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Research Article

Topological Properties of Nano Sheets Based on Octa Graphene

Fozia Bashir Farooq

Department of Mathematics and Statistics, Faculty of Science, Imam Mohammad Ibn Saud Islamic University(IMSIU), P.O. Box 90950, Riyadh 11623, Saudi Arabia

Correspondence should be addressed to Fozia Bashir Farooq; fozia.gc@gmail.com

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Octa Graphene is a carbon nanosheet constructed by octagons and squares to build a unit cell. Topological properties of nano sheets based on Octa graphene are investigated in this article for the first time. Octa Graphene play a role as a Nano carrier for drug delivery and plays vital role in cell base drug delivery applications. Two structures, Namely Octa-Grayphyne and Octa-Graphdine are investigated topically in this article. Mechanically Octa-Grayphyne and Octa-Graphdine are of immense interest in Nano structural batteries and other Nano devices. The ultimate value of these Nano sheets in nano electronics can be interpreted by observing their electronic structure and vibrational properties.

1. Introduction

Nano sheets are two dimensional Nano structures. Carbon Nano sheets from the last decades are of great interest for researchers. These sheets were produced by heating the fibers at a temperature of over 350 F for 24 hours. Graphene is crystal like Nano structure of carbon atoms, with a strong bond and of thickness of one atom. Whereas Graphene Nano sheet is a 2D Nano structure consist of a single lyre of carbon atoms with hexagonal lattices. Wu et al. [1] investigated Graphene oxide sheet can be used as a carrier for Adriamcine, can reverse drug resistance in breast cancer. Moreover, in [2] it was shown that Graphene oxide sheet was also an anti-cancer agent. Without loss of generality we can state that Grephene oxide sheet is a source to decrease death rate due to cancer. The novel 2D Nano sheets have open new horizons to solve the serious environmental water pollution problem of the time by serving as a favorable candidate to reduce heavy metal pollution from water. To read about Nanosheets read [3].

Octa Grephene is a carbon Nano sheet, consisting of octagons and squares. Poupitz at el [4] introduces Octa-Grayphyne and Octa-Graphdine Nano sheets. Before going to further investigation of theses nano sheets, authors would like to recall some history of Topological indices for chemical graphs;

Let G be a graph with vertex set V(G) and edge set E(G) with $|V(G)| = p$ and $|E(G)| = q$ respectively. Degree of a vertex $u \in V(G)$, means number of edges incident with u and symbolically written as d_u. A chemical Graph is the graphical representation of a chemical structure where vertices represents the atoms and edges represents the bond between atoms. In chemical graphs the maximum degree of vertex that can have is 4.

The Molecular Descriptor is a procedure to transform chemical information of a compound to a numerical value. Topological indices are one of the types of molecular descriptors. These indices are numerical values calculated from graphical representation of chemical compounds. Topological indices play an important role for Quantitative Structure Activity relationship (QSAR) by correlating psychochemical and biological properties of chemical compounds. Harry wiener is the founder of chemical graph theory. His contribution towards study of Topological indices is radical and about QSAR methods can be found in [5] commendable. He introduced structure-base invariants correlating boiling points of paraffin. He introduced the Winer index, which plays a major role in nanotechnology.

One of the classical topological index was introduces by Milan Randic in 1975. Randic index is the Sum of bond contribution of the degrees of vertices (atoms) making that bond, defined in [6] as;

$$\chi(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u d_v}}. \tag{1}$$

Idea of general randic index was given by Bollobás and Erdös in [7] as:

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_u d_v)^\alpha. \tag{2}$$

Gutman and Trinajsti´c [8], introduces first and second zegreb indices $M_1(G)$ and $M_2(G)$ as;

$$M_1(G) = \sum_{uv\in E(G)} [d_u + d_v], \quad M_2(G) = \sum_{uv\in E(G)} [d_u d_v]. \tag{3}$$

History of topological index will not complete without discussion of Atom Bond connectivity index(ABC index) as it has extraordinary applications in rationalizing alkanes. ABC index was introduced by Estrada et al. [9] as;

$$I(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{4}$$

Harmonic index $H(G)$ of a graph G is defined [10] as;

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_u + d_v}. \tag{5}$$

For a molecular graph G the forgotten index or F index [11] is;

$$F(G) = \sum_{uv\in E(G)} \left[d_u^2 + d_v^2\right]. \tag{6}$$

Symmetric Division index or SDD for a molecular Graph G is defined [12] as;

$$SDD(G) = \sum_{uv\in E(G)} \left[\frac{\min(d_u, d_v)}{\max(d_u, d_v)} + \frac{\max(d_u, d_v)}{\min(d_u, d_v)}\right]. \tag{7}$$

The reader is encouraged to study [7, 13, 14] for more information on degree-based topological indices.

Deutsch, E.; Deutsch, [15] gave the c[$(\min(d_u, d_v)/\max(d_u, d_v)) + (\max(d_u, d_v)/\min(d_u, d_v))$]oncept of M polynomial of a chemical graph defined as;

$$M(G, x, y) = \sum_{i\leq j} e_{ij}(G)\alpha^i\beta^j, \tag{8}$$

where $e_{ij}(G)$, $(i, j \geq 1)$ denotes the number of the edges $e = uv$ of molecular graph G such that $(d_u, d_v = (\mathrm{i}, \mathrm{j}))$.

For further studies about M-polynomial please read [16–28].

Table 1gives the relation between the degree base topological indices with M polynomial. Where,

$$D_\alpha(f(\alpha,\beta)) = \alpha\cdot\frac{\partial(f(\alpha,\beta))}{\partial\alpha},$$
$$D_\beta(f(\alpha,\beta)) = \beta\cdot\frac{\partial(f(\alpha,\beta))}{\partial\beta},$$
$$J(f) = f(\alpha,\alpha), \tag{9}$$
$$S_\alpha(f(\alpha,\beta)) = \int_0^\alpha \frac{(f(t,\beta))}{t}\mathrm{d}t,$$
$$S_\beta(f(\alpha,\beta)) = \int_0^\beta \frac{(f(\alpha,t))}{t}\mathrm{d}t.$$

2. Main Results

2.1 Let $O_1G_{m\times n}$ be the Octa-Graphyne Nano sheet with m hexagons with each row and n hexagons in each column. From the graphical representation of Octa- Graphyne nano sheet, we can observe that each row of $O_1G_{m\times n}$consists of m hexagons and $m+1$ squares. The Graph of $O_1G_{m\times n}$ can be found in Figure 1. Each successive column of $O_1G_{m\times n}$ also consists n hexagons and $n+1$ squares. Degrees of end vertices for all edges are used to calculate degree-based topological indices. $O_1G_{m\times n}$ has $4mn+2m+2n$ number of vertices, whereas number of edges for $O_1G_{m\times n}$ is $6mn+m+n$. By observing graph of $O_1G_{m\times n}$ one can find that that all vertices of $O_1G_{m\times n}$ has either degree 2 or 3. Depending on the degree of vertices edge partition of $O_1G_{m\times n}$ is given in Table 2.

Partition of edge sets is defined as,

$$E_{(i,j)} = \{uv \in E(O_1G_{m\times n}):\ d_u = i \,\text{and}\, d_v = j, \quad 2\leq i\leq j\leq 3\}. \tag{10}$$

Based on edge partition given in Table 2, we can find M polynomial of Octa-Graphyne Nanosheet $O_1G_{m\times n}$, Whereas Graphical representation of M polynomial of $O_1G_{m\times n}$ is given in Figure 2.

Theorem 1. *Let $O_1G_{m\times n}$ be the graph of Octa- Graphyne Nanosheet, M polynomial for $O_1G_{m\times n}$ is given by,*

$$M(O_1G_{m\times n}) = 2(m+n+2)\alpha^2\beta^2 + 4(m+n-2)\alpha^2\beta^3 + (6mn-5m-5n+4)\alpha^3\beta^3. \tag{11}$$

Proof. Let $O_1G_{m\times n}$ be the graph of Octa- Graphyne Nanosheet, By using Table 2, combined with *equation* 9, we get

$$\begin{aligned} M(O_1G_{m\times n}) &= \sum_{i\leq j} e_{ij}(O_1G_{m\times n})\alpha^i\beta^j \\ &= \sum e_{22}(O_1G_{m\times n})\alpha^2\beta^2 + \sum e_{23}(O_1G_{m\times n})\alpha^2\beta^3 \\ &\quad + \sum e_{33}(O_1G_{m\times n})\alpha^3\beta^3 \\ &= 2(m+n+2)\alpha^2\beta^2 + 4(m+n-2)\alpha^2\beta^3 \\ &\quad + (6mn-5m-5n+4)\alpha^3\beta^3. \end{aligned} \tag{12}$$

□

Table 1: Derivation of topological indices from the M polynomial.

Degree base topological indices	$f(\alpha,\beta)$	Derivation from $M(\mathbb{G},\alpha,\beta)$
$M_1(\mathbb{G})$	$\alpha+\beta$	$(D_\alpha + D_\beta)(M(\mathbb{G},\alpha,\beta))\vert_{\alpha=\beta=1}$
$M_2(\mathbb{G})$	$\alpha\beta$	$(D_\alpha D_\beta)(M(\mathbb{G},\alpha,\beta))\vert_{\alpha=\beta=1}$
$R_\propto(\mathbb{G})$	$(\alpha\beta)^\propto \propto \in N$	$D_\alpha^\propto D_\beta^\propto (M(\mathbb{G},\alpha,\beta))\vert_{\alpha=\beta=1}$
$I(\mathbb{G})$	$\alpha\beta/(\alpha+\beta)$	$2S_\alpha J D_\alpha D_\beta (M(\mathbb{G},\alpha,\beta)\vert_{\alpha=\beta=1}$
$H(\mathbb{G})$	$2/(\alpha+\beta)$	$2S_\alpha J(M(\mathbb{G},\alpha,\beta)\vert_{\alpha=\beta=1}$
$F(\mathbb{G})$	$\alpha^2+\beta^2$	$(D_\alpha^2 + D_\beta^2)M(\mathbb{G},\alpha,\beta)\vert_{\alpha=\beta=1}$
$SDD(\mathbb{G})$	$(\alpha^2+\beta^2)/\alpha\beta$	$(D_\alpha S_\beta + S_\alpha D_\beta)M(\mathbb{G},\alpha,\beta)\vert_{\alpha=\beta=1}$

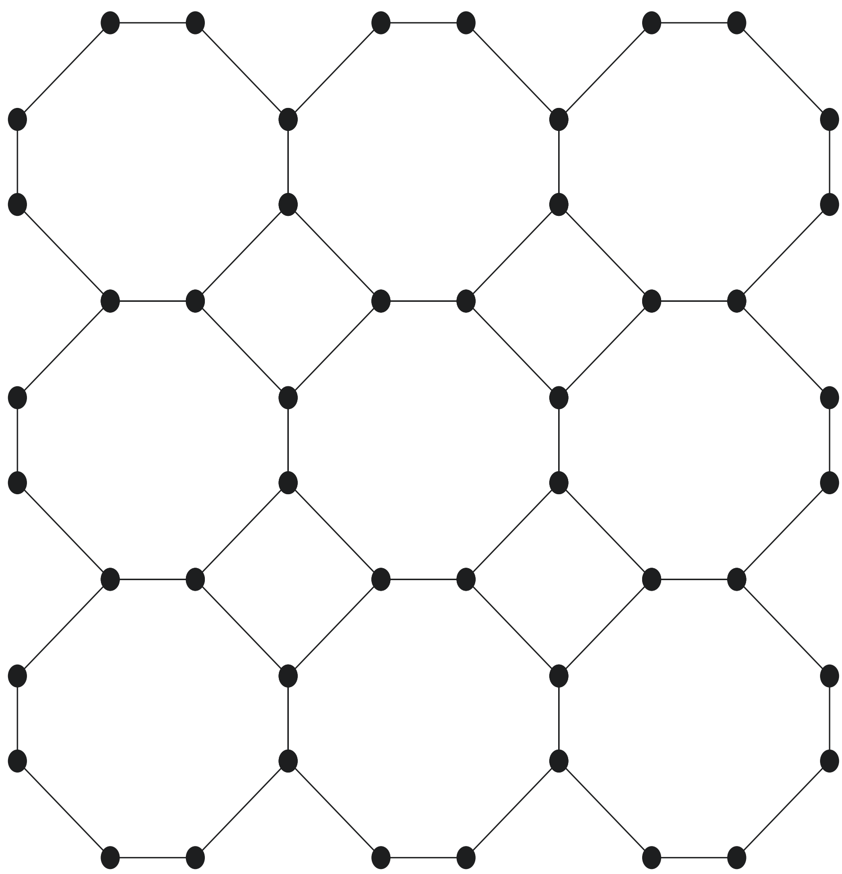

Figure 1: Graph of octa-graphyne nanosheet $O_1\mathbb{G}_{3\times 3}$.

Table 2: Edge Partition of $O_1\mathbb{G}_{m\times n}$ Nano sheet.

Edge types	(d_u, d_v)	Frequency
$E_{(2,2)}$	$(2,2)$	$2(m+n+2)$
$E_{(2,3)}$	$(2,3)$	$4(m+n-2)$
$E_{(3,3)}$	$(3,3)$	$6mn-5m-5n+4$

Theorem 2. *Let $O_1\mathbb{G}_{m\times n}$ be the graph of Octa- Graphyne Nanosheet Let$M(O_1\mathbb{G}_{m\times n}) = 2(m+n+2)\alpha^2\beta^2 + 4(m+n-2)\alpha^2\beta^3 + (6mn-5m-5n+4)\alpha^3\beta^3$ be M polynomial of $O_1\mathbb{G}_{m\times n}$ then we have,*

$$
\begin{aligned}
M_1(O_1\mathbb{G}_{m\times n}) &= 36mn - 2m - 2n,\\
M_2(O_1\mathbb{G}_{m\times n}) &= 54mn - 13m - 13n + 4,\\
R_\propto(O_1\mathbb{G}_{m\times n}) &= 2\left(3^{2\propto+1}\right)mn\\
&\quad + \left(2^{2\propto+1} + 2^{\propto+2}3^\propto - 5\mathbb{G}3^{2\propto}\right)m\\
&\quad + \left(2^{2\propto+1} + 2^{\propto+2}3^\propto - 5\mathbb{G}3^{2\propto}\right)n\\
&\quad + 4\left(2^{2\propto} + 3^{2\propto} - 2\mathbb{G}2^\propto \mathbb{G}3^\propto\right),\\
I(O_1\mathbb{G}_{m\times n}) &= 9mn - 0\cdot 7m - 0\cdot 7n + 0\cdot 4.
\end{aligned}
\tag{13}
$$

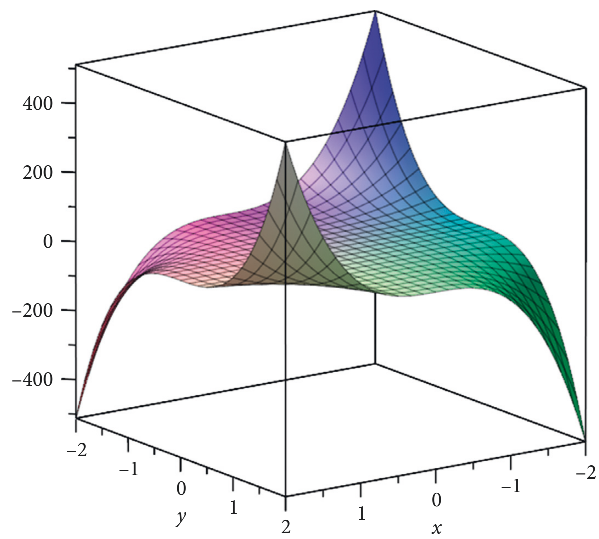

Figure 2: The graph of the M-polynomial of $O_1G_{m\times n}$ Nanosheet.

Proof. Let $M(O_1G_{m\times n},\alpha,\beta)=2(m+n+2)\alpha^2\beta^2+4(m+n-2)\alpha^2\beta^3+(6mn-5m-5n+4)\alpha^3\beta^3$ be M polynomial of Octa- Graphyne Nanosheet then,

$$
\begin{aligned}
\text{(i) } M_1(O_1G_{m\times n}) &= (D_\alpha+D_\beta)M(O_1G,\alpha,\beta)|_{\alpha=\beta=1},\\
D_\alpha M(O_1G_{m\times n},\alpha,\beta) &= 4(m+n+2)\alpha^2\beta^2+8(m+n-2)\alpha^2\beta^3+3(6mn-5m-5n+4)\alpha^3\beta^3,\\
D_\beta M(O_1G_{m\times n},\alpha,\beta) &= 4(m+n+2)\alpha^2\beta^2+12(m+n-2)\alpha^2\beta^3+3(6mn-5m-5n+4)\alpha^3\beta^3,\\
(D_\alpha+D_\beta)M(O_1G_{m\times n},\alpha,\beta) &= 8(m+n+2)\alpha^2\beta^2+20(m+n-2)\alpha^2\beta^3+6(6mn-5m-5n+4)\alpha^3\beta^3,\\
(D_\alpha+D_\beta)M(O_1G,\alpha,\beta)|_{\alpha=\beta=1} &= 8(m+n+2)+20(m+n-2)+6(6mn-5m-5n+4)\\
&= 36mn-2m-2n.\\
\text{(ii) } M_2(O_1G_{m\times n}) &= (D_\alpha D_\beta)M(O_1G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1},\\
(D_\alpha D_\beta)M(O_1G_{m\times n},\alpha,\beta) &= 8(m+n+2)\alpha^2\beta^2+24(m+n-2)\alpha^2\beta^3+9(6mn-5m-5n+4)\alpha^3\beta^3,\\
(D_\alpha D_\beta)M(O_1G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1} &= 8(m+n+2)+24(m+n-2)+9(6mn-5m-5n+4)\\
&= 54mn-13m-13n+4.\\
\text{(iii) } R_\propto(O_1G_{m\times n}) &= D_\alpha^\propto D_\beta^\propto M(O_1G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1},\\
D_\alpha^\propto D_\beta^\propto M(O_1G_{m\times n},\alpha,\beta) &= 2G2^\propto 2^\beta(m+n+2)\alpha^2\beta^2+4G2^\propto 3^\propto(m+n-2)\alpha^2\beta^3+3^\propto 3^\propto(6mn-2^\propto G5m-5n+4)\alpha^3\beta^3,\\
D_\alpha^\propto D_\beta^\propto M(O_1G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1} &= 2G2^\propto 2^\propto(m+n+2)\alpha^2\beta^2\\
&\quad +4G2^\propto 3^\propto(m+n-2)\alpha^2\beta^3+3^\propto 3^\propto(6mn-5m-5n+4)\alpha^3\beta^3\\
&= 2\left(3^{2\propto+1}\right)mn+\left(2^{2\propto+1}+2^{\propto+2}3^\propto-5G3^{2\propto}\right)m+\left(2^{2\propto+1}+2^{\propto+2}3^\propto-5G3^{2\propto}\right)n+4\left(2^{2\propto}+3^{2\propto}\right).
\end{aligned}
$$

$$(\text{iv})\ I(O_1G_{m\times n}) = S_\alpha J\left(D_\alpha D_\beta M(O_1G_{m\times n}, \alpha)\right)|_{\alpha=\beta=1},$$

$$J\left(D_\alpha D_\beta M(O_1G_{m\times n}, \alpha\right) = 8(m+n+2)\alpha^4 + 24(m+n-2)\alpha^5 + 9(6mn-5m-5n+4)\alpha^6,$$

$$S_\alpha J\left(D_\alpha D_\beta M\left(O_1G_{m\times n,\alpha}\right)\right) = 2(m+n+2)\alpha^4 + \frac{24}{5}(m+n-2)\alpha^5 + \frac{3}{2}(6mn-5m-5n+4)\alpha^6,$$

$$S_\alpha J\left(D_\alpha D_\beta M(O_1G_{m\times n}, \alpha)\right)|_{\alpha=\beta=1} = 2(m+n+2) + \frac{24}{5}(m+n-2) + \frac{3}{2}(6mn-5m-5n+4),$$

$$I(O_1G_{m\times n}) = 9mn - 0\cdot 7m - 0\cdot 7n + 0\cdot 4. \tag{14}$$

□

Theorem 3. *Let $O_1G_{m\times n}$ be the graph of Octa- Graphyne Nanosheet Let*

(i) $M(O_1G_{m\times n}) = 2(m+n+2)\alpha^2\beta^2 + 4(m+n-2)\alpha^2\beta^3 + (6mn-5m-5n+4)\alpha^3\beta^3$ be M polynomial of $O_1G_{m\times n}$ then we have,

(ii) $H(O_1G_{m\times n}) = 2mn + 0\cdot 933m + 0\cdot 933n + 0\cdot 133$

(iii) $F(O_1G_{m\times n}) = 108mn - 22m - 22n$

(iv) $SDD(O_1G_{m\times n}) = 12mn + 2\cdot 67m + 2\cdot 67n - 1\cdot 33$

Proof. Let $M(O_1G_{m\times n}, \alpha, \beta) = 2(m+n+2)\alpha^2\beta^2 + 4(m+n-2)\alpha^2\beta^3 + (6mn-5m-5n+4)\alpha^3\beta^3$ be M polynomial of Octa- Graphyne Nanosheet then ,

$$(\text{i})\ H(O_1G_{m\times n}) = 2S_\alpha J(M(O_1G_{m\times n}, \alpha, \beta))|_{\alpha=\beta=1},$$

$$J(M(O_1G_{m\times n}, \alpha, \beta)) = 2(m+n+2)\alpha^4 + 4(m+n-2)\alpha^5 + (6mn-5m-5n+4)\alpha^6,$$

$$2S_\alpha J(M(O_1G_{m\times n}, \alpha, \beta)) = (2mn+m+n+4)\alpha^4 + \frac{8}{5}(4mn+6m+6n)\alpha^5 + \frac{1}{3}mn\alpha^6,$$

$$H(O_1G_{m\times n}) = 2mn + 0\cdot 933m + 0\cdot 933n + 0\cdot 133.$$

$$(\text{ii})\ F(O_1G_{m\times n}) = \left(D_\alpha^2 + D_\beta^2\right)\left(M(O_1G_{m\times n}, \alpha, \beta)\right|_{\alpha=\beta=1},$$

$$D_\alpha^2(M(O_1G_{m\times n}, \alpha, \beta)) = 8(m+n+2)\alpha^2\beta^2 + 16(m+n-2)\alpha^2\beta^3 + 9(6mn-5m-5n+4)\alpha^3\beta^3,$$

$$D_\beta^2(M(O_1G_{m\times n}, \alpha, \beta)) = 8(m+n+2)\alpha^2\beta^2 + 36(m+n-2)\alpha^2\beta^3 + 9(6mn-5m-5n+4)\alpha^3\beta^3,$$

$$F(O_1G_{m\times n}) = 108mn - 22m - 22n.$$

$$(\text{iii})\ SDD(O_1G_{m\times n}) = \left(D_\alpha S_\beta + S_\alpha D_\beta\right)\left(M(O_1G_{m\times n}, \alpha, \beta)\right|_{\alpha=\beta=1}, \tag{15}$$

$$D_\alpha S_\beta(M(O_1G_{m\times n}, \alpha, \beta) = 2(m+n+2)\alpha^2\beta^2 + \frac{8}{3}(m+n-2)\alpha^2\beta^3 + \frac{1}{3}(6mn-5m-5n+4)\alpha^3\beta^3,$$

$$D_\alpha S_\beta(M(O_1G_{m\times n}, \alpha, \beta))|_{\alpha=\beta=1} = 2(m+n+2) + \frac{8}{3}(m+n-2) + \frac{1}{3}(6mn-5m-5n+4),$$

$$S_\alpha D_\beta\left(M(O_1G_{m\times n}, \alpha, \beta)\right) = 2(m+n+2)\alpha^2\beta^2 + 6(m+n-2)\alpha^2\beta^3 + (6mn-5m-5n+4)\alpha^3\beta^3,$$

$$S_\alpha D_\beta(M(O_1G_{m\times n}, \alpha, \beta))|_{\alpha=\beta=1} = 2(m+n+2) + 6(m+n-2) + (6mn-5m-5n+4),$$

$$SDD(O_1G_{m\times n}) = 12mn + 2\cdot 67m + 2\cdot 67n - 1.33.$$

2.2. Let $O_2G_{m\times n}$ be the Octa-Graypdiyne Nanosheet with m)hexadecagones with each row and n hexadecagones in each column. From a graphical representation of the Octa-Grayphdine Nano sheet we can observe that each row of $O_2G_{m\times n}$ consists of m hexadecagones and $m+1$ squares. The graph of Octa-Graypdiyne Nanosheet $O_2G_{m\times n}$ is given in Figure 3. Each successive column of $O_2G_{m\times n}$also consists n hexadecagones and $n+1$ squares. Degrees of end vertices for all edges are used to calculate degree-based topological indices. $O_2G_{m\times n}$ has $8mn+6m+6n+4$ number of vertices, whereas number of edges for $O_2G_{m\times n}$ is $10mn+7m+7n+4$. By observing graph of $O_2G_{m\times n}$ one can find that that all vertices of $O_2G_{m\times n}$has either degree 2 or 3. Depending on the degree of vertices edge partition of $O_2G_{m\times n}$is given in Table 3.

Partition of edge sets is defined as,

$$E_{(i,j)} = \{uv \in E(O_2G_{m\times n}):\ d_u = i\, an\, d\, d_v = j \quad 2 \le i \le j \le 3\}. \tag{16}$$

Based on edge partition given in Table 3, we can find M polynomial of Octa-Grayphdine Nanosheet $O_2G_{m\times n}$, Whereas Graphical representation of M polynomial of $O_2G_{m\times n}$ is given in Figure 4. □

Theorem 4. *Let $O_2G_{m\times n}$ be the graph of Octa-Grayphdine Nanosheet, M polynomial for $O_2G_{m\times n}$ is given by,*

$$M(O_2G_{m\times n}) = (2mn+m+n+4)\alpha^2\beta^2 + (4mn+6m+6n)\alpha^2\beta^3 + 4mn\alpha^3\beta^3. \tag{17}$$

Proof. Let $O_2G_{m\times n}$ be the graph of Octa-Grayphdine Nanosheet, Using Table 3, combined with *equation* 9, we get

$$\begin{aligned} M(O_2G_{m\times n}) &= \sum_{i\le j} e_{ij}(O_2G_{m\times n})\alpha^i\beta^j \\ &= \sum e_{22}(O_2G_{m\times n})\alpha^2\beta^2 + \sum e_{23}(O_2G_{m\times n})\alpha^2\beta^3 + \sum e_{33}(O_2G_{m\times n})\alpha^3\beta^3 \\ &= (2mn+m+n+4)\alpha^2\beta^2 + (4mn+6m+6n)\alpha^2\beta^3 + 4mn\alpha^3\beta^3. \end{aligned} \tag{18}$$

□

Theorem 5. *Let $O_2G_{m\times n}$ be the graph of Octa-Grayphdine Nanosheet Let*

$M(O_2G_{m\times n}) = (2mn+m+n+4)\alpha^2\beta^2 + (4mn+6m+6n)\alpha^2\beta^3 + 4mn\alpha^3\beta^3$ be M polynomial of $O_2G_{m\times n}$ then we have

(i) $M_1(O_2G_{m\times n}) = 52mn+34m+34n+16$.

(ii) $M_2(O_2G_{m\times n}) = 68mn+40m+40n+16$.

(iii) $R_\propto(O_2G_{m\times n}) = 2(4^\propto + 2G6^\propto + 2G9^\propto)mn + (4^\propto + 6^{\propto+1})m + (4^\propto + 6^{\propto+1})n + 4^{\propto+1}$.

(iv) $I(O_2G_{m\times n}) = 12\cdot 8mn + 8\cdot 2m + 8\cdot 2n + 4$.

Proof. Let $M(O_2G_{m\times n},\alpha,\beta) = (2mn+m+n+4)\alpha^2\beta^2 + (4mn+6m+6n)\alpha^2\beta^3 + 4mn\alpha^3\beta^3$ be M polynomial of Octa-Grayphdine Nanosheet then,

$$\begin{aligned} \text{(i) } M_1(O_2G_{m\times n}) &= (D_\alpha + D_\beta)M(O_2G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1}, \\ D_\alpha M(O_2G_{m\times n},\alpha,\beta) &= 2(2mn+m+n+4)\alpha^2\beta^2 + 2(4mn+6m+6n)\alpha^2\beta^3 \\ &\quad + 12mn\alpha^3\beta^3, \\ D_\beta M(O_2G_{m\times n},\alpha,\beta) &= 2(2mn+m+n+4)\alpha^2\beta^2 + 3(4mn+6m+6n)\alpha^2\beta^3 + 12mn\alpha^3\beta^3, \\ (D_\alpha + D_\beta)M(O_2G_{m\times n},\alpha,\beta) &= 4(2mn+m+n+4)\alpha^2\beta^2 + 5(4mn+6m+6n)\alpha^2\beta^3 \\ &\quad + 24mn\alpha^3\beta^3 \\ (D_\alpha + D_\beta)M(O_2G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1} &= 4(2mn+m+n+4) + 5(4mn+6m+6n) + 24mn \\ &= 52mn+34m+34n+16.\ (ii) M_2(O_2G_{m\times n}) \\ (D_\alpha D_\beta)M(O_2G_{m\times n},\alpha,\beta) &= 4(2mn+m+n+4)\alpha^2\beta^2 + 6(4mn+6m+6n)\alpha^2\beta^3 + 36mn\alpha^3\beta^3, \\ (D_\alpha D_\beta)M(O_2G_{m\times n},\alpha,\beta)|_{\alpha=\beta=1} &= 4(2mn+m+n+4) + 6(4mn+6m+6n) + 36mn \\ &= 68mn+40m+40n+16. \end{aligned}$$

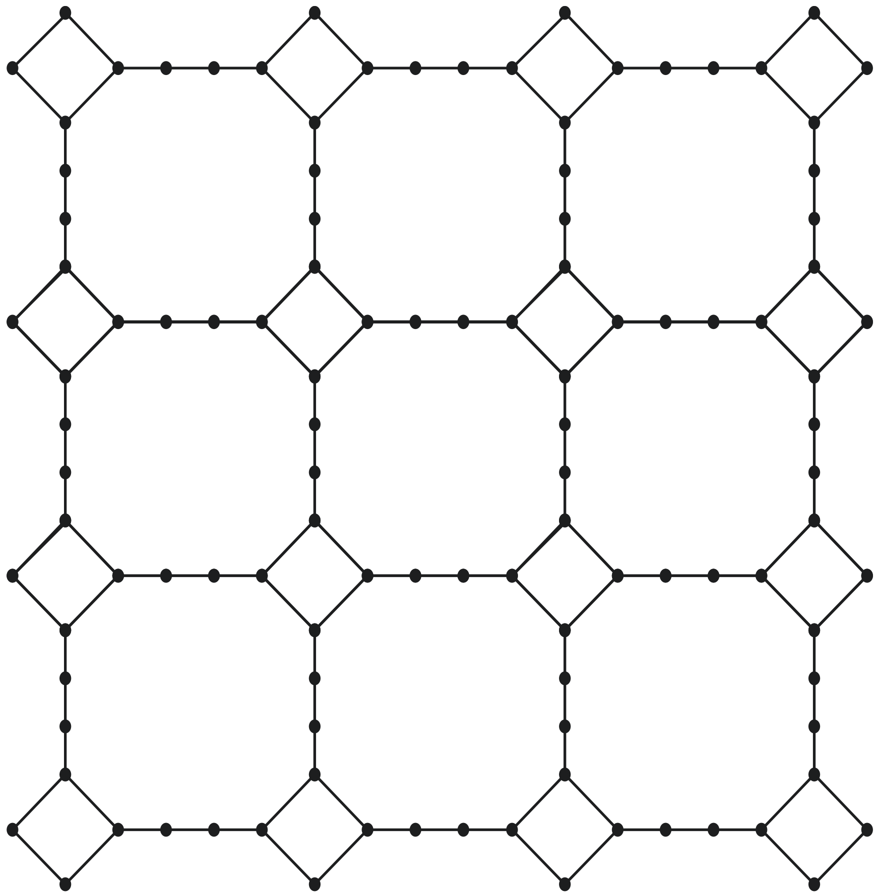

FIGURE 3: Graph of octa-graphdine nanosheet $O_2G_{3\times 3}$.

TABLE 3: Edge Partition of $O_2G_{m\times n}$Nano sheet.

Edge types	(d_u, d_v)	Frequency
$E_{(2,2)}$	$(2, 2)$	$2mn + m + n + 4$
$E_{(2,3)}$	$(2, 3)$	$4mn + 6m + 6n$
$E_{(3,3)}$	$(3, 3)$	$4mn$

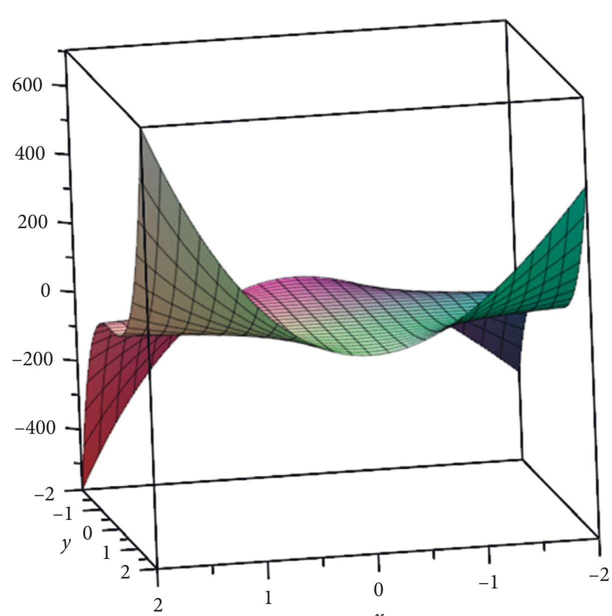

FIGURE 4: The graph of M-polynomial of $O_2G_{m\times n}$ Nanosheet.

$$
\begin{aligned}
&(iii)\ R_{\propto}\left(O_2\mathbb{G}_{m\times n}\right) = D_{\alpha}^{\propto} D_{\beta}^{\propto} M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)|_{\alpha=\beta=1},\\
&D_{\alpha}^{\propto} D_{\beta}^{\propto} M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right) = 2^{\propto}2^{\propto}(2mn+m+n+4)\alpha^2\beta^2\\
&\qquad + 2^{\propto}3^{\propto}(4mn+6m+6n)\alpha^2\beta^3 + 4\times 3^{\propto}3^{\propto}mn\alpha^3\beta^3,\\
&D_{\beta}^{\propto}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)|_{\alpha=\beta=1} = 2^{\propto}2^{\propto}(2mn+m+n+4) + 2^{\propto}3^{\propto}(4mn+6m+6n)\\
&\qquad + 4\times 3^{\propto}3^{\propto}mn,\\
&\qquad = 2\left(4^{\propto} + 2\times 6^{\propto} + 2\times 9^{\propto}\right)mn + \left(4^{\propto} + 6^{\propto+1}\right)m + \left(4^{\propto} + 6^{\propto+1}\right)n + 4^{\propto+1}.\\
&I\left(O_2\mathbb{G}_{m\times n}\right) = S_{\alpha}J\left(D_{\alpha}D_{\beta}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)\right)|_{\alpha=\beta=1},\\
&J\left(D_{\alpha}D_{\beta}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha\right)\right)\right) = 4(2mn+m+n+4)\alpha^4 + 6(4mn+6m+6n)\alpha^5 + 36mn\alpha^6,\\
&S_{\alpha}J\left(D_{\alpha}D_{\beta}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)\right) = (2mn+m+n+4)\alpha^4 + \frac{6}{5}(4mn+6m+6n)\alpha^5 + 6mn\alpha^6,\\
&S_{\alpha}J\left(D_{\alpha}D_{\beta}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)\right)|_{\alpha=1} = (2mn+m+n+4) + \frac{6}{5}(4mn+6m+6n) + 6mn,\\
&I\left(O_2\mathbb{G}_{m\times n}\right) = \frac{64mn+41m+41n}{5} + 4.
\end{aligned}
\tag{19}
$$

□

Theorem 6. *Let $O_2\mathbb{G}_{m\times n}$ be the graph of Octa-Grayphdine Nanosheet Let $M\left(O_2\mathbb{G}_{m\times n}\right) = (2mn+m+n+4)\alpha^2\beta^2 + (4mn+6m+6n)\alpha^2\beta^3 + 4mn\alpha^3\beta^3$ be M polynomial of $O_2\mathbb{G}_{m\times n}$ then we have*

(i) $H\left(O_2\mathbb{G}_{m\times n}\right) = 3.933mn + 2.9m + 2.9n + 2$.

(ii) $F\left(O_2\mathbb{G}_{m\times n}\right) = 140mn + 86m + 86n + 32$.

(iii) $SDD\left(O_2\mathbb{G}_{m\times n}\right) = 20.67mn + 15m + 15n + 8$.

Proof. Let $M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right) = (2mn+m+n+4)\alpha^2\beta^2 + (4mn+6m+6n)\alpha^2\beta^3 + 4mn\alpha^3\beta^3$ be M polynomial of Octa-Grayphdine Nanosheet then,

$$
\begin{aligned}
&(i)\ H\left(O_2\mathbb{G}_{m\times n}\right) = 2S_{\alpha}J\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)|_{\alpha=\beta=1},\\
&J\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right) = (2mn+m+n+4)\alpha^4 + (4mn+6m+6n)\alpha^5 + 4mn\alpha^6,\\
&S_{\alpha}J\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right) = \frac{1}{4}(2mn+m+n+4)\alpha^4 + \frac{1}{5}(4mn+6m+6n)\alpha^5 + \frac{2}{3}mn\alpha^6,\\
&H\left(O_2\mathbb{G}_{m\times n}\right) = 3\cdot 933mn + 2\cdot 9m + 2\cdot 9n + 2.\\
&(ii)\ F\left(O_2\mathbb{G}_{m\times n}\right) = \left(D_{\alpha}^2 + D_{\beta}^2\right)\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)|_{\alpha=\beta=1},\\
&D_{\alpha}^2\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right) = 4(2mn+m+n+4)\alpha^2\beta^2 + 4(4mn+6m+6n)\alpha^2\beta^3 + 36mn\alpha^3\beta^3,\\
&D_{\beta}^2\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right) = 4(2mn+m+n+4)\alpha^2\beta^2\\
&\qquad + 9(4mn+6m+6n)\alpha^2\beta^3 + 36mn\alpha^3\beta^3,\\
&F\left(O_2\mathbb{G}_{m\times n}\right) = 140mn + 86m + 86n + 32.\\
&(iii)\ SDD\left(O_2\mathbb{G}_{m\times n}\right) = \left(D_{\alpha}S_{\beta} + S_{\alpha}D_{\beta}\right)\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)|_{\alpha=\beta=1},\right.\\
&D_{\alpha}S_{\beta}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)|_{\alpha=\beta=1} = (2mn+m+n+4) + \frac{2}{3}(4mn+6m+6n) + 4mn,\\
&S_{\alpha}D_{\beta}\left(M\left(O_2\mathbb{G}_{m\times n}, \alpha, \beta\right)\right)|_{\alpha=\beta=1} = (2mn+m+n+4) + \frac{3}{2}(4mn+6m+6n) + 4mn,\\
&SDD\left(O_2\mathbb{G}_{m\times n}\right) = 20.67mn + 15m + 15n + 8.
\end{aligned}
\tag{20}
$$

□

3. Conclusion

Various forms of degree-based topological indices, such as Randic index variations of Zagreb indices, atom bond connectivity index, Harmonicindex, and Forgotton index and Symmitric Division index are computed using M polynomia for nanosheets based on octa grephene. These discoveries are particularly useful in the theoretical research of nanosheet physical properties, chemical reactivity, and biological activities. The topological indices calculated in this study can be used in quantitative structure activity relations and quantitative structure property relations of nanosheets, which can assist further comprehend the nanosheet's physiochemical properties.

Data Availability

All data is included in Article. There is not hidden data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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