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Research Article

The Pessimistic Diagnosability of Folded Petersen Cubes

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Diagnosability is an important metric parameter for measuring the reliability of multiprocessor systems. The pessimistic diagnosis strategy is a classic diagnostic model based on the PMC model. The class of folded Petersen cubes, denoted by $FPQ_{n,k}$, where $n, k \geq 0$ and $(n, k) \neq (0, 0)$, is introduced as a competitive model of the hypercubes, which is constructed by iteratively applying the Cartesian product operation on the hypercube Q_n and the Petersen graph P. In this paper, by exploring the structural properties of the folded Petersen cubes $FPQ_{n,k}$, we first prove that $FPQ_{n,k}$ is $(n + 3k)$ diagnosable under the PMC model. Then, we completely derive that the pessimistic diagnosability of $FPQ_{n,k}$ is $2n + 6k - 2$ under the PMC model. Furthermore, the diagnosability and the pessimistic diagnosability of the class of folded Petersen cubes, including the hypercube, folded Petersen graph, and hyper Petersen graph, are obtained.

1. Introduction

A multiprocessor system can be modeled as a graph, in which nodes (vertices) and edges correspond to processor and communication links, respectively. Throughout the paper, a graph and a system, a vertex and a processor, and an edge and a link are interchangeable.

The multiprocessor system has been increasingly adopted in the semiconductor technology, and the system reliability is crucial for multiprocessor systems. To maintain high reliability, multiprocessor systems should differentiate between fault-free processors and faulty ones. Determining all faulty processors is known as *fault diagnosis*. When all faulty processors can be evaluated precisely and t is an upper bound of the number of faulty processors, we call the multiprocessor system as t *-diagnosable*. The largest cardinality of the faulty set is named as the *diagnosability* of this system. Diagnosability of many famous networks had been studied; see [1–5] etc.

For the purpose of self-diagnosis of a system, several models have been proposed for diagnosing faulty processors in a multiprocessor system. Among the proposed models, the PMC model [6] was widely used. The PMC model allows each processor to perform diagnosis by testing the neighboring processors and observing their responses.

Observe that under a t-diagnosable system, a node can only be tested by its neighbors. It is impossible to determine whether some processor v is fault free or not when all the neighbors of v are faulty. To improve the diagnosability, Kavianpour and Friedman [7] proposed the pessimistic diagnosis strategy, which is a classic strategy based on the PMC model. In this strategy, all faulty processors can be isolated within a set which has at most one fault-free processor.

Definition 1. Let $G = (V_G, E_G)$ be a system. G is t/t *-diagnosable* if all faulty processors can be isolated within a set of size at most t such that at most, one fault-free processor is mistaken as a faulty one and the number of faulty processors is bounded by t. The *pessimistic diagnosability* of G is $t_p(G) = \max\{t : G \text{ is } t/t\text{- diagnosable}\}$.

Using the PMC model with a pessimistic strategy, the pessimistic diagnosability has been receiving much attention for many well-known multiprocessor systems, such as hypercubes Q_n, Möbius cubes MQ_n, enhanced hypercubes $EQ_{n,s}$, k-ary n-cubes Q_n^k, alternating group graphs AG_n, hypercube-like network HL_n, star graph S_n, and split-star networks S_n^2; see Table 1. More desired results can be found in [8–14] and the references therein.

Table 1: Pessimistic diagnosability of several main multiprocessor systems.

Multiprocessor systems G	Degree	$t_p(G)$
Hypercubes Q_n	n	$2n-2$ [15]
Möbius cubes MQ_n	n	$2n-2$ [16]
Enhanced hypercubes $EQ_{n,s}$	n	$2n-2$ [17]
3-Ary n-cubes Q_n^3	$2n$	$4n-3$ [8]
k-Ary n-cubes Q_n^k, $k\geq 4$	$2n$	$4n-2$ [8]
Alternating group graphs AG_n	$2n-4$	$4n-11$ [18]
Hypercube-like network HL_n	n	$2n-2$ [19]
Star graph S_n	$n-1$	$2n-4$ [20]
Split-star networks S_n^2	$2n-3$	$4n-9$ [11, 21]
Alternating group networks AN_n	$n-1$	$2n-5$ [21]
(n,k) arrangement graphs $A_{n,k}$	$k(n-k)$	$(2k-1)(n-k)-1$ [9]
(n,k) star graphs $S_{n,k}$	$n-1$	$n+k-3$ [9]
Balanced hypercubes BH_n	$2n$	$2n$ [9]
Data center networks $D_{k,n}$	$n+k-1$	$n+2k-2$ [22]
Cayley graphs generated $\Gamma_n(S)$	$\lvert E(A)\rvert$	$2\lvert E(A)\rvert-2$ (A is triangle free) [23]
By transposition graph A		$2\lvert E(A)\rvert-3$ (A has a triangle) [23]
Bubble-sort star graphs BS_n	$2n-3$	$4n-9$ [24]
Augmented cubes AQ_n	$2n-1$	$4n-8$ [25]
Augmented 3-ary n-cubes $AQ_{n,3}$	$4n-2$	$8n-11$ [24]
Augmented k-ary n-cubes $AQ_{n,k}$, $k\geq 4$	$4n-2$	$8n-10$ [24]
…	…	…

Network topology is an important factor because it affects the performance of the network. Hypercubes [26] have been recognized as topologies of multiprocessor systems. The class of folded Petersen cubes, proposed by Ö hring and Das [27], is constructed by iteratively applying the Cartesian product operation on hypercubes and the Petersen graph [28–30]. For recent research about folded Petersen cubes, please refer to [31–33] etc.

Although there are many results about diagnosability and the pessimistic diagnosability of many multiprocessor systems, little is known for folded Petersen cubes. In this paper, by exploring the structural properties of the folded Petersen cubes $\mathrm{FP}Q_{n,k}$, we prove that $\mathrm{FP}Q_{n,k}$ is $(n+3k)$ diagnosasable under the PMC model. Then, we completely determine the pessimistic diagnosability of $\mathrm{FP}Q_{n,k}$ under the PMC model. Furthermore, the diagnosability and the pessimistic diagnosability of the class of folded Petersen cubes, including the hypercube, folded Petersen graph, and hyper Petersen graph, are obtained.

2. Preliminaries

2.1. Terminologies and Notations. We provide Table 2 that contains most of the important notations used in this paper.

Let $G=(V_G,E_G)$ describe the link situation for a simple multiprocessor system. The processors in this system are denoted by an *vertex set* V_G, and the links between each pair of processors are denoted by an *edge set* E_G. Let c be a processor. Denote by $N_G(c)$ the set of processors which have a link to c. For a set $V'\subseteq V_G$, define $N_G(V')=\bigcup_{v\in V'}N_G(v)-V'$, named as the *neighborhood* of V'.

A graph H is a *subgraph* of a graph G if $V_H\subseteq V_G$ and $E_H\subseteq E_G$. We say that a simple graph is *k regular* when each processor has exactly k neighbors. For any $V'\subseteq V_G$ with $|V'|\leq k-1$, if $G-V'$ is still connected, then, G is *k -connected.* The maximally connected subgraphs of a graph G are its *components*. If a component has only one vertex, it is called *trivial*; otherwise, it is called *nontrivial*. Let $V'\subseteq V_G$ be a vertex cut, the biggest component of $G-V'$ is called *a large component*, and the remaining ones are called *small components*. Let $\kappa(G)=\min\{|S|\colon G-S \text{ is disconnected}\}$ be the *connectivity* of G.

Suppose that G and H are two graphs with $|V_G|=|V_H|$. Let M be a perfect matching between the nodes of G and H. Then, $G(G,H:M)$ is the graph with node set $V_G\cup V_H$ and edge set $E_G\cup E_H\cup M$.

2.2. Folded Petersen Cube. Let $G=(V_G,E_G)$ and $H=(V_H,E_H)$ be two graphs. The *Cartesian product* of G and H, denoted by $G\square H$, is the graph with node set $V_G\times V_H=\{(g,h)\colon g\in V_G, h\in V_H\}$, and the vertices (g_1,h_1) and (g_2,h_2) are adjacent if and only if $g_1=g_2$ and $(h_1,h_2)\in E_H$ or $h_1=h_2$ and $(g_1,g_2)\in E_G$. Under isomorphism, the operator

Table 2: Notations.

Symbol	Meaning
$G = (V_G, E_G)$	An undirected graph, where V_G is the set of processors and E_G is the set of communication links between two processors
$N_G(c)$	The set of all nodes adjacent to c in G
$N_G(V') = \bigcup_{v\in V'} N_G(v) - V'$	The neighborhood of a set V' of nodes in G
$G[V']$	The subgraph of G induced by a subset $V' \subseteq V_G$
$G - S$	A graph obtained from G by removing a node (edge) set S
$\kappa(G)$	The connectivity of G
$G\square H$	The Cartesian product of two graphs G and H
$S_1 \Delta S_2$	The symmetric difference of two sets S_1 and S_2
$t(G)$	The diagnosability of G
$t_p(G)$	The pessimistic diagnosability of G

$\square$ is associative and commutative. For any graph G and any positive integer k, we define $G^1 = G$ and $G^k = G^{k-1}\square G$ if $k > 1$. The *hypercube* Q_n (where $n \geq 1$) is defined as $Q_1 = K_2$ and $Q_n = Q_{n-1}\square Q_1$. Thus, we also write $Q_n = K_2^n$.

The Petersen graph P was introduced by Chartrand and Wilson [28]; see Figure 1. Obviously, the Petersen graph has an outer 5 cycle, an inner 5 cycle, and five spokes joining them. For $k \geq 1$, $\mathrm{FP}_k = P^k$ represents the *k -dimensional folded Petersen graph* and $HP_n = P\square Q_{n-3}$ (where $n \geq 3$) represents the *hyper Petersen graph* [34].

For any $i \in \{0, 1, \cdots, 9\}$, denote by $i\mathrm{FP}_{k-1}$ the subgraph of FP_k induced by the node set $\{ix_{k-2}x_{k-3}\cdots x_0 : x_j \in \{0, 1, \cdots, 9\}$ and $0 \leq j \leq k-2\}$. Thus, FP_k is recursively constructed from $i\mathrm{FP}_{k-1}$ for $0 \leq i \leq 9$. For any two nodes x and y of FP_k, suppose that $x = x_1x_2\cdots x_k$ and $y = y_1y_2\cdots y_k$. If $x, y \in V_{i\mathrm{FP}_{k-1}}$, then, x and y are adjacent if and only if $x_2\cdots x_k$ and $y_2\cdots y_k$ are adjacent in FP_{k-1}; otherwise, x and y are adjacent if and only if x_1 is adjacent to y_1 in P and $x_2\cdots x_k = y_2\cdots y_k$.

From the construction of folded Petersen graphs, it is obvious that any node of $V_{i\mathrm{FP}_{k-1}}$ has $3(k-1)$ neighbors in $i\mathrm{FP}_{k-1}$ and other three neighbors (called *extra neighbors*) in $V_{j\mathrm{FP}_{k-1}}$, where i and j are adjacent in P.

Lemma 2 (see [27]). *The folded Petersen graph FP_k is a node and edge transitive regular graph of degree 3k and of node connectivity $\kappa(FP_k) = 3k$. For a node $x \in V_{iFP_{k-1}}$, the three extra neighbors of x are in distinct jFP_{k-1}, where i and j are adjacent in P. Furthermore, for any two nodes $u, v \in V_{iFP_{k-1}}$, $N_{i\bar{FP}_{k1}}(u) \cap N_{i\bar{FP}_{k1}}(u) = \emptyset$, where $i\bar{FP}_{k1} = FP_k - iFP_{k-1}$.*

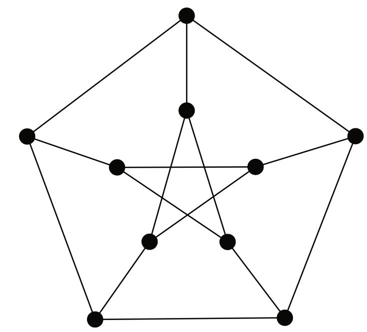

Figure 1: The Petersen graph P.

The class of folded Petersen cubes $FPQ_{n,k} = P^k\square Q_n$, where $n \geq 0$, $k \geq 0$ and $(n, k) \neq (0, 0)$, was introduced as a competitive model of the hypercubes. In particular, $FPQ_{0,k} = P^k$ and $FPQ_{n,0} = Q_n$. Clearly, $FPQ_{n,k}$ is a triangle-free $(n+3k)$-regular graph with 10^k2^n vertices.

Lemma 3 (see [27]). *The folded Petersen cube $FPQ_{n,k}$ is a regular graph with degree $n + 3k$ and connectivity $\kappa(FPQ_{n,k}) = n + 3k$. Furthermore, $FPQ_{n,k}(n \geq 1)$ can be viewed as $G(FPQ_{n-1,k}, FPQ_{n-1,k} : M)$, where M is a perfect matching between two $FPQ_{n-1,k}$'s.*

2.3. PMC Model. In self-diagnosable systems, there are several methods which had been introduced to diagnose faulty processors. The PMC model [6] allows each processor to perform diagnosis by testing the neighboring processors and observing their responses. In the PMC model, a *test syndrome* σ collects all test results. Let $S \subseteq V_G$. S is said to be *compatible* with a syndrome σ if σ can be produced from the condition that all nodes in S are faulty and all nodes in $V_G \setminus S$ are fault free. Let $\sigma_S = \{\sigma : \sigma$ is compatible with $S\}$. Two distinct sets $S_1, S_2 \subseteq V_G$ are *indistinguishable* if $\sigma_{S_1} \cap \sigma_{S_2} \neq \emptyset$ and *distinguishable* otherwise. The *symmetric difference* of two sets L_1 and L_2 is $L_1\Delta L_2 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1)$.

Dahbura and Masson [35] proposed a characterization for a pair of sets to be distinguishable under the PMC model.

Lemma 4 (see [35]). *Let $G = (V_G, E_G)$ be a graph. For any two distinct sets $S_1, S_2 \subseteq V_G$, (S_1, S_2) is a distinguishable pair under the PMC model if and only if there exist two nodes $u \in V_G \setminus (S_1 \cup S_2)$ and $v \in S_1\Delta S_2$ satisfying $(u, v) \in E_G$; see Figure 2.*

3. Main Results

In this section, we first study the structure properties of $FPQ_{n,k}$. Using the structural properties and some basic lemmas, we can obtained the diagnosability and the pessimistic diagnosability of the folded Petersen cube network $FPQ_{n,k}$.

The following inequalities are useful for our proof.

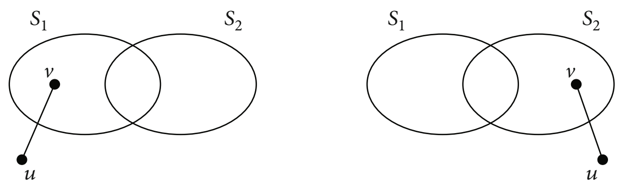

Figure 2: A distinguishable pair (S_1, S_2).

Lemma 5. *Let n and k be non-negative integers with $n \geq 0$, $k \geq 0$ and $(n, k) \neq (0, 0)$. The following inequalities hold.*

$$\begin{aligned} 10^k 2^{n-1} &\geq 3n + 9k - 5, \\ 10^k 2^n &\geq 2n + 6k + 1, \\ 10^k 2^n &\geq 5n + 15k - 6. \end{aligned} \tag{1}$$

Proof. Since the proof for the three statements are similar, we just prove (1), and the proof for (2) and (3) are left for readers. □

The following is the proof by induction on k.

The initial step is as follows: if $k = 0$, then, $2^{n-1} \geq 3n - 5$ holds for any integer $n \geq 1$. If $k = 1$, then, $10 \times 2^{n-1} \geq 3n + 4$ holds for any integer $n \geq 0$.

The induction step is as follows: assume that $k \geq 2$ and the statement holds for $k - 1$, i.e., $10^{k-1} 2^{n-1} \geq 3n + 9(k-1) - 5$. Then,

$$\begin{aligned} &10^k 2^{n-1} - (3n + 9k - 5) \\ &\geq 10(3n + 9(k-1) - 5) - (3n + 9k - 5) \\ &= 27n + 81k - 135 > 0. \end{aligned} \tag{2}$$

Hence, the statement holds for k as well.

3.1. Structure Properties of $FPQ_{n,k}$

Lemma 6 (see [16]). *Let G be a connected graph and $S \subseteq V_G$. If $|V(G) - S| \geq \kappa(G)$, then, $|N_G(S)| \geq \kappa(G)$; otherwise, $|N_G(S)| = |V_G - S|$.*

Lemma 7. *If u and v are two distinct nodes in $FPQ_{n,k}$ with $n \geq 0$, $k \geq 0$, and $(n, k) \neq (0, 0)$, then, $|N_{FPQ_{n,k}}(\{u, v\})| \geq 2n + 6k - 2$.*

Proof. If u and v are adjacent, then, $|N_{\mathrm{FPQ}_{n,k}}(\{u, v\})| = 2(n + 3k) - |\{u, v\}| = 2n + 6k - 2$ by Lemma 3; otherwise, since $\mathrm{FPQ}_{n,k}$ has no triangles, any two nonadjacent nodes have at most one common neighbor, and therefore, $|N_{\mathrm{FPQ}_{n,k}}(\{u, v\})| \geq 2(n + 3k) - 1 > 2n + 6k - 2$. □

Lemma 8. *Let S be a subset of $V(FP_k)$ for $k \geq 1$. If $2 \leq |S| \leq 6k - 4$, then, $|N_{FP_k}(S)| \geq 6k - 2$.*

Proof. The lemma is proved by the induction on k. If $k = 1$, then, $|S| = 2$; the result holds by Lemma 7. Assume that the lemma is true for FP_m, where m is an integer with $2 \leq m \leq k - 1$. In the following, we consider FP_k. Recall that FP_k is constructed by 10 disjoint FP_{k-1}s, denoted by $i\mathrm{FP}_{k-1}$ for $i \in \{0, 1, \cdots, 9\}$. Let $S_i = S \cap V_{i\mathrm{FP}_{k-1}}$ and $i\mathrm{F}\bar{\mathrm{P}}_{k1} = \mathrm{FP}_k - i\mathrm{FP}_{k-1}$ for $i \in \{0, 1, \cdots, 9\}$. W.l.o.g., we may assume that $|S_0| = \max_{i \in \{0,1,\cdots,9\}} |S_i|$. There are the following cases. □

Case 1. $|S_0| = 1$.

In this case, for all $i \in \{0, 1, \cdots, 9\}$, $|S_i| \leq 1$. Clearly, $2 \leq |S| \leq 10$ because of $i \leq 9$. If $|S| = 2$, then, by Lemma 7, we know the result holds. Now, assume that $3 \leq |S| \leq 10$ and $S_i \neq \varnothing$ for $i \in \{0, 1, 2\}$. By Lemma 2, $|N_{0\mathrm{FP}_{k1} \cup 1\mathrm{F}\bar{\mathrm{P}}_{k1} \cup 2\mathrm{FP}_{k1}}(S_0)| \geq 1$. Since FP_k is $3k$ regular and $i\mathrm{FP}_{k-1}$ is isomorphic to FP_{n-1}, we have

$$\begin{aligned} \left|N_{\mathrm{FP}_k}(S)\right| &\geq 3\kappa(i\mathrm{FP}_{k-1}) + \left|N_{0\mathrm{FP}_{k1} \cup 1\mathrm{F}\bar{\mathrm{P}}_{k1} \cup 2\mathrm{FP}_{k1}}(S_0)\right| \\ &= 3(3k - 3) + 1 = 9k - 8 \geq 6k - 2, \end{aligned} \tag{3}$$

for $k \geq 2$.

Case 2. $2 \leq |S_0| \leq 6k - 10$.

By inductive hypothesis in $0\mathrm{FP}_{k-1}$, $|N_{0\mathrm{FP}_{k-1}}(S_0)| \geq 6(k-1) - 2 = 6k - 8$. We distinguish the following two cases.

Case 2.1. $S = S_0$.

By Lemma 2, we know

$$\begin{aligned} \left|N_{FP_k}(S)\right| &= \left|N_{0FP_{k-1}}(S_0)\right| + \left|N_{0F\bar{P}_{k1}}(S_0)\right| \\ &\geq 6k - 8 + 3|S_1| \geq 6k - 2. \end{aligned} \tag{4}$$

Case 2.2. $S \neq S_0$.

There exists a $j \in \{1, 2, \cdots, 9\}$ such that $|S_j| \neq 0$. By Lemma 2,

$$\left|N_{0\mathrm{FP}_{k1} \bar{\cup} j\mathrm{FP}_{k1}}(S_0)\right| \geq 2|S_0| \geq 4. \tag{5}$$

If $|S_j| = 1$, then, $|N_{j\mathrm{FP}_{k-1}}(S_j)| = \kappa(j\mathrm{FP}_{k-1}) = 3k - 3$. Note that $0\mathrm{FP}_{k-1}$ and $j\mathrm{FP}_{k-1}$ are node disjoints; we have

$$\begin{aligned} \left|N_{\mathrm{FP}_k}(S)\right| &\geq \left|N_{0\mathrm{FP}_{k-1}}(S_0)\right| + \left|N_{j\mathrm{FP}_{k-1}}(S_j)\right| \\ &\quad + \left|N_{0\mathrm{FP}_{k1} \bar{\cup} j\mathrm{FP}_{k1}}(S_0)\right| \\ &\geq 6k - 8 + (3k - 3) + 4 = 9k - 7 \geq 6k - 2, \end{aligned} \tag{6}$$

for $k \geq 2$.

Now, consider $2 \leq |S_j| \leq |S_0| \leq 6k - 10$. By induction hypothesis in $j\mathrm{FP}_{k-1}$,

$$\left|N_{j\mathrm{FP}_{k-1}}(S_j)\right| \geq 6(k-1) - 2 = 6k - 8. \tag{7}$$

Thus,

$$\begin{aligned}|N_{\mathrm{FP}_k}(S)| &\geq |N_{0\mathrm{FP}_{k-1}}(S_0)| + |N_{j\mathrm{FP}_{k-1}}(S_j)| \\ &\quad + \left|N_{0\mathrm{FP}_{k1}\bar{\cup} j\mathrm{FP}_{k1}}(S_0)\right| \\ &\geq 2(6k-8)+4 \\ &= 12k-12 \geq 6k-2,\end{aligned} \tag{8}$$

for $k \geq 2$.

Case 3. $6k-9 \leq |S_0| \leq 6k-4$.

Since $\kappa(0\mathrm{FP}_{k-1}) = 3k-3$ and

$$10^{k-1} - |S_0| \geq 10^{k-1} - (6k-4) \geq 3k-3 = \kappa(0\mathrm{FP}_{k-1}). \tag{9}$$

For $k \geq 2$, we have $|N_{0\mathrm{FP}_{k-1}}(S_0)| \geq 3k-3$ by Lemma 6 and $|N_{0\bar{\mathrm{FP}}_{k1}}(S_0)| = 3|S_0|$ by Lemma 2. We distinguish the following two cases.

Case 3.1. $S = S_0$.

By Lemma 2, we know

$$\begin{aligned}|N_{\mathrm{FP}_k}(S)| &= |N_{0\mathrm{FP}_{k-1}}(S_0)| + \left|N_{0\bar{\mathrm{FP}}_{k1}}(S_0)\right| \\ &\geq 3k-3+3(6k-9) = 21k-30 \geq 6k-2,\end{aligned} \tag{10}$$

for $k \geq 2$.

Case 3.2. $S \neq S_0$.

There exists $j \in \{1, 2, \cdots, 9\}$ such that $|S_j| \neq 0$. Since $|S| \leq 6k-4$ and $|S| - |S_0| \leq 5$, we have $1 \leq |S_j| \leq 5$. Recall that $\kappa(j\mathrm{FP}_{k-1}) = 3k-3$. If $|S_j| = 1$, then,

$$|N_{j\mathrm{FP}_{k-1}}(S_j)| = \kappa(j\mathrm{FP}_{k-1}) = 3k-3. \tag{11}$$

If $2 \leq |S_j| \leq 5$, then,

$$10^{k-1} - |S_j| \geq 10^{k-1} - 5 \geq 3k-3 = \kappa(0\mathrm{FP}_{k-1}), \tag{12}$$

for $k \geq 2$, and therefore, $|N_{j\mathrm{FP}_{k-1}}(S_j)| \geq 3k-3$ by Lemma 6. Hence, we have

$$\begin{aligned}|N_{\mathrm{FP}_k}(S)| &\geq |N_{0\mathrm{FP}_{k-1}}(S_0)| + |N_{j\mathrm{FP}_{k-1}}(S_j)| \\ &\quad + \left|N_{0\mathrm{FP}_{k1}\bar{\cup} j\mathrm{FP}_{k1}}(S_0)\right| \\ &\geq (3k-3)+(3k-3)+2|S_0| \\ &\geq 6k-6+2(6k-9) = 18k-24 \geq 6k-2,\end{aligned} \tag{13}$$

for $k \geq 2$.

Lemma 9. *For $S \subseteq V(Q_n)$ with $n \geq 3$ and $2 \leq |S| \leq 2n-4$, it holds that $|N_{Q_n}(S)| \geq 2n-2$.*

Proof. Proof by induction on n.

If $n = 3$, then $|S| = 2$ and the result holds by Lemma 7. Assume that the lemma is true for Q_m with $4 \leq m \leq n-1$. Now, we consider Q_n as follows. Recall that Q_n is constructed by two disjoint Q_{n-1}s, denoted by $Q[0]$ and $Q[1]$. For $i \in \{1, 2\}$, let $S_i = S \cap V_{Q[i]}$. W.l.o.g., we may assume that $|S_0| \geq |S_1|$. Since Q_n is n regular and triangle free, if $|S| = 2$, then, $|N_{Q_n}(S)| \geq 2n-2$. Next, let $3 \leq |S| \leq 2n-4$. It follows that $|S_0| \geq 2$. We distinguish the following two cases. □

Case 1. $2 \leq |S_0| \leq 2n-6$.

By the induction hypothesis in $Q[0]$, $|N_{Q[0]}(S_0)| \geq 2n-4$. If $|S_1| = 0$, thenm $S = S_0$.Thus,

$$|N_{Q_n}(S)| \geq \left|N_{Q[0]}(S_0)\right| + \left|N_{\bar{Q[0]}}(S_0)\right| \geq (2n-4)+2 = 2n-2. \tag{14}$$

If $|S_1| = 1$, then, $|N_{Q[1]}(S_1)| = \kappa(Q[1]) = n-1$.Thus,

$$\begin{aligned}|N_{Q_n}(S)| &\geq \left|N_{Q[0]}(S_0)\right| + \left|N_{Q[1]}(S_1)\right| \\ &\geq (2n-4)+(n-1) = 3n-5 \geq 2n-2.\end{aligned} \tag{15}$$

It remains to assume that $2 \leq |S_1| \leq |S_0| \leq 2n-6$. Then, $|N_{Q[1]}(S_1)| \geq 2n-4$. Thus,

$$\begin{aligned}|N_{Q_n}(S)| &\geq \left|N_{Q[0]}(S_0)\right| + \left|N_{Q[1]}(S_1)\right| \\ &\geq 2(2n-4) = 4n-8 \geq 2n-2.\end{aligned} \tag{16}$$

Case 2. $2n-5 \leq |S_0| \leq 2n-4$.

Since $\kappa(Q[0]) = Q[0]$ is $n-1$ and $|V(Q[0])| - (2n-4) \geq \kappa(Q[0]) = n-1$ for $n \geq 4$, Lemma 6 implies that $|N_{Q[0]}(S_0)| \geq n-1$.

If $S = S_0$, then,

$$\begin{aligned}|N_{Q_n}(S)| &= \left|N_{Q[0]}(S_0)\right| + \left|N_{Q[1]}(S_0)\right| \\ &\geq n-1+(2n-5) = 3n-6 \geq 2n-2.\end{aligned} \tag{17}$$

If $S \neq S_0$, then, $|S_1| = 1$. Thus,

$$\begin{aligned}|N_{Q_n}(S)| &\geq \left|N_{Q[0]}(S_0)\right| + \left|N_{Q[1]}(S_1)\right| \\ &\geq n-1+(n-1) = 2n-2.\end{aligned} \tag{18}$$

Therefore, the lemma is true for Q_n as well.

Lemma 10. *For $S \subseteq V_{FPQ_{n,k}}$ with $2 \leq |S| \leq 2n+6k-4$, $n \geq 0$, $k \geq 0$, and $(n,k) \neq (0,0)$, it holds that $|N_{FPQ_{n,k}}(S)| \geq 2n+6k-2$.*

Proof. Proof by induction on n. □

If $n = 0$, then, $k \geq 1$ and $2 \leq |S| \leq 6k - 4$; the lemma holds by Lemma 8.

Assume $n \geq 1$ and the result holds for $FPQ_{n-1,k}$. We consider the result for $FPQ_{n,k}$ as follows.

If $k = 0$, then, $2 \leq |S| \leq 2n - 4$, the lemma holds by Lemma 9. So, let $k \geq 1$.

By Lemma 3, we know that $FPQ_{n,k}$ can be viewed as $G(FPQ_{n-1,k}, FPQ_{n-1,k} : M)$, where M is a perfect matching between two $FPQ_{n-1,k}$'s. In other words, $FPQ_{n,k}$ contains two copies of $FPQ_{n-1,k}$, denoted by $FPQ^0_{n-1,k}$ and $FPQ^1_{n-1,k}$, respectively. For $i \in \{0, 1\}$, let $S_i = S \cap V_{FPQ^i_{n-1,k}}$. W.l.o.g., we may assume that $|S_0| \geq |S_1|$.

If $|S| = 2$, the result holds by Lemma 7. Next, let $3 \leq |S| \leq 2n + 6k - 4$. It implies that $|S_0| \geq 2$. We distinguish the following two cases.

Case 1. Let $2 \leq |S_0| \leq 2n + 6k - 6$.

By the induction hypothesis in $FPQ^0_{n-1,k}$,

$$\left| N_{FPQ^0_{n-1,k}}(S_0) \right| \geq 2n + 6k - 4. \tag{19}$$

If $|S_1| = 0$, then, $S = S_0$. It leads to

$$\begin{aligned} \left| N_{FPQ_{n,k}}(S) \right| &\geq \left| N_{FPQ^0_{n-1,k}}(S_0) \right| + \left| N_{\overline{FPQ}^0_{n1,k}}(S_0) \right| \\ &\geq (2n + 6k - 4) + 2 = 2n + 6k - 2. \end{aligned} \tag{20}$$

If $|S_1| = 1$, then $|N_{FPQ^1_{n-1,k}}(S_1)| = \kappa(FPQ^1_{n-1,k}) = n + 3k - 1$ .Thus,

$$\begin{aligned} \left| N_{FPQ_{n,k}}(S) \right| &\geq \left| N_{FPQ^0_{n-1,k}}(S_0) \right| + \left| N_{FPQ^1_{n-1,k}}(S_1) \right| \\ &\geq (2n + 6k - 4) + (n + 3k - 1) \\ &\geq 2n + 6k - 2. \end{aligned} \tag{21}$$

It remains to assume that $2 \leq |S_1| \leq |S_0| \leq 2n + 6k - 6$. Then,

$$\left| N_{FPQ^1_{n-1,k}}(S_1) \right| \geq 2n + 6k - 4. \tag{22}$$

Thus,

$$\begin{aligned} \left| N_{FPQ_{n,k}}(S) \right| &\geq \left| N_{FPQ^0_{n-1,k}}(S_0) \right| + \left| N_{FPQ^1_{n-1,k}}(S_1) \right| \\ &\geq 2(2n + 6k - 4) \geq 2n + 6k - 2. \end{aligned} \tag{23}$$

Case 2. Let $2n + 6k - 5 \leq |S_0| \leq 2n + 6k - 4$.

In this case, we have

$$\begin{aligned} &\left| V_{FPQ^0_{n-1,k}} \right| - (2n + 6k - 4) \\ &= 10^k 2^{n-1} - (2n + 6k - 4) \\ &\geq n + 3k - 1 = \kappa\left(FPQ^0_{n-1,k}\right), \end{aligned} \tag{24}$$

where the inequality follows from Lemma 5 (1). By Lemma 6, $|N_{FPQ^0_{n-1,k}}(S_0)| \geq n + 3k - 1$.

If $S = S_0$, then,

$$\begin{aligned} \left| N_{FPQ_{n,k}}(S) \right| &= \left| N_{FPQ^0_{n-1,k}}(S_0) \right| + \left| N_{FPQ^1_{n-1,k}}(S_0) \right| \\ &\geq n + 3k - 1 + (2n + 6k - 5) \\ &= 3n + 9k - 6 \geq 2n + 6k - 2. \end{aligned} \tag{25}$$

If $S \neq S_0$, then, $|S_1| = 1$. Thus,

$$\begin{aligned} \left| N_{FPQ_{n,k}}(S) \right| &\geq \left| N_{FPQ^0_{n-1,k}}(S_0) \right| + \left| N_{FPQ^1_{n-1,k}}(S_1) \right| \\ &\geq n + 3k - 1 + (n + 3k - 1) = 2n + 6k - 2. \end{aligned} \tag{26}$$

3.2. T-Diagnosability of $FPQ_{n,k}$. In what follows, we will discuss the diagnosability of $FPQ_{n,k}$ under the PMC model.

Lemma 11. *Let $FPQ_{n,k}$ be the folded Petersen cube for $n \geq 0$, $k \geq 0$ and $(n, k) \neq (0, 0)$. Then, $t(FPQ_{n,k}) \leq n + 3k$ under the PMC model.*

Proof. Let v_0 be any vertex in $FPQ_{n,k}$. Let $L_1 = N_{FPQ_{n,k}}(v_0)$ and $L_2 = N_{FPQ_{n,k}}(v_0) \cup \{v_0\}$. Assume that both L_1 and L_2 are faulty sets. Obviously, $|L_1| = n + 3k < n + 3k + 1$, $|L_2| = n + 3k + 1$, $L_1 \cup L_2 = L_2 \neq V_{FPQ_{n,k}}$, and $L_1 \Delta L_2 = \{v_0\}$. If L_1 and L_2 are distinguishable under the PMC model, then, there exists some vertex $x \in V(FPQ_{n,k}) \setminus (F_1 \cup F_2)$ such that $(x, v_0) \in E(FPQ_{n,k})$, a contradiction. Thus, L_1 and L_2 are indistinguishable under the PMC model. It leads to $t(FPQ_{n,k}) \leq |L_2| - 1 = n + 3k$. □

Lemma 12. *Let $FPQ_{n,k}$ be the folded Petersen cube for $n \geq 0$, $k \geq 0$ and $(n, k) \neq (0, 0)$. Then, $t(FPQ_{n,k}) \geq n + 3k$ under the PMC model.*

Proof. Let L_1 and L_2 be two distinct faulty sets such that $|L_1|, |L_2| \leq n + 3k$. Since L_1 and L_2 are distinct, $L_1 \Delta L_2 \neq \varnothing$, which implies that $L_2 \setminus L_1 \neq \varnothing$ or $L_1 \setminus L_2 \neq \varnothing$. W.l.o.g., let $L_1 \setminus L_2 \neq \varnothing$. To complete the proof, it is sufficient to show that L_1 and L_2 are distinguishable under the PMC model.

Suppose to the contrary that L_1 and L_2 are indistinguishable. If $V_{FPQ_{n,k}} \setminus (L_1 \cup L_2) = \varnothing$, then, $L_1 \cup L_2 = V_{FPQ_{n,k}}$. Since $|V_{FPQ_{n,k}}| = 10^k 2^n$, we have $|L_1| + |L_2| \geq |L_1 \cup L_2| = 10^k 2^n > 2(n + 3k)$, a contradiction to the assumption. Thus, $V_{FPQ_{n,k}} \setminus (L_1 \cup L_2) \neq \varnothing$. Since the assumption that L_1 and L_2 are indistinguishable under the PMC model, there exist no edges between $L_1 \Delta L_2$ and $V_{FPQ_{n,k}} \setminus (L_1 \cup L_2)$. Note that $FPQ_{n,k}$ is connected. So, $L_1 \cap L_2$ is a vertex cut of $FPQ_{n,k}$ and $|L_1 \cap L_2| \geq \kappa(FPQ_{n,k}) = n + 3k$. Recall that $L_1 \setminus L_2 \neq \varnothing$, i.e., $|L_1 \setminus L_2| \geq 1$. Thus,

$$|L_1| = |L_1 \setminus L_2| + |L_1 \cap L_2| \geq n + 3k + 1, \tag{27}$$

a contradiction. □

The following theorem follows directly from the previous two lemmas.

Theorem 13. *Let $FPQ_{n,k}$ be the folded Petersen cube for $n \geq 0$, $k \geq 0$ and $(n,k) \neq (0,0)$. Then, $t(FPQ_{n,k}) = n + 3k$ under the PMC model.*

The following corollaries are a straightforward consequence of Theorem 13.

Corollary 14. *Let Q_n be the n -dimensional hypercube. Then, $t(Q_n) = n$ under the PMC model.*

Corollary 15. *Let FP_k be the k-dimensional folded Petersen graph. Then, $t(FP_k) = 3k$ under the PMC model.*

Corollary 16. *Let $HP_n = P\square Q_{n-3}$ (where $n \geq 3$) be the hyper Petersen graph. Then, $t(HP_n) = n$ under the PMC model.*

3.3. Pessimistic Diagnosability of $FPQ_{n,k}$. Chwa and Hakimi [36] derived a characterization for a graph to be t/t diagnosability.

Lemma 17 (see [36]). *Let $G = (V_G, E_G)$ be the representation of a system. G is t/t diagnosable if and only if for each integer p with $1 \leq p \leq t - 1$ and each $S^* \subseteq V_G$ with $|S^*| = 2(t-p)$; it holds that $|N_G(S^*)| > p$.*

Tsai and Chen [37] further derived another characterization for a graph to be t/t diagnosability.

Lemma 18 (see [37]). *A graph $G = (V_G, E_G)$ is t/t diagnosable if and only if for each set $S^* \subseteq V_G$ with $0 \leq |S^*| = p \leq t - 1$; the graph $G - S^*$ has at most one trivial component, and each nontrivial component $\mathscr{C}$ of $G - S^*$ satisfies $|V(\mathscr{C})| \geq 2(t-p) + 1$.*

Lemma 19. *Let $FPQ_{n,k}$ be the folded Petersen cube for $n \geq 0$, $k \geq 0$, and $(n,k) \neq (0,0)$. Then, for each $S^* \subseteq V_{FPQ_{n,k}}$ with $|S^*| = 2(2n + 6k - 2 - p)$ and $1 \leq p \leq 2n + 6k - 3$, it holds that $|N_{FPQ_{n,k}}(S^*)| > p$.*

Proof. We consider the following two cases according to the value of p. □

Case 1. Let $1 \leq p \leq n + 3k - 1$.

By the assumption, $2(n + 3k - 1) \leq |S^*| \leq 2(2n + 6k - 3)$. Moreover, $\kappa(\mathrm{FPQ}_{n,k}) = n + 3k$ and $10^k 2^n - 2(2n + 6k - 3) \geq (n + 3k)$ by Lemma 5 (3). Hence, we can deduce that

$$\left|V_{\mathrm{FPQ}_{n,k}}\right| - |S^*| \geq 10^k 2^n - 2(2n + 6k - 3) \geq n + 3k = \kappa(\mathrm{FPQ}_{n,k}). \tag{28}$$

By Lemma 6, $|N_{\mathrm{FPQ}_{n,k}}(S^*)| \geq \kappa(FPQ_{n,k}) = n + 3k > p$.

Case 2. Let $n + 3k \leq p \leq 2n + 6k - 3$.

By the assumption, $2 \leq |S^*| \leq 2n + 6k - 4$. By Lemma 8, $|N_{\mathrm{FPQ}_{n,k}}(S^*)| \geq 2n + 6k - 2 > p$.

Theorem 20. *Let $FPQ_{n,k}$ be the folded Petersen cube for $n \geq 0$, $k \geq 0$ and $(n,k) \neq (0,0)$. Then, $t_p(FPQ_{n,k}) = 2n + 6k - 2$.*

Proof. By Lemmas 17 and 19, $t_p(\mathrm{FPQ}_{n,k}) \geq 2n + 6k - 2$. It remains to show that $t_p(\mathrm{FPQ}_{n,k}) \leq 2n + 6k - 2$ for $n \geq 0$, $k \geq 0$ and $(n,k) \neq (0,0)$. □

Suppose to the contrary that $t_p(\mathrm{FPQ}_{n,k}) \geq 2n + 6k - 1$. Let (u_0, v_0) be an edge in $\mathrm{FPQ}_{n,k}$ and let $S = N_{\mathrm{FPQ}_{n,k}}(\{u_0, v_0\})$. Since $\mathrm{FPQ}_{n,k}$ has no triangles, $|S| = 2n + 6k - 2 \leq t_p(\mathrm{FPQ}_{n,k}) - 1$. The subgraph $\mathscr{C}$ induced by $\{u_0, v_0\}$ is a connected component of $\mathrm{FPQ}_{n,k} - S$. By Lemma 18,

$$\begin{aligned} |V_{\mathscr{C}}| &\geq 2\left(t_p(\mathrm{FPQ}_{n,k}) - |S|\right) + 1 \\ &\geq 2((2n + 6k - 1) - (2n + 6k - 2)) + 1 = 3, \end{aligned} \tag{29}$$

a contradiction.

The following corollaries are obvious from Theorem 20.

Corollary 21. *Let Q_n be the n-dimensional hypercube. Then, $t_p(Q_n) = 2n - 2$.*

Corollary 22. *Let FP_k be the k-dimensional folded Petersen graph. Then, $t_p(FP_k) = 6k - 2$.*

Corollary 23. *Let $HP_n = P\square Q_{n-3}$ (where $n \geq 3$) be the hyper Petersen graph. Then, $t_p(HP_n) = 2n - 2$.*

4. Concluding Remarks

In this paper, by exploring structural properties of the folded Petersen cubes $\mathrm{FPQ}_{n,k}$, we prove that $\mathrm{FPQ}_{n,k}$ is $(n + 3k)$ diagnosasable under the PMC model. Moreover, we study the pessimistic diagnosability of folded Petersen cubes and obtain $t_p(\mathrm{FPQ}_{n,k}) = 2n + 6k - 2$ for $n \geq 0$, $k \geq 0$ and $(n,k) \neq (0,0)$. As corollaries, the diagnosability and the pessimistic diagnosability of hypercube, folded Petersen graph, and the hyper Petersen graph are obtained. Another direction of our study in this paper is to investigate the conditional diagnosability of these graphs.

Data Availability

No data were used to support this study.

Disclosure

This work was accomplished while the first author was visiting Zhejiang Normal University.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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