi}\nu\right) = f(\mu, \nu), \tag{8}$$

for all $\mu, \nu \in \mathbf{O}$. □

Corollary 2. *If a binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is migrative, then $f(-\mu, -\nu) = f(\mu, \nu)$ for all $\mu, \nu \in \mathbf{O}$.*

Theorem 7. *A binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is phase-migrative iff it is the convex sum of a finite family of phase-migrative functions.*

Proof. ($\Rightarrow$) f is the convex sum of itself.

($\Leftarrow$) Let $f(\mu, \nu) = \sum_{i=1}^{n} w_i f_i(\mu, \nu)$ with $\sum_{i=1}^{n} w_i$ and $w_i \in \mathbf{I}$. If f_i $(i = 12, \ldots, n)$ is amplitude-migrative, then for any $\theta \in \mathbb{R}$, $f(e^{j\theta}\mu, \nu) = \sum_{i=1}^{n} w_i f_i(e^{j\theta}\mu, \nu) = \sum_{i=1}^{n} w_i f_i(\mu, e^{j\theta}\nu) = f(\mu, e^{j\theta}\nu)$ for all $\mu, \nu \in \mathbf{O}$.

Similarly, we have the following results. □

Theorem 8. *A binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is amplitude-migrative iff it is the convex sum of a finite family of amplitude-migrative functions.*

Corollary 3. *A binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is migrative iff it is the convex sum of a finite family of migrative functions.*

4. Migrativity and Rotational Invariance

Now we consider the relation between migrativity and rotational invariance [3, 4].

Definition 5. (see [3]). Let $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a binary function, then f is rotationally invariant if

$$f\left(\mu \cdot e^{j\theta}, \nu \cdot e^{j\theta}\right) = f(\mu, \nu) \cdot e^{j\theta}, \tag{9}$$

for any $\theta \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$.

Dick's concept of rotational invariance was generalized as follows:

Definition 6. (see [4]). Let $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a binary function, then f is h-rotationally invariant if, for a function $h: \mathbb{R}^2 \longrightarrow \mathbb{R}$,

$$f\left(\mu \cdot e^{j\theta_1}, \nu \cdot e^{j\theta_2}\right) = f(\mu, \nu) \cdot e^{jh(\theta_1, \theta_2)}, \tag{10}$$

for any $\theta_1, \theta_2 \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$.

Theorem 9. *A binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is h-rotationally invariant iff it is the convex sum of a finite family of h-rotationally invariant functions.*

Proof. ($\Rightarrow$) f is the convex sum of itself.

($\Leftarrow$) Let $f(\mu, \nu) = \sum_{i=1}^{n} w_i f_i(\mu, \nu)$ with $\sum_{i=1}^{n} w_i$ and $w_i \in \mathbf{I}$. If f_i $(i = 12, \ldots, n)$ is h-rotationally invariant, then for any $\theta_1, \theta_2 \in \mathbb{R}$, $f(e^{j\theta_1}\mu, e^{j\theta_2}\nu) = \sum_{i=1}^{n} w_i f_i(e^{j\theta_1}\mu, e^{j\theta_2}\nu) = \sum_{i=1}^{n} w_i f_i(\mu, \nu)e^{jh(\theta_1, \theta_2)} = f(\mu, \nu)e^{jh(\theta_1, \theta_2)}$ for all $\mu, \nu \in \mathbf{O}$. □ □

Corollary 4. *A binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is rotationally invariant iff it is the convex sum of a finite family of rotationally invariant functions.*

First, for binary operations, there is no direct relation between migrativity and Dick's rotational invariance [3]. For example, $f(\mu, \nu) = \mu\nu$ is migrative but not rotational invariance. $f(\mu, \nu) = (\mu + \nu)/2$ is rotational invariance but not migrative.

Theorem 10. *Let $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a migrative binary operation and $g: \mathbf{O} \longrightarrow \mathbf{O}$ be its migrative generator, then g is rotationally invariant iff*

$$f\left(\mu \cdot e^{j\theta_1}, \nu \cdot e^{j\theta_2}\right) = f(\mu, \nu) \cdot e^{j(\theta_1 + \theta_2)}, \tag{11}$$

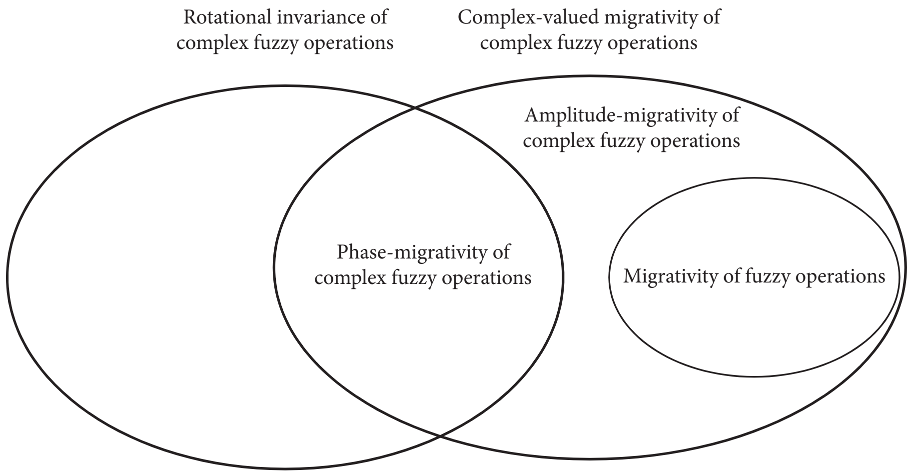

Figure 2: Relations between complex-valued migrativity of complex fuzzy operations, amplitude migrativity of complex fuzzy operations, phase valued migrativity of complex fuzzy operations, rotational invariance of complex fuzzy operations, and the migrativity of fuzzy operations.

for any $\theta_1, \theta_2 \in \mathbb{R}$ *and* $\mu, \nu \in \mathbf{O}$.

Proof. ($\Rightarrow$) g is rotationally invariant, i.e., $g(\mu \cdot e^{j\theta}) = g(\mu) \cdot e^{j\theta}$ for any $\theta \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$. Then $f(\mu \cdot e^{j\theta_1}, \nu \cdot e^{j\theta_2}) = g\ (\mu \cdot e^{j\theta_1} \cdot \nu \cdot e^{j\theta_2}) = e^{j(\theta_1+\theta_2)} \cdot g(\mu\nu) = e^{j(\theta_1+\theta_2)} \cdot f(\mu, \nu)$.

($\Leftarrow$) If f satisfies equation (11), then for any $\theta \in \mathbb{R}$ and $\mu \in \mathbf{O}$, we have $g(\mu \cdot e^{j\theta}) = f(1, \mu \cdot e^{j\theta}) = f(1, \mu) \cdot e^{j\theta} = g(\mu) \cdot e^{j\theta}$.

Moreover, we consider the relation between phase-migrativity and conditional rotational invariance [4]. □

Theorem 11. *A binary operation* $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ *satisfies*

$$f\left(\mu \cdot e^{j\theta_1}, \nu \cdot e^{j\theta_2}\right) = f(\mu, \nu) \cdot e^{j(\theta_1+\theta_2)}, \tag{12}$$

for any $\theta_1, \theta_2 \in \mathbb{R}$ *and* $\mu, \nu \in \mathbf{O}$. *Then it is phase-migrative. But the converse is not true.*

Proof. For any $\theta \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$, $f(e^{j\theta}\mu, \nu) = f(\mu, \nu) \cdot e^{j\theta} = f(\mu, e^{j\theta}\nu)$. Moreover, $f(\mu, \nu) = (\mu\nu)^2$ is phase-migrative but does not satisfy equation (12). □

Corollary 5. *A binary operation* $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ *satisfies*

$$\left.\begin{aligned} f\left(\mu \cdot e^{j\theta_1}, \nu\right) &= f(\mu, \nu) \cdot e^{j\theta_1} \\ f\left(\mu, \nu \cdot e^{j\theta_2}\right) &= f(\mu, \nu) \cdot e^{j\theta_2} \end{aligned}\right\}, \tag{13}$$

for any $\theta_1, \theta_2 \in \mathbb{R}$ *and* $\mu, \nu \in \mathbf{O}$. *Then it is phase-migrative. But the converse is not true.*

Proof. Because equation (12) is equivalent to equation (13), □

Theorem 12. *Let* $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ *be a commutative binary operation, if it satisfies* $f(e^{j\theta}\mu, \nu) = f(\mu, \nu) \cdot e^{j\theta}$ *for all* $\theta \in \mathbb{R}$ *and* $\mu, \nu \in \mathbf{O}$. *Then*

(1) it is phase-migrative;

(2) it is h*–rotationally invariant, where* $h(\theta_1, \theta_2) = \theta_1 + \theta_2$.

Proof

(1) For any $\theta \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$, we have $f(\mu, e^{j\theta}\nu) = f(e^{j\theta}\nu, \mu) = f(\nu, \mu) \cdot e^{j\theta} = f(\mu, \nu) \cdot e^{j\theta} = f(\mu \cdot e^{j\theta}, \nu)$.

(2) For any $\theta_1, \theta_2 \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$, we have $f(\mu \cdot e^{j\theta_1}, \nu \cdot e^{j\theta_2}) = f(\mu, \nu \cdot e^{j\theta_2}) \cdot e^{j\theta_1} = f(\nu \cdot e^{j\theta_2}, \mu) \cdot e^{j\theta_1} = f(\nu, \mu) \cdot e^{j\theta_1} \cdot e^{j\theta_2} = f(\mu, \nu) \cdot e^{j(\theta_1+\theta_2)}$.

We give a binary operation f without commutativity, $f(\mu, \nu) = \mu|\nu|$ satisfies $f(e^{j\theta}\mu, \nu) = f(\mu, \nu) \cdot e^{j\theta}$ for all $\theta \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$. But it is neither commutative nor phase-migrative. Moreover, it is h'–rotationally invariant, where $h'(\theta_1, \theta_2) = \theta_2$.

The relations between complex-valued migrativity of complex fuzzy operations, amplitude migrativity of complex fuzzy operations, phase valued migrativity of complex fuzzy operations, rotational invariance of complex fuzzy operations, and the migrativity of fuzzy operations are shown in Figure 2. □

Theorem 13. *Let* $f\colon \mathbf{O}^2 \longrightarrow \mathbf{O}$ *be a commutative binary operation, if it satisfies* $f(r\mu, \nu) = rf(\mu, \nu)$ *for all* $r \in \mathbf{I}$ *and* $\mu, \nu \in \mathbf{O}$. *Then*

(1) it is amplitude-migrative;

(2) it satisfies $f(r_1\mu, r_2\nu) = r_1r_2f(\mu, \nu)$. *for all* $r_1, r_2 \in \mathbf{I}$ *and* $\mu, \nu \in \mathbf{O}$.

Proof. For any $r \in \mathbf{I}$ and $\mu, \nu \in \mathbf{O}$, we have

(1) $f(\mu, r\nu) = f(r\nu, \mu) = r \cdot f(\nu, \mu) = r \cdot f(\mu, \nu) = f(r\mu, \nu)$.

(2) $f(r_1\mu, r_2\nu) = r_1 \cdot f(\mu, r_2\nu) = r_1 \cdot f(r_2\nu, \mu) = r_2 \cdot r_1 \cdot f(\nu, \mu) = r_1r_2 \cdot f(\mu, \nu)$.

We observe that it is homogeneous of order 2, i.e., $f(r\mu, r\nu) = r^2 f(\mu, \nu)$ when $r_1 = r_2$.

We give a binary operation f without commutativity, $f(\mu, \nu) = \mu \cdot \nu \cdot |\nu|$ satisfies $f(r\mu, \nu) = r \cdot f(\mu, \nu)$ for all $r \in \mathbf{I}$ and $\mu, \nu \in \mathbf{O}$. But it is neither commutative nor amplitude-migrative. Moreover, it is homogeneous of order 3, i.e., $f(r\mu, r\nu) = r^3 f(\mu, \nu)$. □

Corollary 6. *Let $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ be a commutative binary operation, if it satisfies $f(\alpha\mu, \nu) = \alpha \cdot f(\mu, \nu)$ for all $\alpha \in \mathbf{O}$ and $\mu, \nu \in \mathbf{O}$. Then it is migrative*

Theorem 14. *If a binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ is h–rotationally invariant where $h_1(\theta_1, \theta_2) = k(\theta_1, \theta_2)$ for some $k > 0$. Then f is phase-migrative,*

Proof. For any $\theta \in \mathbb{R}$ and $\mu, \nu \in \mathbf{O}$, we have $f(\mu \cdot e^{j\theta}, \nu) = f(\mu, \nu) \cdot e^{jk\theta} = f(\mu, \nu \cdot e^{j\theta})$ for some $k > 0$. □

Theorem 15. *If a binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ satisfies $f(r_1\mu, r_2\nu) = (r_1 \cdot r_2)^k f(\mu, \nu)$ for all $r_1, r_2 \in \mathbf{I}$, all $\mu, \nu \in \mathbf{O}$, and some $k > 0$. Then f is amplitude-migrative.*

Proof. For any $r \in \mathbf{I}$ and $\mu, \nu \in \mathbf{O}$, we have $f(r\mu, \nu) = r^k f(\mu, \nu) = f(\mu, r\nu)$ for some $k > 0$. □

Corollary 7. *If a binary operation $f: \mathbf{O}^2 \longrightarrow \mathbf{O}$ satisfies $f(\alpha_1\mu, \alpha_2\nu) = (\alpha_1 \cdot \alpha_2)^k f(\mu, \nu)$ for all $\alpha_1, \alpha_2 \in \mathbf{O}$, all $\mu, \nu \in \mathbf{O}$, and some $k > 0$. Then f is migrative.*

5. Conclusions

In this paper, we study the migrative binary complex fuzzy operators

$$f(\alpha\mu, \nu) = f(\mu, \alpha\nu), \quad \forall \mu, \nu \in \mathbf{O}, \tag{14}$$

for three cases $\alpha \in \mathbf{I}$, $\alpha \in \overline{\mathbf{O}}$, and $\alpha \in \mathbf{O}$. Interestingly, this equation holds for all $\alpha \in \mathbf{O}$ if and only if it holds for all $\alpha \in \mathbf{I} \cup \overline{\mathbf{O}}$ (see Theorem 1). Note that the size of $\mathbf{I} \cup \overline{\mathbf{O}}$ is much smaller than that of $\mathbf{O}$. Then we give the relationship among phase-migrativity, amplitude-migrativity, migrativity, and rotational invariance for complex fuzzy operations (see Figure 1). We show that phase-migrativity is a special case of conditional rotational invariance (see Theorem 12).

Note that this paper focused on binary complex fuzzy operators. Future research should consider the migrativity of n-dimensional complex fuzzy aggregation operators. Naturally, other properties of complex fuzzy operators are possible topics for future consideration.

In [31], Yager and Abbasov used complex numbers of the form $r \cdot e^{j\theta}$ as Pythagorean membership grades, where $r \in [0, 1]$ and $\theta \in [0, \pi/2]$. These complex numbers are called $\pi - i$ numbers, which belong to the upper-right quadrant of the unit disk in the complex plane. Viewed in this way, studying the migrativity of Pythagorean fuzzy operators is a special case of migrativity of complex fuzzy operators by limiting the domain to $\pi - i$ numbers. Obviously, a more detailed discussion of the migrativity of Pythagorean fuzzy aggregation operators [32], Pythagorean t-norm [33], will be both necessary and interesting.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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