Research Article

Analysis of Metallic Nanoparticles (Cu, Al₂O₃, and SWCNTs) on Magnetohydrodynamics Water-Based Nanofluid through a Porous Medium

P. K. Pattnaik,¹ S. K. Parida,² S. R. Mishra,³ M. Ali Abbas,⁴ and M. M. Bhatti⁵

¹Department of Mathematics, Odisha University of Technology and Research, Bhubaneswar 751029, Odisha, India

²Department of Physics, Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar 751030, Odisha, India

³Department of Mathematics, Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar 751030, Odisha, India

⁴Department of Mathematics, University of Baltistan Skardu, Gilgit-Baltistan 16100, Pakistan

⁵College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong 266590, China

Correspondence should be addressed to M. M. Bhatti; mmbhatti@sdust.edu.cn

Received 15 October 2021; Revised 29 December 2021; Accepted 7 January 2022; Published 14 February 2022

Academic Editor: Fairouz Tchier

Copyright © 2022 P. K. Pattnaik et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this communication, the effect of the addition of the copper (Cu), aluminum oxide (Al₂O₃), and single-wall carbon nanotubes (SWCNTs) metallic nanoparticles on the magnetohydrodynamics (MHD) water-based flow over a porous elastic surface is explored. The objective of the work is to include the radiative effect that interacts with the metallic nanoparticles due to permeability of the surface. The significance of this study stems from the fact that the design of various equipment, such as nuclear power plants, gas turbines, propulsion devices for aircraft, and missiles, is dependent on radiative heat transfer. To formulate the mathematical modelling, similarity transformations were used, and nonlinear differential equations were obtained. To solve the formulated nonlinear differential equations, the Runge–Kutta fourth-order numerical scheme is used in conjunction with the shooting technique. The behavior of velocity profile and temperature profile has been discussed in detail and also engineering quantities such as Nusselt and Sherwood number which are calculated. Furthermore, the addition of metallic nanoparticles enhanced the nanofluid properties for energy transfer enrichment and found many applications in various fields of science and technology.

1. Introduction

Choi introduced a new type of fluid called nanofluid in 1995, which has amazing thermal conductivity properties. The goal of the concept is to saturate nanosized particles in conventional fluids known as base fluids. Nanofluids are extremely important in thermal conductivity, heat transfer enhancement, energy, and other thermos-physical properties for industrial applications [1–5]. The heat transfer capacity of a nanofluid after the addition of metallic and nonmetallic nanoparticles in a conventional base fluid was of particular interest to the researchers. Mohebbi et al. [6] investigated the mathematical model of the Heat Transfer Augmentation Associated with Cu/Water Nanofluid in a

Channel with Surface Mounted Blocks by using Lattice Boltzmann Method. The numerical method is applied for the forced convection flow and heat transfer of a nanofluid flowing inside a straight circular pipe by Saryazdi et al. [7]. Moreover, Baag and Mishra [8] discussed heat and mass transfer analysis on MHD 3D water-based nanofluid. The prominent examples of nanofluids are ethylene glycols, kerosene, and water. It has been observed that conducting nanofluids presents their special attention because of their use in diversified areas such as biomedical solicitation as tuneable optical filters, drug delivery, and cancer therapy. Watanabe and Pop [9] deliberately presented the magnetohydrodynamic flow of particular fluid for the occurrence of applied magnetic field through a flat plate. Numerical

treatment is depicted by Armaghani et al. [10] for the mixed convective flow phenomena of nanofluid within open C-shaped enclosures. For the enhanced properties, they have used CuO nanoparticles which are dispersed within the base fluid water and the enclosure is imposed with constant magnetic field. Furthermore, the influential behavior of the characterizing parameters such as Richardson number and volume concentration affects the flow phenomena as well. The work of Ibrahim and Terbeche [11] leads to bring out the effective properties of the non-Newtonian power-law fluid with due occurrence of the magnetic field. Analytical approach is employed for the solution of the designed problem and numerical methods are useful for the validation of the current result and the convergence criterion.

Fluid flow and heat transfer with non-Newtonian fluids, for example, are a challenge in the modern revolution, particularly in the oil industry, bubble columns and absorption, zymosis, boiling, plastic foam processing [12], etc. However, the possible applications relating to this type of flow can be observed in various industries. The generation of electric power in the corresponding electric power industry is one of the examples that uses the extraction of energy. The governing equations for different non-Newtonian fluid models are amid the utmost complex equations so that the development in mathematical modelling is of great interest nowadays. A time-dependent flow characterized by the several parameters for the nanofluids past an expanding sheet is presented by Andersson et al. [13]. Furthermore, similarity approach for the complex unsteady flow problem past over an expanding sheet is carried out by Elbashbeshy and Bazid [14]. Thermophoresis and Brownian motion effect on the flow of nanofluid through a vertical plate has been studied by Kuznetsov and Nield [15]. They pointed out that the cooling rate of the plate decreases due to decrement in strengthens of thermophoresis and Brownian motion. Heidary and Kermani [16] studied the effect of solid volume fraction of nanofluid and magnetic strength. They examined that existence of magnetic field and nanofluid could significantly enhance heat transfers properties of the flow phenomena. The thermal properties of the base fluids change appreciable after addition of the metallic nanoparticles and calculate the thermos-physical parameters [17]. Masuda et al. [18] reported that, after addition of ultrafine nanoparticles, there is an alteration in the thermal conductivities and viscosities. Mishra et al. [19] recently studied a chemically reactive nano-micropolar fluid with variable heat sink/source and slip conditions. Shutaywi and Shah [20] proposed a numerical and mathematical model of a nanofluid that includes entropy formation.

The application of electrically conductive fluid currents is encircled in the field of nanocomposite and metallurgy. The flows of several fluids under the action of magnetic field such as MHD generators, oil exploration, energy extraction, and boundary layer control have attracted many researchers. Metallurgical requirements consist of continuous cooling belts or filaments such as hardening, disperse, and sketching processes for copper wires. It has been noticed that the effects of Coriolis force are larger than those of viscosity and inertia forces in the hydro-magnetic equations of motion in

a rotating environment. Several researchers have been investigated on MHD with various kinds of fluid and geometries. For example, Ibrahim and Negera [21] investigated the upper-convected Maxwell nanofluid flow with slip and MHD effects through a stretching sheet and chemical reaction. Abdal et al. [22] examine the thermo-diffusion with magnetized mixed convection unsteady nanofluid flow through stretching/shrinking surface with heat source and thermal radiation. Ghasemi and Hatami [23] described the solar radiation effects on magnetized stagnation point nanofluid flow through a stretching surface. Some important references related to the proposed topic can be found in [24–28] and several therein. Recently, Upreti et al. [29, 30] considered carbon nanotube nanofluids for the behavior of various physical quantities in different geometries. They have projected the effect of drag force with an interaction of Joule heating and nonuniform heat source/sink. Also, binary chemical reaction with the impact of radiative heat on the flow phenomena over an expanding surface is considered. Sabu et al. [31] investigated the enhancement of heat transfer caused by a thermal and space dependent heat source, magnetic field, and nanoparticles propagating over an elastic spinning disk. Mahanthesh et al. [32] investigated Reiner–Rivlin nanofluid flow through a rotating disk with multiple slips and a distinct heat source.

Therefore, the primary goal of this research is to determine the presence of three metallic nanoparticles (Cu, Al_2O_3 , and SWCNTs) in an electrically conducting water-based nanofluid propagating through a porous medium. Thermal radiation is important in industrial applications. As a matter of fact, the study's novelty stems from the incorporation of thermal radiation as well as an additional heat source/sink within a permeable medium. The mathematical modelling was developed using similarity transformations. The nonlinear differential equations are solved using the Runge–Kutta and shooting techniques. When compared to other similar methods used for nonlinear problems, the current numerical method yields promising results [33, 34]. The graphical interpretation of the velocity and temperature profiles have been discussed in detail and expected results show the excellent industrial applications.

2. Problem Formulation

The time-dependent electrically conducting flow of nanofluids through a permeable medium is presented in this article. For the enhanced feature in heat transfer attempt is made to consider SWCNTs in the water-based nanofluid along with Cu and Al_2O_3 nanoparticles. Moreover, the novelty of the study arises for the inclusion of radiative heat transfer with additional external heat source/sink that enriches the energy profile. The flow through porous elastic surface along the x -direction and the transverse magnetic field of uniform strength B_0 is proposed along the normal direction of the surface, i.e., y -direction, as given in Figure 1. Due to permeability of the surface, the occurrence of suction/injection has its immense use on the flow phenomena. Following Zhang et al. [35], the proposed assumptions lead to design the model with the boundary conditions as

FIGURE 1: Flow configuration.

$$\begin{aligned}
 \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
 \rho_{nf} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial p}{\partial x} + \mu_{nf} \frac{\partial^2 u}{\partial y^2} \\
 &\quad - \left(\frac{\mu_{nf}}{K} + \sigma_{nf} B_0^2 \right) u, \\
 (\rho c_p)_{nf} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= k_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \\
 &\quad + Q_0 (T - T_\infty),
 \end{aligned} \tag{1}$$

with boundary conditions,

$$\left. \begin{aligned}
 u(x, 0, t) &= 0, \\
 v(x, 0, t) &= v_0(t), \\
 -k_{nf} \frac{\partial T(x, 0, t)}{\partial t} &= q(x), \\
 u(x, \infty, t) &= U(x, t), \\
 T(x, \infty, t) &= T_\infty.
 \end{aligned} \right\} \tag{2}$$

Here, u and v , are the components of velocities along x - and y - direction, T is the temperature of the nanofluid, t is the time taken, p is the fluid pressure, v_0 is a constant, and σ_s and σ_f are the electrical conductivity of the base and nanofluid, respectively.

The physical properties relating to nanofluid such as viscosity, specific heat, density, and conductivity are presented as follows [36]:

$$\left. \begin{aligned}
 \mu_{nf} &= \frac{\mu_f}{(1 - \phi)^{2.5}}, & \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s, & (\rho c_p)_{nf} &= (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s, \\
 \sigma_{nf} &= (1 - \phi)\sigma_f + \phi\sigma_s, & \frac{k_{nf}}{k_f} &= \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)},
 \end{aligned} \right\} \tag{3}$$

where ϕ is the particle concentration, μ_f is the dynamics viscosity, ρ_f and ρ_s are the densities, k_f and k_s are the

thermal conductivities, and the subscripts f and s are for the base fluid and the solid nanoparticles.

$$\rho_{nf} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \left(\frac{\mu_{nf}}{K} + \sigma_{nf} B_0^2 \right) (u - U), \tag{4}$$

where

$$\frac{\partial p}{\partial x} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} - \left(\frac{\mu_{nf}}{K} + \frac{\sigma_{nf}}{\rho_{nf}} B_0^2 \right) U, \tag{5}$$

Here, K is the permeability of the given medium and B_0 is an external magnetic field strength.

The stream function as well as variables for the problem is expressed as (see [36])

$$\begin{aligned}
u &= \frac{\partial \psi}{\partial y}, \\
v &= -\frac{\partial \psi}{\partial x}, \\
\eta &= y \sqrt{\frac{a}{v_f}}, \\
\psi &= \sqrt{av_f} x f(\eta), \\
a &= \frac{1}{s + bt}, \\
T &= T_\infty + \frac{q_0 x}{k_f} \sqrt{\frac{v_f}{a}} \theta(\eta), \\
q(x) &= q_0 x, \\
v_0(t) &= v_i(t).
\end{aligned} \tag{6}$$

Using the aforesaid functional expressions, the governing equations are presented as

$$\begin{aligned}
f''' + A_1 \left[\left(f + \frac{b\eta}{2} \right) f'' - f'^2 + b(f'^2 - 1) + 1 \right] \\
+ (MA_2 + Da)(1 - f') = 0, \\
\frac{1}{A_3 Pr} \left(\frac{k_{nf}}{k_f} + Nr \right) \theta'' + \left(f + \frac{b\eta}{2} \right) \theta' - \left(f' + \frac{b}{2} + \frac{\delta}{A_3} \right) \theta = 0,
\end{aligned} \tag{7}$$

where $M = (\sigma_f / \rho_f b) B_0^2$ is the magnetic parameter, $Pr = v_f / \kappa$ is the Prandtl number, $Nr = 16\sigma_1 T_\infty^3 / 3\kappa k_f$ is the thermal radiation parameter, $Da = v_f / aK$ is the Darcy number with boundary condition, b is the unsteadiness parameter, and $\delta = q_0 x / (\rho c_p)_f U$ is the heat source parameter.

$$\left. \begin{aligned}
f(0) &= fw, \\
f'(0) &= 0, \\
\theta'(0) &= -\frac{k_f}{k_{nf}}, \\
f'(\infty) &\longrightarrow 1, \\
\theta(\infty) &\longrightarrow 0,
\end{aligned} \right\} \tag{9}$$

where

$$\begin{aligned}
A_1 &= (1 - \phi)^{2.5} \left(1 - \phi + \phi \frac{\rho_s}{\rho_f} \right), \\
A_2 &= (1 - \phi)^{2.5} \left(1 - \phi + \phi \frac{\sigma_s}{\sigma_f} \right), \\
A_3 &= 1 - \phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f}.
\end{aligned} \tag{10}$$

The physical quantities are as follows: $C_f = \tau_w / \rho_f U^2$ is called as skin friction coefficient and $Nu_x = q_w x / k_f (T_w - T_\infty)$ is called local Nusselt number:

$$\begin{aligned}
C_f Re_x^{0.5} &= \frac{f''(0)}{(1 - \phi)^{2.5}}, \\
Nu_x Re_x^{-0.5} &= -\left(\frac{k_{nf}}{k_f} + Nr \right) \theta'(0).
\end{aligned} \tag{11}$$

3. Numerical Methodology

For solving equations (10)–(12), a multistep integration method, i.e., the Runge–Kutta method, with shooting technique has been deployed. In this process, equations (7) and (8) are reduced to a set of ordinary differential equations as defined below:

$$\begin{aligned}
f &= y_1, f' = y_2, f'' = y_3, f''' \\
&= -A_1 \left[\left(y_1 + \frac{b\eta}{2} \right) y_3 - y_2^2 + b(y_2^2 - 1) + 1 \right] \\
&\quad - (MA_2 + Da)(1 - y_1), \\
\theta &= y_4, \theta' = y_5, \theta'' = A_3 Pr \left(-\left(y_1 + \frac{b\eta}{2} \right) y_5 \right. \\
&\quad \left. + \left(y_2 + \frac{b}{2} + \frac{\delta}{A_3} \right) y_4 \right) \left(\frac{k_{nf}}{k_f} + Nr \right)^{-1},
\end{aligned} \tag{12}$$

under the boundary condition,

$$\left. \begin{aligned}
y_1 &= fw, \\
y_2 &= 0, \\
y_5 &= -\frac{k_f}{k_{nf}}, \\
y_2 &\longrightarrow 1, \\
y_4 &\longrightarrow 0.
\end{aligned} \right\}. \tag{13}$$

Now, it uses only the initial conditions, i.e., for $\eta = 0$. However, due to the unavailability of initial conditions, the assumed initial conditions y_3 and y_4 are to be determined. Therefore, some initial guesses are incorporated to both these values in order to satisfy the boundary conditions at $\eta \rightarrow \infty$ ($\eta = 4$). These corrections are taken care by a corrective procedure that follows a self-iterative process. This procedure is used to implement a more accurate method, i.e., the RK method, with shooting numerical technique. For the computational purpose, the step size is assumed as $h = 0.01$. Therefore, the accuracy of computation and the convergence criteria are followed.

4. Results and Discussion

An unsteady two-dimensional flow of metallic water-based nanofluids is considered which past a permeable medium for the action of transverse magnetic field is presented. Interaction of Cu and Al_2O_3 nanoparticles along with SWCNTs in base fluid water is dispersed to prepare nanofluid. Incorporation of radiative heat energy enriches the profile in conjunction to the permeable surface. Numerical technique is used to find the solution of the set of equations for the suitable choice of the pertinent parameters. Table 1 displays all the physical properties of both the particles as well as the base fluid. Table 2 present the validation of the present outcomes for the shear rate considering the case of pure fluid as well as the case of nanofluid with the work of Rizwan et al. [36], and this shows a good corroboration. The graphical illustration shows the significant behavior of these parameters associated with the flow phenomena. Furthermore, the tabular simulated results indicate the rate coefficients, i.e., shear rate and Nusselt number. However, throughout the computation, the following values of the parameters are considered as fixed whereas the variation of particular parameters are presented in the corresponding figures, and these are $\phi = 0.2$, $b = 0.1$, $M = 1$, $Da = 1$, $fw = 0.5$, and $\delta = 1$.

The role of particle concentration due to its appearance through the thermo-physical properties is a vital part of this investigation. Figure 2 describes the significance of particle concentration on the velocity for the Cu, Al_2O_3 , and SWCNT-water-based nanofluids. Several characteristics of the suction/injection on each profile are displayed. Here, the parameter $fw > 0$ represents the role of suction whereas $fw < 0$ indicates the injection and $fw = 0$ characterizes the behavior when the flow through impermeable region. The decelerating nature of the profiles shows the increasing width of the bounding surface thickness for the increasing particle concentration. The range of the concentration is treated within $\phi = [0.0, 0.2]$. The impermeability region for the pure fluid is similar to the results obtained by Mishra et al. [19], and it can be obtained by considering $fw = 0$ and $\phi = 0$. Furthermore, the increasing suction enriches the profiles, but the thickness of the bounding surface decreases; however, injection reveals opposite impact on the profile. It reveals the density of the Cu particles and diminishes the profile width in comparison to the particles of Al_2O_3 and SWCNTs. Figure 3 illustrates the behavior of the

TABLE 1: Thermos-physical properties of base fluid and nanoparticles.

	ρ (kg/m ³)	c_p (J/kg K)	k (W/mK)
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401
Aluminum oxide (Al_2O_3)	3970	765	40
SWCNTs	2600	425	6600

TABLE 2: Validation of shear rate.

Nanofluids	ϕ	$f''(0)$ Rizwan et al. [36]	$f''(0)$ Present
Cu-H ₂ O	0.0	1.48113419	1.481021
	0.1	1.71105504	1.711003
	0.2	1.75138728	1.751128
Al_2O_3 -H ₂ O	0.0	1.48113419	1.481021
	0.1	1.43438455	1.431625
	0.2	1.33096758	1.330727
SWCNT-H ₂ O	0.0	1.48113419	1.481021
	0.1	1.45088235	1.450122
	0.2	1.35571879	1.355526

unsteadiness parameter in association with the suction/injection on the nanofluid velocity profiles. Here, $b \neq 0$ indicates the unsteady case on the velocity of the three different water-based nanofluid. An augmentation in the profiles is rendered for the increasing unsteadiness that causes a deceleration in the bounding surface thickness. The profiles of SWCNT nanofluid are lesser than the other nanoparticles of Al_2O_3 and Cu, respectively. Also, for each of the profiles, interestingly, the thickness decelerates more in case of injection in comparison to impermeable and the case of suction successively. Figure 4 portrays the role of magnetic parameter on the nanofluid velocity profiles with the interaction suction/injection. The magnetic field expresses the influence of moving electric charges along with electric current and magnetic materials. In modern technology, there are various applications of magnetic field such as in both the electric motors and generators; the use of rotating magnetic field is important. The profile augments lead to decelerate the thickness of the velocity-bounding surface for the augmented magnetic parameter. This is due to the resistance offered by the resistive force produced with the interaction of magnetic parameter, i.e., the Lorentz force. It is seen that SWCNTs have greater retardation than that of Al_2O_3 and Cu-water nanofluid. However, suction also favors to decelerate the profile significantly than that of injection. Furthermore, Figure 5 examines the significance of the permeability parameter on the nanofluid velocity distribution. Similar to the magnetic parameter, resistive force offered by porosity also causes a similar behavior on the velocity profiles for each of the nanofluids. The influence of the suction/injection has the same tendency on the profiles as described in the earlier description. The control of suction/injection due to the permeable surface is shown in Figure 6 for the velocity distribution of nanofluids. Generally, the pressure differential occurs by the elimination of air from the space. Therefore, the limited pressure is exerted

FIGURE 2: Variation of ϕ on velocity profile. FIGURE 3: Variation of b on velocity profile.

by the external air. The pressure in one part of the system is reduced in comparison to another; there will be force exerts from the fluid of higher pressure region to lower. However, with escalating suction, the pressure increases, and this leads to decelerate the surface thickness, whereas impact is reversed for the case of injection. The case of impermeability is a particular case which validates with the earlier result. The significant characteristics of the controlling parameters on the fluid temperature is observed and presented. The role of these parameters enhances the thermos-physical properties

significantly. Therefore, the current study discloses the properties of particle concentration, magnetic and porosity parameters, suction/injection, and unsteadiness parameter. Figure 7 displays the role of particle concentration on the nanofluid temperature with an interaction of suction/injection. The three-layer variation explains the distribution of different parameters on the Cu, Al_2O_3 , and SWCNT-water-based nanofluids, respectively. Furthermore, the fluid temperature boosts its maximum trend in case of Cu-water nanofluid since it is well known that Cu is a good conductor

FIGURE 4: Variation of M on velocity profile. FIGURE 5: Variation of Da on velocity profile.

of heat. Furthermore, with the increase in volume fraction, the profile decelerates significantly. Moreover, suction produces more energy to boost the profile rather than the impermeability of the surface and the case of injection. Figure 8 demonstrates the role of unsteadiness parameter that has important characteristics on the nanofluid temperature. Again, increasing unsteadiness, the fluid temperature decelerates in an order of preference such as Cu, Al_2O_3 , and SWCNT-water nanofluid. Therefore, it suggests that effectiveness of the Cu nanoparticle is higher than that of

other nanoparticles presented in this study. This gives a suggestive measure for the increasing thermal properties of the Cu-water nanofluid since the proposed thermal conductivity of the nanofluid enhances due to increase in particle concentration. Figure 9 exhibits the effects of the heat source on the nanofluid temperature distribution for different suction/injections. The inclusion of additional heat suppresses the fluid temperature. In a comparative analysis, it is marked that the Cu-water nanofluid exhibits its maximum strength than other nanofluids. However, no

FIGURE 6: Variation of f_w on velocity profile. FIGURE 7: Variation of ϕ on temperature profile.

significant change is marked for the variation of suction/injection. Figure 10 depicts the behavior of the thermal radiation in conjunction to the other contributing parameters on the temperature distributions of the nanofluids. Thermal radiation is due to the release of the electromagnetic waves from the fluid particles that is nothing but the renovation of thermal energy into the electromagnetic energy. With an increase in thermal radiation, the profile rises up and therefore the fluid temperature boosts up. This is because most of the solids and fluids are considered to be the

surface phenomena and the interior molecules help to emit the radiations.

Finally, the simulated results of the rate coefficients for several contributing parameters are obtained and presented in Table 3. The nanoparticle concentration enhances the shear rate coefficients whereas heat transfer rate decreases in magnitude. From the tabular results, it is quite clear to see that the rate coefficients are much higher in case of Cu-water nanofluid in comparison to other nanofluids. Furthermore, the resistive forces such as magnetic and porosity of the

FIGURE 8: Variation of b on temperature profile.

FIGURE 9: Variation of δ on temperature profile.

FIGURE 10: Variation of Nr on temperature profile.

TABLE 3: Numerical computations of physical quantities used in engineering applications.

ϕ	M	Da	b	fw	Pr	δ	Nr	Cu-water		Al_2O_3 -water		SWCNTs-water	
								C_f	Nu_x	C_f	Nu_x	C_f	Nu_x
0.1	1	0.1	0.5	0.5	6.2	1	0.5	2.6073	-4.3891	2.2782	-4.3874	2.1858	-4.2266
0.15								3.0849	-3.7691	2.5915	-3.7728	2.4523	-3.5700
0.2								3.6065	-3.2190	2.9466	-3.2312	2.7597	-3.0029
0.1	2							2.9684	-4.4195	2.6735	-4.4239	2.5920	-4.2637
	3							3.2863	-4.4443	3.0137	-4.4528	2.9387	-4.2927
	1	0.2						2.6450	-4.3924	2.3200	-4.3914	2.2290	-4.2308
		0.3						2.6820	-4.3956	2.3610	-4.3953	2.2713	-4.2348
		0.1	0.1					2.8167	-4.1521	2.4481	-4.1439	2.3432	-3.9932
			0.3					2.7139	-4.2726	2.3645	-4.2678	2.2657	-4.1119
			0.5	-0.5				1.5958	-2.1490	1.5570	-2.1602	1.5442	-2.1508
				0				2.0563	-3.1186	1.8913	-3.1214	1.8433	-3.0520
				0.5	1			2.6073	-1.5644	2.2782	-1.5587	2.1858	-1.5107
					2			2.6073	-2.2670	2.2782	-2.2602	2.1858	-2.1872
					6.2	2		2.6073	-5.0817	2.2782	-5.0971	2.1858	-4.9396
						3		2.6073	-5.6848	2.2782	-5.7129	2.1858	-5.5553
						1	0.1	2.6073	-4.0441	2.2782	-4.0408	2.1858	-3.8926
							0.3	2.6073	-4.2204	2.2782	-4.2180	2.1858	-4.0634

medium favors to enhance the shear rate and opposite trend is rendered for the heat transfer rate. An increase in suction enriches the rate coefficients significantly.

5. Conclusion

The radiative heat transport phenomenon on the two-dimensional flow of water-based nanofluids over an elastic surface is carried out in the current investigation. Here, the electrically conducting nanofluid past a porous surface embedding with porous matrix is presented. The effect of heat source is also included to examine the heat transfer properties. Numerical approach is employed for the solution

of the flow phenomena designed by the proposed model. Furthermore, the important characteristics of the physical parameters are laid down here:

- Comparative analysis shows a pathway for the further investigation of the current problem under study for the behavior of several nanoparticles in the water-based fluid with the interaction of various characterizing parameters.
- Particle concentration decelerates the velocity distributions causing a special effect to enhance the bounding surface thickness whereas the thermal bounding surface behaves in the reverse order, and

it clarifies that Cu nanoparticle has a greater role in both the profile in comparison to Al_2O_3 and SWCNT nanoparticles.

- (iii) The unsteadiness overshoots the velocity profiles for which the thickness of the bounding surface thickness retards; moreover, similar trend is marked for the temperature distribution. However, steady state conditions preserve maximum magnitude for both the profiles.
- (iv) An augmentation in suction enriches the profiles of velocity in comparison to injection, whereas heat source diminishes the fluid temperature significantly.
- (v) The shear rate coefficient rises with increase in particle concentration, whereas heat transfer rate shows its opposite impact.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] Z. H. Khan, S. T. Hussain, and Z. Hammouch, "Flow and heat transfer analysis of water and ethylene glycol-based Cu nanoparticles between two parallel disks with suction/injection effects," *Journal of Molecular Liquids*, vol. 221, pp. 298–304, 2016.
- [2] K. R. V. Subramanian, T. N. Rao, and A. Balakrishnan, *Nanofluids and Their Engineering Applications*, CRC Press, Boca Raton, FL, USA, 2019.
- [3] Y. Ma, M. M. Rashidi, R. Mohebbi, and Z. Yang, "Investigation of magnetohydrodynamics in Ag-TiO₂/water hybrid nanofluid in a Shamse knot shaped cavity," *International Journal of Numerical Methods for Heat and Fluid Flow*, vol. 31, no. 1, pp. 251–272, 2020.
- [4] A. Riaz, T. Abbas, A. Zeeshan, and M. H. Doranehgard, "Entropy generation and MHD analysis of a nanofluid with peristaltic three dimensional cylindrical enclosures," *International Journal of Numerical Methods for Heat and Fluid Flow*, vol. 31, no. 8, pp. 2698–2714, 2021.
- [5] J. Tu, C. Qi, Z. Tang, Z. Tian, and L. Chen, "Experimental study on the influence of bionic channel structure and nanofluids on power generation characteristics of waste heat utilisation equipment," *Applied Thermal Engineering*, vol. 202, Article ID 117893, 2021.
- [6] R. Mohebbi, H. Lakzayi, N. A. C. Sidik, and W. M. A. A. Japar, "Lattice Boltzmann method based study of the heat transfer augmentation associated with Cu/water nanofluid in a channel with surface mounted blocks," *International Journal of Heat and Mass Transfer*, vol. 117, pp. 425–435, 2018.
- [7] A. B. Saryazdi, F. Talebi, T. Armaghani, and I. Pop, "Numerical study of forced convection flow and heat transfer of a nanofluid flowing inside a straight circular pipe filled with a saturated porous medium," *The European Physical Journal Plus*, vol. 131, no. 4, pp. 1–11, 2016.
- [8] S. Baag and S. R. Mishra, "Heat and mass transfer analysis on MHD 3-D water-based nanofluid," *Journal of Nanofluids*, vol. 4, no. 3, pp. 352–361, 2015.
- [9] T. Watanabe and I. Pop, "Thermal boundary layers in magnetohydrodynamic flow over a flat plate in the presence of a transverse magnetic field," *Acta Mechanica*, vol. 105, no. 1, pp. 233–238, 1994.
- [10] T. Armaghani, H. Esmaeili, Y. A. Mohammadpoor, and I. Pop, "MHD mixed convection flow and heat transfer in an open C-shaped enclosure using water-copper oxide nanofluid," *Heat and Mass Transfer*, vol. 54, no. 6, pp. 1791–1801, 2018.
- [11] F. N. Ibrahim and M. Terbeche, "Solutions of the laminar boundary layer equations for a conducting power law non-Newtonian fluid in a transverse magnetic field," *Journal of Physics D: Applied Physics*, vol. 27, no. 4, pp. 740–747, 1994.
- [12] M. Y. Malik, I. Khan, A. Hussain, and T. Salahuddin, "Mixed convection flow of MHD Eyring-Powell nanofluid over a stretching sheet: a numerical study," *AIP Advances*, vol. 5, no. 11, Article ID 117118, 2015.
- [13] H. I. Andersson, J. B. Aarseth, and B. S. Dandapat, "Heat transfer in a liquid film on an unsteady stretching surface," *International Journal of Heat and Mass Transfer*, vol. 43, no. 1, pp. 69–74, 2000.
- [14] E. M. A. Elbashbeshy and M. A. A. Bazid, "Heat transfer over an unsteady stretching surface with internal heat generation," *Applied Mathematics and Computation*, vol. 138, no. 2-3, pp. 239–245, 2003.
- [15] A. V. Kuznetsov and D. A. Nield, "Natural convective boundary-layer flow of a nanofluid past a vertical plate," *International Journal of Thermal Sciences*, vol. 49, no. 2, pp. 243–247, 2010.
- [16] H. Heidary and M. J. Kermani, "Heat transfer enhancement in a channel with block(s) effect and utilizing Nano-fluid," *International Journal of Thermal Sciences*, vol. 57, pp. 163–171, 2012.
- [17] S. U. Choi and J. A. Eastman, *Enhancing Thermal Conductivity of Fluids with Nanoparticles* (No. ANL/MSD/CP-84938, Argonne National Lab., IL, USA, 1995).
- [18] H. Masuda, A. Ebata, K. Teramae, and N. Hishinuma, "Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. Dispersion of Al_2O_3 , SiO_2 and TiO_2 ultra-fine particles," *Netsu Bussei*, vol. 7, no. 4, pp. 227–233, 1993.
- [19] A. K. Mishra, P. K. Pattnaik, S. R. Mishra, and N. Senapati, "Dissipative heat energy on Cu and Al_2O_3 ethylene-glycol-based nanofluid flow over a heated semi-infinite vertical plate," *Journal of Thermal Analysis and Calorimetry*, vol. 145, no. 1, pp. 129–137, 2021.
- [20] M. Shutaywi and Z. Shah, "Mathematical Modeling and numerical simulation for nanofluid flow with entropy optimization," *Case Studies in Thermal Engineering*, vol. 26, Article ID 101198, 2021.
- [21] W. Ibrahim and M. Negera, "The investigation of MHD williamson nanofluid over stretching cylinder with the effect of activation energy," *Advances in Mathematical Physics*, vol. 2020, no. 1, pp. 1–16, 2020.
- [22] S. Abdal, B. Ali, S. Younas, L. Ali, and A. Mariam, "Thermodynamic and multislip effects on MHD mixed convection unsteady flow of micropolar nanofluid over a shrinking/stretching sheet with radiation in the presence of heat source," *Symmetry*, vol. 12, no. 1, p. 49, 2020.
- [23] S. E. Ghasemi and M. Hatami, "Solar radiation effects on MHD stagnation point flow and heat transfer of a nanofluid over a stretching sheet," *Case Studies in Thermal Engineering*, vol. 25, Article ID 100898, 2021.

- [24] F. Saba, N. Ahmed, S. Hussain, U. Khan, S. Mohyud-Din, and M. Darus, "Thermal analysis of nanofluid flow over a curved stretching surface suspended by carbon nanotubes with internal heat generation," *Applied Sciences*, vol. 8, no. 3, p. 395, 2018.
- [25] S. Z. A. Adnan, U. Khan, N. Ahmed, S. T. Mohyud-Din, and I. Khan, "Heat transfer enhancement in H₂O suspended by aluminium alloy nanoparticles over a convective stretching surface," *Advances in Mechanical Engineering*, vol. 12, no. 9, Article ID 1687814020942342, 2020.
- [26] G. Rasool, A. Shafiq, M. S. Alqarni, A. Wakif, I. Khan, and M. S. Bhutta, "Numerical scrutinization of Darcy-Forchheimer relation in convective magnetohydrodynamic nanofluid flow bounded by nonlinear stretching surface in the perspective of heat and mass transfer," *Micromachines*, vol. 12, no. 4, p. 374, 2021.
- [27] N. S. Wahid, N. Md Arifin, M. Turkyilmazoglu, M. E. H. Hafidzuddin, and N. A. Abd Rahmin, "MHD hybrid Cu-Al₂O₃/water nanofluid flow with thermal radiation and partial slip past a permeable stretching surface: analytical solution," *Journal of Nano Research*, vol. 64, pp. 75–91, 2020.
- [28] R. Kalaivanan, N. Vishnu Ganesh, and Q. M. Al-Mdallal, "An investigation on Arrhenius activation energy of second grade nanofluid flow with active and passive control of nanomaterials," *Case Studies in Thermal Engineering*, vol. 22, Article ID 100774, 2020.
- [29] H. Upreti, A. K. Pandey, S. K. Rawat, and M. Kumar, "Modified Arrhenius and thermal radiation effects on three-dimensional magnetohydrodynamic flow of carbon nanotubes nanofluids over bi-directional stretchable surface," *Journal of Nanofluids*, vol. 10, no. 4, pp. 538–551, 2021.
- [30] H. Upreti, A. K. Pandey, M. Kumar, and O. D. Makinde, "Ohmic heating and non-uniform heat source/sink roles on 3D Darcy-forchheimer flow of CNTs nanofluids over a stretching surface," *Arabian Journal for Science and Engineering*, vol. 45, no. 9, pp. 7705–7717, 2020.
- [31] A. S. Sabu, J. Mackolil, B. Mahanthesh, and A. Mathew, "Reiner-Rivlin nanomaterial heat transfer over a rotating disk with distinct heat source and multiple slip effects," *Applied Mathematics and Mechanics*, vol. 42, no. 10, pp. 1495–1510, 2021.
- [32] B. Mahanthesh, N. S. Shashikumar, and G. Lorenzini, "Heat transfer enhancement due to nanoparticles, magnetic field, thermal and exponential space-dependent heat source aspects in nanoliquid flow past a stretchable spinning disk," *Journal of Thermal Analysis and Calorimetry*, vol. 145, pp. 3339–3347, 2020.
- [33] K. A. Al-Utaibi, A. Sohail, Z. Yu et al., "Dynamical analysis of the delayed immune response to cancer," *Results in Physics*, vol. 26, Article ID 104282, 2021.
- [34] A. Riaz, E. Bobescu, K. Ramesh, and R. Ellahi, "Entropy analysis for cilia-generated motion of Cu-blood flow of nanofluid in an annulus," *Symmetry*, vol. 13, no. 12, p. 2358, 2021.
- [35] L. Zhang, M. M. Bhatti, E. E. Michaelides, M. Marin, and R. Ellahi, "Hybrid nanofluid flow towards an elastic surface with tantalum and nickel nanoparticles, under the influence of an induced magnetic field," *The European Physical Journal - Special Topics*, vol. 45, 2021.
- [36] Ul H. Rizwan, S. Nadeem, Z. H. Khan, and N. F. M. Noor, "MHD squeezed flow of water functionalized metallic nanoparticles over a sensor surface," *Physica E*, vol. 73, pp. 45–53, 2015.