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Research Article

A Novel Mathematical Model for Radio Mean Square Labeling Problem

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A radio mean square labeling of a connected graph is motivated by the channel assignment problem for radio transmitters to avoid interference of signals sent by transmitters. It is an injective map h from the set of vertices of the graph G to the set of positive integers $\mathbb{N}$, such that for any two distinct vertices x, y, the inequality $d(x, y) + \lceil (h(x))^2 + (h(y))^2/2 \rceil \geq \dim(G) + 1$ holds. For a particular radio mean square labeling h, the maximum number of $h(v)$ taken over all vertices of G is called its spam, denoted by $\mathrm{rmsn}(h)$, and the minimum value of $\mathrm{rmsn}(h)$ taking over all radio mean square labeling h of G is called the radio mean square number of G, denoted by $\mathrm{rmsn}(G)$. In this study, we investigate the radio mean square numbers $\mathrm{rmsn}(P_n)$ and $\mathrm{rmsn}(C_n)$ for path and cycle, respectively. Then, we present an approximate algorithm to determine $\mathrm{rmsn}(G)$ for graph G. Finally, a new mathematical model to find the upper bound of $\mathrm{rmsn}(G)$ for graph G is introduced. A comparison between the proposed approximate algorithm and the proposed mathematical model is given. We also show that the computational results and their analysis prove that the proposed approximate algorithm overcomes the integer linear programming model (ILPM) according to the radio mean square number. On the other hand, the proposed ILPM outperforms the proposed approximate algorithm according to the running time.

1. Introduction

In wireless networks, each radio station assigns a number called frequency. When different transmitters of district stations send signals, the receiver might get unnecessarily interference of the signals sent by transmitters in particular with close frequencies. This is the channel assignment problem introduced by Hale [1] in 1980 to minimize such interference. In 2001, Chartrand et al. [2] proposed converting this problem to graph theoretical problem using vertex labeling. Many researchers involved with this problem [3–16] and produced different methods to minimize the interference of signals [7]. Recently, Ramesh et al. [8] proposed a new method called radio mean square labeling, which is defined as follows. A radio mean square labeling of a connected graph G is an injective function h from its vertex set V to the set of natural numbers $\mathbb{N}$, such that for any two distinct vertices x and y, $\lceil (h(x))^2 + (h(y))^2/2 \rceil \geq \mathrm{diam}\, G + 1 - d(x, y)$ holds, where $d(x, y)$ denotes the distance between the two vertices, and $\dim G$ represents the diameter of the graph [8]. For a radio mean square labeling h, the maximum number of $h(v)$ taken over all vertices of G is called its spam, denoted by $\mathrm{rsmn}(h)$, and the minimum value of $\mathrm{rmsn}(h)$ taking over all radio mean square labeling h of G is called the radio mean square number of G, denoted as $\mathrm{rmsn}(G)$. The radio mean square number of h, denoted by $\mathrm{rmsn}(h)$ is the maximum number assigned to any vertex of G. Ramesh et al. [8] determined the radio mean square number for some graphs such as in the centric subdivision of spoke wheel graph and biwheel graph.

Due to most of nontrivial coloring models, graph coloring is an NP-hard problem. Therefore, we take into consideration a graph coloring algorithm [9–11]. The judgment of the performance of the used algorithm does include its effectiveness and accuracy for a large number of vertices and the level of complexity regarding suboptimal solutions [12, 13]. Here, we introduce an approximate algorithm that leads to an upper bound of the radio mean square for a large number of vertices. Finally, we turn our attention to reformulate the radio square labeling as a linear programming model and then minimize the suggested linear function. We use of transforming the nonlinear constraint to become a linear constraint by using of some large integers dependably on the coefficient range of the given problem under path calculating conditions. Some illustrated examples and comparison between the techniques will be given.

It should be noted that all the considered graphs in this study are finite, simple, connected, and undirected.

The organization of the study goes as follows: in Section 2, the radio mean square number of cycles and paths are given. Section 3 is devoted to present an approximate algorithm that finds the upper bound of the radio mean square number of a given graph and an illustrative example is included. Section 4 deals with a new mathematical model for finding the upper bound of the radio mean square number of the given graph. Section 5 provides the experimental results. Analysis and statistical tests between the mathematical model and the proposed approximate algorithm are provided. The last section is considered for conclusion.

2. Results

The following theorem is about the rmsn for the path P_n, and next, we will get the rmsn for cycle C_n.

Theorem 1. *For the path P_n, $n \geq 1$, and the radio mean square number is*

$$\operatorname{rmsn}(P_n) = n + k; \begin{cases} 1 \leq n \leq 15 \text{ and } k = 0, \\ 16 \leq n \leq 22 \text{ and } k = 1, \\ 23 \leq n \leq 32 \text{ and } k = 2, \\ 33 \leq n \leq 44 \text{ and } k = 3, \\ 45 \leq n \leq 58 \text{ and } k = 4, \\ 59 \leq n \leq 73 \text{ and } k = 5, \\ 74 \leq n \leq 92 \text{ and } k = 6, \\ \vdots \\ k^2 + 5k + 9 \leq n \leq k^2 + 7k + 14, k \geq 7. \end{cases} \tag{1}$$

Proof. Clearly, diam $(P_n) = n - 1$. Then, one can define h: $V(P_n) \longrightarrow \mathbb{N}$ as follows: □

Case a. For

$$\begin{aligned} 1 \leq n \leq 15, &\quad k = 0, \\ 16 \leq n \leq 22, &\quad k = 1, \\ 23 \leq n \leq 32, &\quad k = 2, \\ 33 \leq n \leq 44, &\quad k = 3, \\ 45 \leq n \leq 58, &\quad k = 4, \\ 59 \leq n \leq 73, &\quad k = 5, \\ 74 \leq n \leq 92, &\quad k = 6. \end{aligned} \tag{2}$$

One may label the vertices of P_n as follows:

Subcase a.1: n is even:

$$\begin{aligned} &h(x_1) = 1, h(x_n) = k + 2, \\ &h(x_{2+i}) = n + k - 2i, 0 \leq i < \frac{n}{2} - 1, \\ &h(x_{n-1-i}) = n + k - 1 - 2j, 0 \leq j < \frac{n}{2} - 1. \end{aligned} \tag{3}$$

Subcase a.1: n is odd:

$$\begin{aligned} &h(x_1) = 1, h(x_n) = k + 2, \\ &h(x_{2+i}) = n + k - 2i, 0 \leq i < \frac{n-1}{2}, \\ &h(x_{n-1-i}) = n + k - 1 - 2j, 0 \leq j < \frac{n-1}{2} - 1. \end{aligned} \tag{4}$$

Case 2. bFor $n \geq 93$ and $k^2 + 5k + 9 \leq n \leq k^2 + 7k + 14, k \geq 7$, one may label the vertices of P_n as the following subcases:

Subcase b.1: n is even:

$$\begin{aligned} &h(x_1) = 1, h(x_n) = k + 2, \\ &h(x_{2+i}) = n + k - 2i, 0 \leq i < \frac{n}{2} - 1, \\ &h(x_{n-1-i}) = n + k - 1 - 2j, 0 \leq j < \frac{n}{2} - 1. \end{aligned} \tag{5}$$

Subcase b.1: n is odd:

$$\begin{aligned} &h(x_1) = 1, h(x_n) = k + 2, \\ &h(x_{2+i}) = n + k - 2i, 0 \leq i < \frac{n-1}{2}, \\ &h(x_{n-1-i}) = n + k - 1 - 2j, 0 \leq j < \frac{n-1}{2} - 1. \end{aligned} \tag{6}$$

Therefore, for any pair $(x_i, x_j), i \neq j, 0 \leq i, j \leq n$, we have $d(x_i, x_j) + \lceil h(x_i)^2 + h(x_j)^2/2 \rceil \geq 1 + n - 1 = 1 + \dim\ (P_n)$.

Hence, h is a valid radio mean square labeling for P_n, and therefore, $\operatorname{rmsn}(P_n) \leq \operatorname{rmsn}(h) = n + k$. Since h is injective, $\operatorname{rmsn}(P_n) = n + k, n \geq 1$ for all radio mean square labeling h, and hence, $\operatorname{rmsn}(P_n) = n + k, n \geq 1$. Therefore, the labeling h defined above satisfies the radio mean square condition.

Example 1. The radio mean square numbers of P_9, P_{10}, P_{96}, and P_{115} are shown in Figure 1. It is clear that $k = 0$ for P_9 and P_{10}, but $k = 7$ for P_{96}, and P_{115}.

Theorem 2. *The radio mean square numbers of the cycles C_n, $n \geq 3$, are given by*

$$\operatorname{rmsn}(C_n) = \begin{cases} n: & 3 \leq n \leq 7, \\ n + k; & \begin{cases} 8 \leq n \leq 15 \text{ and } k = 0, \\ 16 \leq n \leq 27 \text{ and } k = 1, \\ 28 \leq n \leq 43 \text{ and } k = 2, \\ \vdots \\ k^2 + 5k + 9 \leq n \leq k^2 + 7k + 14, k \geq 3. \end{cases} \end{cases} \tag{7}$$

Proof. It is clear that the dimension of $C_n = x_1, x_2, \cdots, x_n$ is $\lfloor n/2 \rfloor$ since its length is n. Then, one can define $h\colon V(C_n) \longrightarrow \mathbb{N}$ as follows: □

Case 3. aFor $3 \leq n \leq 7$, we may label the vertices of C_n by

$$h(x_i) = i; 1 \leq i \leq n. \tag{8}$$

Case 4. bFor $n \geq 8$, $2k^2 + 2k + 4 \leq n \leq 2k^2 + 6k + 7, k \geq 0$. We may label the vertices of C_n by one of the following subcases:

Subcase b.1: n is even:

$$\begin{aligned} h(x_{n/2}) &= k + 1, \\ h(x_{i+1}) &= k + 3 + 2i, 0 \leq i < \frac{n}{2} - 1, \\ h(x_{n-j}) &= k + 2 + 2j, 0 \leq j < \frac{n}{2}. \end{aligned} \tag{9}$$

Therefore, for any pair $(x_i, x_j), i \neq j, 1 \leq i, j \leq n$, we have $d(x_i, x_j) + \lceil h(x_i)^2 + h(x_j)^2/2 \rceil \geq 1 + \lfloor n/2 \rfloor = 1 + \dim(C_n)$.

Subcase b.2: n is odd:

$$\begin{aligned} h(x_{n+1}/2) &= k + 1, \\ h(x_{i+1}) &= k + 2 + 2i, 0 \leq i < \frac{n+1}{2} - 1, \\ h(x_{n-j}) &= k + 3 + 2j, 0 \leq j < \frac{n+1}{2} - 1. \end{aligned} \tag{10}$$

So, for any pair $(x_i, x_j), i \neq j, 1 \leq i, j \leq n$, the following inequality holds $d(x_i, x_j) + \lceil h(x_i)^2 + h(x_j)^2/2 \rceil \geq 1 + \lfloor n/2 \rfloor = 1 + \dim\ (C_n)$.

Hence, h is a valid radio mean square labeling for C_n, and therefore,

$$\operatorname{rmsn}(C_n) \leq \operatorname{rmsn}(h) = \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{jf } n \geq 8 \quad \text{and} \quad \begin{matrix} 2k^2 + 2k + 4 \leq n \leq 2k^2 + 6k + 7, \\ k \geq 0. \end{matrix} \end{cases} \tag{11}$$

Since h is injective,

$$\operatorname{rmsn}(C_n) \geq \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{jf } n \geq 8, \text{and} \quad \begin{matrix} 2k^2 + 2k + 4 \leq n \leq 2k^2 + 6k + 7, \\ k \geq 0. \end{matrix} \end{cases} \tag{12}$$

For all radio mean square labeling h,

$$\operatorname{rmsn}(C_n) = \begin{cases} n, & \text{if } 3 \leq n \leq 7, \\ n + k, & \text{jf } n \geq 8, \text{and} \quad \begin{matrix} 2k^2 + 2k + 4 \leq n \leq 2k^2 + 6k + 7, \\ k \geq 0. \end{matrix} \end{cases} \tag{13}$$

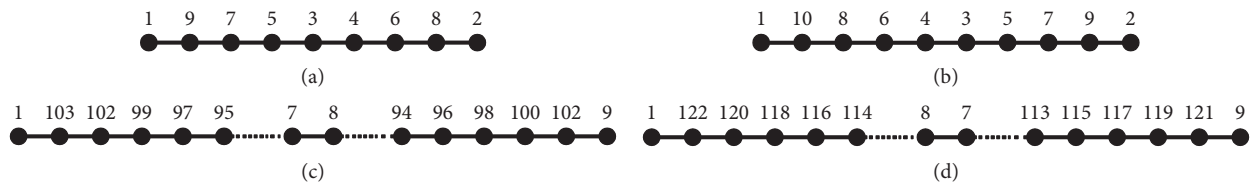

Figure 1: The radio mean square number of P_9, P_{10}, P_{96}, and P_{115}. (a) $\mathrm{rmsn}(P_9) = 9$. (b) $\mathrm{rmsn}(P_{10}) = 10$. (c) $\mathrm{rmsn}(P_{96}) = 103$. (d) rmsn $(P_{115}) = 122$.

Therefore, the labeling $h$defined above satisfies the radio mean square condition.

Example 2. The radio mean square numbers of cycles C_8, C_9, C_{16}, and C_{17} are shown in Figure 2. It is clear that $k = 1$ for C_{16} and C_{17}, but $k = 7$ for C_{95} andC_{112}.

3. A Novel Graph Radio Mean Square Algorithm

Here, we introduce an approximated algorithm. This algorithm finds an upper bound of the radio mean square for arbitrary graph G. The main idea is to labeling some vertices (initial vertices) by $\mathrm{floor}(\sqrt{\mathrm{diam}})$. On the other hand, the algorithm chooses a different vertex as an initial vertex in each iteration.

The time complexity of an algorithm is defined as the number of instructions of this algorithm multiplied by the running time of each instruction. The time complexity is considered as a good metric to evaluate the given algorithm. Thus, Algorithm 1 has nine steps, and both Step 1 and Step 2 have the same instruction. Step 3 has a nested loop that has $O(n^2)$ time complexity. On the other hand, Step 4, Step 5, and Step 6 have $O(n)$ time complexity. Step 7 has one instruction, while Step 8 and Step 9 have $O(n^3)$ and $O(n^4)$, respectively. Therefore, Algorithm 1 has the time complexity $O(n^4)$.

In the coming example, we show and explain how to compute the radio mean square labeling problem for P_5.

Example 3. Suppose that x_i is the label of the vertex v_i, $1 \le i \le 5$. Therefore, 1 explores an upper bound of the radio mean square labeling problem as follows:

It is known that $\mathrm{diam}(P_5) = 4$. We select a vertex x_1 and $\mathrm{col}(x_1) = 2$. Let $S = \{x_1\}$, and for all $v \in V(G) - S$, compute

$$\begin{aligned}
\mathrm{temp}(x_2) &= \max_{x_1}\left\{2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-1}, 1\}}{4}\right)\right\} = 3,\\
\mathrm{temp}(x_3) &= \max_{x_1}\left\{2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-2}, 1\}}{4}\right)\right\} = 3,\\
\mathrm{temp}(x_4) &= \max_{x_1}\left\{2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-3}, 1\}}{4}\right)\right\} = 3,\\
\mathrm{temp}(x_5) &= \max_{x_1}\left\{2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-4}, 1\}}{4}\right)\right\} = 3.
\end{aligned} \tag{14}$$

Let $\min = \min_{v \in V(G)-S}\{\mathrm{temp}(v)\} = 3$; we choose a vertex $x_2 \in V(G) - S$, such that $\mathrm{temp}(x_2) = 3$. Give $\mathrm{col}(x_2) = 3$ and $S = \{x_1, x_2\}$.

$$\begin{aligned}
\mathrm{temp}(x_3) &= \max_{\{x_1,x_2\}}\left\{\begin{array}{l} 2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-2}, 1\}}{4}\right)\\ 3 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-1}, 1\}}{4}\right)\end{array}\right\} = 4,\\
\mathrm{temp}(x_4) &= \max_{\{x_1,x_2\}}\left\{\begin{array}{l} 2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-3}, 1\}}{4}\right)\\ 3 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-2}, 1\}}{4}\right)\end{array}\right\} = 4,\\
\mathrm{temp}(x_5) &= \max_{\{x_1,x_2\}}\left\{\begin{array}{l} 2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-4}, 1\}}{4}\right)\\ 3 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-3}, 1\}}{4}\right)\end{array}\right\} = 4.
\end{aligned} \tag{15}$$

Let $\min = \min_{v \in V(G)-S}\{\mathrm{temp}(v)\} = 4$; we choose a vertex $x_3 \in V(G) - S$, where $\mathrm{temp}(x_3) = 4$. Give $\mathrm{col}(x_3) = 4$ and $S = \{x_1, x_2, x_3\}$.

$$\begin{aligned}
\mathrm{temp}(x_4) &= \max_{\{x_1,x_2,x_3\}}\left\{\begin{array}{l} 2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-3}, 1\}}{4}\right)\\ 3 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-2}, 1\}}{4}\right)\\ 4 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-1}, 1\}}{4}\right)\end{array}\right\} = 5,\\
\mathrm{temp}(x_5) &= \max_{\{x_1,x_2,x_3\}}\left\{\begin{array}{l} 2 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-4}, 1\}}{4}\right)\\ 3 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-3}, 1\}}{4}\right)\\ 4 + \mathrm{ceil}\left(\frac{\max\{(\sqrt{4+1-2}, 1\}}{4}\right)\end{array}\right\} = 5.
\end{aligned} \tag{16}$$

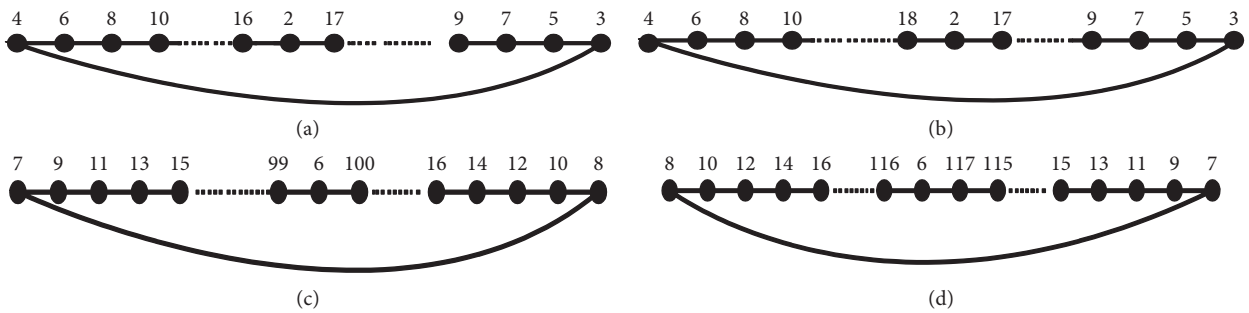

Figure 2: The radio mean square number of cycles C_8, C_9, C_{16}, and C_{17}. (a) $\mathrm{rmsn}(C_{16}) = 17$. (b) $\mathrm{rmsn}(C_{17}) = 18$. (c) $\mathrm{rmsn}(C_{95}) = 100$. (d) $\mathrm{rmsn}(C_{112}) = 117$.

Let $\min = \min_{v \in V(G)-S}\{\mathrm{temp}(v)\} = 5$; we choose a vertex $x_4 \in V(G) - S$, where $\mathrm{temp}(x_4) = 5$. Give $\mathrm{col}(x_4) = 5$ and $S = \{x_1, x_2, x_3, x_4\}$.

$$\mathrm{temp}(x_5) = \max_{x_1}\left\{\begin{array}{l} 2 + \mathrm{ceil}\left(\dfrac{\max\{(\sqrt{4+1-4}, 1\}}{4}\right) \\ 3 + \mathrm{ceil}\left(\dfrac{\max\{(\sqrt{4+1-3}, 1\}}{4}\right) \\ 4 + \mathrm{ceil}\left(\dfrac{\max\{(\sqrt{4+1-2}, 1\}}{4}\right) \\ 5 + \mathrm{ceil}\left(\dfrac{\max\{(\sqrt{4+1-1}, 1\}}{4}\right) \end{array}\right\} = 6 \tag{17}$$

Let $\min = \min_{v \in V(G)-S}\{\mathrm{temp}(v)\} = 6$; we select a vertex $x_5 \in V(G) - S$, where $\mathrm{temp}(x_5) = 6$. Give $\mathrm{col}(x_5) = 6$ and $S = \{x_1, x_2, x_3, x_4, x_5\}$. Plainly, all vertices are labelled and $\mathrm{rmsn}(\mathrm{P}_5) = 6$.

4. Formulation of the Radio Mean Square Labeling as a Mathematical Model

In this section, we present the integer linear programming model (ILPM) for the radio mean square labeling problem.

Let G be a connected graph of order n with $V(G) = \{v_1, v_2, \ldots, v_n\}$ and let $D = [d_{ij}]$ be the distance matrix of G, that is, $d_{ij} = d(v_i; v_j)$ for $1 \le i, j \le n$. Suppose that x_i is the label of the vertex v_i, $1 \le i \le n$. Now, we can propose the mathematical model for the radio mean square labeling problem as the ILPM. Let us suppose the function F is $F = x_1 + x_2 + \ldots + x_n$.

4.1. Formulation 1. Minimize F subject to the $\binom{n}{2}$ constraints $\lceil (x_i)^2 + (x_j)^2/2 \rceil \ge \mathrm{diam} + 1 - d(v_i, v_j)$ for $1 \le i \le n-1$, $2 \le j \le n$, and $i < j$, and as mentioned before, the following steps will transform nonlinear constraints to become linear which is easy to deal with.

4.2. Formulation 2. Since $\lceil (x_i)^2 + (x_j)^2/2 \rceil \le \lceil (x_i) + (x_j)/2 \rceil^2$, we have the following inequalities:

$$\begin{aligned} &\left\lceil \frac{(x_i) + (x_j)}{2} \right\rceil^2 \ge \mathrm{diam} + 1 - d(v_i - v_j), \\ &\left\lceil \frac{(x_i) + (x_j)}{2} \right\rceil \ge \sqrt{\mathrm{diam} + 1 - d(v_i - v_j)}, \\ &\frac{(x_i) + (x_j)}{2} \ge \sqrt{\mathrm{diam} + 1 - d(v_i - v_j)}, \\ &\frac{(x_i) + (x_j)}{2} + 1 \ge \sqrt{\mathrm{diam} + 1 - d(v_i - v_j)}, \\ &M|(x_i - x_j)| \ge \frac{(x_i) + (x_j)}{2} + 1 \ge \sqrt{\mathrm{diam} + 1 - d(v_i - v_j)}. \end{aligned} \tag{18}$$

M is a large integer number which depends on the coefficients range of the problem.

The absolute value notation is used to get distinct values for x_i, where $i = 1, 2, \ldots, n$.

Now, we can reformulate the radio mean square problem as follows:

$$\min F = x_1 + x_2 + \cdots + x_n. \tag{19}$$

$$\begin{aligned} &\text{Subject to } M|(x_i - x_j)| \ge \left\lfloor \sqrt{\mathrm{diam} + 1 - d(v_i - v_j)} \right\rfloor, \\ &\qquad 1 \le i \le n-1 \\ &\qquad 2 \le j \le n \text{ and } i < j. \end{aligned} \tag{20}$$

Here, the floor function is used because the values of $x_i, 1 \le i \le n$, are integers.

Example 4. The details of the ILPM to compute the radio mean square labeling for P_3. Assume that x_i is the label of the vertex v_i, such that $1 \le i \le 3$. Thus, the mathematical model for the radio mean square labeling problem as the ILPM is prepared as follows:

$$\min f = x_1 + x_2 + x_3. \tag{21}$$

Subject to

Input: G be an n-vertex graph, simple connected graph, and the diameter of (diam)
Output: an upper bound of radio mean square number of G
Begin
Step 1: choose a vertex u and $\mathrm{col}(u) = \mathrm{floor}(\sqrt{\mathrm{diam}})$
Step 2: $S = \{u\}$
Step 3: for all $v \in V(G) - S$, compute
$\mathrm{temp}(v) = \max_{t \in s}\left\{col(t) + \mathrm{ceil}\left(\max\left\{\sqrt{(D + 1 - d(u, v), 1}\right\}/\mathrm{diam}\right)\right\}$
Step 4: let $\min = \min_{v \in V(G)-S}\{\mathrm{temp}(v)\}$
Step 5: choose a vertex $v \in V(G) - S$, such that$\mathrm{temp}(v) = \min$
Step 6: give $\mathrm{col}(v) = \min$
Step 7: $S = S \cup \{v\}$
Step 8: repeat **Step 3–Step 6** until all vertices are labelled
Step 9: repeat **Step 1–Step 7** for every vertex $x \in V(G)$
End

Algorithm 1: Finding an upper bound of the radio mean square number of a graph G.

Table 1: Description of the computing environment.

CPU	Intel (R) Core (TM) i5-2430 CPU at 2.40 GHz
RAM size	4 GB RAM
MATLAB version	R2019a (9.6.0.1072779)

Table 2: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the path graph.

		Path graph			
		Proposed algorithm		Integer linear programming	
N	Standard RMS	rmsn (P_n)	CPU time	rmsn (P_n)	CPU time
1	1	—	—	—	—
2	2	2	0.0001565	2	0.0135961
3	3	3	0.0007495	3	0.0137607
4	4	4	0.0009242	4	0.0142838
5	5	6	0.0009259	9	0.0148360
6	6	7	0.0011930	11	0.0149701
7	7	8	0.0028154	13	0.0151607
8	8	9	0.0031659	15	0.0153437
9	9	10	0.0086583	17	0.0153530
10	10	12	0.0092995	28	0.0153602
11	11	13	0.0178153	31	0.0154630
12	12	14	0.0179739	34	0.0158122
13	13	15	0.0195242	37	0.0159264
14	14	16	0.0214671	40	0.0160664
15	15	17	0.0323685	43	0.0165312
16	17	18	0.0397062	46	0.0165756
17	18	20	0.0488390	65	0.0166756
18	19	21	0.0521773	69	0.0168593
19	20	22	0.0635412	73	0.0168673
20	21	23	0.0773853	77	0.0173017
21	22	24	0.0926470	81	0.0173982
22	23	25	0.1082337	85	0.0174649
23	25	26	0.1321028	89	0.0176519
24	26	27	0.1680530	93	0.0178209
25	27	28	0.1819503	97	0.0178319
26	28	30	0.2174177	126	0.0178408
27	29	31	0.2423215	131	0.0178502
28	30	32	0.2838989	135	0.0179323
29	31	33	0.3306090	142	0.0181244
30	32	34	0.3907487	146	0.0182150
50	54	56	3.3044964	344	0.0295531

Table 3: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the cycle graph.

		Cycle graph			
		Proposed algorithm		Integer linear programming	
N	Standard RMS	rmsn (C_n)	CPU time	rmsn (C_n)	CPU time
1	—	—	—	—	—
2	—	—	—	—	—
3	3	3	0.0002946	3	0.0121703
4	4	4	0.0006198	4	0.0128697
5	5	5	0.0013164	5	0.0130696
6	6	6	0.0017746	6	0.0134300
7	7	7	0.0023419	7	0.0137379
8	8	9	0.0037564	15	0.0137439
9	10	10	0.0050932	17	0.0140673
10	10	11	0.0060342	19	0.0143174
11	11	12	0.0088211	21	0.0143427
12	12	13	0.0148844	23	0.0147269
13	13	14	0.0172248	25	0.0147985
14	14	15	0.0203240	27	0.0149231
15	15	16	0.0251720	29	0.0151018
16	17	17	0.0323380	31	0.0151929
17	18	18	0.0397318	33	0.0155649
18	19	20	0.0492787	52	0.0156036
19	20	21	0.0612156	55	0.0156856
20	21	22	0.0765414	58	0.0157774
21	22	23	0.0898619	61	0.0160212
22	23	24	0.1191102	64	0.0162420
23	24	25	0.1328831	67	0.0163022
24	25	26	0.1583415	70	0.0165312
25	26	27	0.1758159	73	0.0165648
26	27	28	0.2122978	76	0.0165116
27	28	29	0.2491384	79	0.0166234
28	30	30	0.2822584	82	0.0167352
29	31	31	0.3290932	85	0.0168470
30	32	32	0.3655289	88	0.0171844
50	53	54	3.0468593	246	0.0250638

$$\begin{aligned} &|x_1 - x_2| \geq \left\lfloor \sqrt{\text{diam} + 1 - d(v_1, v_2)} \right\rfloor, \\ &|x_1 - x_3| \geq \left\lfloor \sqrt{\text{diam} + 1 - d(v_1, v_3)} \right\rfloor, \\ &|x_2 - x_3| \geq \left\lfloor \sqrt{\text{diam} + 1 - d(v_2, v_3)} \right\rfloor, \\ &x_1, x_2, x_3 \geq 0. \end{aligned} \tag{22}$$

Since $\text{diam} = n - 1$ and $\text{diam} = 2$ for $P_3, M = 1$, the distance matrix of P_3 is

$$D = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}. \tag{23}$$

Therefore, the above mathematical model can be written as follows:

$$\min f = x_1 + x_2 + x_3. \tag{24}$$

Subject to

$$|x_1 - x_2| \geq 1; |x_1 - x_3| \geq 1; |x_2 - x_3| \geq 1;\ x_1, x_2, x_3 \geq 0. \tag{25}$$

The solution of the above model equals to 3.

5. Computational Study

In this article, we propose the analysis of the computational results that show the superiority of Algorithm 1 on the ILPM according to the radio mean square number. On the other hand, the proposed ILPM outperforms Algorithm 1 according to CPU time.

Paths and cycles are used to evaluate the proposed models. The computation environment is given in Table 1. MATLAB solver is used to solve the ILPM. In Tables 2 and 3, the following symbols standard RMS, rmsn, and CPU times are used to indicate the exact radio mean square number, the calculated mean square number, and the running time for path and cycles, respectively. The convergence between the calculated and exact upper bounds of the radio mean square number of paths is given in Table 2. Figures 3 and 4 show that the superiority of the proposed Algorithm 1 on the ILPM according to the radio mean square number. For example, the standard radio mean square number for P_{50} is 54, but it is 56 and 344 by Algorithm 1 and the ILPM, respectively. Figures 5 and show the superiority of the

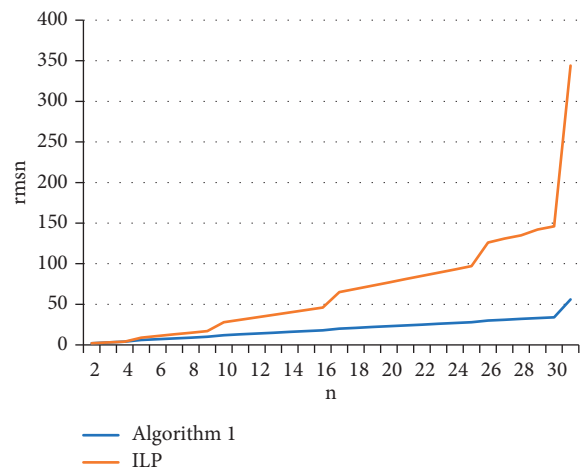

Figure 3: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the path graph.

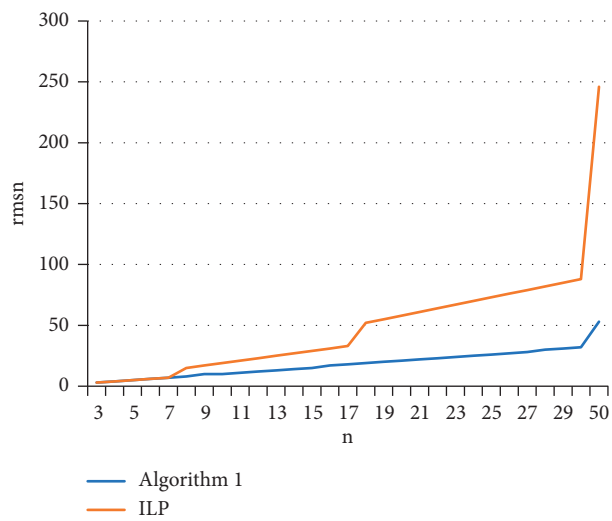

Figure 4: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the cycle graph.

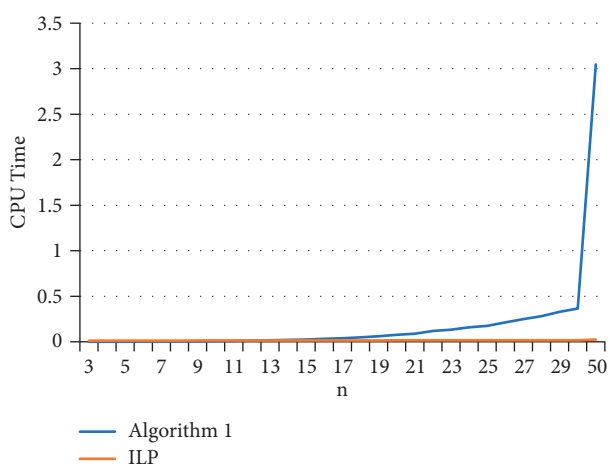

Figure 5: Comparison between standard radio mean square number, algorithm, and integer linear programming for the upper bound of radio mean square number for the path graph according to CPU time.

proposed Algorithm 1 on the ILPM according to CPU time. Table 3 provides that the gap between the ILPM and the proposed Algorithm 1 is large according to the radio mean square number. It is clear that 1 is better than the ILPM according to the radio mean square number. According to the CPU time, Tables 2 and 3 explain the superiority of the proposed ILPM on Algorithm 1.

6. Conclusions

In this work, we determined the radio mean square numbers $\mathrm{rmsn}(P_n)$ and $\mathrm{rmsn}(C_n)$ for paths and cycles. Then, the proposed approximate algorithm is introduced to obtain $\mathrm{rmsn}(G)$ for graph G. In addition, a new mathematical model is proposed in order to find the upper bound of $\mathrm{rmsn}(G)$ for graph G, and a comparison between the proposed approximate algorithm and the proposed mathematical model is introduced. Finally, the computational results and their analysis have proved that the proposed approximate algorithm overcomes the ILPM according to the radio mean square number.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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