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Research Article

Sensitivity and Bifurcation Analysis of Fuzzy SEIR-SEI Dengue Disease Model

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Dengue is one of the most serious mosquito-borne infectious diseases in the world. The number of dengue cases is increasing every year worldwide. In this work, we discuss the fuzzy epidemic SEIR-SEI compartmental model with the intervention of bed nets and fumigation to describe the transmission dynamics of dengue disease. We consider the biting rate, transmission rate, and recovery rate of the disease as fuzzy numbers. With different amounts of virus loads, we discuss the dynamical behavior of the system. The sensitivity analysis of the model is performed to compare the relative importance of the model parameters and to discuss the importance of fumigation, use of bed nets, and the effectiveness of bed nets. We demonstrate the bifurcation of the equilibrium point of the system with and without fumigation, with and without bed net user, and with different levels of effectiveness of bed nets. Numerical simulations are made to illustrate the mathematical results graphically. The infectivity of the disease depends on the amount of virus loads. The mathematical and simulated result shows that the intervention strategies, use of fumigation and bed nets, reduces the value of the basic reproduction number. Thus, this study suggests that the endemic situation of the disease can be brought under control by the effective use of the combination of fumigation and bed nets.

1. Introduction

Dengue is a mosquito-borne fastest-growing tropical viral disease, which can affect humans of almost all ages worldwide. Dengue was experienced only in 9 countries before 1970. Now dengue is endemic in more than 120 countries in the Americas, Africa, the Middle East, Asia, and the Pacific Islands [1]. Approximately 3.9 billion people are living in the dengue risk areas, which is more than one third of the world's population [1]. Annually, up to 400 million new dengue infections are occurring worldwide [1]. There is no licensed vaccine and no specific antiviral drugs for the disease till date.

Dengue fever is infected by one of the four serologically different viruses DENV-1 to DENV-4, which circulate simultaneously in the endemic areas. It is transmitted to humans by the day-feeding mosquitoes called Aedes Aegypti and Aedes Albopictus [2].

In recent years, the number of cases of dengue fever and the area of outbreaks has been increased significantly in Nepal. The first dengue cases were reported in 2004. In Nepali's history of dengue from 2004 to 2013, major outbreaks have occurred in 2010 and 2013. However, from 2014 to 2019, the yearly outbreak of dengue has been increased significantly than in previous years [3, 4]. Between 2014 and 2019, the annual number of confirmed dengue incidence varied from 336 to 14,662 [5]. In the year 2019, the largest-ever outbreak was reported, more than 14,662 cases conformed DENV infection from 67 districts of Nepal, and a total of six people were reported to die due to dengue disease infection [6]. In this year, the largest number of dengue cases was reported globally [1]. Now, more than 70% of the total population of Nepal are at the risk of dengue infection [7].

The mathematical compartmental model helps to understand the transmission dynamics of the infectious disease from various angles [8–12]. It is a more effective

tool to identify the influential parameters in spreading the disease and to propose strategies for the control of the disease. To study the evolution and transmission dynamics of infectious diseases, there is a long and distinguished history of using mathematical models. Kermack and McKendrick (1927) formulated a SIR compartmental model to study infectious diseases mathematically [11]. It is remodeled by Esteva and Vargas to use for vector host dynamics of dengue disease taking constant [13] and variable human population [14]. Since then, many researchers have studied dengue disease transmission dynamics using the compartmental model. The impact of awareness on the spread of dengue infection in human population was studied by different researchers, Gakkhar and Chavda, Phaijoo, and Gurung [15, 16]. The mobility of human population causes the spread of the disease in new human populations, so through mathematical models, the impact of these mobility parameters has been studied in [17].

In the mathematical studies of infectious diseases, a sensitivity analysis is very important. From this analysis, we can determine the relative importance of the model parameters. By the study of sensitivity analysis, Chitnis et al. determined important parameters in the spread of malaria [18]. Phaijoo and Gurung discussed the sensitivity of model parameters [10] in the study of dengue disease transmission dynamics. Ndii performed a sensitivity analysis using the combination of a Latin hypercube sampling and partial rank correlation coefficient in his work on the use of vaccine and Wolbachia on the spread of dengue disease [19]. Later, Ndii et al. performed a global sensitivity analysis in their work on optimal vaccination strategy for dengue transmission in Kupang City, Indonesia. They suggested to vaccinate the seropositive individuals to reduce the proportion of severe dengue cases [20]. Biting rate and transmission probability of dengue are dependent on the fumigation, number of bet net user, and effectiveness of bed nets. In this work, we perform the sensitivity analysis of these parameters with the help of basic reproduction number.

Most of the researchers [10–13, 21] have proposed the SEIR-SEI epidemic model with constant model parameters while studying dengue disease transmission dynamics mathematically. Generally, they assumed that each individual can transmit the disease and recover from the disease at a constant rate. However, these assumptions are not true in the real epidemic situation. Some model parameters such as transmission rates, biting rates, and recovery rates are uncertain.

Lotfi A. Zadeh [22] had introduced the uncertainty in the biological model and defined the fuzzy set and fuzzy theory to study this uncertainty mathematically. By considering the disease transmission parameter and treatment control parameter as fuzzy number, Mondal et al. modified the epidemic SIS model [23]. Considering different degrees of infectivity, De Barros et al. applied the fuzzy theory technique to SI epidemiological model. They used the transmission coefficient as a fuzzy set [24]. Nandi et al. described the dynamical behavior of SIS epidemic model with transmission rate and treatment control as fuzzy numbers. They derived a threshold condition at which the system undergoes a backward bifurcation [25].

In recent times, the fuzzy theory has been used to study the diagnosis of diseases and is being introduced in many models of engineering, banking, public health, and biology. With fuzzy transmission parameter, some researchers [23, 24, 26] have developed the general epidemic models (SI, SIR) of infectious disease for human-to-human transmitted disease. Dengue is an infectious disease, which is transmitted to humans through the bite of the infected Aedes mosquito and cannot be transmitted from humans to humans directly. Bhuju et al. have studied the dengue disease transmission dynamics considering the model parameters as fuzzy numbers [27]. In this work, we study the transmission dynamics of dengue by considering some model parameters as fuzzy numbers. We compute the fuzzy basic reproduction number to observe the stability of the equilibrium points and discuss the bifurcation of the disease-free equilibrium point [28, 29]. We perform a sensitivity analysis to observe the impact of the basic reproduction on the parameters of the model.

There are many methods of controlling mosquitoes, which transmit dengue disease, such as the use of bed net and fumigation [1]. In recent studies, the researchers in [30–33] have constructed mathematical models to analyze the transmission of infectious diseases including climatic factors, bed net, and fumigation. Agusto et al., Chitnis et al., and Ngonghala et al. used climatic factors and mosquito nets in the mathematical models of malaria distribution, and Xiunan and Xiao-Qiang used mosquito nets in the mathematical models of climate-based malaria [33].

The study is organized as follows. In Section 2, we formulate SEIR-SEI model to describe the dengue disease transmission dynamics assuming some model parameters as fuzzy numbers. The membership function of these fuzzy numbers is described in Subsection 2.2. In Section 3, we analyze the stability of the model. Section 4 studies the sensitivity analysis with and without fumigation and bed net user with effectiveness of bed nets. Bifurcation of the system is discussed in Section 5. In Section 6, we describe the numerical results and conclusion of the work is presented in Section 7.

2. Model Formation and Description

We divide the total human population into four epidemiological classes, namely susceptible, exposed, infected, and recovered; and mosquito population into three epidemiological classes, namely susceptible, exposed, and infected. The transmission dynamics of dengue between these classes is described by the system of nonlinear ordinary differential equations (10). In the deterministic model [10] as means of intervention to eliminate dengue, we use fumigation and bed nets with the assumption that no mosquitoes are resistant to fumigation. The transmission and recovery of the infection of the disease are uncertain. We assume that the disease transmission coefficient β_h between susceptible and infected individuals, the recovery rate γ_h, and biting rate of mosquito b are fuzzy numbers. To describe the virus load on the

parameters β_h and γ_h, we use the membership function $\beta_h(v)$ and $\gamma_h(v)$ for the transmission rate and recovery rate, respectively. To describe the impact of the bed net user k and effectiveness of bed nets r on the parameter b, we use the membership function $b(k,r)$ for biting rate. Then, the fuzzy SEIR-SEI model of dengue with fumigation and bed net is described by the following system of differential equations.

For human population,

$$\begin{aligned}
\frac{dS_h}{dt} &= \mu_h N_h - \frac{\beta_h(v)b(k,r)}{N_h} S_h I_m - \mu_h S_h,\\
\frac{dE_h}{dt} &= \frac{\beta_h(v)b(k,r)}{N_h} S_h I_m - (k_h + \mu_h)E_h,\\
\frac{dI_h}{dt} &= k_h E_h - (\mu_h + \gamma_h(v))I_h,\\
\frac{dR_h}{dt} &= \gamma_h(v) I_h - \mu_h R_h.
\end{aligned} \tag{1}$$

For mosquito population,

$$\begin{aligned}
\frac{dS_m}{dt} &= A - \frac{\beta_m b(k,r)}{N_m} I_h S_m - (\mu_m + u)S_m,\\
\frac{dE_m}{dt} &= \frac{\beta_m b(k,r)}{N_h} S_m I_h - (k_m + \mu_m + u)E_m,\\
\frac{dI_m}{dt} &= k_m E_m - (\mu_m + u)I_m.
\end{aligned} \tag{2}$$

Here, $N_h = S_h + E_h + I_h + R_h$ and $N_m = S_m + E_m + I_m$,We have

N_h: host (human) population size,
S_h: size of susceptible in the host population,
E_h: size of exposed host population,
I_h: size of infective in the host population,
R_h: size of immunes (recovered) in the host population,
N_m: vector (mosquito) population size,
S_m: size of susceptible in the vector population,
E_m: size of exposed vector population,
I_m: size of infective in the vector population,
μ_h: birth/death rate in the host population,
μ_m: death rate in the vector population,
β_h: transmission coefficient from vector to host,
β_m: transmission coefficient from host to vector,
γ_h: recovery rate in the host population,
$b(k,r)$: biting rate of vector,
k_h: host's incubation rate,
k_m: vector's incubation rate,
u: fumigation rate. (3)

Assume that without fumigation intervention, total human population and total mosquito population are constant, and we have the following:

$$\begin{aligned}
\frac{dN_h}{dt} &= 0,\\
\frac{dN_m}{dt} &= A - \mu_m N_m,\\
\Rightarrow 0 &= A - \mu_m N_m,\\
\Rightarrow A &= \mu_m N_m.
\end{aligned} \tag{4}$$

When $u \neq 0$, we have the following:

$$\begin{aligned}
\frac{dN_m}{dt} &= A - \mu_m N_m - uN_m\\
&= -uN_m < 0.
\end{aligned} \tag{5}$$

This implies that the total number of mosquitoes decreases with respect to time using fumigation and bed nets.

2.1. Fuzzy Set. Let X be a nonempty crisp set. A fuzzy subset S of X, denoted by $\widetilde{S}$, is defined as follows:

$$\widetilde{S} = \left\{\left(x, \mu_{\widetilde{S}}(x)\right):\ x \in X\right\}, \tag{6}$$

where $\mu_{\widetilde{S}}: X \longrightarrow [0,1]$ is a membership function associated with a fuzzy set $\widetilde{S}$, which describes the degree of belongingness of x with X.Here, we use the membership function $\mu(x)$ to indicate the fuzzy subset $\widetilde{S}$. Also, $\mu(x)$ is called fuzzy number, when X is the set of real number.

2.2. Membership Functions. Mosquitoes' biting rate can be reduced using the bed nets. The membership function of mosquito biting rate is defined from [34] as follows:

$$b(k,r) = \begin{cases} b_0, & \text{if } k = 0,\\ b_0 kr + b_0(1-k), & \text{if } 0 < k < 1,\\ b_0 r, & \text{if } k = 1, \end{cases} \tag{7}$$

where k and r indicate the proportion of humans who use bed net and the effectiveness of bed nets, respectively. When k increases, the number of people using bed nets increases and increase in r indicates poorer prevention of mosquito bites or increase in the total number of mosquito bites. Note that $r \in (0,1)$ and $b_0 \in [0,1]$, where b_0 is the maximum value of the biting rate. The diagram of the membership function $b(r,k)$ is given in Figure 1.

The fuzzy membership function of the transmission parameter $\beta_h(v)$, which depends on the amount of virus load v, is given as follows [24, 35]:

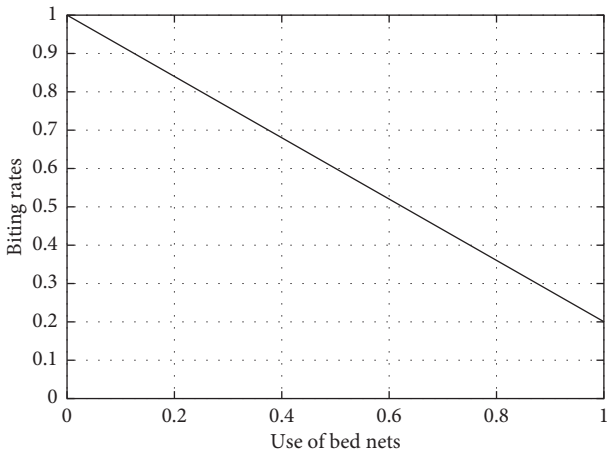

Figure 1: Membership function of biting rate with respect to k when $r = 0.2$.

$$\beta_h(v) = \begin{cases} 0, & \text{if } v < v_{\min}, \\ \dfrac{v - v_{\min}}{v_M - v_{\min}}, & \text{if } v_{\min} \le v \le v_M, \\ 1, & \text{if } v_M \le v \le v_{\max}, \end{cases} \tag{8}$$

where $v_{\min}$ represents the minimum amount of virus needed to occur the disease transmission. When the amount of virus in an individual is less than $v_{\min}$, the chance of transmission of disease is negligible. Moreover, for the certain amount of virus v_M, the transmission rate of the disease is maximum and equal to 1. Furthermore, we suppose that for the dengue disease, the individual's amount of virus is always limited by $v_{\max}$. The diagram of the membership function $\beta_h(v)$ is given in Figure 2.

Here, $\gamma_h(v)$ represents the recovery rate from the infection of the disease, which depends on the amount of virus load. When the virus load is higher, it will take the longer time to recover from the disease. Thus, the fuzzy membership function of recovery rate $\gamma_h(v)$ is given as follows [26]:

$$\gamma_h(v) = \frac{(\gamma_0 - 1)}{v_{\max}} v + 1, \quad \text{if } 0 < v < v_{\max}, \tag{9}$$

where $0 < \gamma_0 < 1$ is the lowest recovery rate. The diagram of $\gamma_h(v)$ is given in Figure 3.

2.3. Nonnegativity and Boundedness of Solutions

Theorem 1. *The solutions of system (1) are nonnegative for all $t > 0$.*

Proof. Suppose $W = \{(S_h, E_h, I_h, R_h, S_m, E_m, I_m) \in \mathbb{R}^7 : 0 \le S_h, E_h, I_h, R_h, S_m, E_m, I_m\}$.

We show that W should be positively invariant. For this, we prove that the state variables are nonnegative at the boundaries of W.

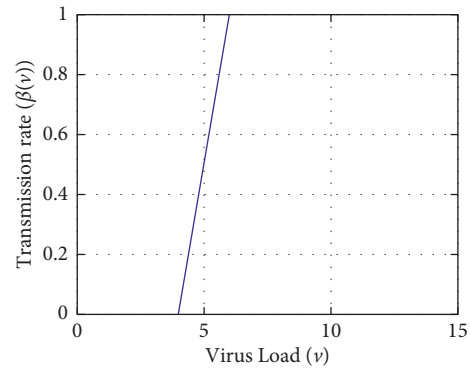

Figure 2: Membership function $\beta_h(v)$.

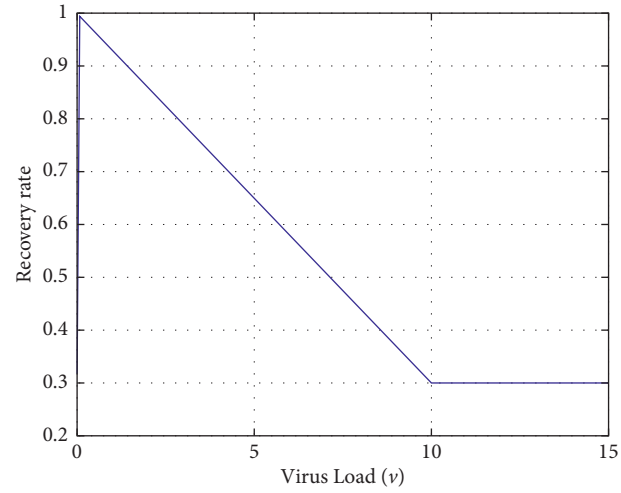

Figure 3: Membership function $\gamma_h(v)$.

(**a**) At the boundary $S_h = 0$, then

$$S_h' = \mu_h N_h > 0. \tag{10}$$

Thus, the solution cannot cross the boundary $S_h = 0$.

(**b**) At the boundary $E_h = 0$, then we get the following:

$$E_h' = \frac{\beta_h(v) b(k, r)}{N_h} S_h I_m. \tag{11}$$

Case 1: if $E_h = 0$, $S_h > 0$, $I_m(t) > 0$, then $E_h' > 0$.

Case 2: if $E_h = 0$, $S_h > 0$, $I_m = 0$, then $E_h' = 0$.

Case 3: if $E_h = 0$, $S_h = 0$, $I_m > 0$, then $E_h' = 0$.

In each of these cases, $E_h' \ge 0$, so the solution cannot cross the boundary $E_h = 0$.

(**c**) At the boundary $I_h = 0$, we have $I_h' = k_h E_h$.

If $I_h = 0$, $E_h > 0$, then $I_h' > 0$.

Thus, the solution cannot cross the boundary $I_h = 0$.

In a similar manner, we can show that the solution of the system cannot exit W by crossing the boundary of any of the state variables. □

Theorem 2. *The solutions of system (1) are bounded on* $[0, d)$ *for some* $d > 0$.

Proof. From system (1), we have $N_h = S_h + E_h + I_h + R_h$ and $(dN_h/dt) = 0$. Thus, N_h is constant for all $t \in [0, d)$ for some $d > 0$. Therefore, all the state variables $S_h(t), E_h(t), I_h(t)$, and $R_h(t)$ are bounded on $[0, d)$.

Again, we have the following:

$$N_m = S_m + E_m + I_m. \tag{12}$$

This implies

$$\frac{dN_m}{dt} = A - \mu_m N_m + uN_m,$$
$$N_m = \frac{A}{(\mu_m + u)} + \left(N_m(0) - \frac{A}{(\mu_m + u)}\right)e^{-(\mu_m+u)t}. \tag{13}$$

Hence,

$$\limsup_{t\longrightarrow\infty} N_m \leq \frac{A}{(\mu_m + u)}. \tag{14}$$

Therefore, $S_m(t), E_m(t)$, and $I_m(t)$ are bounded above by $(A/(\mu_m + u))$ on $[0, d)$ for some $d > 0$. Since all the variables are nonnegative, these are bounded below by 0. Hence, the solution of system (1) is bounded on $[0, d)$ for some $d > 0$ [36]. □

2.4. Existence and Uniqueness of Solution. We assume that the system has the initial conditions as follows:

$$S_h(0) > 0,\ E_h(0) \geq 0,\ I_h(0) > 0,\ R_h(0) \geq 0,$$
$$S_m(0) > 0,\ E_m(0) \geq 0,\ I_m(0) \geq 0. \tag{15}$$

Theorem 3. *Consider system (1) with nonnegative initial condition (15). Solutions to system (1) with initial conditions (15) exist and are unique for all* $t \geq 0$.

Proof. Let $z(t) = (S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t)) \in \mathbb{R}^7$. System (1) is written in the form $z' = f(z)$. For $i = 1, 2, 3, 4, 5, 6, 7$, suppose g_i denotes the components of the vector field g and we have the following:

$$g_1 = \mu_h N_h - \frac{\beta_h(v)b(k,r)}{N_h}S_h I_m - \mu_h S_h,$$
$$g_2 = \frac{\beta_h(v)b(k,r)}{N_h}S_h I_m - (k_h + \mu_h)E_h,$$
$$g_3 = k_h E_h - (\gamma_h(v) + \mu_h)I_h,$$
$$g_4 = \gamma_h(v)I_h - \mu_h R_h, \tag{16}$$
$$g_5 = A - \frac{\beta_m b(k,r)}{N_h}S_m I_h - (\mu_m + u)S_m,$$
$$g_6 = \frac{\beta_m b(k,r)}{N_h}S_m I_h - (k_m + \mu_m + u)E_m,$$
$$g_7 = k_m E_m - (\mu_m + u)I_m.$$

The vector field g consists of the algebraic polynomials of state variables. Thus, each g_i is continuous autonomous function on $\mathbb{R}^7$ and partial derivatives $(\partial g_i/\partial S_h)$, $(\partial g_i/\partial E_h)$, $(\partial g_i/\partial I_h)$, $(\partial g_i/\partial R_h)$, $(\partial g_i/\partial S_m)$, $(\partial g_i/\partial E_m)$, and $(\partial g_i/\partial I_m)$ exist and are continuous. Hence, by existence and uniqueness theorem, a unique solution of the system $z' = f(z)$ exists for any initial condition $z(0) \in \mathbb{R}^7$ [37]. □

3. Stability Analysis of the Model

3.1. Basic Reproduction Number. Basic reproduction number is defined as the average number of secondary infections caused by single infectious individual during their entire infectious lifetime. The number is denoted by R_0.

Assume that F is the matrix of transmission terms and V is the matrix of transition terms of system (1). R_0 is defined as the spectral radius of the matrix FV^{-1}, i.e., $\rho(FV^{-1})$. R_0 is obtained using the next-generation matrix method [38–40]. For model (1),

$$F = \begin{bmatrix} 0 & 0 & 0 & \dfrac{\beta_h(v)b(k,r)}{N_h}S_h \\ 0 & 0 & \dfrac{\beta_m b(k,r)}{N_h}S_m & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$V = \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ -k_h & 0 & q & 0 \\ 0 & -k_m & 0 & \mu_m + u \end{bmatrix}, \tag{17}$$

where $p = k_h + \mu_h$, $\alpha = k_m + \mu_m + u$, $q = \gamma_h(v) + \mu_h$.

Thus, the basic reproduction number is as follows:

$$R_0(v) = \rho\left(FV^{-1}\right) = \sqrt{\frac{\beta_h(v)\beta_m b^2(k,r)k_m k_h S_h S_m}{(\mu_m + u)\alpha p q N_h^2}}. \tag{18}$$

3.2. Equilibrium Points. There are two equilibrium points of the system of differential equation (1), the disease-free equilibrium point $P_0(N_h, 0, 0, 0, (A/\mu_m + u), 0, 0)$ and endemic equilibrium point $P_1(S_h^*, E_h^*, I_h^*, R_h^*, S_m^*, E_m^*, I_m^*)$. Here,

$$S_h^* = \frac{N_h\left[R_0^2\mu_h(\mu_m+u)N_h\alpha+\beta_h(v)b(k,r)Ak_m\right]}{R_0^2\left[\mu_h(\mu_m+u)N_h\alpha+\beta_h(v)b(k,r)Ak_m\right]},$$

$$E_h^* = \frac{\left(R_0^2-1\right)\mu_h(\mu_m+u)^2N_h^2q\alpha}{\beta_m b(k,r)k_h\left(\beta_h(v)b(k,r)Ak_m+\mu_h(\mu_m+u)N_h\alpha\right)},$$

$$I_h^* = \frac{\left(R_0^2-1\right)\mu_h(\mu_m+u)^2N_h^2\alpha}{\left(\beta_h(v)b(k,r)^2A\beta_m k_m+\mu_h(\mu_m+u)N_h\alpha\beta_m b(k,r)\right)},$$

$$R_h^* = \frac{\left(R_0^2-1\right)\gamma_h(v)(\mu_m+u)^2N_h^2\alpha}{\beta_m b(k,r)\left(\beta_h(v)b(k,r)Ak_m+\mu_h(\mu_m+u)N_h\alpha\right)},$$

$$S_m^* = \frac{A\left(\beta_h(v)b(k,r)Ak_m+\mu_h(\mu_m+u)N_h\alpha\right)}{R_0^2\mu_h(\mu_m+u)^2N_h\alpha+(\mu_m+u)\beta_h(v)b(k,r)Ak_m},$$

$$E_m^* = \frac{\left(R_0^2-1\right)\mu_h N_h A(\mu_m+u)}{\left[R_0^2\mu_h(\mu_m+u)N_h\alpha+\beta_h(v)b(k,r)Ak_m\right]},$$

$$I_m^* = \frac{\left(R_0^2-1\right)\mu_h N_h Ak_m}{\left[R_0^2\mu_h(\mu_m+u)N_h\alpha+\beta_h(v)b(k,r)Ak_m\right]}. \tag{19}$$

The basic reproduction number at P_0 is as follows:

$$R_0(v) = \sqrt{\frac{\beta_h(v)\beta_m b^2(k,r)k_m k_h A}{(\mu_m+u)^2\alpha pqN_h}}. \tag{20}$$

When $k = 0$,

$$R_0(v) = \sqrt{\frac{\beta_h(v)\beta_m b_0^2 k_m k_h A}{(\mu_m+u)^2\alpha pqN_h}}. \tag{21}$$

When $0 < k < 1$,

$$R_0(v) = \sqrt{\frac{\beta_h(v)\beta_m\left(b_0rk+b_0(1-k)\right)^2k_m k_h A}{(\mu_m+u)^2\alpha pqN_h}}. \tag{22}$$

When $k = 1$,

$$R_0(v) = \sqrt{\frac{\beta_h(v)\beta_m(b_0r)^2k_m k_h A}{(\mu_m+u)^2\alpha pqN_h}}. \tag{23}$$

Theorem 4. *The disease-free equilibrium point* $P_0(N_h, 0, 0, 0, (A/\mu_m+u), 0, 0)$ *is locally asymptotically stable when* $R_0 < 1$ *and unstable when* $R_0 > 1$.

Proof. The Jacobian matrix for the system of equations (1) has the block matrix about the disease-free equilibrium point:

$$J = \begin{bmatrix} M_1 & M_2 \\ 0 & F-V \end{bmatrix}. \tag{24}$$

If all the eigenvalues of the Jacobian matrix J have negative real parts, then the disease-free equilibrium is asymptotically stable [39]. Since J is upper triangular matrix, the stability of the system of (1) depends on the eigenvalues of the matrices on the diagonal, namely M_1 and $F-V$. The eigenvalues of matrix M_1 are $-\mu_h < 0$ and $-(\mu_m+u) < 0$. Now, the stability of the disease-free equilibrium depends on the eigenvalues of $F-V$, where F is nonnegative and V is non-singular M matrix [41]. Again, all the eigenvalues of $F-V$ have negative real parts if and only if $\rho(FV^{-1}) < 1$ [39]. Here, $R_0 = \rho(FV^{-1})$, and therefore, the disease-free equilibrium is locally asymptotically stable if $R_0 < 1$.

If $\rho(FV^{-1}) > 1$, then $s(F-V) > 0$ [39], which means that, if $R_0 = \rho(FV^{-1}) > 1$, spectral abscissa of the matrix $F-V$ is positive. It shows that at least one eigenvalue of $F-V$ has positive real part, and so, the disease-free equilibrium point is unstable. Hence, the disease-free equilibrium is unstable if $R_0 > 1$. □

Theorem 5. *The system has an endemic equilibrium point* $P_1(S_h^*, E_h^*, I_h^*, R_h^*, S_m^*, E_m^*, I_m^*)$, *which exists only when* $R_0 > 1$.

Proof. We have endemic equilibrium point $P_1(S_h^*, E_h^*, I_h^*, R_h^*, S_m^*, E_m^*, I_m^*)$, where

$$I_h^* = \frac{\left(R_0^2-1\right)\mu_h(\mu_m+u)^2N_h^2\alpha}{\left(\beta_h(v)b^2(k,r)A\beta_m k_m+\mu_h(\mu_m+u)N_h\alpha\beta_m b(k,r)\right)}. \tag{25}$$

(a) When $v < v_{\min}$, $\beta_h(v) = 0$ and $R_0 = 0$, which implies (from (25)),

$$I_h^* = \frac{-(\mu_m+u)N_h}{\beta_m b(k,r)} < 0. \tag{26}$$

It shows that the endemic equilibrium point P_1 does not exist when $v < v_{\min}$ and $R_0 = 0$.

(b) When $v_{\min} \le v \le v_M$, we have the following:

$$\beta_h(v) = \frac{v-v_{\min}}{v_M-v_{\min}} > 0. \tag{27}$$

So, $I_h^* > 0$ if $R_0 > 1$ (from (25)). It shows that the endemic equilibrium point P_1 exits when $v_{\min} \le v \le v_M$ if $R_0 > 1$.

(c) When $v_M \le v \le v_{\max}$, $\beta_h(v) = 1$.

So, $I_h^* > 0$ if $R_0 > 1$ (from (25)). It shows that the endemic equilibrium point P_1 exits when $v_M \le v \le v_{\max}$ if $R_0 > 1$.

Hence, the endemic equilibrium point P_1 exits when virus load $v \ge v_{\min}$ if $R_0 > 1$. □

4. Sensitivity Analysis

In the sensitivity analysis, we investigate the impact on basic reproduction (R_0), when the associated parameters vary. To understand the relative importance of model parameter, we perform the sensitivity analysis:

(1) The first analysis is the R_0 sensitivity towards the transmission rate $\beta_h(v)$ and recovery rate $\gamma_h(v)$ of dengue disease when $k = 0$ and $u \neq 0$.

$$\frac{\partial R_0}{\partial \beta_h(v)} = \frac{1}{2}\sqrt{\frac{b_0^2 k_h k_m \beta_m A}{N_h(\mu_m + u)^2 \beta_h(v) pq\alpha}} > 0, \tag{28}$$

$$\frac{\partial R_0}{\partial \gamma_h(v)} = -\frac{1}{2}\sqrt{\frac{b_0^2 k_h k_m \beta_h(v) \beta_m A}{N_h(\mu_m + u)^2 pq^{3/2}\alpha}} < 0. \tag{29}$$

When virus load is minimum, that is, $v < v_{\min}$, we have $\beta_h(v) = 0$. It shows that the model parameters are not sensitive to disease transmission.

When virus load is medium, that is, $v_{\min} < v < v_M$, we have $\beta_h(v) = (v - v_{\min})/(v_M - v_{\min}) > 0$, and when virus load is strong, we have $\beta_h(v) = 1$. Also, $\gamma_h(v) > 0$ for all virus loads v. So, the sign of $(\partial R_0/\partial \beta_h(v))$ and $(\partial R_0/\partial \gamma_h(v))$ is unchanged for all values of v as in (28) and (29).

Thus, the expression on the right-hand side of (28) is always positive and that on the right-hand side of (29) is always negative, which means that the curve of the transmission coefficient $\beta_h(v)$ towards R_0 is monotonically increasing and the curve of the recovery rate $\gamma_h(v)$ towards R_0 is monotonically decreasing for medium and strong amount of virus loads. Thus, as $\beta_h(v)$ increases, R_0 also increases, and as $\gamma_h(v)$ increases, R_0 decreases.

When all parameter values from Table 1 except $\beta_h(v)$ and $\gamma_h(v)$ are substituted for $R_0 = 1$,

$$0.8234\sqrt{\beta_h(v)} = \sqrt{0.000046 + \gamma_h(v)}. \tag{30}$$

It can be seen from Figure 4 that when $\beta_h(v) < 0.0013$, the disease dies out since $R_0 < 1$, so the fumigation is not needed. When $\beta_h(v) > 0.0013$, we get $R_0 > 1$, so fumigation is required when the transmission rate is more than 0.0013.

Similarly, without fumigation rate, that is, $u = 0$, Figure 5 shows that, when $\beta_h(v) < 1.3847 \times 10^{-6}$, we get $R_0 < 1$ (dengue-free situation), and when $\beta_h(v) > 1.3847 \times 10^{-6}$, we get $R_0 > 1$, so we need fumigation to control the disease.

(2) Secondly, we analyze the R_0 sensitivity of biting rate $b(k, r)$. The another analysis is the R_0 sensitivity towards the fumigation rate u. We have the following:

Table 1: Parameters and their values [12].

Parameters	Value	Dimension
μ_h	0.000 046	day^{-1}
k_h	0.166 7	day^{-1}
γ_h	0.328 833	day^{-1}
N_h	5071 126	Dimensionless
μ_m	0.025	day^{-1}
b_0	0.5	day^{-1}
β_m	0.375	Dimensionless
β_h	0.75	Dimensionless
A	2500 000	Mosquito $\times$ day^{-1}
k_m	0.142 8	day^{-1}

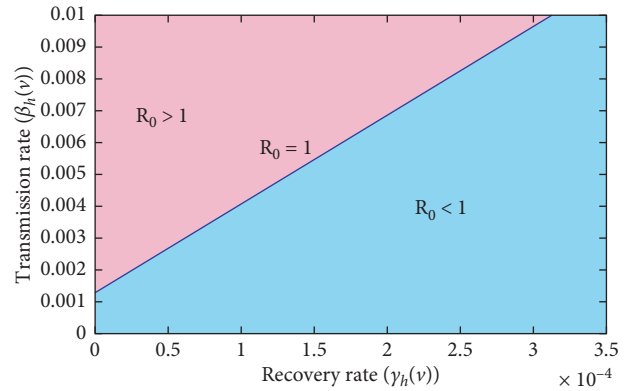

Figure 4: Sensitivity diagram of parameters β_h and γ_h with $u = 0.15$ and the critical line $R_0 = 1$. The red colored region defines endemic condition of dengue, and the blue colored area defines dengue-free condition.

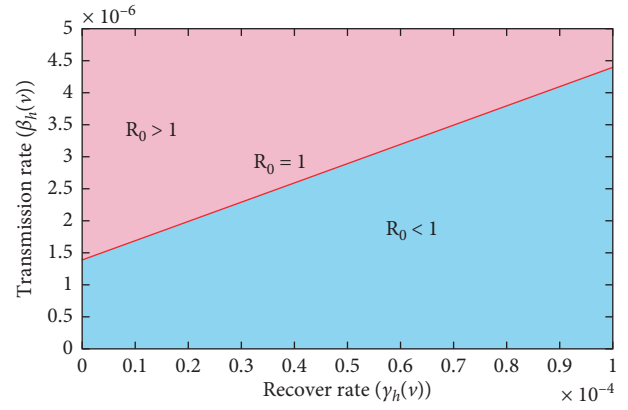

Figure 5: Sensitivity diagram of parameters β_h and γ_h with $u = 0$ and the critical line $R_0 = 1$. The red colored region defines endemic condition of dengue, and the blue colored area defines dengue-free condition.

$$\frac{\partial R_0}{\partial b(k, r)} = \sqrt{\frac{\beta_h(v)\beta_m k_h k_m A}{N_h(\mu_m + u)^2 pq\alpha}} > 0. \tag{31}$$

$$\frac{\partial R_0}{\partial u} = -\frac{1}{2}\left(\frac{2\alpha + \mu_m + u}{(\mu_m + u)^2}\right)\sqrt{\frac{\beta_h(v)\beta_m b^2(k, r) k_h k_m A}{N_h pq\alpha^3}} < 0. \tag{32}$$

The expression on the right-hand side of (31) is always positive and that on right-hand side of (32) is always negative for medium and strong amount of virus loads, which means that the curve of the parameter biting rate b towards R_0 is increasing monotonically and the curve of the parameter fumigation rate u towards R_0 is decreasing monotonically. Thus, as b increases, R_0 also increases, and as u increases, R_0 decreases.

However, the biting rate b of mosquito is dependent on the proportion of the bed net user k and effectiveness of bed net r. Thus, we have three cases: $k = 0$, $0 < k < 1$, and $k = 1$.

4.1. Case

(i) When $k = 0$ (no bed nets are used), on substituting all parameter values except u and b_0 from Table 1 for $R_0 = 1$, we get the following:

$$0.1664b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \quad (33)$$

(a) When $u = 0$, we get $b_0 = 0.0615$. Based on (33), Figure 6 explains how R_0 relies on the values of b_0 and u qualitatively. We can see that when $b_0 < 0.0615$, we get $R_0 < 1$. When $b_0 > 0.0615$, we get $R_0 > 1$, so fumigation is required to reach $R_0 < 1$.

(b) When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 0.5929$. So, when $b_0 < 0.5929$, we get $R_0 < 1$, disease will die out, and the fumigation is not required. When $b_0 > 0.5929$, we get $R_0 > 1$, and disease will be endemic, so the fumigation is not sufficient. We have the following:

$$0.1664b_0 = \left(0.025 + u_{\min}\right)\sqrt{\left(0.1678 + u_{\min}\right)}. \quad (34)$$

Therefore, to achieve the condition $R_0 < 1$ we need $u > u_{\min}$ when $b_0 > 0.5929$. It shows that when infection rate is less than the minimum boundary of u, we get $R_0 < 1$, the disease dies out, and fumigation is not necessary.

4.2. Case (ii). Let $0 < k < 1$; that is, some hosts use bed nets. In this case, the sensitivity of biting rate of mosquito is dependent on the effectiveness of bed nets r. Substituting all parameter values except u and b from Table 1 for $R_0 = 1$, we get the following:

$$0.1664\left(b_0 rk + b_0(1 - k)\right) = (0.025 + u)\sqrt{(0.1678 + u)}. \quad (35)$$

Suppose $k = 0.5$.

(i) For good quality of bed net, when $r = 0.1$ from (35), we get the following:

$$0.0915b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \quad (36)$$

When $u = 0$, that is, without use of fumigation, we get $b_0 = 0.1119$. Thus, when $b_0 > 0.1119$, we get $R_0 > 1$, so to achieve $R_0 < 1$ fumigation is required.

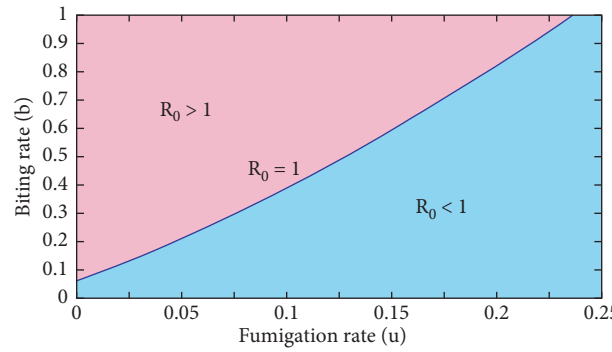

Figure 6: Sensitivity diagram of parameters b_0 and u with the critical line $R_0 = 1$. The red colored region defines endemic condition of dengue, and the blue colored area defines dengue-free condition.

When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 1.0782$. So, when $b_0 < 1.0782$, we get $R_0 < 1$; that is, disease will die out, and the fumigation is not necessary. When $b_0 > 1.0782$, we get $R_0 > 1$, and disease will be endemic, so we need more fumigation than 0.15.

(ii) For medium quality of bed net, when $r = 0.5$ from (35), we get the following:

$$0.1248b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \quad (37)$$

When $u = 0$, we get $b_0 = 0.0821$. Thus, when $b_0 > 0.0821$, we get $R_0 > 1$, and disease will be endemic, so fumigation is required to eliminate the disease. When $b_0 < 0.0821$, we get $R_0 < 1$, and the disease dies out.

When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 0.7905$. So, when $b_0 < 0.7905$, we get $R_0 < 1$. Disease will die out, and the fumigation is not necessary. When $b_0 > 0.7905$, we get $R_0 > 1$. Disease will be endemic.

(iii) For poor quality of bed net, when $r = 0.9$ from (35), we get the following:

$$0.1581b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \quad (38)$$

When $u = 0$, we get $b_0 = 0.1390$. Thus, when $b_0 > 0.1390$, we get $R_0 > 1$, so fumigation is needed to achieve $R_0 < 1$, and when $b_0 < 0.1390$, we get $R_0 < 1$.

When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 0.0648$. So, when $b_0 < 0.0648$, we get $R_0 < 1$ and the disease dies out. So, the fumigation is not required. When $b_0 > 0.0648$, we get $R_0 > 1$, and disease will be endemic.

Figure 7 shows that, when $0 < k < 1$ to control the disease, we need better quality of bed nets. The area of $R_0 < 1$ increases with increasing effectiveness of bed nets. Thus, if we have better quality of bet nets, we need less fumigation to control the disease.

4.3. Case (iii). When $k = 1$, substituting all parameter values except u and b from Table 1 for $R_0 = 1$, we get the following:

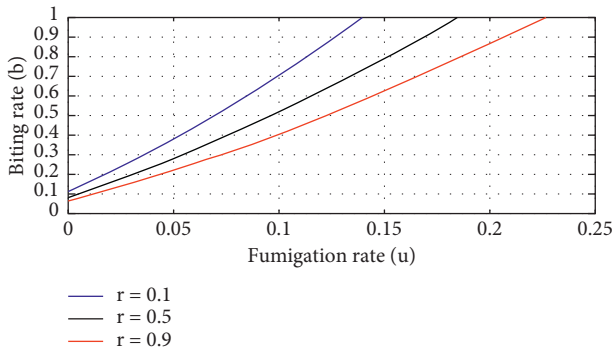

Figure 7: Parameters b and u with the critical line $R_0 = 1$ with different values of r.

$$0.1664 b_0 r = (0.025 + u)\sqrt{(0.1678 + u)}. \tag{39}$$

(i) When $r = 0.1$ in (39),

$$0.0166 b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \tag{40}$$

When $u = 0$, we get $b_0 = 0.6154$. Thus, when $b_0 > 0.6154$, we get $R_0 > 1$, so to eliminate the disease, fumigation is required. When $b_0 < 0.6154$, we get $R_0 < 1$; that is, the disease dies out.

When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 5.9287$. So, when $b_0 < 5.9287$, we get $R_0 < 1$; that is, the disease dies out, and the fumigation is not necessary. When $b_0 > 5.9287$, we get $R_0 > 1$, and disease will be endemic, so more fumigation is required.

(ii) When $r = 0.5$ in (39), we get the following:

$$0.0832 b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \tag{41}$$

When $u = 0$, we get $b_0 = 0.1231$. Thus, when $b_0 > 0.1231$, we get $R_0 > 1$, and disease will be endemic, so we need fumigation, and when $b_0 < 0.1231$, we get $R_0 < 1$, and fumigation is not necessary in this case.

When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 1.1857$. So, when $b_0 < 1.1857$, we get $R_0 < 1$. Disease dies out, and the fumigation is not required. When $b_0 > 1.1857$, we get $R_0 > 1$, and disease will be endemic. So, the fumigation is required to control disease.

(iii) When $r = 0.9$ in (39), we get the following:

$$0.1489 b_0 = (0.025 + u)\sqrt{(0.1678 + u)}. \tag{42}$$

When $u = 0$, we get $b_0 = 0.0684$. Thus, when $b_0 > 0.0684$, we get $R_0 > 1$, and we must use fumigation to achieve $R_0 < 1$, and when $b_0 < 0.0684$, we get $R_0 < 1$.

When $u \neq 0$, suppose $u = 0.15$, we get $b_0 = 0.6586$. So, when $b_0 < 0.6586$, we get $R_0 < 1$ and the disease dies out. So, the fumigation is not required. When $b_0 > 0.6586$, we get $R_0 > 1$, disease will be endemic, and we need more fumigation.

Figure 8 shows that the region of $R_0 < 1$ is decreasing with increasing the value of the effectiveness of bed nets r, when $k = 1$. On increasing the quality of bed nets, that is, decreasing the value of r, there is reduction in the transmission of disease. So, disease will die out very fast. Thus, the proportion of bed net user and effectiveness of bed nets are very sensitive towards R_0.

(3) Now, we analyze R_0 sensitivity of the bed net parameters when $0 < k < 1$, which is the proportion of the bed net users k and the proportion of effectiveness of bed nets r.

$$\frac{\partial R_0}{\partial r} = b_0 k \sqrt{\frac{\beta_h(v)\beta_m k_h k_m A}{N_h(\mu_m + u)^2 pq\alpha}} > 0, \tag{43}$$

$$\frac{\partial R_0}{\partial k} = -b_0(1 - r)\sqrt{\frac{\beta_h(v)\beta_m k_h k_m A}{N_h(\mu_m + u)^2 pq\alpha}} < 0. \tag{44}$$

Since $r \in (0, 1)$ and $k \in [0, 1]$, we get $(r - 1) < 0$ and $(1 - k) \geq 0$. Thus, (43) and (44) show that R_0 increases monotonically with respect to r and decreases monotonically with respect to k. That is, as r increases, R_0 also increases and as k increases, R_0 decreases.

Substituting all the parameter values except k and r for $R_0 = 1$, we get the following:

$$2.3664\sqrt{\{0.5(kr - k + 1)\}^2} = 1. \tag{45}$$

Based on the above equation, Figure 9 explains how R_0 can be determined by the values of r and k. It can be seen from Figure 9 that when $k < 0.1548$, we get $R_0 > 1$, so the proportion of bed net user cannot eliminate the disease. It shows that the number of bed net user should be increased to achieve $k > 0.1548$; that is, $R_0 < 1$. In other words, among the endemic population of dengue more than 15.48% need to use bed net. The bed nets are not effective, if portion of bed net user is less than 15.48%. Note that the effectiveness of bed net is very important to achieve $R_0 < 1$ when we use the bed nets. When $k > 0.1548$, from above equation we get the following:

$$r_{\min} = \frac{k - 0.1548}{k}. \tag{46}$$

Therefore, to achieve the condition $R_0 < 1$, we need $r > r_{\min}$ when $k > 0.1548$, which means that when we provide the bed nets, we must consider the high-quality bed nets. The better quality of bed nets, the high chance to decrease the number of mosquito bites as in Figure 10.

5. Bifurcation

When basic reproduction number R_0 increases through unity, the stability of disease-free equilibrium changes from stable to unstable forms. The disease-free equilibrium point P_0 is locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$. So, the system attains a bifurcation at $R_0 = 1$ about P_0, which is called the backward bifurcation. The control variables such as fumigation, bed net user, and effectiveness of bed nets affect the transmission of the disease significantly. So, the weights of these variables influence the

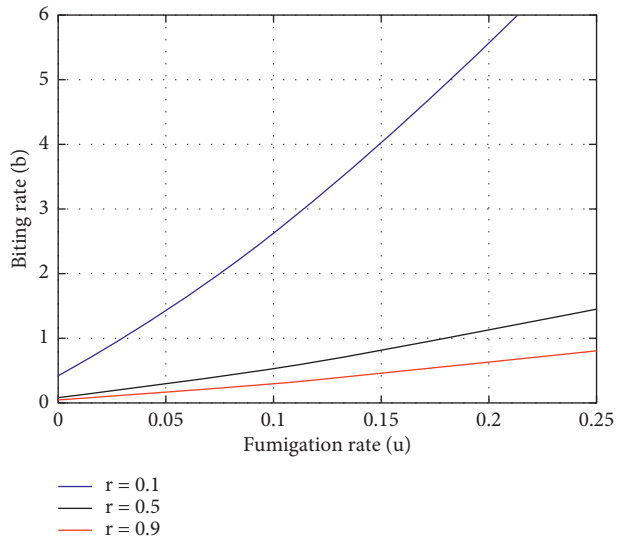

Figure 8: Parameters b and u with the critical line $R_0 = 1$ and different values of r when $k = 1$.

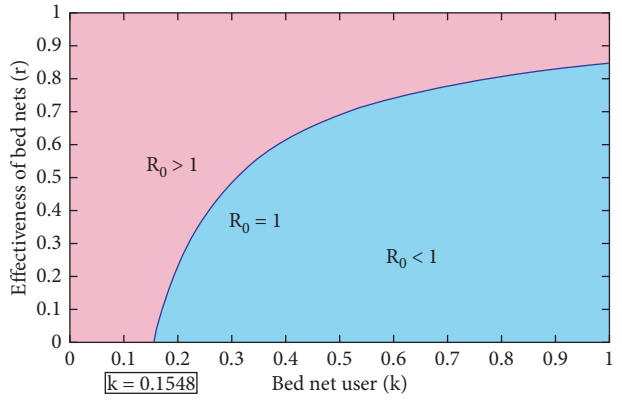

Figure 9: Sensitivity diagram of parameters k and r with the critical line $R_0 = 1$. The red colored region defines endemic condition of dengue, and the blue colored area defines dengue-free condition.

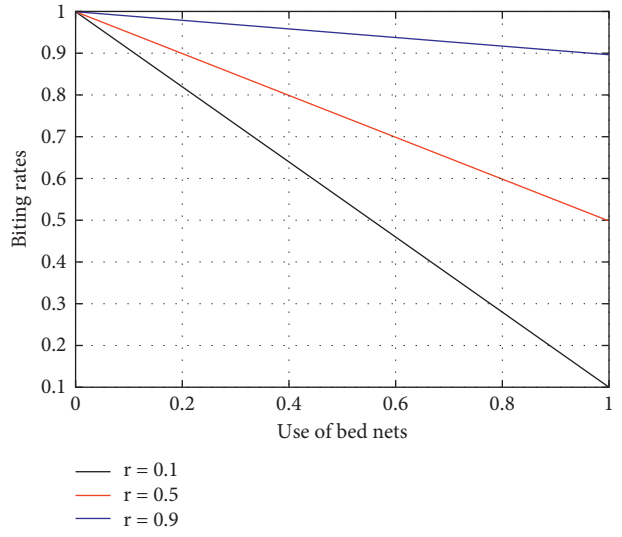

Figure 10: Biting rate with different values of r.

stability and bifurcation of the system. Now, we have two cases to study bifurcation one with fumigation and another without fumigation.

5.1. Case (i)

(a) When $u \neq 0$ (with fumigation) and $k = 0$ (without bed net user) at $R_0 = 1$, we have the following:

$$\beta_h(v)\beta_m k_h k_m b_0^2 A = (\mu_m + u)^2 N_h pq\alpha. \tag{47}$$

Putting the values of parameters on Table 1 in (47), we get $\beta_h = 0.0157$. Thus, when there is fumigation $u = 0.15$, P_0 is asymptotically stable provided $\beta_h < 0.0157$, and if $\beta_h > 0.0157$, P_0 becomes unstable, whereas P_1 is asymptotically stable (from Figure 11). The study of stability of the fuzzy model of dengue (1) shows that P_1 is unstable when P_0 is asymptotically stable for $\beta_h < 0.0157$. Therefore, P_0 has bifurcation at $\beta_h = 0.0157$ when fumigation $u = 0.15$.

Now, we can find a particular value v^* of the virus load v such that the system undergoes a backward bifurcation at v^*, but value of v^* depends on the value of the virus load v. So, we have three cases.

(i) When $v < v_{\min}$, we have the following:

$$\begin{aligned} \beta_h(v) &= 0, \\ R_0(v) &= 0. \end{aligned} \tag{48}$$

Thus, there is no change with change in parameter values.

(ii) When $v_{\min} \leq v \leq v_M$, we have the following:

$$\begin{aligned} \beta_h(v) &= \frac{v - v_{\min}}{v_M - v_{\min}}, \\ \gamma_h(v) &= \frac{1 - \gamma_0}{v_{max}} v + 1. \end{aligned} \tag{49}$$

Thus,

$$\begin{aligned} &\frac{v - v_{\min}}{v_M - v_{\min}} \beta_m k_h k_m b_0^2 A = (\mu_m + u)^2 N_h p\alpha \left[\frac{1 - \gamma_0}{v_{\max}} v + 1 + \mu_h \right] \\ &\Rightarrow (v - v_{\min}) v_{\max} = M(v_M - v_{\min})[(1 - \gamma_0)v + (1 + \mu_h)v_{\max}] \\ &\Rightarrow v = \frac{[v_{\min} + M(v_M - v_{\min}) + (1 + \mu_h)]v_{\max}}{v_{\max} - M(v_M - v_{\min})(1 - \gamma_0)}, \\ &\text{where, } M = \frac{(\mu_m + u)^2 N_h p\alpha}{\beta_m k_h k_m b_0^2 A}. \end{aligned} \tag{50}$$

Therefore, when $v_{\min} \leq v \leq v_M$ at

$$v^* = \frac{[v_{\min} + M(v_M - v_{\min}) + (1 + \mu_h)]v_{\max}}{v_{\max} - M(v_M - v_{\min})(1 - \gamma_0)}, \tag{51}$$

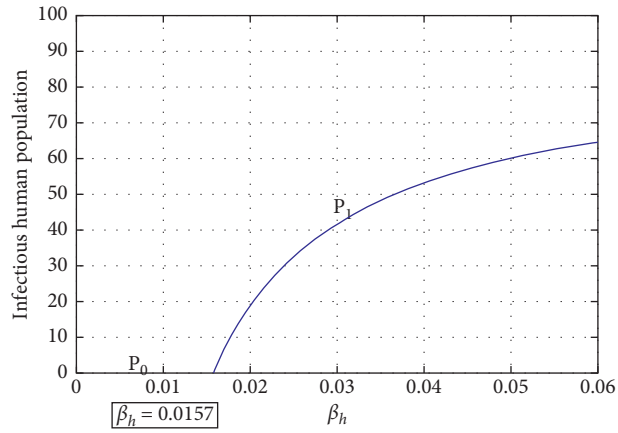

Figure 11: Bifurcation diagram with fumigation.

the system has backward bifurcation at $v = v^* = 1.1027 \times 10^7$.

(iii) When $v_M < v < v_{\max}$, we have the following:

$$\beta_h(v) = 1,$$
$$\gamma_h(v) = \frac{1-\gamma_0}{v_{\max}} v + 1. \tag{52}$$

Thus,

$$\beta_m k_h k_m b^2 A = \mu_h^2 N_h p\alpha \left[\frac{1-\gamma_0}{v_{\max}} v + 1 + \mu_h\right]$$
$$\Rightarrow (1-\gamma_0)v + (1+\mu_h)v_{\max} = \frac{\beta_m k_h k_m b_0^2 A v_{\max}}{(\mu_m+u)^2 N_h p\alpha} \tag{53}$$
$$\text{Therefore, } v = \frac{v_{\max}}{(1-\gamma_0)}\left[\frac{\beta_m k_h k_m b^2 A - (1+\mu_h)(\mu_m+u)^2 N_h p\alpha}{(\mu_m+u)^2 N_h p\alpha}\right].$$

(b) When $u \neq 0$ and $0 < k < 1$, that is, partial hosts use bed nets with fumigation, we have (at $R_0 = 1$):

$$\beta_h(v)\beta_m k_h k_m (b_0 rk + b_0(1-k))^2 A = (\mu_m+u)^2 N_h pq\alpha. \tag{54}$$

The stability of the equilibrium point P_0 of system (1) depends on the value the effectiveness of bed nets r (Figure 12). When $k = 0.5$ and $r = 0.1$, from equation (12), we get $\beta_h = 0.4869$. Thus, the equilibrium point P_1 is unstable and P_0 is asymptotically stable for $\beta_h < 0.4869$. If $\beta_h > 0.4869$, P_0 is unstable. Therefore, the equilibrium point P_0 of the system has bifurcation at $\beta_h = 0.4869$ (Figure 12).

Similarly, when $r = 0.5$, P_0 of the system has bifurcation at $\beta_h = 0.2621$ and, when $r = 0.9$, bifurcation at $\beta_h = 0.1634$ (Figure 12).

(c) When $u \neq 0$ and $k = 1$, that is, all hosts used bed nets with fumigation, we have (at $R_0 = 1$) the following:

$$\beta_h(v)\beta_m k_h k_m (b_0 r)^2 A = (\mu_m+u)^2 N_h pq\alpha. \tag{55}$$

In this case, as above, the bifurcation point of P_0 of system (1) is different for the different values of effectiveness of bed nets r (from Figure 13). When $r = 0.1$, using values of the parameters in Table 1, in equation (55) except β_h, we get $\beta_h = 0.3685$. Thus, the equilibrium point P_0 is asymptotically stable for $\beta_h < 0.4869$ and P_1 is unstable. P_0 is unstable, when $\beta_h > 0.3685$. Therefore, P_0 of the system has bifurcation at $\beta_h = 0.3685$ (Figure 13).

Similarly, when $r = 0.5$, P_0 of the system has bifurcation at $\beta_h = 0.0590$ and, when $r = 0.9$, bifurcation at $\beta_h = 0.0246$ (Figure 13).

5.2. Case (ii)

(a) When $u = 0$ (without fumigation) and $k = 0$ (without use of bed nets), we have (at $R_0 = 1$) the following:

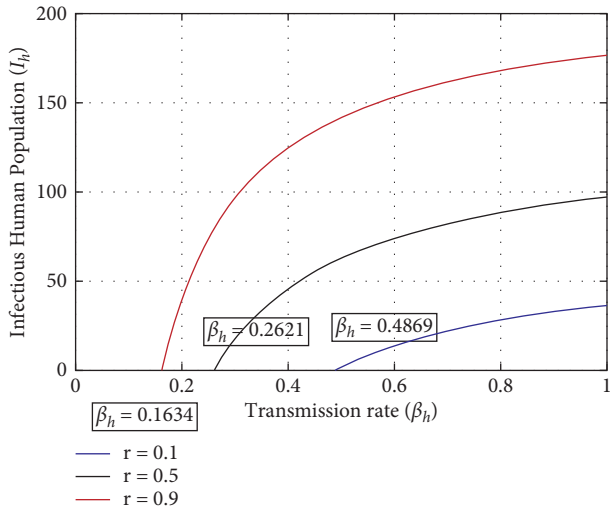

Figure 12: Bifurcation diagram with different values of r when $0 < k < 1$.

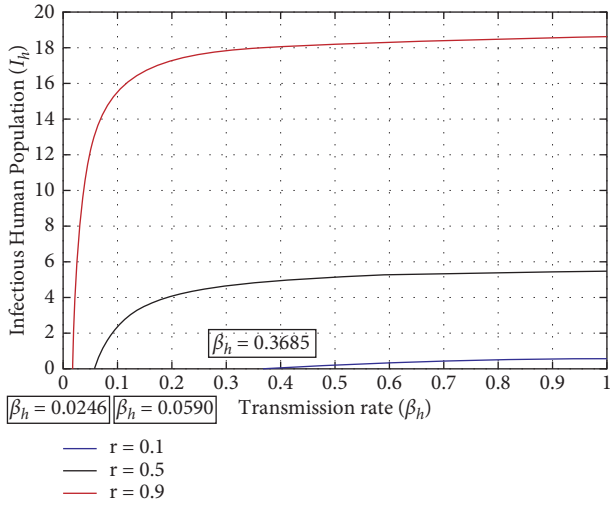

Figure 13: Bifurcation diagram with different values of r when $k = 1$.

$$\beta_h(v)\beta_m k_h k_m b_0^2 A = (\mu_m)^2 N_h pq\alpha. \tag{56}$$

From equation (56), using values of the parameters in Table 1 we get $\beta_h = 0.0016$. So, the equilibrium point P_0 is asymptotically stable provided $\beta_h < 0.0016$ and unstable when $\beta_h > 0.0016$, and fumigation is required to control the disease (from Figure 14). Therefore, P_0 of the system has bifurcation at $\beta_h = 0.0016$.

(b) When $u = 0$ and $0 < k < 1$, some hosts use bed nets. At $R_0 = 1$, we have the following:

$$\beta_h(v)\beta_m k_h k_m \left(b_0 kr + b_0(1-k)\right)^2 A = (\mu_m)^2 N_h pq\alpha. \tag{57}$$

In this case, the effectiveness of bed nets r is very important, and the transformation of disease is changed with changing the values of r. Using the values of the parameters from Table 1 in (57) when $r = 0.1$, we get $\beta_h = 0.0052$. So, P_0 is asymptotically stable provided $\beta_h < 0.0052$ and unstable if $\beta_h > 0.0052$, whereas equilibrium point P_1 is stable. So, P_0 of the system has bifurcation at $\beta_h = 0.0052$. Similarly, when $r = 0.5$, P_0 has bifurcation at $\beta_h = 0.0028$ and, when $r = 0.9$, bifurcation at $\beta_h = 0.0018$ (from Figure 15).

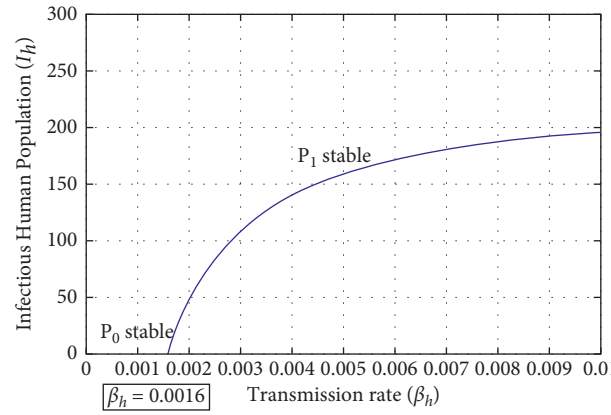

Figure 14: Bifurcation diagram without fumigation.

(c) When $u = 0$ and $k = 1$, all hosts use bed nets, and we have the following:

$$\beta_h(v)\beta_m k_h k_m (b_0 r)^2 A = (\mu_m)^2 N_h pq\alpha. \tag{58}$$

When all people use bed nets, the effectiveness of bed nets r is very important. The transformation of disease is different for different qualities of bed nets r. From equation (58), using values of the parameters in Table 1 when $r = 0.1$, we get $\beta_h = 0.0400$. Thus, P_0 is asymptotically stable when $\beta_h < 0.0400$ and unstable when $\beta_h > 0.0400$, whereas equilibrium point P_1 is stable. Thus, P_0 has bifurcation at $\beta_h = 0.0400$. Similarly, when $r = 0.5$, P_0 has bifurcation at $\beta_h = 0.0064$, and when $r = 0.9$, P_0 has bifurcation at $\beta_h = 0.0020$ (from Figure 16).

Therefore, when the value of r increases, the value of the bifurcation point of P_0 decreases; that is, the effectiveness of bed nets influences the bifurcation of P_0 of system (1).

6. Numerical Results and Discussion

In this work, we use the SEIR-SEI model for transmission dynamics of dengue disease with control variables such as fumigation, use, and effectiveness of bed nets. The numerical values for the simulation are presented in Table 1.

Figures 17 and 18 show the transmission of the susceptible and infectious human population with and without control variable u. Figure 17 shows that, when control variables are used, the susceptible human remains uninfected for long time. Only few people get infected from the

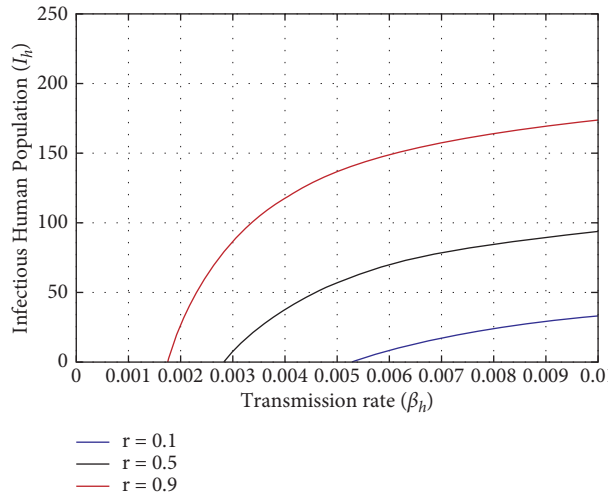

Figure 15: Bifurcation diagram without fumigation with different values of r when $0 < k < 1$.

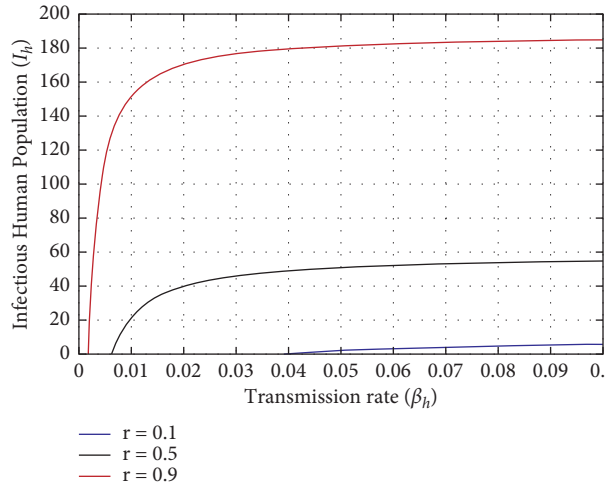

Figure 16: Bifurcation diagram without fumigation with different values of r when $k = 1$.

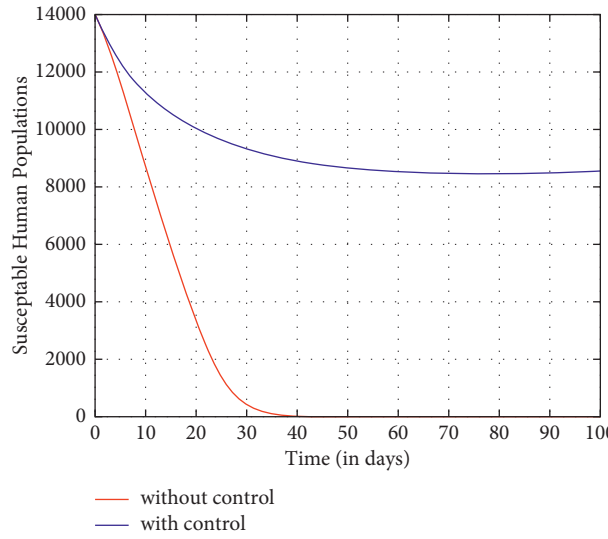

Figure 17: Susceptible human population with and without control variable u.

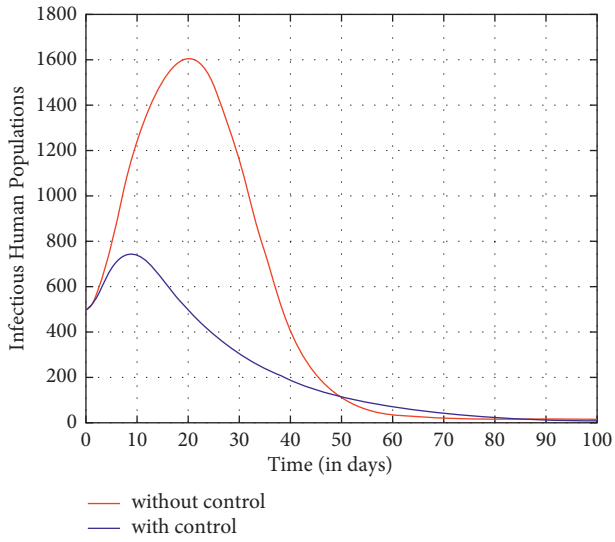

Figure 18: Infectious human population with and without control variable u.

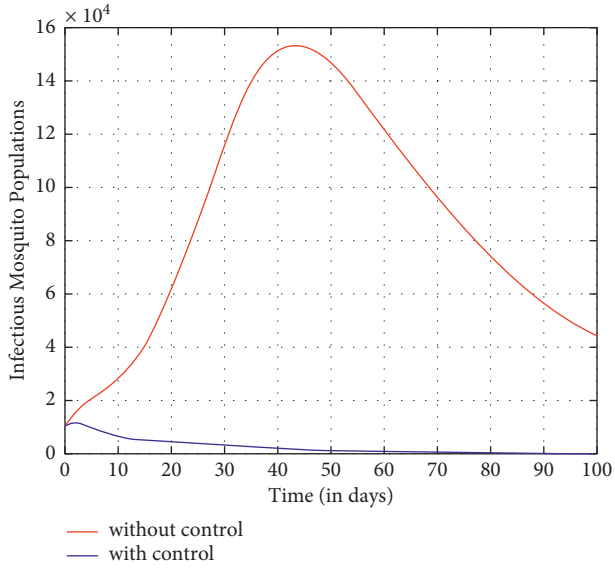

Figure 19: Infectious mosquito population with and without control variable u.

disease, when they use control variables such as fumigation and bed nets (Figure 18). Figure 19 shows that only few mosquitoes get infected when we use the control variable fumigation. Figures 17–19 show that the proper use of control variables can save humans from the infection.

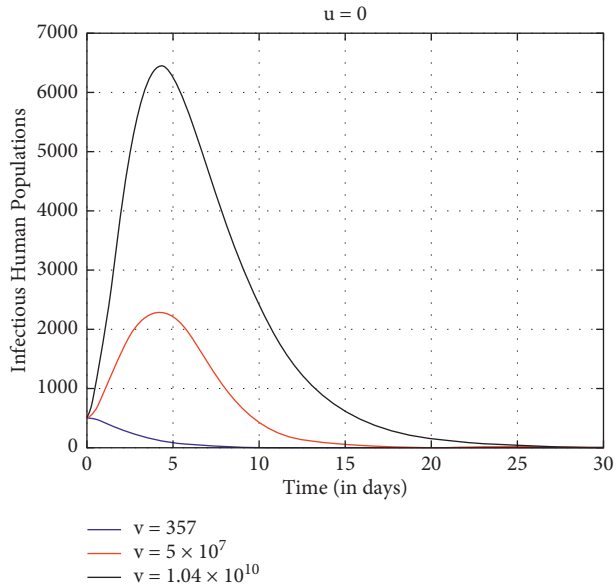

Figure 20: Infectious human population with different virus loads without fumigation.

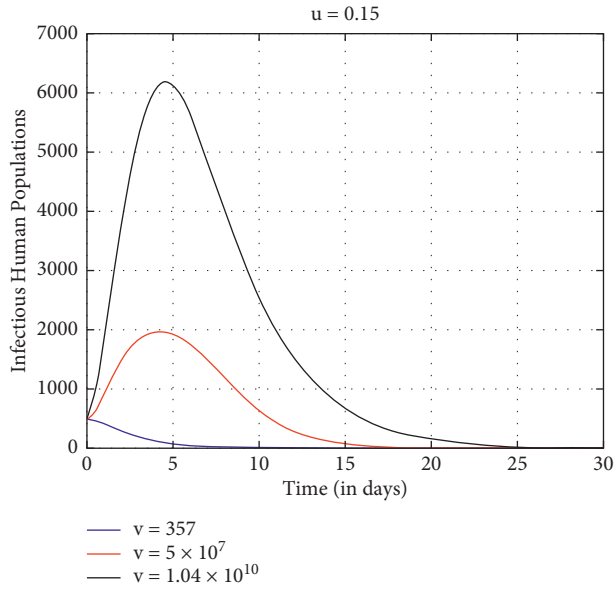

Figure 21: Infectious human population with different virus loads when $u = 0.15$.

Figures 20–22 describe the nature of infectious human population with different virus loads with different fumigation rates. These figures show that, if we increase the amount of control variable, that is, the amount of fumigation and quality of the bed nets, infectious human population can

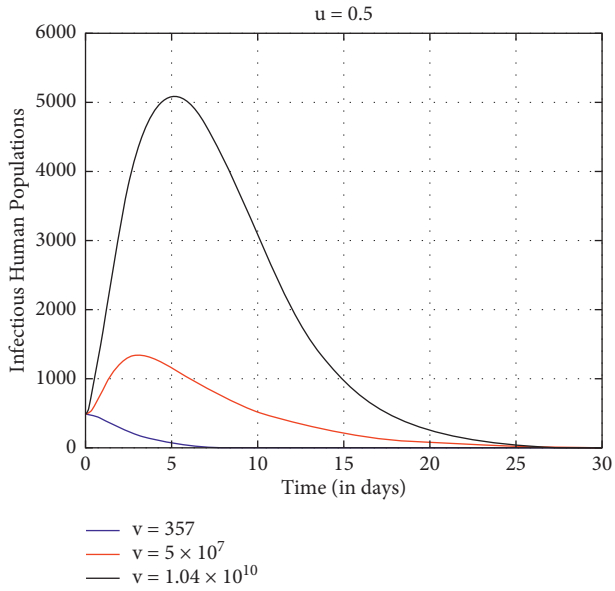

Figure 22: Infectious human population with different virus loads when $u = 0.5$.

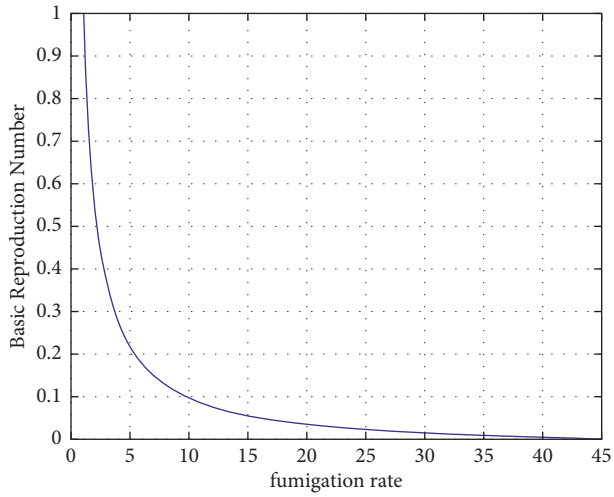

Figure 23: Basic reproduction number with fumigation.

decrease. When virus load is very low, fumigation is not necessary. When there is no fumigation, infectious human population is around 6500 (from Figure 20). When $u = 0.15$, infectious human population becomes 6000 (from Figure 21), and when $u = 0.5$, infectious human population is 5000 (from Figure 22). Thus, fumigation helps in decreasing the prevalence of dengue disease. Further, the basic reproduction number decreases with increasing fumigation rate (Figure 23).

7. Conclusion

The fuzzy epidemic model of dengue with bed nets and fumigation intervention has two equilibrium points: disease-free equilibrium point and endemic equilibrium point. Disease-free equilibrium point exists and stable when $R_0 < 1$, the endemic equilibrium point exists only when $R_0 > 1$. Simulation shows that, when virus load is less than $v_{\min}$, the chance of transmission of disease is negligible, so fumigation is not required. In the bed net intervention, we conclude that, to eliminate the infection of dengue, the proportion of bed net user needs to be evaluated, since if the proportion of bed net user is less, then it cannot control dengue prevalence. Additionally, the chances of mosquito bites depend on the proportion of bed net user and effectiveness of bed nets. The better quality of bed nets has more chances of reducing the mosquito bites. Simulated results show that more susceptible hosts are infected, when fumigation is not used.

In both cases with and without fumigation, the sensitivity of the model parameter transmission coefficient β_h is opposite to recovery rate γ_h with R_0 and biting rate b is also opposite to fumigation rate u with R_0. When biting rate and transmission coefficient increase, R_0 increases, and when recovery rate and fumigation rate increase, R_0 decreases. The proportion of bed net user and effectiveness of bed nets also affect the sensitivity of these parameters. Mosquito biting rate and disease transmission rate can be reduced using better quality of bed nets.

The stability of the disease-free equilibrium P_0 changes from stable to unstable, when the basic reproduction number changes from less than unity to greater than unity. Thus, at $R_0 = 1$ the system attains a bifurcation. In system (2.1), the control variable fumigation, proportion of bed net user, and proportion of effectiveness of bed nets are very important parameters for transmission of dengue. The bifurcation values of the system for P_0 are also different for different values of the control variables.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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