Hindawi
Journal of Mathematics
Volume 2022, Article ID 2635564, 8 pages
https://doi.org/10.1155/2022/2635564

Research Article

Assignment Computations Based on $C_{\exp}$ Average in Various Ladder Graphs

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Received 20 February 2022; Revised 29 March 2022; Accepted 7 April 2022; Published 11 May 2022

Academic Editor: M. T. Rahim

This study introduces the $C_{\exp}$ average assignments and investigates its properties using various ladder graphs. The ladder graphs can be found in every communication networks. Ladder networks are increasingly being used in everyday life for monitoring and environmental applications such as domestic, military, surveillance, industrial, medical applications, and traffic management. These datasets are afflicted by the average representation of the graph structure. It aids in the visualisation and comprehension of data analysis. The $C_{\exp}$ labeling is used in sensor networks, adhoc networks, and other applications. It also efficiently creates a communication network after using noise reduction methods to remove salt and pepper noise.

1. Introduction

A graph labeling is the assignment of labels, conventionally indicated by integers, to edges and/or vertices of a graph in the mathematical domain of graph theory. The concept of labeling may be applied to many areas of graph theory, for example, in automata theory and formal language theory. We use [1–5] for notations and nomenclature. We recommend [6] for a thorough examination of graph labeling. Let P_n be a path on n nodes denoted by $u_{1,\mu}$, where $1 \le \mu \le n$, and with $n-1$ lines denoted by $e_{1,\delta}$, where $1 \le \delta \le n-1$, where e_μ is the line joining the vertices $u_{1,\mu}$ and $u_{1,\mu+1}$. On each edge e_δ, erect a ladder with $n-(\mu-1)$ steps including the edge e_μ, for $\mu = 1, 2, 3, \ldots, n-1$. The resulting graph is called the one-sided step graph, and it is denoted by ST_n. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices, respectively. Then, $G_1 \times G_2$ is the Cartesian product of two graphs. A ladder graph L_n is the graph $P_2 \times P_n$. The graph $G^\circ S_m$ is obtained from G by attaching m pendant vertices to each vertex of G. The triangular ladder TL_n, for $n \ge 2$, is a graph obtained from two paths by $u_1, u_2, \ldots u_n$ and $v_1, v_2, \ldots v_n$ by adding the edges $u_\mu v_\mu$, $1 \le \mu \le n$ and $u_\mu v_{\mu+1}$, $1 \le \mu \le n-1$. The slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \ldots u_n$ and $v_1, v_2, \ldots v_n$ by joining each v_μ, with $u_{\mu+1}, 1 \le \mu \le n-1$. The graph D_n^* having the vertices $\{a_{\mu,\delta}\colon 1 \le \mu \le n, \delta = 1, 2, 3, 4\}$ and its edge set is $\{a_{\mu,1}a_{\mu+1,1}, a_{\mu,3}a_{\mu+1,3}\colon 1 \le \mu \le n-1\} \cup \{a_{\mu,1}a_{\mu,2}, a_{\mu,2}a_{\mu,3}, a_{\mu,3}\ a_{\mu,4}, a_{\mu,4}a_{\mu,1}\colon 1 \le \mu \le n\}$.

2. Literature Survey

In [7], the authors talked about the F-root square mean labeling for line graph of the path, cycle, star, $P_n{}^\circ S_1, P_n{}^\circ S_2$, $[P_n; S_1]$, $S(P_n{}^\circ S_1)$, ladder, slanting ladder, the crown graph $C_n{}^\circ S_1$, and the arbitrary subdivision of S_3. The authors in [8] discussed $(1,1,0)$ F-face mean labeling some planar graphs and, in [9], face labelings of type $(1,1,1)$ for generalized prism.

Alanazi et al. explained the classical meanness of the graphs, the one-sided step graph ST_n, double-sided step graph $2ST_{2n}$, planar grid $P_m \times P_n$, ladder graph L_n, graph $L_n \circ S_m$ for $m \le 2$, triangular ladder graph TL_n, graph $TL_n \circ S_m$ for $m \le 2$, graph $SL_n \circ S_m$ for $m \le 2$, slanting ladder graph SL_n, graph $SL_n \circ S_m$ for $m \le 2$, graph D_n^*, diamond ladder graph Dl_n, and latitude ladder graph LL_n in [10–12]. Dafik slamin et al. highlighted the super (a, d)-edge-antimagic total properties of triangular book and diamond ladder graphs in [13]. Moussa and Badr discussed the odd gracefulness of few ladder graphs and proved that ladder and subdivision of ladder graphs with pendent edges are odd graceful in [14]. In [15], the authors emphasized the significance of exponential mean labeling of graphs, and they examined the exponential mean labeling of some graphs obtained from duplicating operations. Inspired by such tremendous works of researchers in the region of graph assignments in [16–25], we defined $C_{\exp}$ average assignment of graphs. A $C_{\exp}$ average of two integers is not always an integer. Consequently, $C_{\exp}$ average assignment must be an integer; we may get ceiling function by considering the integral part. In this study, our conversation and attempt is to examine the various assignment techniques on $C_{\exp}$ average assignment for few ladder graphs.

3. Methodology

A function Ψ is known as an $C_{\exp}$ average assignment of G if $\Psi\colon V(G) \longrightarrow N - \{q+2, q+3, \ldots, \infty\}$ is one to one and the instigated bijective function $\Psi^*\colon E(G) \longrightarrow N - \{1, q+2, q+3, \ldots, \infty\}$ characterized by

$$\Psi^*(uv) = \lceil \frac{1}{e} \left(\frac{X(v)}{X(u)} \right)^{1/Y} \rceil, \tag{1}$$

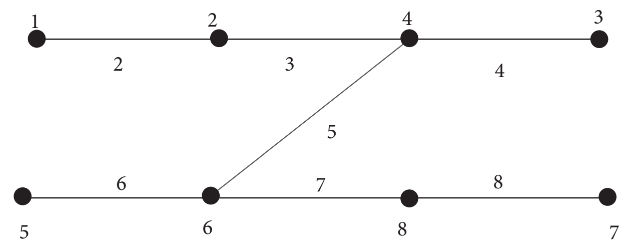

Figure 1: An $C_{\exp}$ average assignment of the graph, $SL_2 \circ S_1$.

where $X(w) = \Psi(w)^{\Psi(w)}$, $Y = \Psi(v) - \Psi(u)$, q is the number of edges, and N is the set of all natural numbers. A graph that concedes an $C_{\exp}$ average assignment is known as a $C_{\exp}$ average assignment graph.

Figure 1 shows the $C_{\exp}$ average assignment of the graph $SL_2 \circ S_1$.

4. Main Results

Theorem 1. *The one-sided step graph ST_n is an $C_{\exp}$ average assignment graph, for $n \ge 2$.*

Proof. Make the vertex assignment $\Psi\colon V(ST_n) \longrightarrow N - \{n^2+n, n^2+n+1, \ldots, \infty\}$, $\Psi(u_{1,\mu}) = n^2 - 1 + \mu$, for $2 \le \mu \le n$, and $\Psi(u_{\lambda,\mu}) = (1+n-\lambda)^2 + \mu - 1$, for $2 \le \lambda \le n$ and $1 \le \mu \le n + 2 - \lambda$.

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned}
&\Psi^*\left(u_{\lambda,\mu}u_{\lambda+1,\mu}\right) = -2n\lambda + n - \lambda + \lambda^2 + n^2 + \mu, \text{ for } n-1 \ge \lambda \ge 1 \text{ and } 1 - \lambda + n \ge \mu \ge 1,\\
&\Psi^*\left(u_{1,\mu}u_{1,\mu+1}\right) = n^2 + \mu, \text{ for } 1 \le \mu \le n-1, \text{ and}\\
&\Psi^*\left(u_{\lambda,\mu}u_{\lambda,\mu+1}\right) = (1+n-\lambda)^2 + \mu, \text{ for } 2 \le \lambda \le n \quad \text{and } 1 \le \mu \le n+1-\lambda.
\end{aligned} \tag{2}$$

As a result, for $n \ge 2$, Ψ is an $C_{\exp}$ average assignment and the one-sided step graph ST_n is an $C_{\exp}$ average assignment graph. □

Theorem 2. *The graph $P_m \times P_n$ is an $C_{\exp}$ average assignment graph, for $m \le 4$ and $n \ge 2$.*

Proof

Case (i): $m = 2$.

Make the vertex assignment, $\Psi\colon V(P_2 \times P_n) \longrightarrow N - \{3n, 3n+1, \ldots, \infty\}$.

$\Psi(v_{\lambda\mu}) = \lambda - 3 + 3\mu$, for $1 \le \lambda \le 2$ and $1 \le \mu \le n$.

Consequently, the instigated edge assignment Ψ^* is acquired as follows.

$\Psi^*(v_{\lambda\mu} \quad v_{\lambda(\mu+1)}) = \lambda - 1 + 3\mu$, for $1 \le \lambda \le 2$ and $1 \le \mu \le n - 1$.

$\Psi^*(v_{1\mu}v_{2\mu}) = -13\mu$, for $1 \le \mu \le n$.

Case (ii): $m = 3$.

Make the vertex assignment, $\Psi\colon V(P_3 \times P_n) \longrightarrow N - \{5n-1, 5n+1, \ldots, \infty\}$.

$\Psi(v_{\lambda\mu}) = \lambda - 5 + 5\mu$, for $1 \le \lambda \le 3$ and $2 \le \mu \le n$.

$$\Psi(v_{\lambda 1}) = \begin{cases} \lambda, & 1 \le \lambda \le 2, \\ 4, & \lambda = 3. \end{cases}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$\Psi^*(v_{\lambda\mu}v_{\lambda(\mu+1)}) = \lambda - 2 + 5\mu$, for $1 \le \lambda \le 3$ and $2 \le \mu \le n - 1$

$\Psi^*(v_{\lambda\mu}v_{(\lambda+1)\mu}) = \lambda - 4 + 5\mu$, for $1 \le \lambda \le 2$ and $2 \le \mu \le n$

$\Psi^*(v_{\lambda 1}v_{\lambda 2}) = \lambda + 3$, for $1 \le \lambda \le 3$,

$\Psi^*(v_{\lambda 1}v_{(\lambda+1)1}) = 1 + \lambda$, for $1 \le \lambda \le 2$,

Case (iii): $m = 4$ and $n \ge 3$.

Make the vertex assignment:

Ψ: $V(P_4 \times P_n) \longrightarrow N - \{7n-2, 7n-1, \ldots, \infty\}$

$\Psi(v_{\lambda\mu}) = \lambda - 7 + 7\mu$, for $1 \le \lambda \le 4$ and $3 \le \mu \le n$

$\Psi(v_{\lambda 2}) = \lambda + 7$, for $1 \le \lambda \le 4$

$$\Psi(v_{\lambda 1}) = \begin{cases} \lambda, & 1 \le \lambda \le 2 \\ \lambda + 1, & 3 \le \lambda \le 4 \end{cases}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned}
&\Psi^*\left(v_{\lambda\mu}v_{(\lambda+1)\mu}\right) = \lambda - 6 + 7\mu, \text{ for } 1 \le \lambda \le 3 \text{ and } 3 \le \mu \le n,\\
&\Psi^*\left(v_{\lambda 2}v_{(\lambda+1)2}\right) = 8 + \lambda, \text{ for } 1 \le \lambda \le 3,\\
&\Psi^*\left(v_{\lambda 1}v_{(\lambda+1)1}\right) = \begin{cases} \lambda + 1, & 1 \le \lambda \le 2, \\ 5, & \lambda = 3, \end{cases}\\
&\Psi^*\left(v_{\lambda\mu}v_{\lambda(\mu+1)}\right) = \lambda - 3 + 7\mu \text{ for } 2 \le \lambda \le 4 \text{ and } 1 \le \mu \le n-1, \text{ and}\\
&\Psi^*\left(v_{1\mu}v_{1(\mu+1)}\right) = \begin{cases} 8\mu - 4, & 1 \le \mu \le 2, \\ 7\mu - 2, & 3 \le \mu \le n-1. \end{cases}
\end{aligned} \tag{3}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the graph $P_m \times P_n$ is an $C_{\exp}$ average assignment graph, for $m \le 4$. □

Corollary 1. *Every Ladder graph $L_n = P_2 \times P_n$ is an $C_{\exp}$ average assignment graph.*

Theorem 3. *The graph $L_n \circ S_m$ is an $C_{\exp}$ average assignment graph, for $n \ge 2$ and $m \le 2$.*

Proof

Case (i): $m = 1$.

Make the vertex assignment: Ψ: $V(L_n \circ S_1) \longrightarrow$ N$-\{5n, 5n+1, \ldots, \infty\}$,

$$\begin{aligned}
&\Psi\left(x_1^{(\lambda)}\right) = \begin{cases} 2, & \lambda = 1, \\ -1 + 5\lambda, & 2 \le \lambda \le n, \end{cases}\\
&\Psi\left(u_1^{(\lambda)}\right) = -4 + 5\lambda, \text{ for } 1 \le \lambda \le n,\\
&\Psi(v_\lambda) = \begin{cases} 4, & \lambda = 1, \\ -2 + 5\lambda, & 2 \le \lambda \le n \text{ and,} \end{cases}\\
&\Psi(u_\lambda) = \begin{cases} 3, & \lambda = 1, \\ -3 + 5\lambda, & 2 \le \lambda \le n. \end{cases}
\end{aligned} \tag{4}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned}
&\Psi^*\left(v_\lambda x_1^{(\lambda)}\right) = \begin{cases} 3, & \lambda = 1, \\ -1 + 5\lambda, & 2 \le \lambda \le n. \end{cases}\\
&\Psi^*\left(u_\lambda w_1^{(\lambda)}\right) = 5\lambda - 3, \text{ for } 1 \le \lambda \le n,\\
&\Psi^*(u_\lambda v_\lambda) = \begin{cases} 4, & \lambda = 1, \\ -2 + 5\lambda, & 2 \le \lambda \le n, \end{cases}\\
&\Psi^*(v_\lambda v_{\lambda+1}) = 1 + 5\lambda, \text{ for } 1 \le \lambda \le n-1 \text{ and}\\
&\Psi^*(u_\lambda u_{\lambda+1}) = 5\lambda, \text{ for } 1 \le \lambda \le n-1.
\end{aligned} \tag{5}$$

Case (ii): $m = 2$.

Make the vertex assignment: Ψ: $V(L_n \circ S_2) \longrightarrow$ N$-\{7n, 7n+1, \ldots, \infty\}$,

$$\begin{aligned}
&\Psi\left(x_2^{(\lambda)}\right) = \begin{cases} 8, & \lambda = 1, \\ -5 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -2 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi\left(x_1^{(\lambda)}\right) = \begin{cases} 2\lambda + 3, & 1 \le \lambda \le 2, \\ -6 + 7\lambda, & 3 \le \lambda \le n, \lambda = 2k+2, k \in \mathbb{N}, \\ -3 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k+1\ k \in \mathbb{N}, \end{cases}\\
&\Psi\left(w_2^{(\lambda)}\right) = \begin{cases} 2, & \lambda = 1, \\ -1 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -4 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi\left(w_1^{(\lambda)}\right) = \begin{cases} 1, & \lambda = 1, \\ -3 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -6 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi(v_\lambda) = \begin{cases} 4, & \lambda = 1, \\ -4 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -1 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi(u_\lambda) = \begin{cases} 3, & \lambda = 1, \\ -2 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -5 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}. \end{cases}
\end{aligned} \tag{6}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned}
&\Psi^*\left(v_\lambda x_2^{(\lambda)}\right) = \begin{cases} 7\lambda - 4, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -1 + 7\lambda, & 1 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi^*\left(v_\lambda x_1^{(\lambda)}\right) = \begin{cases} 7\lambda - 5, & 1 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -2 + 7\lambda, & 4 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi^*\left(u_\lambda w_2^{(\lambda)}\right) = \begin{cases} 7\lambda - 1, & 1 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -4 + 7\lambda, & 1 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi^*\left(u_\lambda w_1^{(\lambda)}\right) = \begin{cases} 2, & \lambda = 1, \\ -2 + 7\lambda, & 2 \le \lambda \le n, \lambda = 2k, k \in \mathbb{N}, \\ -5 + 7\lambda, & 1 \le \lambda \le n, \lambda = 2k+1, k \in \mathbb{N}, \end{cases}\\
&\Psi^*(u_\lambda v_\lambda) = 7\lambda - 3, \text{ for } 1 \le \lambda \le n,\\
&\Psi^*(v_\lambda v_{\lambda+1}) = 7\lambda + 1, \text{ for } 1 \le \lambda \le n-1, \text{ and}\\
&\Psi^*(u_\lambda u_{\lambda+1}) = \begin{cases} 8, & \lambda = 1, \\ 7\lambda, & 2 \le \lambda \le n-1. \end{cases}
\end{aligned} \tag{7}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the graph $L_n \circ S_m$ is an $C_{\exp}$ average assignment graph, for $n \ge 2$ and $m \le 2$. □

Theorem 4. *The triangular ladder graph TL_n is an $C_{\exp}$ average assignment graph, for $n \geq 2$.*

Proof. Make the vertex assignment; $\Psi: V(TL_n) \longrightarrow N - \{4n-1, 4n+1, \ldots, \infty\}$,

$$\begin{aligned} \Psi(v_\lambda) &= \begin{cases} 4\lambda - 1, & 1 \leq \lambda \leq n-1, \\ 4n-2, & \lambda = n, \text{ and,} \end{cases} \\ \Psi(u_\lambda) &= 4\lambda - 3, \text{ for } 1 \leq \lambda \leq n. \end{aligned} \tag{8}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned} \Psi^*(v_\lambda u_{\lambda+1}) &= 4\lambda, \text{ for } 1 \leq \lambda \leq n-1, \\ \Psi^*(u_\lambda v_{\lambda+1}) &= 1 + 4\lambda, \text{ for } 1 \leq \lambda \leq n-1, \\ \Psi^*(u_\lambda v_\lambda) &= -2 + 4\lambda, \text{ for } 1 \leq \lambda \leq n, \text{and} \\ \Psi^*(u_\lambda u_{\lambda+1}) &= -1 + 4\lambda, \text{ for } 1 \leq \lambda \leq n-1. \end{aligned} \tag{9}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the triangular ladder graph TL_n is an $C_{\exp}$ average assignment graph, for $n \geq 2$. □

Theorem 5. *The graph $TL_n \circ S_m$ is an $C_{\exp}$ average assignment graph, for $n \geq 2$ and $m \leq 2$.*

Proof

Case (i): $m = 1$.

Make the vertex assignment; $\Psi: V(TL_n \circ S_1) \longrightarrow N - \{6n-1, 6n, \ldots, \infty\}$,

$$\begin{aligned} \Psi\left(x_1^{(\lambda)}\right) &= \begin{cases} 3, & \lambda = 1, \\ -3 + 6\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi\left(u_1^{(\lambda)}\right) &= \begin{cases} -6 + 7\lambda, & 1 \leq \lambda \leq 2, \\ -5 + 6\lambda, & 3 \leq \lambda \leq n, \end{cases} \\ \Psi(v_\lambda) &= 6\lambda - 2, \text{ for } 1 \leq \lambda \leq n, \text{ and} \\ \Psi(u_\lambda) &= \begin{cases} -3 + 5\lambda, & 1 \leq \lambda \leq 2, \\ -4 + 6\lambda, & 3 \leq \lambda \leq n. \end{cases} \end{aligned} \tag{10}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned} \Psi^*\left(v_\lambda x_1^{(\lambda)}\right) &= \begin{cases} 3, & \lambda = 1, \\ -2 + 6\lambda, & 2 \leq \lambda \leq n. \end{cases} \\ \Psi^*\left(u_\lambda w_1^{(\lambda)}\right) &= -4 + 6i, \text{ for } 1 \leq \lambda \leq n \text{ and} \\ \Psi^*(u_\lambda v_\lambda) &= \begin{cases} 4, & \lambda = 1, \\ -3 + 6\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi^*(v_\lambda u_{\lambda+1}) &= 6\lambda, \text{ for } 1 \leq \lambda \leq n-1, \\ \Psi^*(v_\lambda v_{\lambda+1}) &= 1 + 6\lambda, \text{ for } 1 \leq \lambda \leq n-1, \text{ and} \\ \Psi^*(u_\lambda u_{\lambda+1}) &= -1 + 6\lambda, \text{ for } 1 \leq \lambda \leq n-1. \end{aligned} \tag{11}$$

Case (ii): $m = 2$.

Make the vertex assignment; $\Psi: V(TL_n \circ S_2) \longrightarrow N - \{8n-1, 8n, \ldots, \infty\}$,

$$\begin{aligned} \Psi\left(x_2^{(\lambda)}\right) &= \begin{cases} 9, & \lambda = 1, \\ -6 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi\left(x_1^{(\lambda)}\right) &= \begin{cases} 4, & \lambda = 1, \\ -7 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi\left(w_2^{(\lambda)}\right) &= \begin{cases} 3, & \lambda = 1, \\ -2 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi\left(w_1^{(\lambda)}\right) &= \begin{cases} 1, & \lambda = 1, \\ -4 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi(v_\lambda) &= \begin{cases} 6, & \lambda = 1, \\ -5 + 8\lambda, & 2 \leq \lambda \leq n, \text{ and} \end{cases} \\ \Psi(u_\lambda) &= \begin{cases} 2, & \lambda = 1, \\ -3 + 8\lambda, & 2 \leq \lambda \leq n. \end{cases} \end{aligned} \tag{12}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned} \Psi^*\left(v_\lambda x_2^{(\lambda)}\right) &= \begin{cases} 8, & \lambda = 1, \\ -5 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi^*\left(v_\lambda x_1^{(\lambda)}\right) &= \begin{cases} 5, & \lambda = 1, \\ -6 + 8\lambda, & 2 \leq \lambda \leq n \end{cases} \\ \Psi^*\left(u_\lambda w_2^{(\lambda)}\right) &= \begin{cases} 3, & \lambda = 1, \\ -2 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi^*\left(u_\lambda w_1^{(\lambda)}\right) &= \begin{cases} 2, & \lambda = 1, \\ -3 + 8\lambda, & 2 \leq \lambda \leq n, \end{cases} \\ \Psi^*(v_\lambda u_{\lambda+1}) &= \begin{cases} 6, & \lambda = 1, \\ 8\lambda, & 2 \leq \lambda \leq n-1, \end{cases} \\ \Psi^*(u_\lambda v_\lambda) &= 8\lambda - 4, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(v_\lambda v_{\lambda+1}) &= \begin{cases} 9, & \lambda = 1, \\ -1 + 8\lambda, & 2 \leq \lambda \leq n-1, \text{ and} \end{cases} \\ \Psi^*(u_\lambda u_{\lambda+1}) &= \begin{cases} 7, & \lambda = 1, \\ 1 + 8\lambda, & 2 \leq \lambda \leq n-1. \end{cases} \end{aligned} \tag{13}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the graph $TL_n \circ S_m$ is an $C_{\exp}$ average assignment graph, for $n \geq 2$ and $m \leq 2$. □

Theorem 6. *The slanting ladder graph SL_n is an $C_{\exp}$ average assignment graph, for $n \geq 2$.*

Proof. Make the vertex assignment; $\Psi: V(SL_n) \longrightarrow N - \{3n-1, 3n, \ldots, \infty\}$,

$$\begin{aligned} \Psi(v_n) &= 3n - 2, \\ \Psi(v_\lambda) &= 3\lambda, \text{ for } 1 \leq \lambda \leq n-1, \\ \Psi(u_\lambda) &= 3\lambda - 4, \text{ for } 2 \leq \lambda \leq n, \quad \text{and} \\ \Psi(u_1) &= 1. \end{aligned} \tag{14}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned}
&\Psi^*(v_\lambda u_{\lambda+1}) = 3\lambda, \text{ for } 1\le\lambda\le n-1,\\
&\Psi^*(v_{n-1}v_n) = -2+3n,\\
&\Psi^*(v_\lambda v_{\lambda+1}) = 3\lambda+2, \text{ for } 1\le\lambda\le n-2, \text{ and}\\
&\Psi^*(u_\lambda u_{\lambda+1}) = \begin{cases} 2, & \lambda=1,\\ 3\lambda-2, & 2\le\lambda\le n-1.\end{cases}
\end{aligned}\tag{15}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the graph SL_n is an $C_{\exp}$ average assignment graph. □

Theorem 7. *The graph $SL_n{\circ}S_m$ is an $C_{\exp}$ average assignment graph, for $n\ge 2$ and $m\le 2$.*

Proof

Case i: $m=1$ and $n\ge 3$.

Make the vertex assignment, Ψ: $V(SL_n{\circ}S_1)\longrightarrow N-\{5n-1,5n,\ldots,\infty\}$.

$$\begin{aligned}
&\Psi\left(x_1^{(\lambda)}\right) = \begin{cases} 7, & \lambda=1,\\ 1+5\lambda, & 2\le\lambda\le n-1,\\ -3+5n, & \lambda=n,\end{cases}\\
&\Psi\left(w_1^{(\lambda)}\right) = \begin{cases} 3\lambda-2, & 1\le\lambda\le 2,\\ -7+5\lambda, & 3\le\lambda\le n,\end{cases}\\
&\Psi(v_\lambda) = \begin{cases} 6, & \lambda=1,\\ 5\lambda, & 2\le\lambda\le n-1,\\ 5n-2, & \lambda=n, \text{ and}\end{cases}\\
&\Psi(u_\lambda) = \begin{cases} 1+\lambda, & 1\le\lambda\le 2,\\ -6+5\lambda, & 3\le\lambda\le n.\end{cases}
\end{aligned}\tag{16}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned}
&\Psi^*\left(v_\lambda x_1^{(\lambda)}\right) = \begin{cases} 7, & \lambda=1,\\ 1+5\lambda, & 2\le\lambda\le n-1,\\ -2+5n, & \lambda=n,\end{cases}\\
&\Psi^*\left(u_\lambda w_1^{(\lambda)}\right) = \begin{cases} 2, & \lambda=1,\\ -6+5\lambda, & 2\le\lambda\le n,\end{cases}\\
&\Psi^*(v_\lambda u_{\lambda+1}) = 5\lambda, \text{ for } 1\le\lambda\le n-1,\\
&\Psi^*(v_\lambda v_{\lambda+1}) = \begin{cases} 5\lambda+3, & 1\le\lambda\le n-2,\\ -3+5n, & \lambda=n-1,\end{cases}\\
&\Psi^*(u_\lambda u_{\lambda+1}) = \begin{cases} 3\lambda, & 1\le\lambda\le 2,\\ 5\lambda-3, & 3\le\lambda\le n-1.\end{cases}
\end{aligned}\tag{17}$$

Case (ii): $m=2$ and $n\ge 3$.

Make the vertex assignment; Ψ: $V(SL_n{\circ}S_2)\longrightarrow N-\{7n-1,7n,\ldots,\infty\}$,

$$\begin{aligned}
&\Psi\left(x_2^{(\lambda)}\right) = \begin{cases} 11, & \lambda=1,\\ 1+7\lambda, & 2\le\lambda\le n-3, \lambda=2k, k\in\mathbb{N},\\ -2+7\lambda, & 2\le\lambda\le n-3, \lambda=2k+1, k\in\mathbb{N},\\ -9+7n, & \lambda=n-2, n=2k, k\in\mathbb{N},\\ -16+7n, & \lambda=n-2, n=2k+1, k\in\mathbb{N},\\ -6+7n, & \lambda=n-1, \text{ and}\\ -2+7n, & \lambda=n,\end{cases}\\
&\Psi\left(x_1^{(\lambda)}\right) = \begin{cases} 9, & \lambda=1,\\ 7\lambda, & 2\le\lambda\le n-3, \lambda=2k, k\in\mathbb{N},\\ -3+7\lambda, & 2\le\lambda\le n-3, \lambda=2k+1, k\in\mathbb{N},\\ -12+7n, & \lambda=n-2, n=2k, k\in\mathbb{N},\\ -17+7n, & \lambda=n-2, n=2k+1, k\in\mathbb{N},\\ -8+7n, & \lambda=n-1, n=2k, k\in\mathbb{N},\\ -7+7n, & \lambda=n-1, n=2k+1, k\in\mathbb{N},\\ -4+7n, & \lambda=n,\end{cases}\\
&\Psi\left(w_{(2)}^{(\lambda)}\right) = \begin{cases} 7\lambda-5, & 1\le\lambda\le 2,\\ -5+7\lambda, & 3\le\lambda\le n-1, \lambda=2k+2, k\in\mathbb{N},\\ -8+7\lambda, & 3\le\lambda\le n-1, \lambda=2k+1, k\in\mathbb{N},\\ -7+7n, & \lambda=n, n=2k, k\in\mathbb{N},\\ -8+7n, & \lambda=n, n=2k+1, k\in\mathbb{N},\end{cases}\\
&\Psi\left(w_1^{(\lambda)}\right) = \begin{cases} 1, & \lambda=1,\\ -5+5\lambda, & 2\le\lambda\le 3,\\ -7+7\lambda, & 4\le\lambda\le n-1, \lambda=2k+2, k\in\mathbb{N},\\ -10+7\lambda, & 4\le\lambda\le n-1, \lambda=2k+3, k\in\mathbb{N},\\ -11+7n, & \lambda=n \text{ and } \lambda=n-1,\\ -10+7n, & \lambda=n \text{ and } \lambda=n,\end{cases}\\
&\Psi(v_\lambda) = \begin{cases} 8, & \lambda=1,\\ 2+7\lambda, & 2\le\lambda\le n-3, \lambda=2k, \quad k\in\mathbb{N},\\ -1+7\lambda, & 2\le\lambda\le n-3, \lambda=2k+1, \quad k\in\mathbb{N},\\ -13+7n, & \lambda=n-2, n=2k, \quad k\in\mathbb{N},\\ -15+7n, & \lambda=n-2, n=2k+1, \quad k\in\mathbb{N},\\ -5+7n, & \lambda=n-1,\\ -3+7n, & \lambda=n,\end{cases}\\
&\Psi(u_\lambda) = \begin{cases} \lambda+2, & 1\le\lambda\le 2,\\ -6+7\lambda, & 3\le\lambda\le n-1, n=2k+2, \quad k\in\mathbb{N},\\ -9+7\lambda, & 3\le\lambda\le n-1, n=2k+1, \quad k\in\mathbb{N},\\ -10+7n, & \lambda=n, \quad k\in\mathbb{N},\\ -9+7n, & \lambda=n, n=2k+1, \quad k\in\mathbb{N}.\end{cases}
\end{aligned}\tag{18}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$
\begin{aligned}
&\Psi^*\left(v_\lambda x_2^{(\lambda)}\right) = \begin{cases} 10, & \lambda = 1, \\ 2+7\lambda, & 2 \le \lambda \le n-3, \lambda = 2k, \quad k \in \mathbb{N}, \\ -1+7\lambda, & 2 \le \lambda \le n-3, \lambda = 2k+1, \quad k \in \mathbb{N}, \\ -11+7n, & \lambda = n-2, n = 2k, \quad k \in \mathbb{N}, \\ -15+7n, & \lambda = n-2, n = 2k+1, \quad k \in \mathbb{N}, \\ -5+7\lambda, & \lambda = n-1, \\ -2+7n, & \lambda = n, \end{cases} \\
&\Psi^*\left(v_\lambda x_2^{(\lambda)}\right) = \begin{cases} 10, & \lambda = 1, \\ 2+7\lambda, & 2 \le \lambda \le n-3, \lambda = 2k, \quad k \in \mathbb{N}, \\ -1+7\lambda, & 2 \le \lambda \le n-3, \lambda = 2k+1, \quad k \in \mathbb{N}, \\ -11+7n, & \lambda = n-2, n = 2k, \quad k \in \mathbb{N}, \\ -15+7n, & \lambda = n-2, n = 2k+1, \quad k \in \mathbb{N}, \\ -5+7\lambda, & \lambda = n-1, \\ -2+7n, & \lambda = n, \end{cases} \\
&\Psi^*\left(v_\lambda x_1^{(\lambda)}\right) = \begin{cases} 9, & \lambda = 1, \\ 1+7\lambda, & 2 \le \lambda \le n-3, \lambda = 2k, \quad k \in \mathbb{N}, \\ -2+7\lambda, & 2 \le \lambda \le n-3, \lambda = 2k+1, \quad k \in \mathbb{N}, \\ -12+7n, & \lambda = n-2, n = 2k, \quad k \in \mathbb{N}, \\ -16+7n, & \lambda = n-2, n = 2k+1, \quad k \in \mathbb{N}, \\ -6+7n, & \lambda = n-1, \\ -6+7n, & \lambda = n, \end{cases} \\
&\Psi^*\left(u_\lambda w_2^{(\lambda)}\right) = \begin{cases} 4\lambda-1, & 1 \le \lambda \le 2, \\ -5+7\lambda, & 3 \le \lambda \le n-1, \lambda = 2k+2, \quad k \in \mathbb{N}, \\ -8+7\lambda, & 3 \le \lambda \le n-1, \lambda = 2k+1, \quad k \in \mathbb{N}, \\ -8+7n, & \lambda = n, \end{cases} \\
&\Psi^*\left(u_\lambda w_1^{(\lambda)}\right) = \begin{cases} 2, & \lambda = 1, \\ -7+6\lambda, & 2 \le \lambda \le 3, \\ -6+7\lambda, & 4 \le \lambda \le n-1, \lambda = 2k+2, \quad k \in \mathbb{N}, \\ -9+7\lambda, & 4 \le \lambda \le n-1, \lambda = 2k+3, \quad k \in \mathbb{N}, \\ -10+7n, & \lambda = n, n = 2k, \quad k \in \mathbb{N}, \\ -9+7n, & \lambda = n, n = 2k, \quad k \in \mathbb{N}, \end{cases} \\
&\Psi^*\left(v_\lambda v_{\lambda+1}\right) = \begin{cases} 12, & \lambda = 1, \\ 7\lambda+4, & 2 \le \lambda \le n-3, \\ 7n-9, & \lambda = n-2, n = 2k, \quad k \in \mathbb{N}, \\ 7n-10, & \lambda = n-2, n = 2k+1, \quad k \in \mathbb{N}, \\ 7n-4, & \lambda = n-1, \end{cases} \\
&\Psi^*\left(v_\lambda u_{\lambda+1}\right) = \begin{cases} 6, & \lambda = 1, \\ 7\lambda, & 2 \le \lambda \le n-1, \end{cases} \\
&\Psi^*\left(u_\lambda u_{\lambda+1}\right) = \begin{cases} 4\lambda, & 1 \le \lambda \le 2, \\ 7\lambda-4, & 3 \le \lambda \le n-2, \\ 7n-13, & \lambda = n-1, n = 2k, \quad k \in \mathbb{N}, \\ 7n-11, & \lambda = n-1, n = 2k, \quad k \in \mathbb{N}. \end{cases}
\end{aligned}
\tag{19}
$$

Case (iii): $m = 1, 2$ and $n = 2$. An $C_{\exp}$ average assignment of the graphs $SL_2 \circ S_1$ and $SL_2 \circ S_2$ are shown in Figures 1 and 2.

As a result, Ψ is an $C_{\exp}$ average assignment and the graph $SL_n \circ S_m$ is an $C_{\exp}$ average assignment graph, for $n \ge 2$ and $m \le 2$. □

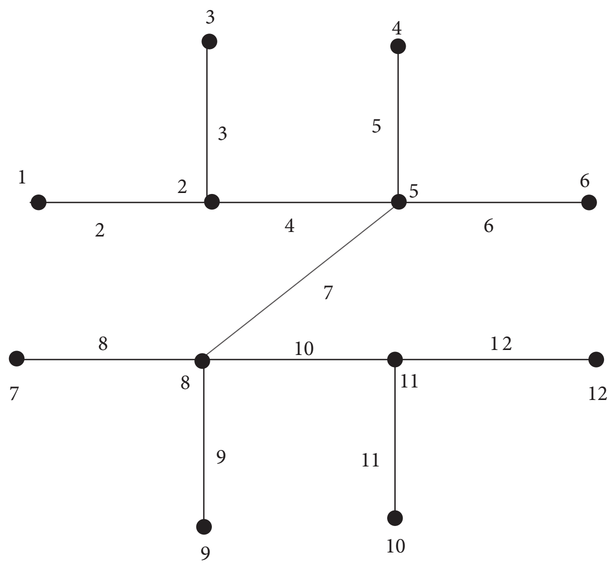

Figure 2: An $C_{\exp}$ average assignment of the graph $SL_2 \circ S_2$.

Theorem 8. *The graph D_n^* is an $C_{\exp}$ average assignment graph, for $n \geq 2$.*

Proof. Make the vertex assignment; $\Psi: V(D_n^*) \longrightarrow N - \{6n, 6n+1, \ldots, \infty\}$,

$$\begin{aligned} \Psi(a_{\lambda,4}) &= -1 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi(a_{\lambda,3}) &= -3 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi(a_{\lambda,2}) &= -5 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \text{ and} \\ \Psi(a_{\lambda,1}) &= -2 + 6\lambda, \text{ for } 1 \leq \lambda \leq n. \end{aligned} \tag{20}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned} \Psi^*(a_{\lambda,4}a_{\lambda,1}) &= -1 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(a_{\lambda,3}a_{\lambda,4}) &= -2 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(a_{\lambda,2}a_{\lambda,3}) &= -4 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(a_{\lambda,1}a_{\lambda,2}) &= -3 + 6\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(a_{\lambda,3}a_{\lambda+1,3}) &= 6\lambda, \text{ for } 1 \leq \lambda \leq n-1, \quad \text{and} \\ \Psi^*(a_{\lambda,1}a_{\lambda+1,1}) &= 1 + 6\lambda, \text{ for } 1 \leq \lambda \leq n-1. \end{aligned} \tag{21}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the graph D_n^* is an $C_{\exp}$ average assignment graph, for $n \geq 2$. □

Theorem 9. *The diamond ladder graph Dl_n is an $C_{\exp}$ average assignment graph, for $n \geq 1$.*

Proof. Make the vertex assignment; $\Psi: V(Dl_n) \longrightarrow N - \{8n-1, 8n, \ldots, \infty\}$,

$$\Psi(z_\lambda) = \begin{cases} 1, & \lambda = 1, \\ -2 + 4\lambda - \left(\dfrac{(-1)^{\lambda+1} + 1}{2}\right), & 2 \leq \lambda \leq 2n \text{ and } \lambda \text{ is even}, \\ -2 + 4\lambda - \left(\dfrac{(-1)^{\lambda+1} + 1}{2}\right), & 3 \leq \lambda \leq 2n \text{ and } \lambda \text{ is odd}, \end{cases}$$

$$\Psi(y_\lambda) = -3 + 8\lambda, \text{ for } 1 \leq \lambda \leq n, \quad \text{and}$$

$$\Psi(x_\lambda) = -5 + 8\lambda, \text{ for } 1 \leq \lambda \leq n. \tag{22}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\begin{aligned} \Psi^*(y_\lambda z_{2\lambda}) &= -2 + 8\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(y_\lambda z_{2\lambda-1}) &= -5 + 8\lambda, \text{ for } 1 \leq \lambda \leq n, \text{ and} \\ \Psi^*(x_\lambda z_{2\lambda}) &= -3 + 8\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(x_\lambda z_{2\lambda-1}) &= -6 + 8\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(z_{2\lambda}z_{2\lambda+1}) &= 8\lambda, \text{ for } 1 \leq \lambda \leq n-1, \\ \Psi^*(x_\lambda y_\lambda) &= -4 + 8\lambda, \text{ for } 1 \leq \lambda \leq n, \\ \Psi^*(y_\lambda y_{\lambda+1}) &= 1 + 8\lambda, \text{ for } 1 \leq \lambda \leq n-1, \text{ and} \\ \Psi^*(x_\lambda x_{\lambda+1}) &= -1 + 8\lambda, \text{ for } 1 \leq \lambda \leq n-1. \end{aligned} \tag{23}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the diamond ladder graph Dl_n is an $C_{\exp}$ average assignment graph, for $n \geq 1$. □

Theorem 10. *The latitude graph is an $C_{\exp}$ average assignment graph.*

Proof. Make the vertex assignment; $\Psi: V(G) \longrightarrow N - \{3n/2 + 1, 3n/2 + 2, \ldots, \infty\}$,

$$\Psi(u_\lambda) = \begin{cases} -2 + 3\lambda, & 1 \leq \lambda \leq \dfrac{n}{2}, \\ -1 + 3\lambda 1, & \lambda = \dfrac{n}{2}, \\ \dfrac{3n}{2}, & \lambda = \dfrac{n}{2} + 1, \\ 3 + 3n - 3\lambda, & \dfrac{n}{2} + 2 \leq \lambda \leq n - 1, \\ 3, & \lambda = n. \end{cases} \tag{24}$$

Consequently, the instigated edge assignment Ψ^* is acquired as follows:

$$\Psi^*\left(u_\lambda u_{n+2-\lambda}\right) = -2 + 3\lambda, \text{ for } 2 \le \lambda \le \frac{n}{2},$$

$$\Psi^*\left(u_n u_1\right) = 2, \text{ and}$$

$$\Psi^*\left(u_\lambda u_{\lambda+1}\right) = \begin{cases} 3\lambda, & 1 \le \lambda \le \dfrac{n}{2}, \\ -1 + \dfrac{3n}{2}, & \lambda = \dfrac{n}{2} + 1, \\ 2 + 3n - 3\lambda, & \dfrac{n}{2} + 2 \le \lambda \le n - 1. \end{cases} \tag{25}$$

As a result, Ψ is an $C_{\exp}$ average assignment and the latitude graph is an $C_{\exp}$ average assignment graph. □

5. Conclusion

The significant properties on $C_{\exp}$ average assignment of several ladder networks are discovered in this work. Analysis of the $C_{\exp}$ average assignment of other networks can be discussed further.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] J. Gross and J. Yellen, *Graph Theory and its Applications*, CRC Press, London, UK, 1999.

[2] F. Harary, *Graph Theory*, Narosa Publishing House Reading, New Delhi, India, 1988.

[3] G. Chartrand, L. Lesniak, and P. Zhang, *Graphs and Digraphs*, Taylor & Francis Group, Boca Raton, New York, USA, 6th edition, 2016.

[4] F. Buckley and F. Harary, *Distance in Graphs*, Addison-Wesley Reading, 1990.

[5] G. S. Bloom and S. W. Golomb, "Numbered complete graphs, unusual rulers, and assorted applications," *Lecture Notes in Mathematics*, vol. 642, pp. 53–65, 1978.

[6] J. A. Gallian, "A dynamic survey of graph labeling," *Electronic Journal of Combinatorics*, vol. 24, p. #DS6, 2021.

[7] S. Arockiaraj, A. Durai Baskar, and A. Rajesh, "Kannan, F-root square mean labeling of line graph of some graphs," *Utilitas Mathematica*, vol. 112, pp. 11–32, 2019.

[8] A. Meena Kumari and S. Arockiaraj, "On (1,1,0)- F-Face magic mean labeling of some graphs," *Utilitas Mathematica*, vol. 103, pp. 139–159, 2017.

[9] S. I. Butt, M. Numan, I. A. Shah, and S. Ali, "Face labelings of type (1,1,1) for generalized prism," *Ars Combinatoria*, vol. 137, pp. 41–52, 2018.

[10] A. M. Alanazi, G. Muhiuddin, A. R. Kannan, and V. Govindan, "New perspectives on classical meanness of some ladder graphs," *Journal of Mathematics*, vol. 2021, pp. 1–14, 2021.

[11] G. Muhiuddin, A. M. Alanazi, A. R. Kannan, and V. Govindan, "Preservation of the classical meanness property of some graphs based on line graph operation," *Journal of Mathematics*, vol. 2021, pp. 1–10, 2021.

[12] H. U. Afzal, M. Javaid, A. M. Alanazi, and M. G. Alshehri, "Computing edge weights of symmetric classes of networks," *Mathematical Problems in Engineering*, vol. 2021, Article ID 5562544, 22 pages, 2021.

[13] S. Dafik, R. Fitriana Eka, and L. Sya' Diyah, *Super Anti-magicness of Triangular Book and Diamond Ladder Graphs*, Indoms-UGM Yogyakarta, Indonesia, 2013.

[14] M. I. Moussa and E. M. Badr, "Ladder and subdivision of ladder graphs with pendent edges are odd graceful," *International Journal on Applications of Graph Theory in Wireless Ad hoc Networks and Sensor Networks*, vol. 8, no. 1, pp. 1–8, 2016.

[15] A. Rajesh Kannan, P. Manivannan, and A. Durai Baskar, "Exponential mean labeling of some graphs obtained from duplicating operations," *Journal of Physics: Conference Series*, vol. 1597, no. 1, Article ID 012028, 2020.

[16] G. V. Ghodasara and I. I. Jadav, "New grid related cordial graphs," *International Journal Applied Mathematics.*vol. 28, no. 2, pp. 1244–1248, 2013.

[17] Y. Kurniawati and I. H. Agustin, "On the super edge local antimagic total labeling of related ladder graph," *Journal de Physique: Conf. Ser.*vol. 1465, no. 1, Article ID 012027, 2020.

[18] L. Ratnasari and Y. Susanti, "Total edge irregularity strength of ladder-related graphs," *Asian-European Journal of Mathematics*, vol. 13, no. 4, Article ID 2050072, 2020.

[19] A. Ahmed and M. G. Ruxandra, "Radio labeling of some ladder related graphs," *Mathematical Reports*, vol. 19, no. 69, pp. 107–119, 2017.

[20] R. Mohamed, "Zeen El Deen, Edge graceful labeling for some cyclic related graphs," *Advances in Mathematical Physics*, vol. 2020, pp. 1–8, 2020.

[21] L. Ratnasari and Y. Susanti, "Total edge irregularity strength of ladder-related graphs," *Asian-European Journal of Mathematics*, vol. 13, no. 4, Article ID 2050072, 2020.

[22] A. A. Elsonbaty and S. N. Daoud, "Edge even graceful labeling of cylinder grid graph," *Symmetry*, vol. 11, no. 4, pp. 584–630, 2019.

[23] P. Deb and N. B. Limaye, "On elegant labelings of triangular snakes," *Journal of Combinatorics, Information and System Sciences*, vol. 25, pp. 163–172, 2000.

[24] N. Diefenderfer, D. C. Ernst, M. G. Hastings et al., "Prime vertex labelings of several families of graphs," *Involve, a Journal of Mathematics*, vol. 9, no. 4, pp. 667–688, 2016.

[25] A. Elsonbaty and S. N. Daoud, "Edge even graceful labeling of some path and cycle related graphs," *Ars Combinatoria*, vol. 30, pp. 79–96, 2017.