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Research Article

A Study on C-Exponential Mean Labeling of Graphs

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A function h is mentioned as a C-exponential mean labeling of a graph $G(V, E)$ that has s vertices and r edges if $h: V(G) \longrightarrow \{1, 2, 3, \cdots, r+1\}$ is injective and the generated function $h^*: E(G) \longrightarrow \{2, 3, 4, \cdots, r+1\}$ defined by $h^*(ab) = \lceil 1/e ((h(b)^{h(b)})/(h(a)^{h(a)}))^{1/(h(b)-h(a))} \rceil$, for all $ab \in E(G)$, is bijective. A graph which recognizes a C-exponential mean labeling is defined as C-exponential mean graph. In the following study, we have studied the exponential meanness of the path, the graph triangular tree of T_n, $C_m^{P_n}$, cartesian product of two paths $P_m \square P_n$, one-sided step graph of ST_n, double-sided step graph of $2ST_{2n}$, one-sided arrow graph of A_r^s, double-sided arrow graph of DA_r^s, and subdivision of ladder graph $S(L_t)$.

1. Introduction

In the field of mathematics, along with some areas of sciences, graph theory has become an interesting topic of study. A graph labeling is considered as an integer's assignment to the edges or vertices, or vice versa, subjected to particular conditions. Many mathematicians and scientists have contributed and introduced different kinds of labeling [1–6].

In the present study, the graphs considered here are undirected, simple, and finite graphs $G = (V, E)$ that have s vertices and r edges. Referring to the graph labeling introduced by Gallain, a detailed survey is conducted on graph labeling [4]. Somasundaram and Ponraj [7] originated the theory of mean labeling of graphs. Many mathematicians introduced different aspects of mean labeling. The study of Kannan et al. on the exponential mean labeling of a few different graphs studied through duplicate operations is examined for the present study [8]. Barrientos' study on alpha graphs has demonstrated the presence of α-labeling of a tree using various vertices and lengths of base path and proved that these trees can be utilized to demonstrate unicycle graphs with α-labeling [9]. Studies on cordial labeling between paths and cycles for a Cartesian product have demonstrated that these Cartesian products, under any conditions, are always cordial and even proved that two path Cartesian products are always cordial [10].

Sumathi and Rathi introduced the quotient labeling number for a wide family of ladder graphs, namely, closed triangular ladder, open triangular ladder, closed ladder, open ladder, step ladder, slanting ladder, and open diagonal ladder [11]. Baskar, referring to the flooring function edge labels, defined the logarithmic mean labeling on graphs and studied the logarithmic meanness of different ladder-related graphs [12].

Traditionally, the logarithmic mean of any two positive integers is not necessary to be an integer. And, if the logarithmic mean is considered an integer, the flooring of ceiling function is used. The edge label is set through flooring or a ceiling function, which is defined to be the logarithmic mean labeling of graphs. Baskar defined logarithmic mean labeling on graphs by setting the edge labels from flooring function [12]. A graph is considered a logarithmic mean graph if it recognizes logarithmic mean labeling. In 1967, Rosa proposed graceful labeling, known as β-valuation [13], and later, Golomb represented it as graceful labeling [1]. Kaneria et al., in 2010, introduced arrow graph (A_n^k) and double arrow graph (DA_n^k) [14]. And, in 2015, step grid (St_n) graph and double step grid graph (DSt_n) were introduced [15]. These graphs were defined to be graceful graphs.

Motivated by such works, in this study, we aimed to work to introduce a new class of C-exponential mean labeling for different ladder graphs, looking at the ceiling function. A graph which recognizes a C-exponential mean labeling is defined as C-exponential mean graph. In the present study, we have examined the exponential meanness of the path, the graph triangular tree of T_n, $C_m^{P_n}$, cartesian product of two paths $P_m \square P_n$, one-sided step graph of ST_n, double-sided steps graph of $2ST_{2n}$, one-sided arrow graph of A_r^s, double-sided arrow graph of DA_r^s, and subdivision of ladder graph $S(L_t)$.

A function h is mentioned as a C-exponential mean labeling of a graph $G(V, E)$ that possess s vertices and r edges if $h^* : V(G) \longrightarrow \{1, 2, 3, \cdots, r+1\}$ is injective and the generated function $h^* : E(G) \longrightarrow \{2, 3, 4, \cdots, r+1\}$ defined by

$$h^*(ab) = \left\lceil \frac{1}{e} \left(\frac{h(b)^{h(b)}}{h(a)^{h(a)}} \right)^{1/(h(b)-h(a))} \right\rceil, \qquad (1)$$

for all $ab \in E(G)$, is bijective.

1.1. Preliminaries. The below-mentioned definitions are essential for the present study.

Definition 1. Let $v_1, v_2, \cdots, v_n$ be the consecutive vertices of P_n; a *triangular tree* is calculated by amalgamating each v_i with a leaf (or vertex of degree 1) of P_i. We denote this tree by T_n and refer to P_n as the base of T_n. Note that T_n has size $n(n+1)/2$, which means that its order is a triangular number. We say that the first vertex of P_i leaf is amalgamated with the vertex of P_n.

Definition 2. Let P_n be a path on n vertices represented by $u_{1,1}, u_{1,2}, u_{1,3}, \cdots, u_{1,n}$ and with $n-1$ edges signified by $e_1, e_2, e_3, \cdots, e_{n-1}$, where e_i represents the edge connecting $u_{1,i}$ and $u_{1,i+1}$, the vertices. On every edge e_i, erect a ladder that has $n-(n-1)$ steps counting the edge e_i, for $i = 1, 2, 3, \cdots, n-1$. The graph hence drawn is defined as *a one-sided step graph*, and it is represented by ST_n.

Definition 3. Let P_{2n} be a path on $2n$ vertices $w_{1,1}, w_{1,2}, w_{1,3}, \cdots, w_{1,2n}$ with $2n-1$ edges $u_1, u_2, u_3, \cdots, u_{2n-1}$, where u_i represents the edge connecting $w_{1,i}$ and $w_{1,i+1}$, the vertices; on every edge u_i, we erect a ladder that has $i+1$ steps counting the edge u_i, for $i = 1, 2, 3, \cdots, n$, and on every u_i, erect a ladder that has $2n+1-i$ steps counting u_i, for $i = n+1, n+2, \cdots, 2n-1$. The graph hence drawn is defined as *double-sided step graph*, and it is represented by $2ST_{2n}$.

Definition 4. An *arrow graph* A_r^s with breadth s and length r is acquired by joining a vertex w with superior vertices of $P_s \times P_r$ by r new edges from one end.

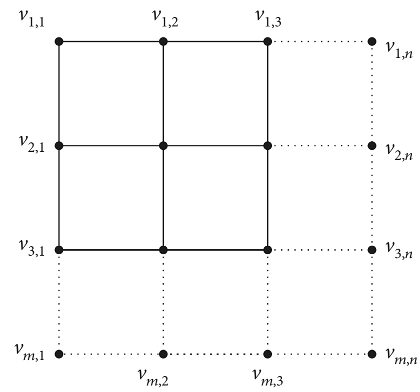

Figure 1: Representation of superior vertex graph.

Note. In the graph, $P_s \times P_r$ (grid graph on mn vertices) vertices $v_{1,1}, v_{2,1}, v_{3,1}, \cdots, v_{m,1}$ and vertices $v_{1,n}, v_{2,n}, v_{3,n}, \cdots, v_{m,n}$ are known as superior vertices (Figure 1) from both ends.

Definition 5. A *double arrow graph* DA_r^s with breadth s and length r is calculated by joining a vertex w with superior vertices of $P_s \times P_r$ by $s+r$ new edges from both ends.

Definition 6. A graph, which can be formed from an identified graph G G by dividing up each edge into exactly two segments by positioning intermediate vertices between its two ends, is called a *subdivision graph*. It is represented by $S(G)$.

2. Main Results

Theorem 8. *Each triangular tree T_n is a C-exponential mean graph, for n ≥ 1.*

Proof. Assume $v_1, v_2, \cdots, v_n$ denote the vertices of the path P_n and each vertex adjoining the path that is represented by v_{ij}, for $i = 1, 2, \cdots, n, j < i$.

Define $f : V(T_n) \longrightarrow \{1, 2, \cdots, n(n+1)/2\}$ as follows:

$$\begin{aligned} f(v_i) &= \begin{cases} \dfrac{i(i+1)}{2}, & \text{if i = odd}, \\ \dfrac{i^2 - i + 2}{2} & \text{if i = even}, \end{cases} \\ f(v_{ij}) &= \begin{cases} \dfrac{i^2 + i - 2}{2} - (j-1), & \text{if i = odd}, j < i, \\ \dfrac{i^2 - i + 4}{2} + (j = 1), & \text{if i = even}, j < i. \end{cases} \end{aligned} \qquad (2)$$

Then, the generated edge labeling is calculated as follows:

Define $f^*: E(T_n) \longrightarrow \{2, 3, 4, \cdots, n(n+1)/2\}$ as follows:

$$
\begin{aligned}
f^*(v_i, v_{i+1}) &= \begin{cases} v_{i(i+1)} + 1, & \text{if i = even}, \\ v_i + 1, & \text{if i = odd}, \end{cases} \\
f^*(v_i, v_{ij}) &= \begin{cases} v_i + 1, & \text{if i = even}, j < i, \\ v_i - (j-1), & \text{if i = odd}, i < j. \end{cases}
\end{aligned}
\tag{3}
$$

□

Hence, f is a C-exponential mean labeling of the triangular tree graph T_n, for $n \geq 1$. A typical example is illustrated in Figure 2.

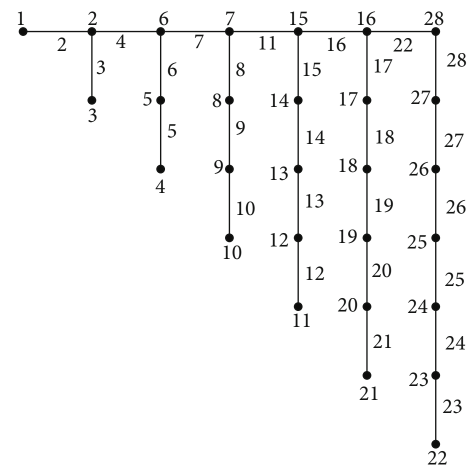

Figure 2: A C-exponential mean labeling of T_7

Theorem 9. *The graph $C_m^{P_n}$ is a C-exponential mean graph, for $m \geq 3$, $n \geq 1$.*

Proof. Assume $v_{1,j}, v_{2,j}, v_{3,j} \cdots, v_{m,j}$ denote the vertices of the cycle C_n for $j = 1, 2, \cdots, n+1$. Then, path P_n extended from the cycle through the vertices $v_{i,1}, v_{i,2}, v_{i,3} \cdots, v_{i,n+1}$ for $i = 1, 2, \cdots, m$.

Define $f: V(C_m^{P_n}) \longrightarrow \{1, 2, \cdots, m(n+1)+1\}$ as follows:

$$
f(v_i, v_j) = \begin{cases} (n+1)i - (j-1), & \text{if } i \text{ is odd, and } i \leq \left[\dfrac{m-1}{2}\right], \\ (n+1)i + 1 - (j+1), & \text{if } i \text{ is odd, and } i > \left[\dfrac{m-1}{2}\right], \\ n(i-1)i + i + (j-1), & \text{if } i \text{ is even, and } i < \left[\dfrac{m}{2}\right], \\ n(i-1) + i + 1 + (j-1), & \text{if } i \text{ is even, and } i \geq \left[\dfrac{m}{2}\right]. \end{cases}
\tag{4}
$$

Hence, the generated edge labeling is calculated as follows:

Define $f^*: E(C_m^{P_n}) \longrightarrow \{2, 3, 4, \cdots, m(n+1)+1\}$ as follows:

$$
\begin{aligned}
f^*(v_{i+1}, v_{i+1,1}) &= \begin{cases} (i+1)i + 1, & \text{if } i \leq \left[\dfrac{m-1}{2}\right], \\ (n+1)i + 2, & \text{if } i > \left[\dfrac{m-1}{2}\right] \text{ and } \left[\dfrac{m-1}{2}\right] < i \leq m-1, \end{cases} \\
f^*(v_{m,1}, v_{1,1}) &= \left[\dfrac{m+1}{2}\right](n+1) + 1,
\end{aligned}
\tag{5}
$$

when $j = 1, 2, \cdots, n$.

$$
f^*(v_{i,j}, v_{ij+1}) = \begin{cases} n(i+1) + i + j, & \text{if } i \text{ is even, and } i \leq \left[\dfrac{m-1}{2}\right], \\ n(i-1) + i + j + 1, & \text{if } i \text{ is even, and } i > \left[\dfrac{m-1}{2}\right], \\ (n+1)i - (j-1), & \text{if } i \text{ is odd, and } i \leq \left[\dfrac{m}{2}\right], \\ (n+1)i - j + 2, & \text{if } i \text{ is even, and } i > \left[\dfrac{m}{2}\right]. \end{cases}
\tag{6}
$$

Hence, f is a C-exponential mean labeling of the graph $C_n^{P_n}$, for $n \geq 3$. Figure 3 depicts an example of the aforementioned labeling. □

Theorem 10. *The graph $P_m \square P_n$ is a C-exponential mean graph, for $m, n \geq 1$*

Proof. The Cartesian product of the graphs G and H is the graph $G \square H$ that has vertex set $V(G)$ and edge set $E(G)$. In this article, the vertices of the graph $P_m \square P_n$ are presented as a matrix with n rows and m columns. Moreover, we denote $v_{i,j}$ as the vertex which lies at the i-th row and j-th column where $1 \leq i \leq m$ and $1 \leq j \leq n$.

Define $f: V(P_m \square P_n) \longrightarrow \{1, 2, \cdots, m(2n-1) - n + 1\}$ as follows:

$$
f(v_i, v_j) = 2m(j-1) - (j-2) + (i-1), \quad \text{for } 1 \leq i \leq m \text{ and } 1 \leq j \leq n.
\tag{7}
$$

Then, the generated edge labeling is calculated as follows:

Define $f^*: E(P_m \square P_n) \longrightarrow \{2, 3, 4, \cdots, m(2n-1) - n + 1\}$ as follows:

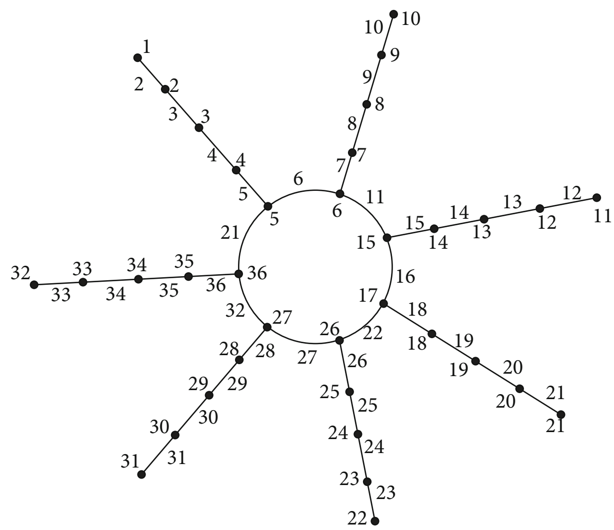

FIGURE 3: A C-exponential mean labeling of $C_n^{P_n}$

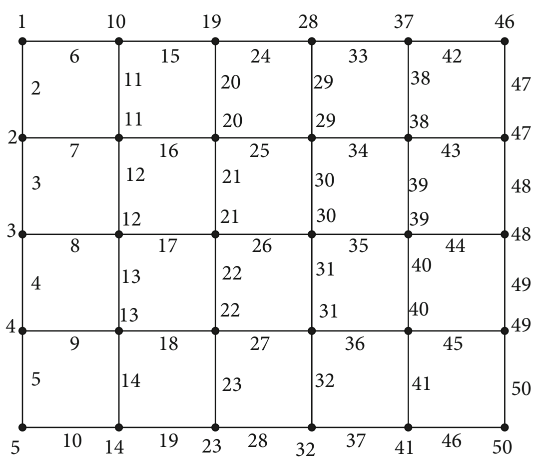

FIGURE 4: A C-exponential mean labeling of $P_m \square P_n$

$$f^*\left(v_{ij}, v_{ij+1}\right) = 2m(j-1) - (j-3) + (i-1), \quad \text{for } 1 \le i \le m \text{ and } 1 \le j \le n.$$
$$f\left(v_{ij}, v_{ij+1}\right) = m(2j-1) - (j-2) + (i-1), \quad \text{for } 1 \le i \le m \text{ and } 1 \le j \le n. \tag{8}$$

Hence, f is a C-exponential mean labeling of the graph $P_m \square P_n$, for $n \geq 2$. Figure 4 illustrates a representative example of the labeling described above. □

Theorem 11. *The graph ST_n is a C-exponential mean graph, for $n \geq 2$.*

Proof.
Assume $v_{1,1}, v_{1,2}, v_{1,3}, \cdots \cdots v_{1,n}, v_{2,1}, v_{2,2}, v_{2,3}, \cdots \cdots v_{2,n}, v_{3,1}, v_{3,2}, v_{3,3}, \cdots \cdots v_{3,n-1}, v_{4,1}, v_{4,2}, v_{4,3}, \cdots \cdots v_{4,n-2}, \cdots . v_{n,1}, v_{n,2}$ denote the vertices of the step graph ST_n.

Let $G = ST_n$ be the step ladder graph with $n-(i-1)$ steps for $1 \le i \le n$.

Let $v_{1,j}$ be the n vertices on the base where $1 \le j \le n$.

Let $v_{2,j}$ be the n vertices on the second stage above the base for $1 \le j \le n$.

Let $v_{3,j}$ be the $n-1$ vertices on the third step for $1 \le j \le n-1$.

Proceeding like this, we have vertices for $n-(i-1)$ steps.

Now the vertices of ST_n is denoted by $v_{i,j}$.

In $v_{i,j}$, i signifies the row (calculated through bottom to top) and j signifies the column (calculated through left to right) in which the vertex occurs.

Now, the graph ST_n of vertices and edges are n^2+n-1 with $\deg(v_{1,1}) = \deg(v_{1,n}) = \deg(v_{2,n}) = \deg(v_{n,1}) = 2$; $\deg(v_{i,n-i+2}) = 2$ for $3 \le i \le n$; $\deg(v_{i,1}) = 3$ for $2 \le i \le n-1$; $\deg(v_{1,j}) = 3$ for $2 \le j \le n-1$; and $\deg(v_{1,j}) = 4$ for $1 \le i \le n-1, 1 \le j \le n-1$, and $j \ne n-i+2$.

But, G, $S(G) = 2$ and $\Delta(G) = 4$.

Define $f : V(ST_n) \longrightarrow \{1, 2, \cdots, n^2+n-1\}$ as follows:

$$f(v_{i,1}) = \{n-(i-1)\}, \text{ for } 1 \le i \le n,$$
$$f\left(v_{ij1}\right) = \left\{3n - 3j + 5 + 2\sum_{j\ge 3}^{n}(n-j+3) - (i-1)\right\}, \quad \text{for } 1 \le i \le n \text{ and } 2 \le j \le n. \tag{9}$$

Hence, the generated edge labeling is calculated as follows:

Define $f^* : E(ST_n) \longrightarrow \{2, 3, 4, \cdots, n^2+n-1\}$ as follows:

$$f^*\left(v_{i,1}, v_{i+1,1}\right) = n-(i-1), \text{ for } 1 \le i \le n,$$
$$f^*\left(v_{i,j}, v_{i+1,j}\right) = 3n - 3j + 5 + 2\sum_{j\ge 3}^{n}(n-j+3) - (i-1), \quad \text{for } 1 \le i \le n-1 \text{ and } 2 \le j \le n. \tag{10}$$

Hence, f is a C-exponential mean labeling of the graph ST_n. Thus, the graph ST_n is a C-exponential graph for $n \geq 2$. A characteristic example of the labeling mentioned above is shown in Figure 5. □

Theorem 12. *The graph $2ST_{2n}$ is a C-exponential mean graph, for $n \geq 2$.*

Proof. Assume $w_{1,1}, w_{1,2}, w_{1,3}, \cdots, w_{1,n}, w_{2,n}, w_{3,n}, \cdots, w_{2,2n}, w_{3,1}, w_{3,2}, w_{3,3}, \cdots, w_{3,2n-2}, \cdots, w_{4,1}, w_{4,2}, w_{4,3}, \cdots, w_{4,2n-4}, \cdots \cdots, w_{n+1,1}, w_{n+1,2}$ denote the vertices of the double-sided step graph $2ST_{2n}$. In $w_{i,j}$, i signifies the row (calculated through bottom to top) and j signifies the column (calculated through left to right) in which the vertex occurs.

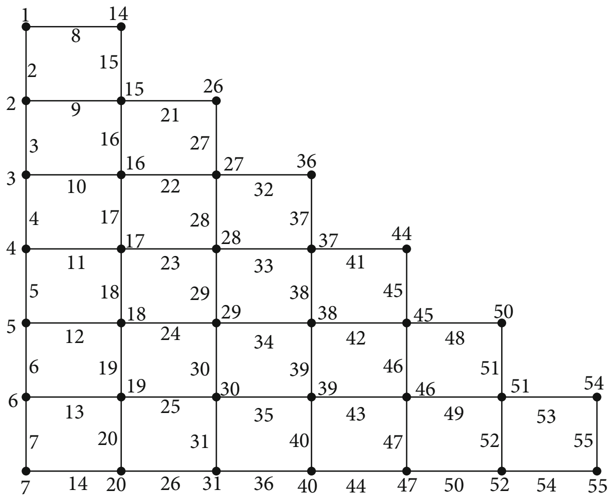

Figure 5: A C-exponential mean labeling of ST_7

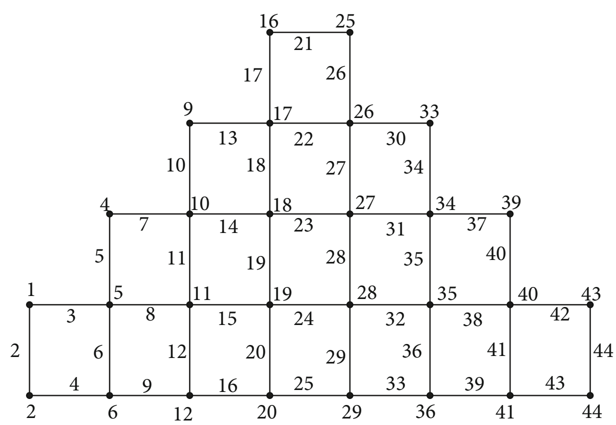

Figure 6: A C-exponential mean labeling of $2ST_8$

Define $l : V(2ST_{2n}) \longrightarrow \{1, 2, \cdots, n(2n+3)\}$ as follows:

$$l(w_{1,j}) = \begin{cases} 2, & j = 1, \\ w_{i,j-1} + n + 2(j-3), & 1 < j \le n, \\ w_{i,j-1} + 4n + 3 - 2j, & 1 < j \le n, \end{cases}$$
$$l(w_{2,j}) = \{w_{i-1,j} - (i-1), \text{for } 1 \le j \le 2n, 2 < i \le n+1 \text{ and } 1 \le j \le 2n+4-2i,$$
$$l(w_{i,j}) = w_{i-1,j+1} - 1. \tag{11}$$

The above-defined labeling pattern gives rise to l as an injective map and defines $l^* : E(2ST_{2n}) \longrightarrow \{2, 3, \cdots, n(2n+3)\}$ as follows:

$$f^*(uv) = \left\lceil \frac{1}{e}\left(\frac{f(v)^{f(v)}}{f(u)^{f(u)}}\right)^{1/(f(v)-f(u))} \right\rceil, \tag{12}$$

for all $uv \in E(2ST_{2n})$, is defined as bijective.

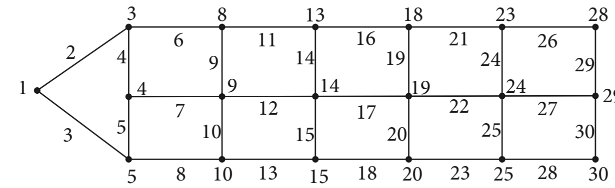

Figure 7: A C-exponential mean labeling of A_6^2

$$l^*(w_{1,j}, w_{i,j+1}) = \begin{cases} w_{i,j} + (j+1), & 1 \le j \le n, \\ w_{i,j} + 2n - j + 1, & n+1 \le j \le 2n, \end{cases}$$
$$l^*(w_{2,j}, w_{i,j+1}) = \begin{cases} w_{i,j}w_{1,j+1} - 1, & \text{for } 1 \le j < 2n, \\ w_{i,j} + 2n - j + 1, & n+1 \le j \le 2n, \\ \text{for } i > 2, & 1 \le j < 2n+4-2i, \end{cases}$$
$$l^*(w_{i,j}w_{i,j+1}) = w_{i-1,j+1}w_{i-1,j+2} - 1,$$
$$l^*(w_{1,j}, w_{2,j}) = \begin{cases} w_{1,j}, & \text{for } 1 \le j \le 2n, \\ w_{i,j} + 2n - j + 1, & n+1 \le j < 2n, \\ \text{for } i > 1, & 1 \le i < n, 2 \le j \le 2n+4-2i, \end{cases}$$
$$l^*(w_{i,j}w_{i+1,j-1}) = w_{i,j}. \tag{13}$$

Hence, l is a C- exponential mean labeling of $2ST_{2n}$, Hence, the graph, for $2ST_{2n}$, is a C-exponential mean graph for $n \ge 2$. Figure 6 displays a distinctive illustration of the labeling stated before. □

Theorem 13. *The graph A_r^s is a C-exponential mean graph, where $r \ge 2$ and $s \ge 2$.*

Proof. Assume $H = A_r^s$ is an arrow graph calculated by connecting a vertex w with superior vertices of $P_s \times P_r$ by two new edges.

Let $x_{i,j} (i = 1, 2; j = 1, 2, \cdots, m)$ be vertices of $P_s \times P_r$.

Join w with $x_{i,1} (i = 1, 2)$ by 2 new edges to obtain $H.V(H)| = 2m + 1$ and $|E(H)| = 3m$.

$\xi : V(A_r^s) \longrightarrow \{1, 2, \cdots, 2rs - r - s + 3\}$ by using C-exponential mean labeling formula for all $uv \in E(H)$ is defined as bijective.

$$\xi(w) = 1,$$
$$\xi(x_{i,j}) = 3 + (i-1) + (2s-1)(j-1), i = 1, 2, \cdots, s \text{ and } j = 1, 2, \cdots, r-1. \tag{14}$$

The above-defined labeling pattern gives rise to ξ as an injective map and defines $\xi^* : E(A_r^s) \longrightarrow \{2, 3, \cdots, 2rs - r - s + 3\}$ as follows:

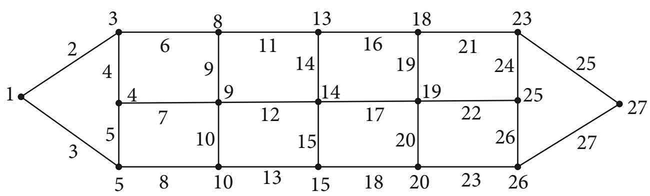

Figure 8: A C-exponential mean labeling of DA_5^2

$$\begin{aligned}
&\xi^*(w, x_{1,1}) = 2, \xi^*(w, x_{2,1}) = 3,\\
&\xi^*\left(x_{i,j}, x_{i+1,j}\right) = 3 + i + (2s-1)(j-1), i\\
&\quad = 1, 2, \cdots, s \text{ and } j = 1, 2, \cdots, r,\\
&\xi^*\left(x_{i,j}, x_{i,j+1}\right) = 3 - s + j(2s-1) + i, i\\
&\quad = 1, 2, \cdots, s \text{ and } j = 1, 2, \cdots, r-1.
\end{aligned} \tag{15}$$

Hence, the graph A_r^s is a C-exponential mean graph, for $r \geq 2$ and $s \geq 2$. A sample example of the previously mentioned labeling is shown in Figure 7. □

Theorem 14. *The graph DA_r^s is a C-exponential mean graph, where $r \geq 2$ and $s \geq 2$.*

Proof. Assume $H = DA_r^s$ is a double arrow graph calculated by joining two vertices w, y with $P_s \times P_r$ by four new edges on both sides.

Let $x_{i,j}(i = 1, 2, \cdots, s; j = 1, 2, \cdots, r)$ be vertices of $P_s \times P_r$. Join w with $x_{i,1}(i = 1, 2, \cdots, s)$ and y with $x_{i,r}(i = 1, 2, \cdots, s)$ by four new edges to obtain H. $|V(G)| = 2rs - r - s + 5$ and $|E(G)| = 2rs - r - s + 5$.

Define $\xi : V(DA_r^s) \longrightarrow \{1, 2, \cdots, 2rs - r - s + 5\}$ as follows:

$$\begin{aligned}
&\xi(w) = 1,\\
&\xi(y) = 2rs - r - s + 5,\\
&\xi\left(x_{i,j}\right) = 3 + (i-1) + (2s-1)(j-1), i\\
&\quad = 1, 2, \cdots, s \text{ and } j = 1, 2, \cdots, r-1,\\
&\xi(x_{1,r}) = 2rs - r - 2s + 4,\\
&\xi(x_{i,r}) = 2rs - r - 2s + 4 + i, i = 2, 3, \cdots, s.
\end{aligned} \tag{16}$$

Then, the generated edge labeling is calculated as follows:

$\xi^* : E(DA_r^s) \longrightarrow \{2, 3, 4, \cdots, 2rs - r - s + 5\}$by using C-exponential mean labeling formula, for all $uv \in E(H)$, is defined as bijective.

$$\begin{aligned}
&\xi^*(w, x_{1,1}) = 2, \xi^* = (w, x_{s,1}) = 3,\\
&\xi^*\left(x_{i,j}, x_{i+1,j}\right) = 3 + i + (2s-1)(j-1), i\\
&\quad = 1, 2, \cdots, s-1; j = 1, 2, \cdots, r-1,\\
&\xi^*(x_{1,r}, \mathrm{x}_{2,r}) = 2rs - r - 2s + 5,\\
&\xi^*\left(x_{i,r}, x_{i+1,r}\right) = 2rs - r - 2s + 7 + (i-2), i = 2, 3, \cdots, r,\\
&\xi^*\left(x_{i,j}, x_{i,j+1}\right) = 3 - s + j(2s-1) + i, i\\
&\quad = 1, 2, \cdots, s; j = 1, 2, \cdots, r-1,\\
&\xi^*(x_{1,r}, y) = \xi(x_{1,r}) + 2,\\
&\xi^*(x_{s,r}, y) = \xi(x_{s,r}) + 1.
\end{aligned} \tag{17}$$

Hence, the graph DA_r^s is a C-exponential mean graph, for $r \geq 2$. Figure 8 displays an illustration of the labeling from earlier as an example. □

Theorem 15. *The subdivision of ladder graph $S(L_t)$ is a C-exponential mean graph, for $t \geq 2$.*

Proof. Assume $H = L_t$. The ladder graph L_t is defined as $L_t = P_t \times K_2$, where P_t is a path with $\times$ signifing the cartesian product. Let $r_1, r_2, \cdots, r_t, s_1, s_2, s_3, \cdots, s_t$ be the ladder vertices L_t. Let s_i' be the lately added vertex joining s_i and s_{i+1}, r_i' be the newly added vertex between r_i and r_{i+1} and q_i be the lately added vertex joining r_i and s_i. Clearly, $G = S(L_t)$ has $5t - 2$ vertices and $6t - 4$ edges.

Define $\psi : V(S(L_t)) \longrightarrow \{1, 2, \cdots, 6t - 4 + 1\}$ as follows:

$$\begin{aligned}
&\psi(r_1) = 1,\\
&\psi(r_1) = \begin{cases} 6(i-1) & i = \text{odd and } 1 < i < t,\\ 2(3i-2) & i = \text{even and } 1 \leq i \leq t, \end{cases}\\
&\psi(s_1) = 3,\\
&\psi(r_1) = \begin{cases} 6(i-1) & i = \text{even and } 1 \leq i \leq t,\\ 2(3i-2) & i = \text{odd and } 1 < i \leq t, \end{cases}\\
&\psi(q_1) = 2, \psi(q_i) = 6i - 5, 1 < i \leq t,\\
&\psi\left(r_i'\right) = \begin{cases} 3(2i+1), & i = \text{odd and } 1 \leq i < t,\\ 6i - 1, & i = \text{even and } 1 \leq i < t, \end{cases}\\
&\psi\left(s_i'\right) = \begin{cases} 6i - 1, & i = \text{odd and } 1 \leq i < t,\\ 3(2i+1), & i = \text{even and } 1 \leq i < t. \end{cases}
\end{aligned} \tag{18}$$

Hence, the generated edge labeling is calculated as follows:

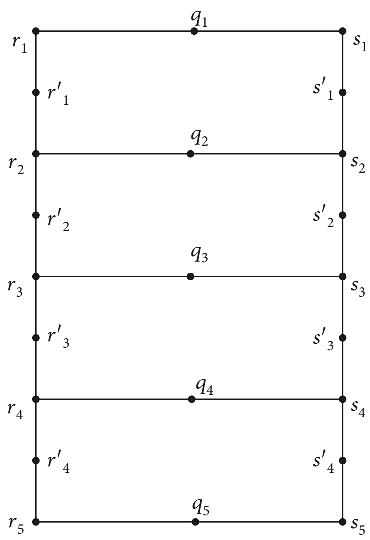

FIGURE 9: $S(L_5)$ is a C-exponential mean labeling

Define $\psi^* : E(S(L_t)) \longrightarrow \{2, 3, \cdots, 6t - 4 + 1\}$ as follows, for $1 \le i < t$.

$$\begin{aligned}
\psi\left(r_i, r_i'\right) &= \begin{cases} 6i - 1, & i = \text{odd}, \\ 6i - 2, & i = \text{even}, \end{cases} \\
\psi\left(r_i', r_{i+1}\right) &= \begin{cases} 6i, & i = \text{even}, \\ 3(2i + 1), & i = \text{odd}, \end{cases} \\
\psi\left(s_i, s_i'\right) &= \begin{cases} 2(3i - 1), & i = \text{odd}, \\ 6i - 1, & i = \text{even}, \end{cases} \\
\psi^*\left(s_i', s_{i+1}\right) &= \max\left\{\psi\left(s_i'\right), \psi(s_{i+1})\right\}, \\
\psi^*(r_i, q_i) &= \max\left\{\psi(r_i), \psi(q_i)\right\} \text{ for } i \le i \le t, \\
\psi^*(q_i, s_i) &= \max\left\{\psi(q_i), \psi(s_i)\right\} \text{ for } i \le i \le t.
\end{aligned} \tag{19}$$

Hence, ψ is a C-exponential mean graph $S(L_t)$, for $t \ge 2$. A representation of the prior labeling is shown as an example in Figure 9. □

3. Conclusion

The C-exponential mean labeling of tree, cycle-path, step graph, ladder graphs, arrow graphs, and subdivision of ladder graph was introduced and discussed in this work using graph operations.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest

There is no conflict of interest to declare.

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